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Patterns for Simple Cellular Automata in a Universe of Dense-Packed Spheres

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Abstract. Rules for describing certain simple cellular automata in a universe of dense-packed spheres are presented and explored. Specifically investigated are oscillators and other stable forms for a rule introduced in [2] as a candidate for a game of Life. The "signature" of a form is defined, and the signatures are given for all known smaller forms for the above rule. Rates of growth are discussed and computer implementation techniques are mentioned.

1. Introduction

In a three-dimensional universe of close-packed spheres, which is also called a hexahedral tesselation [4], each sphere, or cell, has 12 immediate neighbors. The spatial relationship between neighbors is best illustrated by placing a cell at the center of a cuboctahedron. The 12 corners then show the location of neighbor cells (see figure 1-A), which are arranged in four intersecting planes, each containing a hexagonal ring of six neighbors touching the central cell. For example, in figure 1-A, the four planes are defined by (1,2,3,4,5,6), (1,7,8,4,12,10), (2,7,9,5,12,11), and (3,8,9,6,10,11). Figure 1-B shows the neighbors of a central cell as spheres; here, the center cell is surrounded by its neighbors and hence is barely visible.

Although one can easily envision space when it has been packed full (see figure 1-C), a display of just a few cells arranged in some configuration can be confusing. To help alleviate this problem, all illustrations of configurations are shown from the same perspective: namely, the viewer is always looking straight down, in a direction perpendicular to plane (1,2,3,4,5,6).

1.1 Some notation

Let Ω denote a universe of dense-packed spheres; R^3 will be used to denote our occasional reference to a three-dimensional Cartesian universe. A cell can have two states, which we will call *living* and *dead*. The next generation state of a cell is determined by the current state of the cell and its 12 immediate neighbors. Define *environment*, E, as the number of living neighbors of a given living cell, c, so that c remains alive the next generation. *Fertility*, F, is the number of living neighbors of a given dead cell, d, so that d becomes alive next generation. The valid fertility and environment ranges are given as $E_l \leq E \leq E_u$ and $F_l \leq F \leq F_u$. Thus, all rules, R, under consideration can be written as 4-tuples of the form $R = (E_l, E_u, F_l, F_u)$. These rules are *semi-totalistic* [1] in that the next generation outcome depends solely upon c and the quantity of living neighbors and not their relative positions.

Throughout this paper, we shall deal mainly with Ω. Computer experimentation was done with various finite universe sizes, occasionally wrapping coordinates in order to simulate an infinite universe. Because of the method of computer implementation, the finite values for Ω appear more or less cubic in shape and may be considered as being composed of successive, approximately square layers. The number of such layers is referred to as the grid size.

Many rules yield forms called *oscillators*; that is, they are periodic. An oscillator of period one is called a *stable* form. An oscillator that translates in some manner through space is, by tradition, called a *glider*.

In reference [2], the author introduces a totalistic cellular automaton, defined by $R = (3,3,3,3)$, that appears to satisfy criteria for consideration as a "game of Life". This rule can also be specified verbally:

> If a cell touches three living cells, it becomes (remains) alive for the next generation; otherwise it dies (remains dead).

Due to its importance as a Life game, the rule (3,3,3,3) was investigated thoroughly in order to determine the nature and quantity of its small oscillators and stable forms. It is important to note that many other rules of the form (E_l, E_u, F_l, F_u) support similar quantities of such structures, although an extensive search has so far revealed no glider for any semi-totalistic rule in Ω, and for only two such rules in R^3.

2. Oscillators and stable forms for $R = (3,3,3,3)$

Three methods of discovering forms were employed: a) condensation from random "soup", b) condensation from some symmetric pattern, and c) construction. A Macintosh Plus was used for all experimentation.

Method (a) revealed most of the common oscillators, but eventually ceased to yield any new discoveries. This method was also used to discover both gliders (see figure 2, N and O) by starting with a small cluster of about 30 randomly created cells at the center of a fairly large finite universe. Any configuration which struck the "edge" of the universe was saved.

Method (b) yielded the vast majority of forms. For each experiment under this method a small symmetric shape, S, (not necessarily stable under (3,3,3,3)) was placed at the center of a large finite universe. The shape was "stirred" twice with randomly created rules of the form

$$2 \leq E_l,\ E_u \leq 4;\ 2 \leq F_l,\ F_u \leq 5.$$

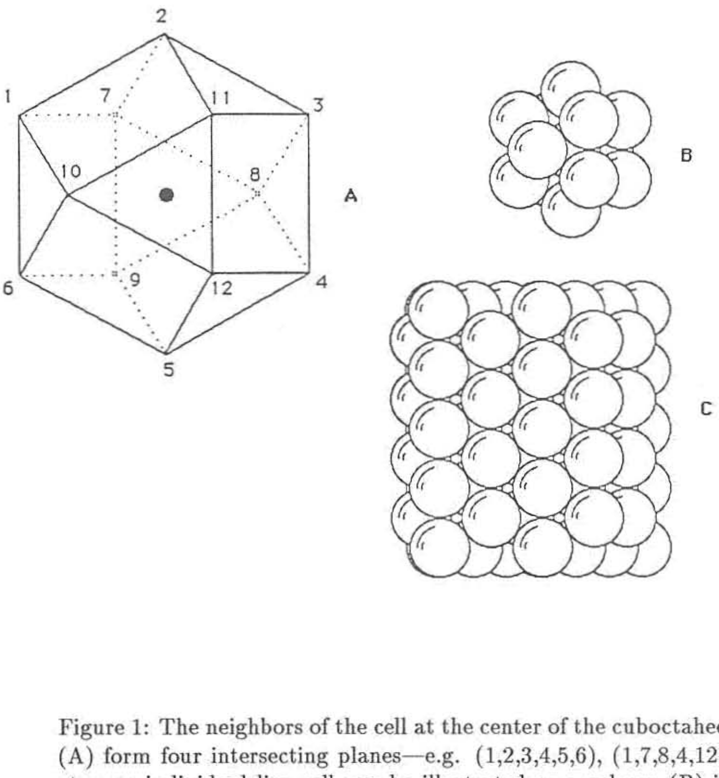

Figure 1: The neighbors of the cell at the center of the cuboctahedron (A) form four intersecting planes—e.g. (1,2,3,4,5,6), (1,7,8,4,12,10), etc.; an individual live cell can be illustrated as a sphere; (B) illustrates the 12 neighbors of a cell along with the cell, which can be seen in the middle; (C) depicts a universe of grid size = 4, since 4 planes of cells parallel to plane (1,2,3,4,5,6) have been constructed.

After two stirrings, the (frequently larger) shape was saved as the starting shape for the next experiment, and R was switched to (3,3,3,3). If the starting shape had become too large, it was discarded and shape S was used again.

Most experiments conducted under methods (a) and (b) yielded no debris; all cells usually died after 2 to 20 generations depending upon the size of the starting form. If an object lasted more than 70 generations, it was considered stable and was saved on a disk file. After a maximum of 25 such shapes were found, the program was stopped; the files were examined and new discoveries were noted. Due to the viewing limitations imposed by the program, it was sometimes very difficult to determine when two forms were different or merely the same form viewed from another angle (see, for example, figure 3-F). Furthermore, there was no way to prevent different files from being created for similar forms. Both of these problems were alleviated by incorporating *signatures*, described below.

2.1 Signatures

The signature, S, of any configuration, is defined by the 25-tuple

$$\{A_{12}\, A_{11}\, A_{10}\, \ldots A_2\, A_1\, A_0 \diamond D_1\, D_2\, D_3\, \ldots\, D_{10}\, D_{11}\, D_{12}\}$$

where A_i gives the total number of living cells in the configuration with exactly i living neighbors and D_j gives the total number of dead cells in the vicinity of the configuration with exactly j living neighbors. Note that D_0 represents "space" and hence is omitted. Also, leading and trailing zeros may be omitted when writing a signature; the "$\diamond$" acts in a manner analogous to a decimal point. Some fairly obvious characteristics of signatures are given below.

1. Two identical configurations with different orientations have the same signature.
2. $A_3 + D_3$ (at generation k) $= \sum A_i$ (at generation $k+1$).
3. Any stable form under (3,3,3,3) has a signature whose A_i are zero except for A_3, and whose D_3 must be zero.
4. Oscillators whose phases are symmetric with each other in some way have identical signatures for these phases.

   Several oscillators exhibit (4); for example, figure 2-A through E. Other interesting observations may be made; unfortunately, the observation that we would most like to make—that different oscillators are guaranteed to have different signatures—is not true. This fact is stated below.

5. It is possible to construct as many distinct oscillators as we wish, with each oscillator bearing the same signature.

Clearly, we only include as "distinct oscillators" those shapes where all cells play an integral part in the overall form. More formally, if the convex hulls of two (or more) forms always remain at least two cells apart, than we would say that these forms are separated and hence do not affect each other. Thus, we would not consider several separated copies of, say, figure 2-A as being another oscillator; here, we would merely have several copies of 2-A. With this in mind, a proof of (5) can be obtained by observing figure 11.

In spite of (5), under (3,3,3,3) no two distinct small oscillators (or stable forms) with identical signatures have yet been found. Hence, we have been able (so far) to classify all oscillators discovered to date by their signatures, or in cases where different phases have different signatures, the lowest signature alone. Table 1 is a complete list of such signatures for all known forms under (3,3,3,3). These forms have been illustrated in figures 2 through 9 and of course do not include "constructed" configurations—that is, configurations unlikely to be discovered by primordial experimentation on a computer. If different phases of a given oscillator have different signatures, the lower signature was given.

Although work with signatures of oscillators under rules other than (3,3,3,3) has been limited, nevertheless no two identical signatures for different forms have been found—with the interesting exception of the (4,6,3,3) oscillator shown in figure 13.

One should note that the signature can be obtained at little expense during most computer simulations, as all the neighbors must usually be checked anyway. Of course the signature concept could be extended to include the 42 neighbors that are a distance of two from a particular cell (or greater distances if desired). Naturally, this would increase the execution time and would cause the signatures to lengthen, although signature length is obviously bounded by 2*[number of cells in the configuration]. No work has been done with such "extended" signatures, nor has work yet been done with signatures in R^3.

2.2 Characteristics of $(3,3,3,3)$ oscillators

As mentioned in [2], two distinct gliders have been discovered, both with a period of two. These gliders move in a straight line along any of the 12 "neighbor touching directions." The little glider (see figure 2-N) has a total of 24 orientations, and the more symmetric big glider (see figure 2-O) has 12. A lengthy search has failed to turn up any other glider; if one does exist, it will probably have to be manufactured rather than discovered from primordial experiments.

All but four of the oscillators discovered exhibit a period of two. The exceptions are three period-four oscillators, figures 2-D, 7-G and 8-H; and one period-six oscillator, figure 5-C. The relative frequency of the most common forms have been tallied in table 2.

Almost all stable and oscillating forms that have been discovered exhibit symmetry of some sort. The few oscillators that appear to be asymmet-

12 ◊ 48 30 12	5-A
16 ◊ 56 22 16 11	9-G
18 ◊ 90 36 18	9-B
24 ◊ 120 48 24	9-C
4 2 ◊ 34 4 6 2	2-F
4 4 ◊ 36 12 8 2	6-D
4 6 ◊ 46 16 10 2	8-I
4 6 ◊ 54 12 10 2	7-F
4 6 ◊ 54 16 10	8-E
6 6 ◊ 42 30 12	6-C
8 3 ◊ 53 11 11 4	6-E
12 0 ◊ 68 18 12	8-F
12 6 ◊ 78 24 18 6	9-A
1 4 3 ◊ 33 12 8 1 1	6-G
2 2 6 ◊ 48 10 10 4	7-C
2 4 2 ◊ 30 12 8 0 2	8-D
2 6 2 ◊ 38 16 10 0 2	7-E
3 0 1 ◊ 24 3 4	2-E
3 4 2 ◊ 35 14 9 2	2-L
4 4 2 ◊ 38 16 10 2	9-I
4 4 2 ◊ 42 18 10	7-A
4 8 0 ◊ 44 18 12 3	8-G
4 8 4 ◊ 48 28 16 1 4	6-A
4 12 0 ◊ 48 24 16 7	9-H
6 0 2 ◊ 38 6 8 1 0 1	2-M
6 4 0 ◊ 38 8 10 0 4	8-B
10 0 0 ◊ 30 20 10	7-B
12 0 4 ◊ 24 48 16	4-H
1 0 3 0 ◊ 21 6 3	2-C
1 2 1 0 ◊ 19 6 3	2-A
1 5 1 1 ◊ 35 9 7 2	6-F
2 0 4 0 ◊ 26 8 4 2	2-G
2 0 6 0 ◊ 30 9 6 3 0 1	3-F
2 0 6 0 ◊ 30 14 6 2	8-C
2 0 6 6 ◊ 54 12 18 6	8-J
2 2 0 0 ◊ 16 8 2	2-B
2 2 12 0 ◊ 42 26 12 2 2	7-D
2 4 2 0 ◊ 26 13 6 1 0 1	2-I
4 0 0 0 ◊ 12 12	4-A
4 0 8 0 ◊ 44 14 8 7	2-H
4 0 8 0 ◊ 48 14 8 6	2-J
4 3 0 0 ◊ 22 12 3 0 1 1	2-N
4 4 2 0 ◊ 34 14 6 2 2 0	2-O
4 4 0 0 ◊ 22 16 4 1 0 1	3-B
4 4 8 0 ◊ 44 22 12 7 0 2	3-E
6 3 0 0 ◊ 21 24 3 0 0 1	3-A
6 4 2 2 ◊ 40 21 8 5	6-H
6 9 6 1 ◊ 48 45 16 9	5-B
8 0 8 4 ◊ 76 28 12 10	5-C
8 2 2 0 ◊ 30 22 4 2 4	8-H
8 4 0 0 ◊ 32 22 4 6	2-K
8 6 0 0 ◊ 32 30 8 4	8-A
12 6 0 0 ◊ 42 42 6 6	3-C
12 12 0 0 ◊ 36 66 12 6	9-F
12 12 12 0 ◊ 120 36 24 24	9-E
18 0 0 0 ◊ 30 45 0 6 0 3	4-B
20 0 0 0 ◊ 24 36 0 12 0 6	9-J
24 0 0 0 ◊ 24 48 0 12 0 8	4-D
24 0 0 0 ◊ 28 44 0 13 0 8	4-C
24 12 0 0 ◊ 60 54 12 24 0 6	9-D
32 4 0 0 ◊ 40 68 4 16 4 4 0 4	6-B
48 24 0 0 ◊ 96 120 24 36 0 20	4-E
144 24 0 0 ◊ 144 192 24 96 24 56 0 12	4-G
4 12 0 0 4 ◊ 64 24 12 10	7-G
12 0 12 0 4 ◊ 24 78 28	4-F
1 0 3 2 0 1 ◊ 22 13 4 0 0 1	2-D

Table 1: The signatures for all small stable forms discovered to date for $R = (3,3,3,3)$ are given here. When different phases have different forms (and therefore different signatures), the lowest signature is given. The right-hand column gives the figure illustrating the form.

OBJECT	SIGNATURE (LEFT PART ONLY)	NUMBER OF OCCURRENCES
2-A	1 2 1 0 ◇	139
2-B	2 2 0 0 ◇	106
2-E	3 0 1 ◇	47
2-C	1 0 3 0 ◇	42
4-A	4 0 0 0 ◇	32
2-D	1 0 3 2 0 1 ◇	31
2-N	4 3 0 0 ◇	4
2 × 2-B	4 4 0 0 ◇	2
2-A, 2-C	2 2 4 0 ◇	1
2-G	2 0 4 0 ◇	1
2-F	4 2 ◇	1

Table 2: Each experiment was initiated by randomly placing about 32 cells in a cluster at the very center of a size-19 grid. The relative density of the cluster was around .35. Most experiments resulted in the death of all cells after about ten generations. If a form still existed after 75 generations, its signature was examined. Gliders were detected when they hit the edge of the grid. The above table lists every object that was found when 18,000 experiments were conducted. In two cases, pairs of objects occurred.

ric usually have another phase which is a reflection of the first. Extensive random soup experiments have produced only two completely asymmetric oscillators—figures 6-F and 6-G. Probably other small asymmetric oscillators exist, and it is certainly possible to construct an infinite number of large ones (e.g. figure 11).

There is wide variation in the number of distinct orientations possible for a form. Specifically, forms that exhibit the least symmetry have the highest number of distinct orientations. For example, the form in figure 6-F has 48 distinct orientations while 4-D has only one.

Some oscillators and stable forms can be extended into larger configurations—see figure 3-D and figure 10. All such forms appear to be quite rigid in that they cannot be bent or twisted, but they can be made arbitrarily large.

3. Growth of random primordial populations under $(3,3,3,3)$

Random finite configurations of different densities and sizes were created and were allowed to evolve. Although there was considerable variation in the rate of evolution, eventually all forms either stabilized or disappeared (see figure 12). The maximum amount of stable residue was produced with a primordial density of about .07. At this density, successive generations evolved into small, isolated clusters, some of which yielded stable objects, which nevertheless were few and far between. At higher densities, a large, more-or-less

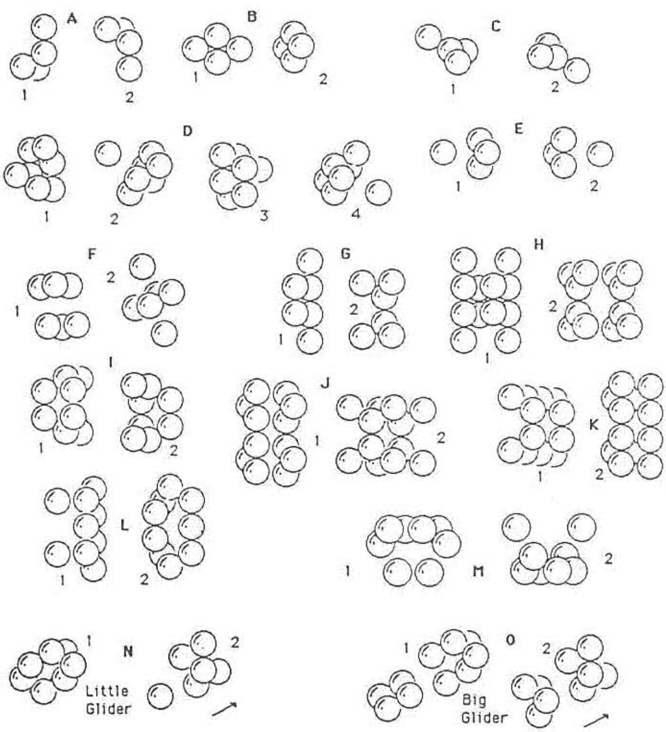

Figure 2: The top two rows (A through E) illustrate oscillators that commonly condense from primordial soup experiments. Oscillator D is rather interesting—at high speeds it appears to revolve about an axis, with a lone disconnected cell leading the way. The two gliders (N and O) occasionally appear, although not frequently. Oscillators G through L were discovered by starting with soup that was symmetric about a plane perpendicular to plane (1,2,3,4,5,6) and passing through line (8,10) (see figure 1-A). Most of these oscillators would rarely condense from soup that was completely random. All objects in figures 2 through 11 are under $R = (3,3,3,3)$.

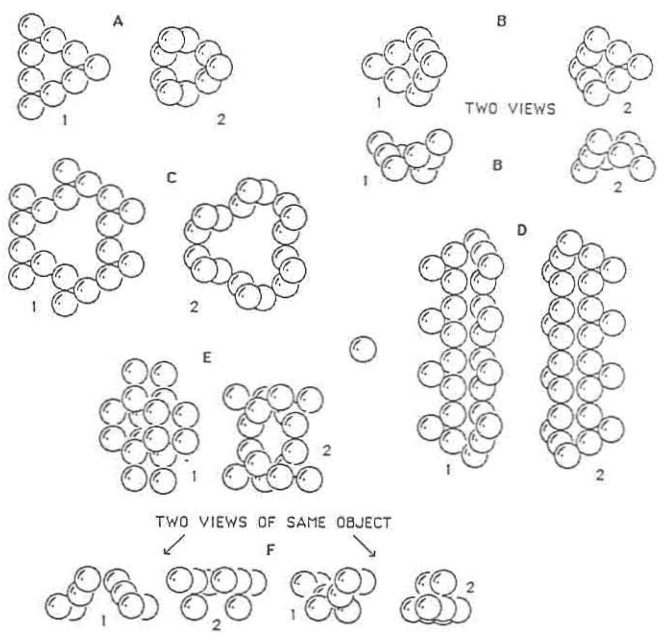

Figure 3: Most oscillators have a period of two. Object B is shown here in two different orientations. We can add indefinitely to the length of object B and thereby obtain object D. Note the importance of orientation—the two views of object F require close scrutiny to see that both views are of the same oscillator.

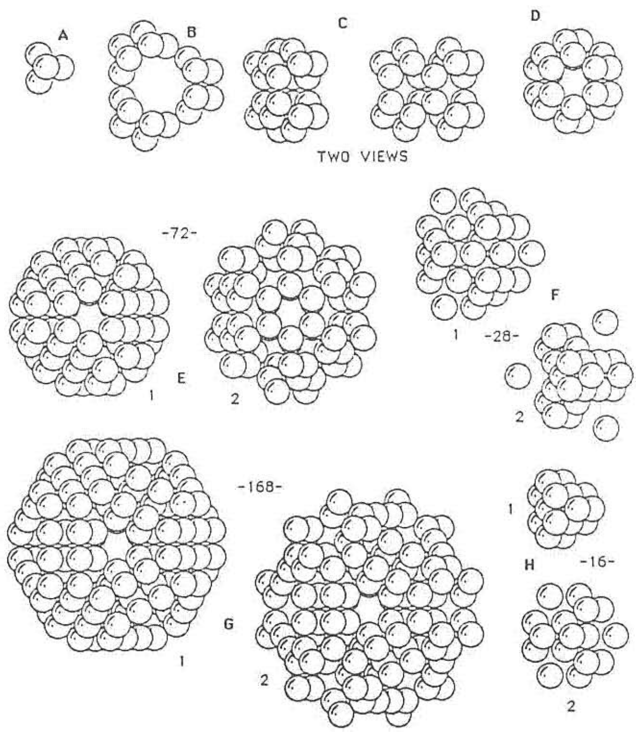

Figure 4: Stable forms are rare. Object A appears frequently in random soup experiments. Objects B, C, and D can be found by starting with symmetric blobs. The -168- and other similarly placed values give the number of living cells in the object.

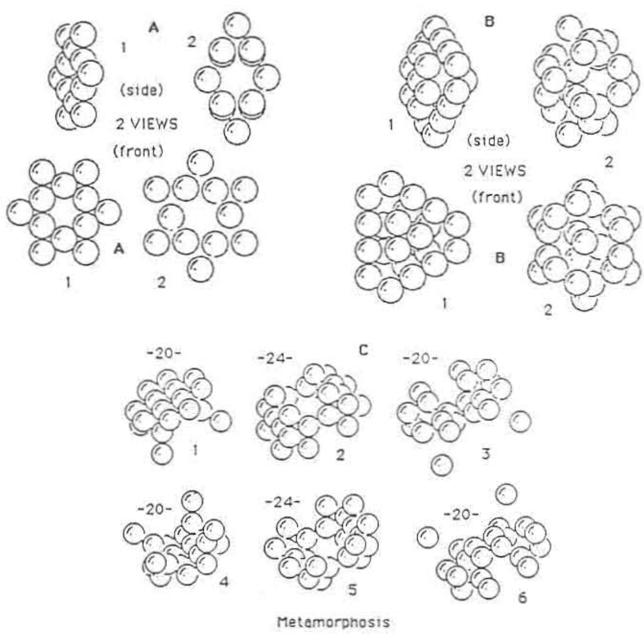

Figure 5: Note the difficulty in discerning the precise placement of cells for both views of state two, object A. This is one of the unavoidable difficulties of representing forms in Ω. The form "metamorphosis" is a remarkable oscillator of period 6. It exhibits a total of three different rotations; other objects have from 1 to 48 orientations, depending upon the type of symmetry inherent in the structure and the pattern of oscillation for the object.

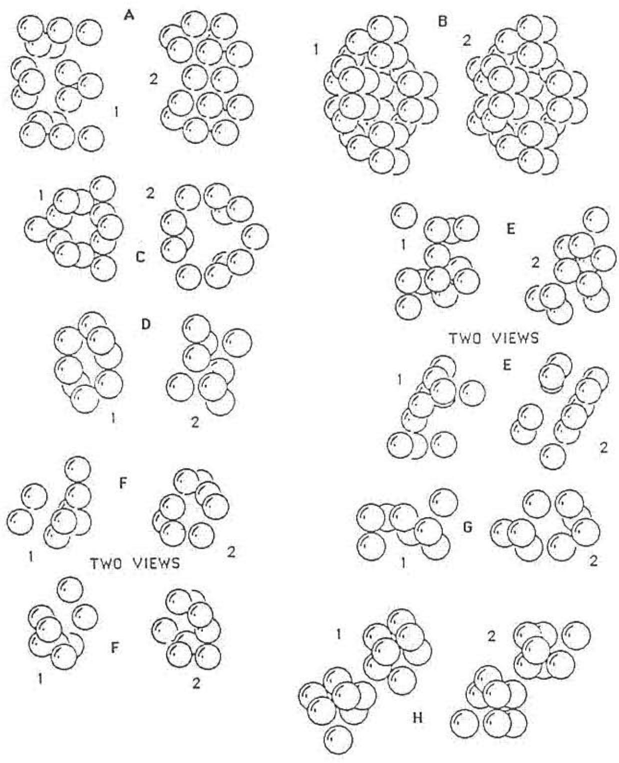

Figure 6: All previously illustrated forms exhibit some sort of symmetry; hence the surprise when the 11-cell oscillator E and the 8-cell oscillators F and G were discovered. Object E exhibits symmetry about a plane, and although objects F and G are asymmetric overall, parts of the oscillators (if considered separately) exhibit symmetry. The two halves of object H are symmetric but "twisted" with respect to each other.

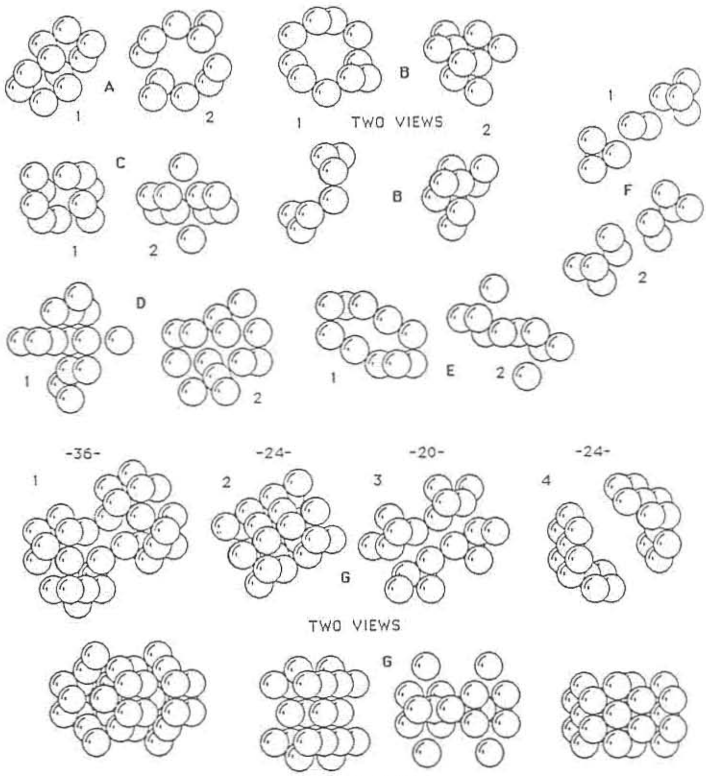

Figure 7: The oscillators at the top are confusingly similar, but they are distinct. The period-four oscillator "mitosis" contains 36, 24, 20, and 24 cells.

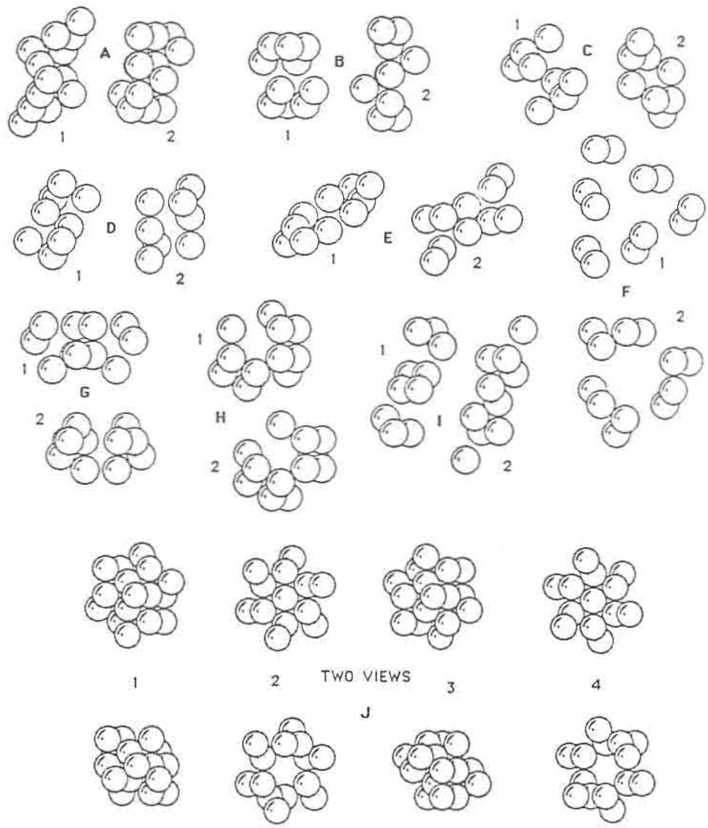

Figure 8: All the oscillators in figures 6 through 8 were discovered by starting with a known form (e.g. figure 3-B) and applying two or more randomly generated rules. Then the rule (3,3,3,3) was applied. Usually, all cells died; occasionally new forms were discovered and were saved in a file. Duplicate appearances were eliminated by comparing signatures.

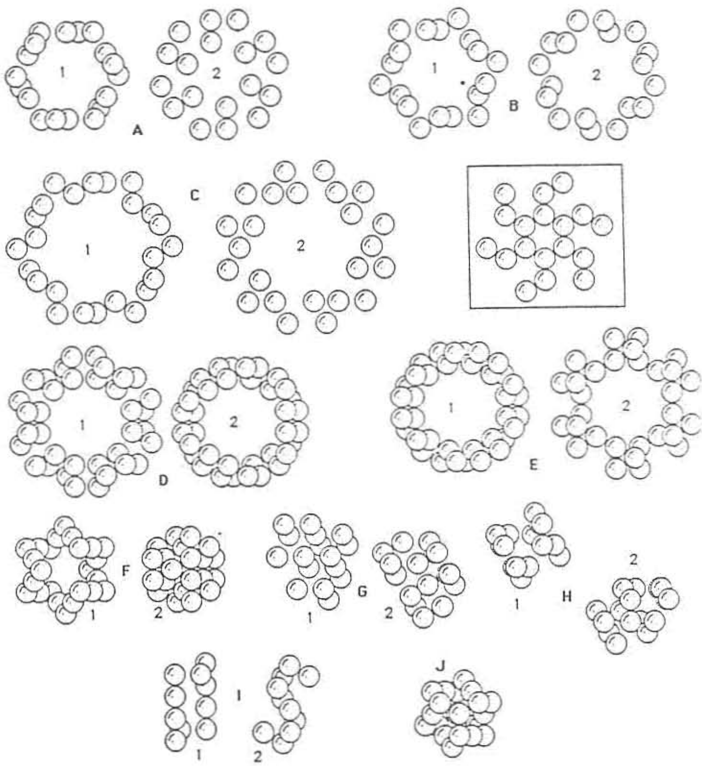

Figure 9: The oscillators A through E were discovered by starting with the inset pattern (not stable), applying randomly generated rules for a few generations, then reverting to $R = (3,3,3,3)$. Other starting patterns were used for the remaining forms. Note the similarity between form I and figure 8-D and between stable form J and period-four form 8-J.

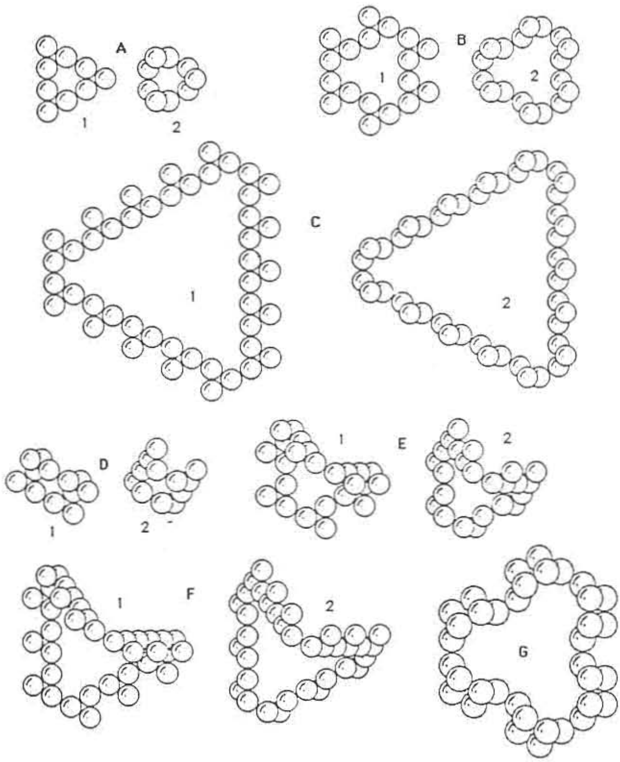

Figure 10: Some oscillators, such as figure 3-B, can be extended into arbitrarily large forms. The forms shown here extend to closed loops of aribtrary size. Object A (see figures 3-A, 3-C, which are repeated here for convenience) can be expanded to create objects such as B and C. Object D (see figure 2-K) extends into forms such as E and F. Also, the stable form in figure 4-B can be extended into shapes similar to G.

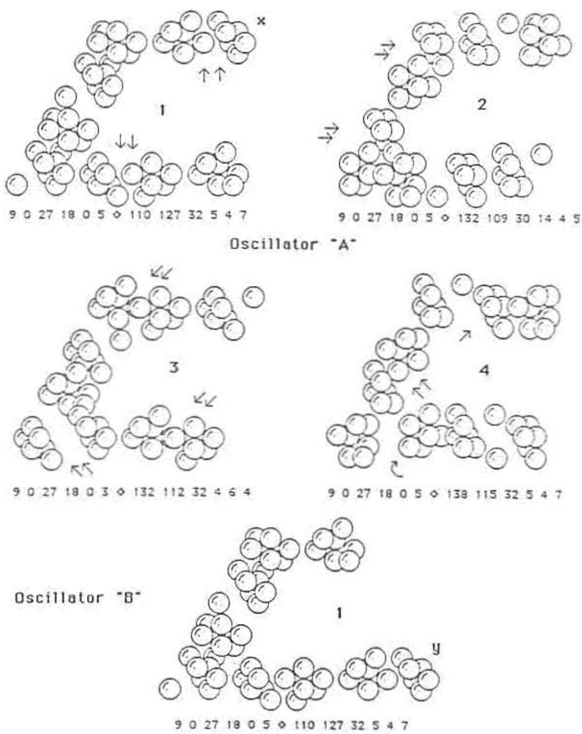

Figure 11: Proof that we may create as many oscillators as we wish that have the same signatures. The oscillators here have been constructed by stringing together nine figure-2D oscillators in close proximity (i.e., their convex hulls intersect). Specifically, the lone revolving cell (see caption, figure 2) disappears from time to time. (Without this interaction we would simply have individual 2-D oscillators spaced apart.) Oscillator "B" is constructed by removing the portion of oscillator "A" marked with an "x" and placing it at position "y." The arrows indicate where the lone revolving cells mentioned above have disappeared, due to the proximity of the 2-D oscillators. Note that we can extend the arms of this U-shaped oscillator as far to the right as we wish, and repeat the above procedure as many times as the length of the arms permits. Thereby, we obtain as many oscillators as we wish, and all will have identical signatures.

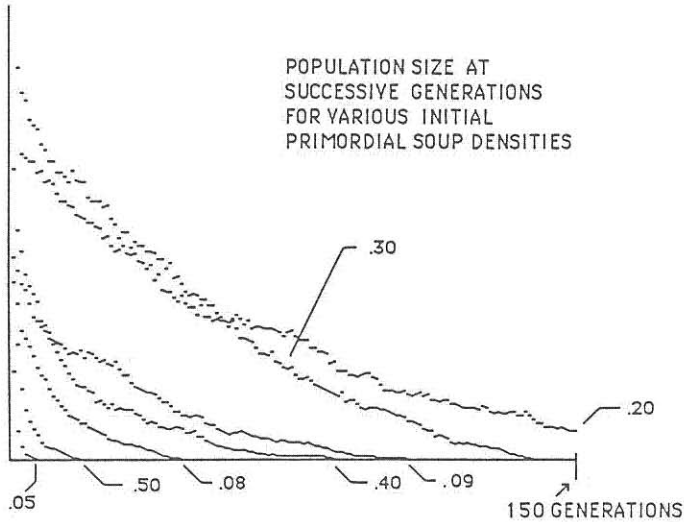

Figure 12: Plot showing the population evolution for various starting densities of random primordial soup. The maximum final residue of objects was realized when the starting density was about .07. In this case, many individual clusters formed and some of these condensed into objects. Between .15 and .35, the entire mass gradually contracted and usually disappeared. There was considerable variation in the behavior of the population mass over this range; i.e., the plots for .20 and .35 could be easily interchanged. When the initial density was larger than about .35, behavior reverted to that of the small densities—after a first generation where most cells died. For these experiments, a grid size of 25 was used.

spherical mass formed; this mass gradually diminished and eventually disappeared, usually leaving no residue. The effect was most pronounced at initial densities of about .2 to .4. This interesting phenomenon is currently under investigation.

4. Other rules

By no means should one conclude that (3,3,3,3) is the only rule worthy of investigation. One rationale for the extensive exploration of (3,3,3,3) was that it appears to be the only rule that might be considered to be a game of Life [2]. Other rules of the form (E_l, E_u, F_l, F_u), though apparently not supporting gliders, nevertheless exhibit remarkable oscillators; in particular, rules of the form $(E_l, E_u, 3, 3)$ seem to be most interesting.

One such rule is (4,6,3,3), which allows unbounded growth and for which

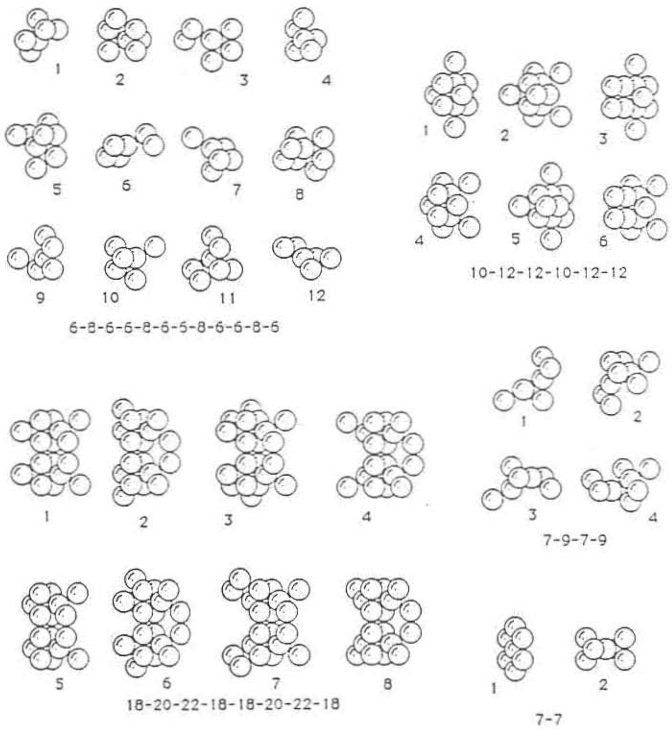

Figure 13: Here are just a few of the many oscillators for (4,6,3,3) with a period greater than two. The period-two (4,6,3,3) oscillator at the lower right is the only example discovered to date where distinct phases have identical signatures. The signature is 1 0 0 6 0 0 0 ◇ 18 12 6. The numbers separated by dashes give the live cell counts for each phase.

no glider has been discovered. Some of the more fascinating oscillators for this rule are illustrated in figure 13, including one mentioned earlier that has distinct phases with identical signatures. These were found with only a very limited search; undoubtedly other interesting forms could easily be discovered for (4,6,3,3) and other rules. For this rule, unlimited growth can be easily initiated with sufficiently large starting random blobs.

5. Methods of computer implementation

The non-orthogonality of Ω presents certain problems when we want to implement a computer simulation. The simplest approach is given in [4], where R^3 is utilized and neighbors of a cell at (i,j,k) are situated at $(i,j,k-1)$, $(i,j+1,k-1)$, $(i+1,j,k-1)$, $(i,j+1,k)$, $(i+1,j,k)$, $(i,j-1,k)$, $(i-1,j,k)$, $(i+1,j-1,k)$, $(i-1,j+1,k)$, $(i,j,k+1)$, $(i,j-1,k+1)$, $(i-1,j,k+1)$. When this implementation is used, the (finite) universe is shaped like a rhombohedron and must be adjusted to fit more perfectly on the typically square or rectangular computer screen. One may either adjust the viewing angle, or the coordinates utilized (i.e., chop off part of the rhombohedron with successive k-planes).

Another method is particularly attractive if one has already at hand a working implementation of a program that operates on three dimensional cellular automata in R^3—e.g., the Life games mentioned in [2,3]. Here we arrange the universe as shown in figure 14, with alternate locations (and alternate rows) left vacant in the (i,j) planes. The neighbors of a cell at (i,j,k) are at $(i+1,j-1,k+1)$, $(i+1,j+1,k+1)$, $(i-1,j,k+1)$, $(i-1,j-1,k-1)$, $(i-1,j+1,k-1)$, $(i+1,j,k-1)$, $(i,j+2,k)$, $(i,j-2,k)$, $(i+2,j-1,k)$, $(i+2,j+1,k)$, $(i-2,j-1,k)$, $(i-2,j+1,k)$. For the display, we change the cubes to spheres and make other minor adjustments. The universe will be approximately cubic in appearance (see figure 1-C for a small example). Unfortunately, wrapping the coordinates in this implementation is a nightmare.

If the particular implementation requires that we look at every cell, we must alter the manner in which we sweep through space. The following bit of code will suffice.

```
DIM A(GRIDSIZE*2,·GRIDSIZE*2, GRIDSIZE)
I_INIT_ARRAY = [1,2,1,2]
J_INIT_ARRAY = [2,1,1,2]
FOR K = 1 TO GRIDSIZE
    KMOD4 = K(MOD 4) +1 {set up starting points for row/column}
    ISTART = I_INIT_ARRAY(KMOD4)
    JSTART = J_INIT_ARRAY(KMOD4)
    FOR I = ISTART TO GRIDSIZE*2 STEP 2
        JSTART = JSTART + 1 MOD 2 {for next column}
        FOR J = JSTART TO GRIDSIZE*2 STEP 2
            {examine cell I,J,K in array A etc.}
        NEXT J
    NEXT I
NEXT K
```

Note that the above method requires four times as much memory as is actually used. With megabyte memories, this is not really a problem. Furthermore, if we employ the hashing method described in [2], then our memory requirement will depend only upon the number of live cells: we do not need the above code at all and only need to access the neighbors. Execution times for the various methods are discussed in [2].

6. Future and current work

An effort is currently under way to devise a classification scheme according to rate and pattern of growth for three-dimensional cellular automata. Part of the difficulty in devising such a scheme is that when we allow non-totalistic rules, or expand the neighborhood to include non-adjacent cells, even if we reject a large percentage of the 2^{8192} possible rules in Ω as "illegal" (see [5]), the possible number of rules is still vast. Hence, meaningful empirical studies would probably be difficult to conduct.

Nevertheless, it appears that the $(13 + 12 + 11 + \ldots + 1)^2 = 8281$ rules of the form (E_l, E_u, F_l, F_u) fall into three general classes according to growth patterns. Work is currently under way to determine in a more precise fashion the characteristics of these three categories, and their relation to the four types described by Wolfram [5].

Future work involves the application of signatures to known oscillators in other universes—most notably the two games of Life that exist in R^3, $R = (4,5,5,5)$, and $R = (5,7,6,6)$. Perhaps a type of signature could be devised that would guarantee that different forms will have different signatures. For example, one could design a method where we find the convex hull of a form and map the N cells contained in the hull to the integers 1 through N. Then, find the product $P_{i1} * P_{i2} * P_{i3} * \ldots$, where the subscripts represent live cells and P_k is the k^{th} prime. This "signature" would certainly be unique, but probably of little practical value. Then there is the unexplored world of

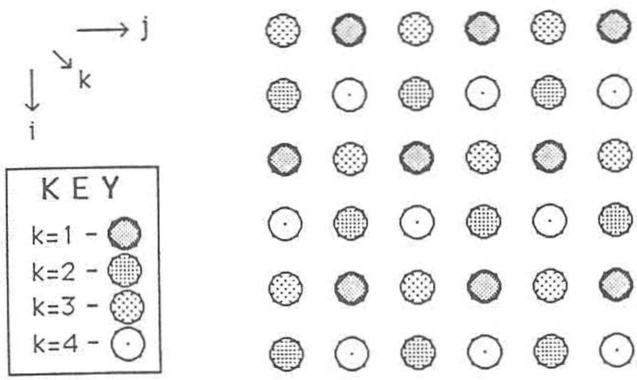

Figure 14: Three-dimensional Cartesian coordinates may be used to emulate the hexahedral tessellation in a roughly cubic universe. Here, only one in four cells is actually utilized. The valid cells are indicated by the various shaded circles.

$R = (3,3,3,3)$: glider collisions, glider guns, etc. This is work best left for the computer hobbyist or hacker and will undoubtedly keep some individuals busy for long hours.

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