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Demystifying Quantum Mechanics: A Simple Universe with Quantum Uncertainty

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Abstract. An artificial universe is defined that has entirely deterministic laws with exclusively local interactions, and that exhibits the fundamental quantum uncertainty phenomenon: superposed states mutually interfere, but only to the extent that no observation distinguishes among them. Showing how such a universe could be elucidates interpretational issues of actual quantum mechanics. The artificial universe is a much-simplified version of Everett's real-world model, the so-called *multiple-worlds* formulation of quantum mechanics. In the artificial world, as in Everett's model, the trade-off between interference and observation is deducible from the universe formalism. Artificial-world examples analogous to the quantum double-slit experiment and the Einstein-Podolsky-Rosen (EPR) experiment are presented.

1. Statement of the paradox

Isaac Newton's objective, mechanical world, grounded in an elegant collection of precise rules, seemed for a time to realize an ideal that had been sought for two millennia. But the preemption of classical physics by quantum mechanics is widely regarded, not least by physicists themselves, as a fundamental retreat from this ideal. Physics, which was once the best exemplar of the mechanical paradigm, now seems to be its most formidable detractor.

The well-known apparent nondeterminism of quantum mechanics is the least of its oddities; probabilistic laws still afford a straightforwardly mechanical model. Far stranger is the apparent observer-dependency of nature. Of the several states that a particle might be in, it seems that all coexist—as is shown, statistically, by their mutual interference—unless we try to observe this so-called *superposition* of states. Paradoxically, any such observation always reveals just one of the thitherto-coexisting states. Which of the states we observe is unpredictable in principle, hence the apparent nondeterminism.

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This paper first highlights the seeming paradox of quantum mechanics, then presents a simple model that, using little more than high-school mathematics, illustrates Everett's solution to the paradox—a solution that rescues the mechanical paradigm, restoring determinism and observer-independent reality to quantum physics.

1.1 The double-slit experiment

The classic double-slit experiment highlights the quantum paradox. We aim an electron at a pair of adjacent, narrow slits in a barrier (imagine this happening in just two dimensions). Beyond the barrier lies a backdrop with a row of densely-packed electron detectors each of the same resolution as the width of each of the slits; the distance between the two slits is much greater than this resolution. If the electron passes through the barrier via the slits, we find that one and only one detector soon registers the arrival of the electron.

Suppose we block one of the two slits and conduct many trials of this experiment, plotting the distribution of electron-arrivals at the various detectors. Not surprisingly, we see a smooth curve with a peak opposite the unblocked slit. If instead we unblock the other slit, then of course the distribution curve has a peak opposite that other slit. If we conduct a number of trials, half with one slit blocked and half with the other blocked, the distribution curve is just the sum of the two single-slit curves. All this is consistent with an electron particle that is smaller than the width of each slit, and that passes through the currently unblocked slit.

But now, suppose we try the experiment with both slits unblocked. Bizarrely, the distribution curve is not the expected sum of the single-slit curves; rather, the curve shows an interference pattern. At some points along the backdrop, the frequency of an electron's arrival is not only less than what the sum of the single-slit curves predicts, it is less than what either single-slit curve *alone* would predict. The distribution seen over a large enough number of trials must approximate the sum of the probability distributions of the individual trials; hence, by providing an additional path by which an electron might arrive at a certain point along the backdrop, we have *reduced* the probability of its arriving there on a given trial.

This result is inexplicable if the electron indeed passes through just one slit or the other. If a given electron encounters just slit A, opening slit B could not reduce the likelihood of the electron's reaching a given destination through slit A. But the interference is just what we would expect if the electron were not a spatially localized particle, but rather an expansive wave that passes through both slits, creating usual wave-like interference on the other side of the barrier. Indeed, the observed interference pattern accords quantitatively with the predictions of wave mechanics. The wave's amplitude at a given point corresponds to the probability (it is actually the square root of the probability) that the electron arrives there, as seen by a detector at that point.

But this raises an apparent paradox. If the electron spreads out in a wave-like fashion, why does the backdrop detect only a local, discrete arrival for each electron? Why does only a single detector react, rather than many adjacent ones? As noted above, the statistical distribution over a large number of trials warrants an inference about what occurs during *each* trial. We can thus infer from the statistical evidence that the electron passes through both slits on each trial. Thus, the universe seems to be playing hide-and-seek: whenever we detect the electron, we see a localized particle; but when we do not observe it, the electron is a wave, passing simultaneously through two widely separated slits (widely separated compared to the size of the particle), and exhibiting interference on the other side.

We might seek to clarify the situation by shining a light source on the barrier to see the electron as it passes through. In that case, we unambiguously see the electron emerge from just one slit or the other. But then, the distribution curve over many such trials no longer shows interference; instead, it simply equals the sum of the single-slit curves.

1.2 The interference-observation duality

Thus we have the fundamental, paradoxical duality:

- There are coexisting, mutually interfering states, so long as the states are not distinguished by observation. (Here there is a continuum of such states that propagate in a wave-like fashion.)
- Whenever an observation is made, only one of the superposed states is seen. (Here a conventional particle, much smaller than the wave, is all we see when we look.)

This is known as the quantum-mechanical *wave-particle* duality. A standard understatement of this duality is that an electron (or other physical entity) acts sometimes like a wave, sometimes like a particle. More strikingly, we have here an *interference-observation* duality: there are many superposed, mutually interfering states whenever we are not "looking," but only one such state whenever we do look. Heisenberg's uncertainty principle says, moreover, that no matter how precise an observation we perform, some superposition must remain. Indeed, the more precisely we measure a given attribute, the more superposition there is with respect to some other attribute.

To see how dramatic the interference-observation duality really is, consider Wheeler's *delayed-choice* modification of the double-slit experiment: one does not decide until after the electron passes the barrier whether to collect the electron against the backdrop or pull the backdrop out of the way and observe which slit the electron came through (by using a pair of "telescopes," each focused on one slit). If we choose to remove the backdrop and make the observation, we see that the electron passed through just one of the slits. If we choose not to observe, the distribution we see over many such trials is once again consistent with the "particle" having passed, wave-like, through *both* slits on *each* trial, the two parts of the wave then mutually interfering. What, then, does the electron do when it reaches the barrier prior to the decision whether to observe where it comes from? Does it pass through one slit or both? It seems that the answer is determined *in retrospect* when the distinguishing observation is made, or when the electron instead reaches the backdrop.

1.3 Interpretations: Copenhagen and Everett

The standard interpretation of such phenomena, the Copenhagen interpretation, shows the profound effect of this paradox on physicists' sense of reality. According to the Copenhagen interpretation, no physical phenomenon is real until it has been observed. Nothing real passes through both slits of the apparatus; there is a potential for a real particle to pass through either slit, but that potential is not realized unless the passing-through is observed. This interpretation does, indeed, accord with the fact that the particle cannot simply pass through just one of the slits (else the interference would not be seen statistically), and with the fact that that is just what the particle has done whenever we look. But it gains this accord at the price of denying the observer-independent existence of the building blocks of reality.

Thus, quantum mechanics seems to challenge not just the world's determinism, but the very objectivity of its existence. Indeed, the Copenhagen interpretation provides no way to express the state of the universe as a whole, since a system's state is real only with respect to an external observer, and the universe as a whole has no external observer.

The Copenhagen interpretation exhibits the usual rigor of physics to say what happens to the world *between observations*. This is given by Schrödinger's equation, which governs the (fully deterministic) propagation of a (wave-like) quantum state of the universe. This state is a superposition of many individual, sometimes mutually interfering states, such as the state of an electron being at one slit or the other. When an observation occurs, Copenhaganists insist that the superposition of states *collapses*, leaving just one member of the previous superposition. Schrödinger's equation itself does not predict any such event as this collapse.

The Copenhagen interpretation has no formal criterion for what constitutes an observation, and hence for when the putative collapse occurs. Is the detection of a quantum event by a laboratory instrument an observation? In [10], it is shown that the same prediction is made whether one stipulates a collapse at that point or, on the contrary, one regards the superposition as persisting[1] so that the macroscopic instrument is itself in a superposition of more than one detection state. Von Neumann's conclusion: only when a conscious being observes the state of the instrument and sees that it is unambiguously in one state or the other does it become clear that only one outcome really occurred. Thus was von Neumann led to conclude that hu-

[1] Actually, the same prediction is made only when some trace of the observation persists. See section 5.3 for elaboration.

man consciousness plays a fundamental role in physics: conscious observation precipitates the collapse of the quantum superposition.

Most physicists, unlike von Neumann, accept that inanimate observation suffices to bring about the collapse. Still, a number of eminent theoretical physicists share von Neumann's version of the Copenhagen interpretation—quantum mechanics' most profound departure from the mechanical paradigm.

However, there is an alternative interpretation of quantum mechanics that restores a mechanical understanding of the universe. Quantum phenomena such as the double-slit experiment show that, prior to observation, the superposed states have symmetric status; that is, no one of the superposed states is already the unique real one. (*Hidden-variable* theories try to deny this, but such theories are provably wrong; see section 5.4.) Logically, then, there are two ways to achieve this symmetry: either none of the superposed states is real, or all of them are. The Copenhagen interpretation says none of the yet-unobserved states are yet real. Everett's so-called *multiple-worlds* interpretation [7] says all of them are real.

In Everett's formulation, the quantum collapse never occurs. Superposed states remain in superposition even after observation (whether by inanimate objects or by conscious observers). It remains to account for the *apparent* collapse—the fact that we see only one outcome of the quantum observation. Everett's crucial insight is that the deterministic Schrödinger formalism already predicts an apparent collapse, even while denying an actual one. According to the formalism, observing a superposed state results in different versions of the observer in different versions of the universe, each version of the observer seeing a different outcome to the exclusion of all other outcomes. Of course, it makes no difference whether the observer is animate. Thus, versions of the observers themselves are in superposition, but they are mutually isolated so each sees a seemingly unique outcome. Following Everett, I argue here that this interpretation is actually the more parsimonious, but it takes a formal model to demonstrate that claim.

In this paper, I try to make sense of the quantum-mechanical universe. Often, the best way to understand a thing is to build one. Hence, I build a universe, a qualitative model of quantum mechanics. That is, I define a universe whose physics are quite different from (and much simpler than) our own world's, and I demonstrate that this universe exhibits an interference-observation duality analogous to that of real physics. We can call this model *quantish physics*. The analogy runs deep enough to support a comparison between the "Everett" and "Copenhagen" interpretations with respect to the qualitative model, and this comparison elucidates an interpretation of real physics.

I present three artificial "universes": U1, U2, and the quantish-physics model. The first of these universes, U1, has straightforwardly "classical" mechanics. U2 attempts to incorporate quantum-like uncertainty in its physics, but fails in instructive ways. Finally, the quantish-physics model, building from the U2 attempt, succeeds in reconstructing the fundamental quantum interference-observation duality.

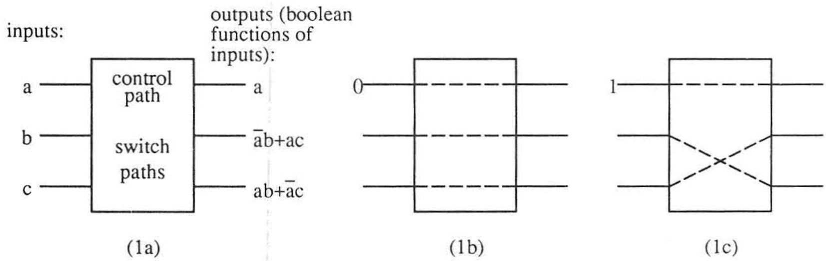

Figure 1: A Fredkin gate.

2. U1: "Classical" physics, configuration-space representation

Let us define a universe consisting of a circuit built from Fredkin gates [8]. A Fredkin gate has three binary (0 or 1) inputs and outputs. Each output computes a boolean function of the inputs, as specified by figure 1(a). But the gate is more easily understood as having a *control path* going across the top of the gate, and two *switch paths* below. If the first input (the control input) has a 0, then the second and third inputs (the switch inputs) simply propagate to the second and third outputs respectively, as suggested by figure 1(b). If instead the control wire has a 1, then the two switch wires "cross," so the second input comes out at the third output, and vice versa (figure 1(c)). The control wire simply propagates its input to its output. All three paths through a gate impose a delay of one time unit between the appearance of an input value and its propagation to the corresponding output,[2] and all gates in the circuit are synchronized. Fredkin gates, unlike some logic gates, do not allow fan-in or fan-out; rather, each output must connect to exactly one input.

Fredkin gates, like NAND gates, are *universal*. Loosely speaking, their universality means that any logic circuit that can be built at all can be built using only Fredkin gates. Fredkin gates have the further property of *conserving ones and zeros*—that is, the number of ones (or zeros) that leave a gate equals the number that entered the gate one time unit earlier, hence the total number of ones (or zeros) coursing through the circuit remains constant.

For a given "universe" (that is, a given Fredkin-gate circuit), one might represent the state of the universe at a given time by listing, for each wire, whether that wire has a one or a zero. Hence the state can be represented by a vector $\mathbf{v}_1, \ldots, \mathbf{v}_n$, where $\mathbf{v}_i$ is 0 or 1 according to the state of the ith wire, and n is the number of wires in the universe. (A wire goes from an output to an input; a gate's output wires are distinct from its input wires.)

Alternatively, because Fredkin gates conserve ones and zeros, we can index the world-state the other way around: for each 1—think of 1s as "particles"—we can say in which wire it currently resides. We will construe a particle as passing through a gate in the obvious fashion: a particle

[2] In [8], delays occur in the wires rather than in the gates.

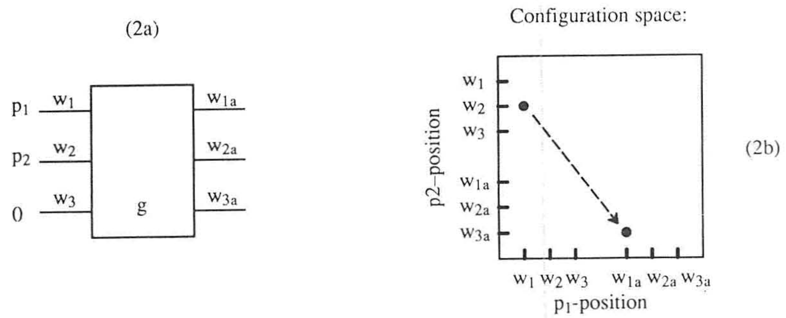

Figure 2: A state moves through configuration space.

on a gate's control-wire input emerges from the gate's control-wire output; a particle at one of the two switch inputs either proceeds straight across or crosses over, depending on the control-wire state. To specify which wire a particle is in is to fully specify the particle's position. No gradations of position along a wire are recognized.

Let us now present the particle-indexed state geometrically. If the universe has k particles, we define a k-dimensional space, and each dimension has discrete coordinates ranging from 1 to n (the number of wires in the universe). For a given point $(p_1, \ldots, p_k)$ in this space, the point's ith dimension says which wire the ith particle is in. Call this space *configuration space*.[3] A single point in configuration space represents the entire state of the universe. Rephrasing the physics of this universe in terms of configuration space, we get a rule for moving from one point in this space to another at each unit-time interval.

Figure 2 illustrates this formulation. Suppose gate g appears in the Fredkin circuit that defines our model universe, and suppose for now that there exist just two particles, p_1 and p_2. Particle p_1 appears at g's control wire, p_2 at g's upper switch wire. Figure 2 shows the configuration space point s_0 that designates this state of the universe. At the next time unit, the state of the universe becomes s_1. In that state, p_1 has moved to w_{1a} and p_2 has crossed over to w_{3a}.

The configuration space representation is equivalent to, but more cumbersome than, the more obvious wire-vector representation. But in the following sections, we shall see how this representation supports the introduction of quantum-like phenomena to our Fredkin-gate universes.

[3]Configuration space is analogous to *phase space* in real-world classical physics. For a system with k objects, phase space has $6k$ dimensions: three dimensions for each particle's position and momentum. Thus, a single point in phase space specifies the position and momentum of every object.

3. U2: A universe with non-interfering superpositions

Suppose we modify the classical physics to allow a superposition of states to coexist. Rather than representing the state of the universe by a single point in configuration space, we assign a *weight* (in $[0, 1]$) to each configuration-space point, where the weights sum to unity. In U1, a single point changed its configuration-space coordinates at each unit-time interval. In U2, all weighted points move simultaneously, carrying their respective weights along; each moves according to the same rules that governed the single point in U1.

To avoid ambiguity, we now say that each point in configuration space represents a *classical* state of the universe, and the entire set of weight assignments in configuration space is a *quantum* state. In U2, the state of the universe is the quantum state, which we say is a superposition of its nonzero-weighted classical states. (When no ambiguity results, I will continue to speak of a "state," with "classical" or "quantum" left implicit.)

We may think of the weights in configuration space as probabilities. The set of weight assignments specifies a probability distribution as to what classical state the universe is in. But note that the physical laws of U2 are not, in fact, probabilistic. They are deterministic laws that push weights through configuration space, though it will be helpful to think of these weights as probability measures.

Figure 3(a) shows a fragment of a Fredkin-gate circuit. (Here and throughout, unconnected wires are understood to connect to gates not shown.) Particle p_1 is in a superposition of two positions, w_1 and w_2; particle p_2 is at p_3; and p_3 is at w_4. Figure 3(b) shows this situation from a three-dimensional cross-section of configuration space, with dimensions corresponding to the positions of the three particles. States s_{1a} and s_{1b}, each with weight .5, correspond to the superposed positions of p_1.

Initially, the three particles' positions are mutually independent. In particular, p_2's position and p_3's position are the same whether p_1 is at w_1 or w_2. One time unit later, though, the gates have correlated p_1 with p_2 and p_3 (as shown in figure 3(c)). There is still a superposition of two world states, s_{2a} and s_{2b}; in each, p_2 is at w_{3a} and p_3 at w_{5a} if and only if p_1 is at w_{1a}. Hence, the position of p_1 has been "observed" by p_2 and p_3. Although the universe still contains a superposition of two states for p_1, p_1's state *relative to* p_2's (to use Everett's terminology) is unambiguous: p_1 is at w_{1a} relative to p_2 at w_{3a}; p_1 is at w_{2a} relative to p_2 at w_{6a}. Similarly, p_1's state is unambiguous with respect to p_3's state.

Note the consistency of the two observations of p_1. There are only two possible outcomes: one state where p_2 crosses over and p_3 does not, so that only p_2 is diverted by p_1; and, symmetrically, a state where only p_3 is diverted by p_1. Hence, either state is consistent with p_1 being at w_1 or w_2, but not both. Moreover, it is easily verified that any subsequent observations of p_1, p_2, or p_3 will maintain this consistency. By virtue of this consistent repeatability, the interactions with p_1 are what Everett calls *good observations*.

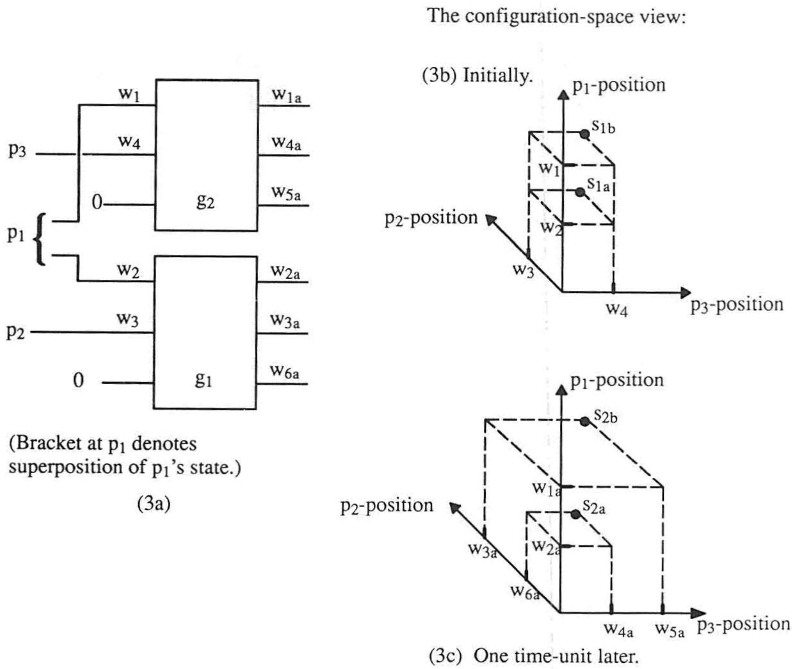

Figure 3: Particles p_2 and p_3 observe p_1's position.

Prior to the observation, p_1 was in a superposition of two states. Subsequently, although this superposition continues, there are two branches of the universe, each consistently and unambiguously showing one state of p_1. Thus, we might try to construe this interaction to model the apparent collapse of the quantum superposition—apparent, that is, from the standpoint of any observer embodied in U2.[4]

But that construal would be wrong. In fact, from within U2 there was never any apparent superposition to begin with. Hence the observation did not appear to collapse any superposition. The problem is that there is no "interference"—no interaction at all—among the superposed classical states. Each such state has a unique immediate predecessor as well as a unique immediate successor (because, as is readily seen, a Fredkin gate's outputs uniquely specify what the inputs must have been, as well as vice versa).

[4]Here, in a large leap of imagination, I suppose a Fredkin-gate circuit that implements—or, if one prefers, simulates—a universe vaguely like our own, with complicated physical objects, including those that have the machinery of intelligence. Hence, that universe could embody intelligent observers. This should seem possible in principle to those who accept the possibility of artificial intelligence, or who believe that intelligence has a mechanistic explanation. For present purposes, nothing need be specified about the workings of the hypothetical embodied intelligence other than that it is implemented by some sort of computer program or program-like mechanism.

Thus, two superposed classical states never converge; each evolves entirely independently of the other, moving through configuration space without interfering with the other. Therefore, the superposition is evident only to an observer external to the entire universe who can examine configuration space directly. To any observer embodied in any "branch" of the universe (any element of the superposition), there is never any evidence of the existence of any other branch. Hence, as seen from within this universe, the universe appears entirely classical and is indistinguishable from U1. In particular, the 1s behave like ordinary "particles," just as in U1.

4. The laws of quantish physics

In this section I present laws of physics that are analogous to real quantum mechanics under the Everett interpretation. Indeed, this section largely recapitulates Everett's relative-state formulation of quantum mechanics, but with Fredkin-gate mechanics substituted for quantum-wave mechanics. The interference-observation duality of real-world physics, that superposed states interfere with one another if and only if no observation has distinguished among them, is a property of quantish physics as well.

The quantish-physics model extends and modifies the U2 model. Quantish physics has three characteristics that distinguish it from U2 physics: multiple successor and predecessor states, complex rather than real-valued weights, and a binary-valued *gender* associated with each particle. A particle's gender is analogous to spin in real quantum mechanics.

In U2, each classical state has a unique successor and predecessor, so distinct states do not interfere. In the quantish model, a classical state can have multiple immediate successors and predecessors. The weight of a configuration-space point splits into components that each contribute to one of the point's immediate successors; the contributions of multiple predecessor points to a common successor simply add.

To facilitate interference, quantish classical states are assigned complex weights rather than real-valued weights. The probability measure associated with a classical state is the squared magnitude of its weight, and in every quantum state the probability measures of the classical states sum to unity. When a classical state splits into two successors, its weight splits into two orthogonal components of the original weight, so the sum of the successors' probability measures equals the predecessor's probability. When several configuration-space points contribute to a common successor point, the sum of the contributing weights has a squared magnitude that may be less than, equal to, or greater than the sum of the contributing squared magnitudes. This provides for destructive and constructive interference.

Each quantish particle has a gender whose value is either *female* or *male*, and each gate has a *measurement angle*. A gate's measurement angle cannot change; like the circuit topology, it is simply built into the universe. But a particle's gender can change, so each particle's gender is part of each quantish classical state and must be represented in quantish configuration space. Therefore, quantish configuration space has *two* dimensions for each particle: one, as in U2, for the particle's position, and the other for the particle's gender. Each gender-dimension has just two discrete coordinates, one corresponding to female, the other to male.

4.1 Definition of quantish physics

As with U2, quantish physics is defined by laws that say, for any classical state, where each particle next moves to (and, now, what its next gender is). As in the previous model, these laws translate into a rule that specifies the coordinates of a classical state's successor point in configuration space. The weight associated with the predecessor point moves to the new point.

But in the quantish model, a given particle in a given classical state can have two next positions and two next genders, rather than just one of each. This multiplicity of destinations and genders corresponds to a four-fold split in the given classical state. That is, the given state has four successor states rather than a single successor: there is one successor state for each of the four combinations of destination and gender for the given particle. Thus, no successor state shows the particle simultaneously at more than one position or with more than one gender. Rather, there is a distinct classical state for each of the alternatives.

The given state's weight divides among the four successors, as described below. More generally, in a given classical state there may be n particles with two next positions and genders each. Then there are 4^n successor states, one for each combination of the binary next-position and next-gender choices for each of the n particles.

Defining quantish physics, then, requires specifying:

- How particles move through gates—the rule for a particle's next position (or positions) and next gender (or genders); and, in the event of multiple destinations or genders, the rule by which the weight of a configuration-space point divides among its successor points.
- The rule by which weights combine when multiple predecessors have one or more successor points in common.

How particles move through gates is explained just below. The rule for combining weights is trivial: as mentioned above, when several configuration-space points each contribute a portion of their weight to a common successor point, the contributed weights simply add. This, together with the fact that a classical state's successors are a function of that state alone (regardless of any other classical states superposed in the quantum state), ensures that the quantish state-succession, like real-world quantum-state evolution, is *linear*. That is,

$$\text{successor } (q_1) + \text{successor } (q_2) = \text{successor } (q_1 + q_2)$$

where the successor function maps a quantum state onto its successor quantum state, q_1 and q_2 are quantum states, and $q_1 + q_2$ is the quantum state

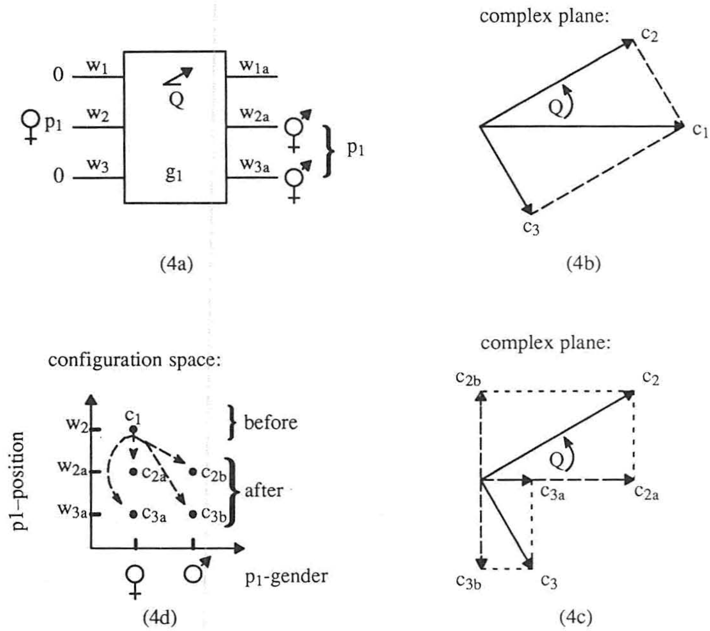

Figure 4: A classical state with weight c_1 splits into four successors.

whose weight at each configuration-space point is the sum of the weights of q_1 and q_2 at that point.

A particle at the control-wire input to a quantish gate simply passes through to the control-wire output, as in U1 and U2; its gender remains the same. However, a particle that is at a gate's switch-wire in a given classical state behaves differently than in U1 and U2. Roughly speaking, the particle emerges at *both* of the gate's switch-wire outputs with *both* genders at each destination, as suggested by the bracket notation beside gate g_1 in figure 4(a). (The gate's measurement angle is Q, as depicted in the figure.) More precisely, as mentioned above, the different destinations and genders occupy four distinct successor states. The original weight c_1 splits among those as follows:

- First, we define a *measurement vector* in the complex plane. If, as in figure 4, the switch-wire particle is female, then the measurement vector is the weight c_1 rotated in the complex plane by the gate's measurement angle Q (figure 4(b)). If, instead, the switch-wire particle is male, the measurement vector is the weight rotated by $Q - pi/2$. The rationale for this orthogonal twist will become apparent in the following section.

- The weight c_1 divides into two orthogonal components, c_2 and c_3. One is parallel, the other perpendicular, to the measurement vector in the complex plane (figure 4(b)). Call these the *measurement-parallel* and *measurement-perpendicular* components of the original weight. They are rotated by Q and $Q + pi/2$ respectively from c_1, and their magnitudes are $|c_1| \cos Q$ and $|c_1| \sin Q$, respectively.

- If, as in figure 4, the classical state has no particle at the gate's control wire, the measurement-parallel component subdivides further (as specified just below), dividing itself between the successor states in which the switch-wire particle passes straight across. The measurement-perpendicular component similarly divides between the successors in which the switch-wire particle crosses over. If instead a control-wire particle is present, the opposite correspondence holds: the measurement-parallel component corresponds to crossing over, and the measurement-perpendicular component corresponds to passing straight across.

- The measurement-parallel and measurement-perpendicular components each subdivide into two components, one parallel and one perpendicular to the original weight c_1 (figure 4(c)). The parallel components move to the successors in which the switch-wire particle has the same gender it had in the original classical state. The perpendicular components move to the other successors, in which the particle's gender has changed (figure 4(d)).

Thus, the weight-splitting rule twice decomposes a weight into a pair of orthogonal components. The sum of the components therefore equals the original weight: $c_1 = c_2 + c_3 = c_{2a} + c_{2b} + c_{3a} + c_{3b}$. Also, at both steps the probability measure, defined as a weight's squared magnitude, is conserved: $c_1^2 = c_2^2 + c_3^2 = (c_{2a}^2 + c_{2b}^2)^2 + (c_{3a}^2 + c_{3b}^2)^2$. Finally, note that in the special case of the measurement angle being zero, the above rule is equivalent to U1 and U2 state-succession. Since the measurement-orthogonal component is zero, no state-splitting occurs and the particle entirely passes straight across or entirely crosses over, depending on whether a control-wire particle is present. The next section shows that a measurement angle of zero is not privileged in this respect; any measurement angle can fail to produce state-splitting under certain circumstances.

The above description specifies the four-fold split of a classical state for a single switch-wire particle in that classical state. When a classical state has n particles at switch wires, there are 4^n successor states, as noted above. The n four-way splits are applied in succession, in any order.[5] As the reader may verify, each of the four successor weights split apart for a given switch-wire particle equals the original weight multiplied by a complex factor. Since such multiplication is commutative and associative, one may think of the n splits as occurring in any order, or simultaneously. This 4^n-fold splitting also conserves both probability and weight, and is equivalent to n successive four-fold splits, each conserving probability and weight.

[5]If a gate has particles at both switch-wire inputs, this formulation allows some successor states that have two particles at the same position. However, that does not occur in any of the examples in this paper.

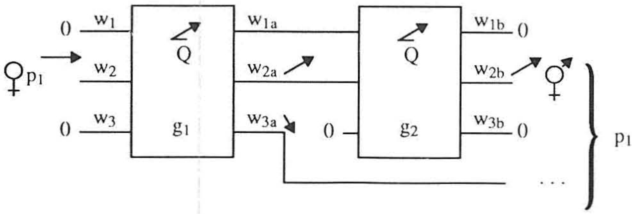

Figure 5: Using the same measurement angle twice in a row causes no further state-splitting.

4.2 Successive measurements

The laws of quantish physics are now completely specified.[6] A brief look at the effects of passing a particle through the switch-wire inputs of successive gates will elucidate important properties of these laws in preparation for examining their quantum-like properties.

Figure 5 extends figure 4: in figure 5, g_1's upper switch-wire output connects to g_2's upper switch-wire input, and g_2 has the same measurement angle as g_1. (The gate's other switch-wire output diverts to some other gate, not shown.) The arrow at wire w_2 designates the weight c_1 (from figure 4(a)) associated with the state in which p_1 is at that wire, with female gender. The arrow at wire w_{2a} designates the measurement-parallel component weight c_2, which is divided between the two successor states in which p_1 reaches w_{2a}; analogously for the arrows at w_{3a} and w_{2b}. It turns out, as explained just below, that the second gate causes no further state-splitting. That is, p_1 proceeds straight across to w_{2b} with no change in the weights assigned to the states that assign its genders, and p_1 never emerges from wire w_{3b}.

Actually, the two states (with weights c_{2a} and c_{2b}) in which p_1 reaches w_{2a} each have the usual four successor states, two for each of p_1's next position and gender. But both states have the same four successors, so each of those successors receives a component of c_{2a} and of c_{2b}. In each of the two successor states in which p_1 crosses over to w_{3b}, the two components sum to zero. In the other two successors, the components sum to recreate c_{2a} and c_{2b}. That this happens can be verified by applying the state-splitting rule in detail, but there is also a more intuitive explanation:

- The second gate, g_2, decomposes each of the weights c_{2a} and c_{2b} with respect to the same measurement vector that was used for g_1's decomposition of c_1 into c_{2a} and c_{2b}. This is so for c_{2a} because c_{2a} is parallel to c_1. On the other hand, c_{2b} is perpendicular. But p_1 has become male

[6]We must also specify an admissible set of initial quantum states. Clearly, an initial quantum state must at least satisfy the constraint that the squared magnitudes of its weights sum to unity. Moreover, it turns out that for some initial quantum states, the quantish laws do not conserve probability. A sufficient condition for conservation of probability is that the initial quantum state have just one nonzero-weighted classical state. (There are alternative conditions, less restrictive but more complicated, that also ensure conservation.)

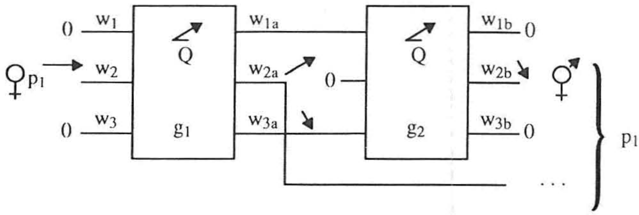

Figure 6: Again, reusing the same measurement angle causes no further state-splitting.

in the state whose weight is c_{2b}, so the hitherto unmotivated rule that adds an orthogonal twist to the measurement vector for a male particle now cancels c_{2b}'s perpendicularity, so that the measurement vectors of both states are the same.

- Once the components have been decomposed with respect to the same measurement vector as before, the measurement-parallel component moves to configuration-space points that have p_1 passing straight across, and the measurement-orthogonal component moves to points that have p_1 crossing over. But the measurement-orthogonal component is zero; that component of c_1 was diverted away by the first gate. Thus, the measurement-parallel component undergoes no further decomposition due to the second gate.

- Finally, the states that distinguish p_1's genders keep the same respective weights. The state-splitting rule either leaves a particle's gender as well as the orientation of the corresponding weight unchanged, or complements the particle's gender along with making an orthogonal twist to the corresponding weight. A sequence of two such complements and twists both restores the original gender and reestablishes parallelism with the original weight. Thus, all resulting weights are either parallel to the original and assign the same gender to the particle, or are perpendicular and assign the opposite gender. Thus, the reconstructed measurement-parallel weight c_1 must decompose into the same components as before, respectively assigning the same genders to p_1.

In the alternative circuit of figure 6, g_1's lower switch wire, rather than its upper one, connects to g_2. By reasoning similar to the above, g_2 again causes no further state-splitting. In this case, it is only the measurement-orthogonal component of the original weight that reaches the configuration-space points corresponding to p_1 reaching g_2. Rather than entirely passing straight across, here p_1 entirely crosses over, arriving back at the top at switch wire w_{2b}.

Figure 7, which combines the results of figures 5 and 6, shows the result of connecting both of g_1's switch-wire outputs to the corresponding inputs of g_2. Since quantish state-succession is linear, the weights reaching the four states in which p_1 emerges from g_2 are the sums of those weights in the previous two

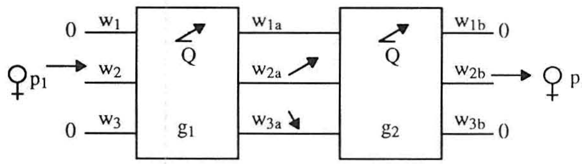

Figure 7: Connecting *both* switch wires to a gate with the same measurement angle undoes the first gate's splitting.

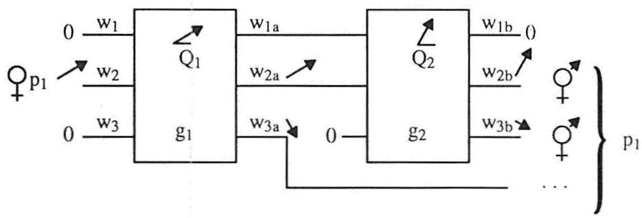

Figure 8: A succession of different measurement angles makes states split.

examples. The measurement-parallel component of c_1 follows the states that have p_1 passing straight across the upper switch path at both gates. The measurement-perpendicular component follows p_1 crossing over at the first gate, then back again at the second, thus also arriving at g_2's upper switch-wire output. The two components sum there to recreate the original weight (and with p_1 restored exclusively to its original gender). Thus, a quantish gate separates and reassembles components of a state's weight in a manner that is *symmetrically invertible*: the prior quantum state is the same function of the subsequent state as the subsequent state is of the prior state.

Finally, figure 8 illustrates the effect of a succession of different measurement angles. For the states in which p_1 appears at g_2's upper switch wire, g_2 divides the corresponding weights into measurement-parallel and measurement-perpendicular components, but with respect to a different measurement vector than at g_1 (in figure 8, the weight corresponding to the arrow at w_{2a} divides into the orthogonal components shown at w_{2b} and w_{3b}). The measurement vector at the second gate differs from that at the first by $Q_2 - Q_1$; hence, at the second gate the the measurement-parallel and measurement-perpendicular components have squared magnitudes of $\cos^2(Q_2 - Q_1)$ and $\sin^2(Q_2 - Q_1)$, respectively. Figure 5 was the special case in which $Q_1 = Q_2$.

5. Quantum-like properties of quantish physics

The laws of quantish physics, like the laws of U1 and U2, are *local*. The destinations (and new genders) of a particle at a switch wire of some gate in some classical state depend only on the particle's current gender, the gate's

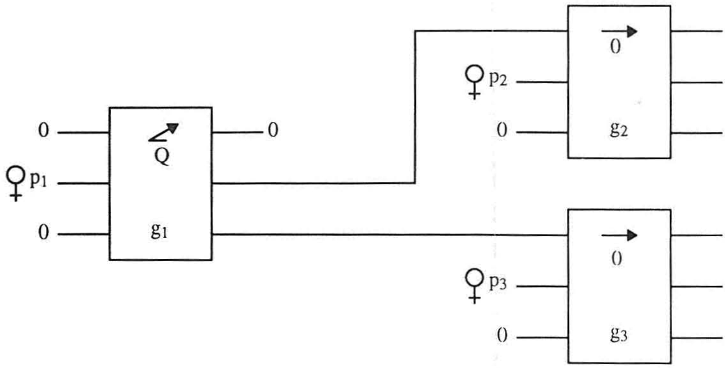

Figure 9: Particles p_2 and p_3 observe p_1's position.

measurement angle, and whether there is a particle at the control wire of the same gate in the same classical state. Similarly, the destination and gender of a control-wire particle at some gate in some state depend only on that gate and that particle in that state.

Thus, there is no action at a distance with respect to circuit-topology space, or with respect to configuration space. And, of course, the quantish-physics laws are entirely deterministic. I now demonstrate that these local, deterministic laws support phenomena like those of the real quantum world: apparent indeterminacy of quantum states, interference of superposed outcomes, and interference-observation duality.

5.1 Apparently nondeterministic outcomes and the uncertainty principle

In figure 9, particle p_1 "splits" at g_1 (as in figure 4(a)), and is then observed at gates g_2 and g_3 (as in figure 3). (Here and throughout, when I show gates wired in series, inputs shown at gates later in the series are synchronized by circuitry not shown to arrive there simultaneously with inputs from earlier in the series. Thus, in figure 9, p_2 and p_3 arrive at gates g_2 and g_3 simultaneously with p_1.)

In the successor states that have p_1 arriving at g_2's control wire, p_2 entirely crosses over. There is no state-splitting because g_2's measurement angle is zero. In those same states, particle p_3 passes straight across since the states in which p_1 arrives at g_2's control wire do not have p_1 arriving at g_3's. Similarly, in states in which p_1 does arrive at g_3, p_3 crosses over and p_2 passes straight across. Thus, as in figure 3, the two observations are consistent: p_1 is always observed at exactly one of its two possible destinations.

From within the quantish universe, then, it appears that p_1 arrives at one gate or the other, but never both. Every successor state is consistent with there being just one destination. Although different successors with different destinations remain in superposition, they have no effect on one

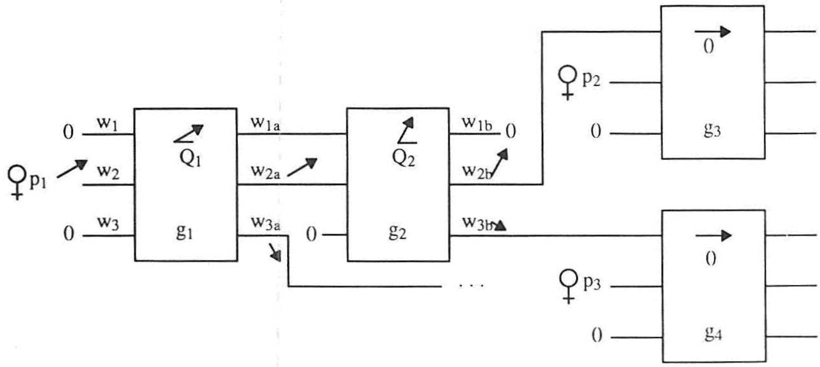

Figure 10: An observation distinguishes between two outcomes of passing p_1 through a succession of gates with distinct measurement angles.

another (unless they later reconverge in configuration space, as addressed in the next section). However, because g_1's measurement angle is oblique, *which* destination the particle will have cannot be specified in advance because, in reality, it will have both destinations, notwithstanding appearances to the contrary from the point of view of any superposed classical state in the quantish universe. If observers embodied in the quantish universe conduct a number of trials with an apparatus such as in figure 9 and record the result of each trial, the cumulative records (in most states) will show a mixture of results. Statistically, by virtue of such cumulative records, the outcome of such trials appears from within the quantish universe to be nondeterministic.

Moreover, the apparent nondeterminism is quantifiable. Given enough trials, almost all the weight in configuration space will be assigned to states whose cumulative records show that p_1 passed straight across in approximately $\cos^2 Q$ of the trials, and crossed over in approximately $\sin^2 Q$ of the trials. If we think of the weights as being the actual stuff of the quantum universe, each weight being an actual quantity of universe-branch, then in almost all of the universe-stuff there is a distribution of trials in which the particle has passed straight across or crossed over approximately $\cos^2 Q$ or $\sin^2 Q$ of the time. Those, then, are the apparent probabilities of the two outcomes as seen from almost everywhere within the quantum universe. (This argument for quantifying apparent nondeterminism by appeal to cumulative records is adapted directly from Everett.)

Figure 10 extends figure 8, observing (as in figure 9) whether p_1 emerges at w_{2b} or w_{3b}. Over many such trials (counting only those occasions on which p_1 passes through g_2 at all), the typical cumulative record would show p_1 emerging at w_{2b} with frequency $\cos^2(Q_2 - Q_1)$, and from w_{3b} with frequency $\sin^2(Q_2 - Q_1)$.

Thus, if both gates have the same measurement angle, p_1 will always be observed to emerge at g_2's upper switch wire. We may therefore say that p_1, having passed through g_1, has a *definite state* with respect to g_1's measurement angle Q_1—meaning that there is no apparent nondeterminism (in reality, no multiplicity of outcomes) as to p_1's next destination if p_1 runs through another gate with that measurement angle.

But having a definite state with respect to one measurement angle always means having an indefinite state with respect to all angles oblique to that angle. Quantish configuration space does not separately encode (i.e., provide a distinct configuration-space dimension for) a particle's state with respect to each possible measurement angle. Rather, configuration space designates a single binary attribute for each particle, namely its gender. That attribute corresponds to a definite state with respect to particular measurement angles ($\pi/2$ and its multiples) but not with respect to other angles. Alternatively, for any other angle there is a superposition of genders that creates a definite state with respect to that angle (and angles parallel or orthogonal to it), but not with respect to oblique angles. Thus, a particle's inclination to cross over at the next gate cannot be made definite with respect to all possible measurement angles at the next gate. Eliminating apparent nondeterminism by observing a particle's inclination to cross over with respect to one measurement angle thereby creates apparent nondeterminism with respect to other angles. This fact recapitulates Heisenberg's uncertainty principle in the quantish universe.

Similarly, in real-world quantum physics, configuration space does not separately encode a particle's position and momentum; only position is encoded (or, equivalently, only momentum, or only some linear combination of the two). The basic physical law of motion says that an undisturbed particle spreads in all directions at light speed—or rather, that a weight in configuration space spreads at light speed (with no change in phase) into a filled-in sphere along the three configuration-space dimensions corresponding to the particle's position. The particle thus has a maximal superposition of momenta. But the spread can be confined to a smaller envelope by arranging a superposition of appropriately phased weights for the particle's position. The weights assign a superposition of positions to the particle, but interference among them constrains their spread, limiting the superposition of the particle's momenta. The sharing of a single configuration space dimension for a given particle's position and momentum along a given spatial dimension creates an unavoidable trade-off between superposition of positions and superposition of momenta.

5.2 Interference of superposed states

Having seemingly nondeterministic outcomes is a step towards having quantum-like phenomena; but it falls short of the fundamental quantum duality, which requires superposed states that mutually interfere unless distinguished from one another by observation. I now demonstrate such interference in the quantish-physics model.

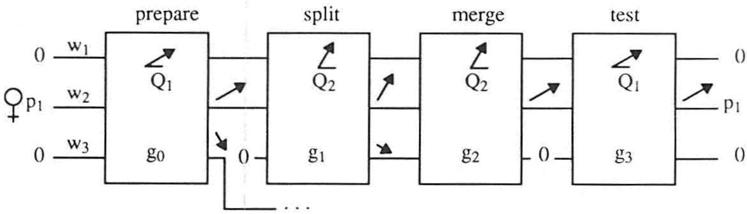

Figure 11: Superposed states remerge and interfere.

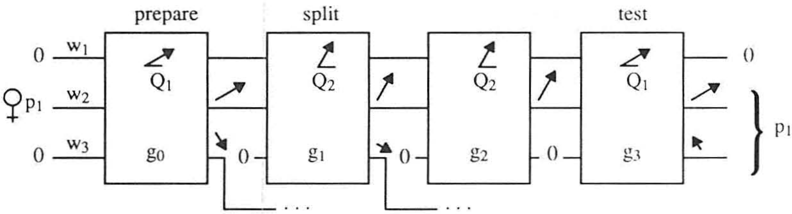

Figure 12: Here, one path diverts from merging at g_2.

In figure 11 the first gate, g_0, *prepares* particle p_1 by putting p_1 in a definite state with respect to Q_1 before sending p_1 to g_1. Particle p_1 also diverts away from g_1, but the discussion that follows only addresses the case in which it reaches g_1. The states in which p_1 diverges do not interfere with the states under discussion since they are separated along their p_1-position dimension.

At g_1, p_1 splits using measurement angle Q_2. Then at g_2, p_1 remerges (as in figure 7), reconstructing the weight with which p_1 entered g_1, thereby reestablishing p_1's definite state with respect to Q_1. Finally, g_3 verifies that p_1 has a definite state with respect to Q_1. Suppose p_1 were then observed emerging from g_3. (This observation is not shown here, but would be similar to the observation of p_1's emergence from g_2 in figure 10). Over many such trials, particle p_1 would always be observed to arrive at g_3's upper switch-wire output.

In figure 12, one path to g_2 is disconnected (as in figure 6), circumventing the merging. In those states in which p_2 does reach g_2, p_2 already has a definite state with respect to Q_2, so p_2 entirely passes straight across and keeps its definite state with respect to Q_2. Thus, p_2 does *not* have a definite state with respect to Q_1 so, unlike in figure 11, p_1 is split by g_3.

We are now in a position to see the effects of interference between superposed states in the quantish-physics model. Contrasting figures 11 and 12, we see a genuine quantum interference phenomenon: figure 11, compared to figure 12, provides an additional path by which p_1 might reach g_3's lower switch-wire output, yet p_1 emerges there *less* often (in fact, never) with the extra path provided than without that path. This contrast is inexplicable on the assumption—which otherwise seems correct from within the quantish universe, as seen in the previous section—that p_1 is a particle-like entity that exists at just one wire at a time.

Only by acknowledging the simultaneous reality of p_1's superposed positions at *both* of g_1's switch-wire outputs can we (or any observer embodied in the quantish universe) account for the possibility that those states can interfere with one another when a path is provided to convey the interfering influence. The interference is achieved, of course, by the addition of complex weights at common successor states, as discussed in section 4.2; opposite weights cancel when added. In figure 12, diverting p_1 from reconverging to the same position thereby diverts the corresponding configuration-space path from reconverging, thus circumventing its interference.

The setup of figures 11 and 12 is analogous to the real-world double-slit experiment, in which a particle is in a superposition of states (passing through slit 1 or slit 2).[7] *Destructive* interference among the superposed states reduces the likelihood of the particle's arrival at certain points along the backdrop, but blocking one of the two possible paths thereby blocks that interference, returning the probability to normal. (The two slits are like the two switch-wire inputs to g_2 in figures 11 and 12. The diversion away from the lower input to g_2 in figure 12 is like blocking one of the two slits.) A less dramatic paradox, *constructive* interference increases the probability of arrival at certain points so that the probability exceeds what the sum of the two single-slit curves would predict. Correspondingly the frequency of arrival at g_3's upper switch-wire output is greater with both paths provided than the sum of the probabilities when just one or the other is provided.

5.3 Blocking interference via observation

If inhabitants of a quantish-physics universe perform the above experiments, they face the same apparent paradox as physicists in the real universe. When a "split" particle is observed as in figure 9, the results consistently and unambiguously show that the particle reached one destination or the other, but not both. Yet, comparing the behavior of the figure 11 circuit with that of figure 12, there is a demonstrable interference effect that is explicable only on the assumption that the particle indeed reaches both destinations (which is indeed the case, as we privileged observers of configuration space, looking from outside the quantish universe, can see).

Let us sharpen the "paradox" further. Suppose inhabitants of the quantish universe try to observe p_1 on its way to g_2, that is, after its path splits and before it remerges. Figure 13 shows a setup in which p_2, at gate g_4, crosses over depending on whether p_1 passes through g_4. Particle p_1 is then routed into g_2 as before, and *delay* gates (labelled D) are inserted at the other

[7] We might also take this setup as an analog of the Stern-Gerlach experiment (see, for example, [3]). Particle p_1's gender is analogous to a real-world particle's spin; g_1 and g_2 together correspond to a Stern-Gerlach module that diverges and then reconverges paths of the particles according to their spins with respect to a certain axis (analogous to the gates' common measurement angle).

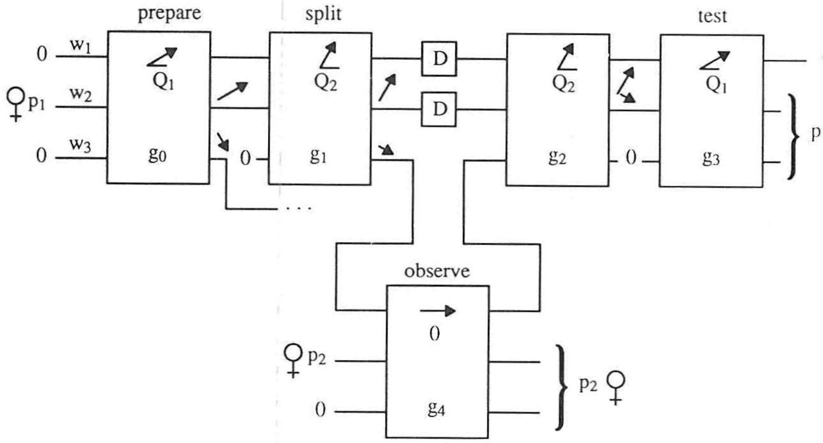

Figure 13: An observation circumvents subsequent interference.

two paths to g_2 to maintain synchronization. (A delay gate is an ordinary Fredkin gate. The wire shown is its control wire. The switch-wire inputs, not shown, have no particles present.)

Particle p_1 is unaltered by the observation. Classically, then, the observation should not change the outcome of the experiment. But in quantum physics, making an observation to distinguish two superposed states blocks any subsequent interference between those states. And that is precisely what happens here.

We find the same bizarre result as in the real universe when we observe which slit the electron came through: the interference disappears, and p_1 can emerge from either of g_3's switch-wire outputs. The configuration-space explanation of this phenomenon is straightforward. Although p_1 reconverges after passing through g_2, occasioning a reconvergence on the corresponding states' p_1-position dimension in configuration space, the states remain separated along their p_2-position dimension because p_2 does not reconverge. Since the states thus fail to reconverge, their weights do not add together and interfere. (The pair of vectors shown at g_2's upper switch-wire output represents the superposed weights separated along the p_2-position dimension.) The outcome, as seen from any of the successor states, is just as though p_1 had traversed just one path or the other (as the classical view would have it), but not both.

Note, by the way, that even a so-called *negative observation* results in the absence of interference. In the states in which p_1 does not reach g_4, p_1 does not interact with p_2. But that very absence of interaction—that is, a negative observation—is fully informative as to p_1's whereabouts: if p_2 does not cross over, p_1 must be on g_1's upper switch-wire output. Accordingly, even the states in which the observation at g_4 was negative have successors that exhibit no interference, as shown by the fact that p_1 emerges from g_3's lower switch-wire output with the expected nonzero frequency following a negative observation at g_4.

Renninger (see [4]) cites negative observation to demonstrate the incorrectness of one naïve account of eliminating interference via observation—the account that attributes this elimination to the inevitable disturbance of an observed entity by the observer. But a negative observation can cause no such disturbance (since there is no interaction at all), yet the interference disappears all the same. Looking at the situation from configuration space, this is just as we would expect. The fact that p_2 encounters p_1 in *one* of two superposed states makes those two states differ along their p_2-position dimension, moving them out of "interference range" of one another and thus circumventing interference in both states.

At this point, the quantum interference-observation duality becomes a comprehensible—indeed, deducible—property of the quantish universe. The quantish physical laws say that the configuration-space destination of a classical state's weight is determined only by that state; other superposed states are irrelevant. Therefore states that are separated from one another along some particle-position dimension in configuration space can interfere with one another only by reconverging to the same point in configuration space (as happens, for example, in figure 11). Any observation that distinguishes the superposed states must (as in figure 13) create a corresponding separation along a distinct dimension in configuration space, and any additional such observations, or any observations of the observations, compound the separation along still other dimensions. Then, reversing the original separation creates no interference, since there is still separation in one or more other dimensions. (But if those separations are also reversed, interference is reestablished, as in figure 14.) Thus, given the laws of quantish physics, there is a *necessary* trade-off between an interfering superposition and any observation that distinguishes among the superposed states.

Thus the quantish universe, like the real quantum universe, behaves classically to just the extent that we try to catch it in the act of behaving otherwise. The quantish-physics formalism shows how such behavior can be exhibited by deterministic mechanical laws that support only local interactions and that have no peculiarity with respect to there being a definite, objective, observer-independent (quantum) state of the universe. The following section shows that the quantish formalism also supports an analog of the crucial EPR experiment.

5.4 Coupled particles: the EPR experiment

As a final example, this section presents the quantish parallel of the Einstein-Podolsky-Rosen (EPR) experiment [6]. The experiment disproves all so-called *hidden-variable* accounts, which postulate that there is no superposition of distinct states, but rather a definite state that is merely unknown.

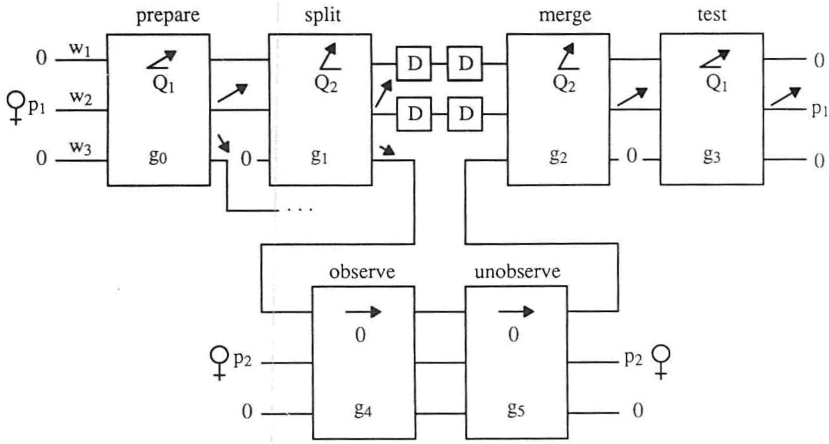

Figure 14: Particle p_2 must cross over 0 or 2 times, always emerging at g_5's upper switch wire, erasing the observation made at g_4 and thus reestablishing interference at g_2, as manifested by the definite outcome at g_3.

In figure 15, p_3 compares p_1's position to p_2's. If both particles have emerged from the upper switch-wire outputs of the splitting gates g_1 and g_2, p_3 encounters both particles and crosses over at both g_3 and g_4. If p_1 and p_2 both emerge from the splitting gates' lower outputs, p_3 encounters neither and passes straight across g_3 and g_4. In either case, p_3 emerges from g_4's upper switch-wire output. But if p_1 and p_2 do not emerge from the corresponding outputs of their respective gates, p_3 emerges from g_4's lower output.

Let us say that p_1 and p_2 are *coupled* in those states in which p_3 has emerged from g_4's upper switch wire. The following discussion concerns only the states in which p_1 and p_2 are coupled (which are not affected by the other states due to separation in configuration space along the p_3-position dimension). In the coupled states, neither p_1 nor p_2 has a definite position; rather, each is in a superposition of positions. But that superposition is definite as to the correspondence of the particles' positions: each is on an upper wire if and only if the other is also.

At gates g_5 and g_6, p_1 and p_2 encounter measurement angle Q_1. The outcome is remarkable: regardless of the value of the shared measurement angle Q_1, p_1 and p_2 remain coupled, both emerging from the upper wires or both from the lower wires of their respective gates g_5 and g_6. The continued coupling is explained as follows (this explanation is optional; the subsequent discussion rests only on the conclusion):

- Consider the combined weight c_{upper} of the states in which p_1 and p_2 are both on upper wires before passing through g_5 and g_6, and the similarly defined weight c_{lower} of the states in which p_1 and p_2 are on the lower

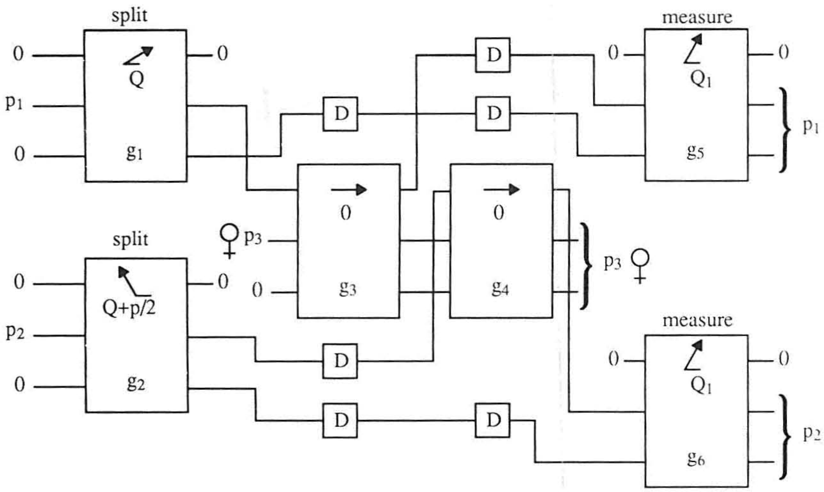

Figure 15: Particle p_3's observation couples p_1 and p_2.

wires. The weight c_{upper} is the measurement-parallel component with respect to Q of the measurement-parallel component with respect to $Q+pi/2$ of the original weight c_{original}. Hence, it equals c_{original} rotated by $2Q + pi/2$ (the sum of the two measurement-parallel rotations), with magnitude $|c_{\text{original}}| + \cos Q \cos(Q + pi/2)$ (the product of the two attenuations). Similarly, c_{lower}, a doubly measurement-perpendicular component of c_{original}, is c_{original} rotated by $2Q{+}3pi/2$ (the extra rotation by p is due to the two orthogonal projections), and its magnitude is $|c_{\text{original}}| + \sin Q \sin(Q + pi/2)$. Hence, the magnitudes are equal and the directions opposite, so $c_{\text{upper}} = -c_{\text{lower}}$.

- At g_5 and g_6, the two superposed states with weights c_{upper} and $-c_{\text{upper}}$ each undergoes a four-fold decomposition into measurement-parallel and measurement-perpendicular components for angle Q_1. The outcome in which p_1 emerges from the upper wire and p_2 from the lower has two predecessors: one in which p_1 and p_2 arrived at g_5's and g_6's upper wires and only p_2 crossed over; and one in which they arrived at the lower wires and only p_1 crossed over. Since g_5 and g_6 share measurement angle Q_1, both outcomes correspond to the same decomposition of exactly opposite weights, so the weights are exactly opposite. Thus, they converge in configuration space and sum to zero. Similarly for the other outcome in which p_1 and p_2 emerge from opposite wires.

- Finally, consider the outcome in which p_1 and p_2 both emerge from the upper wires of g_5 and g_6. This outcome has two predecessors, one corresponding to a doubly measurement-parallel component from the states in which p_1 and p_2 entered g_5 and g_6 at the upper wires,

> the other to a double measurement-perpendicular component from the states in which they entered at the lower wires. By an analysis similar to the above, the rotation of the doubly-perpendicular component is just opposite the rotation of the doubly-parallel component, and the magnitudes remain the same. But the weights being rotated started out opposite, so they end up equal. Thus, these components converge to double rather than cancel, and similarly for the outcome in which p_1 and p_2 both emerge from their lower wires. Thus, p_1 and p_2 remain coupled.

The state-splitting achieved by g_5 and g_6 simultaneously is the same as if p_1 and p_2 encountered one gate before the other. Hence, either g_5 or g_6 alone gives *both* particles the same definite state with respect to Q_1 (recall the discussion of figure 10 in section 5.1). Checking the other particle's state with respect to the same measurement angle is like checking the same particle twice with respect to that angle—the outcomes are always consistent.

That the particles remain coupled after g_5 and g_6 can be demonstrated from within the quantish universe by observing the positions of both particles over a large number of trials, using a different Q_1 on each trial. But the indefiniteness of their positions is harder to show. Proponents of a classical world view—a view that denies the reality of multiple superposed states of the universe—would want to explain the demonstrated correspondence between the positions by postulating that, from the outset of the experiment and prior to the comparison performed by p_3, p_1 and p_2 already had a definite (albeit unknown) state for every measurement angle, thus violating the quantish analog of Heisenberg's uncertainty principle, as discussed in section 5.1. In particular, on each trial both particles start with the same definite state with respect to that trial's Q_1, which explains the observed correspondence.

From our privileged vantage point, we know that the hidden-variable account is false. We see that configuration space provides a superposition of outcomes at both gates, not a single, definite outcome at each. But can the hidden-variable account be disproved from *within* the quantish universe? A subtle theorem due to Bell [2] facilitates such a proof.

Let us say that g_5 *measures* p_1 with respect to Q_1; p_1's binary state for that measurement is whether its inclination is to pass straight across or to cross over. Suppose we modify the experiment by substituting a distinct angle Q_2 for Q_1 at g_6, so g_6 now measures p_2 with respect to Q_2. The *discrepancy rate* between the measurements at g_5 and g_6 is the probability that, after passing through those gates, p_1 and p_2 will not be both on upper or both on lower wires. Since, as noted above, g_5 puts both p_1 and p_2 in the same definite state with respect to Q_1, p_2's measurement with respect to Q_2 is effectively the same as measuring p_1 with respect to Q_2 (having just measured it with respect to Q_1). As in figure 10, that sequence of measurements has a discrepancy rate of $\sin^2(Q_2 - Q_1)$, which of course is zero if $Q_1 = Q_2$.

Let us consider whether the observed discrepancy rates for various values of Q_1 and Q_2 are explicable by postulating that the particles have prior definite states for the Q_1 and Q_2 measurements. Bell's theorem states that, if each pair of coupled particles already has a single definite state for each of three arbitrary measurement angles Q_a, Q_b, and Q_c, and if we perform measurements on many pairs of coupled particles, then the discrepancy between Q_a and Q_c measurements (that is, the discrepancy rate over trials in which one coupled particle is measured with respect to Q_a and the other with respect to Q_c) cannot exceed the sum of the discrepancy between Q_a and Q_b measurements and the discrepancy between Q_b and Q_c measurements. This is *Bell's inequality.* The inequality follows simply from the fact that any particle with a different state with respect to Q_a than with respect to Q_c must also have a difference between its Q_a and Q_b states or between its Q_b and Q_c states, since its Q_b state cannot match both its Q_a state and its Q_c state if its Q_a and Q_c states differ.

Let us take Q_a to be 0, Q_b to be $pi/8$, and Q_c to be $pi/4$. If we perform a series of experiments in the quantish universe using the setup of figure 15 and variously choosing the values of Q_1 and Q_2 from Q_a, Q_b, and Q_c, we will find that the discrepancy between Q_a and Q_c is $\sin^2 pi/4 = .5$, and the discrepancy between Q_a and Q_b, and also between Q_b and Q_c, is $\sin^2 pi/8$, which is about 0.146. This clearly violates Bell's inequality.

Therefore, the observed correlation between paired particles' measurements with respect to angles 0, $pi/8$, and $pi/4$ cannot possibly be explained by saying that on each trial, the two particles already had, prior to their measurement, a single definite state for each possible measurement angle (the states for different angles Q_1 and Q_2 being the same on $\sin^2(Q_2 - Q_1)$ of the trials). By Bell's theorem, that interpretation is impossible. If one were to deny the reality of multiple superposed states of the universe, the only remaining way to account for the observed correlation among the coupled particles' measurements with respect to the three angles would be to postulate that the indefinite (i.e., unpredictable) outcome of measuring one particle is then communicated to the other coupled particle—by some unknown, unexplained mechanism—in such a way as to force the other particle into the same state with respect to whatever measurement angle was used for the first particle.

In fact, given quantish physical laws, no such mechanism is or could be involved when there is no circuitry between the two measuring gates to communicate the outcome from one gate to the other. The quantish-physics model instead accounts for the correlation by saying that there is a superposition of appropriately weighted entire classical states of the universe, and each superposed state shows the coupled particles having corresponding positions. Interference among these states creates correlations that would be impossible by Bell's theorem if there were only one such state.

The foregoing is adapted from the proposed EPR experiment, later carried out in modified form by several investigators (e.g., [1]). These experiments reveal correlations that Bell's analysis proves impossible if each particle already has a single, definite state with respect to each possible measurement, and if the two particles cannot communicate with one another at the moment of measurement. Since the two measurements can be performed arbitrarily far from one another and arbitrarily close in time, physicists who reject the reality of multiple superposed states of the universe are thereby forced to postulate an unexplained, faster-than-light interaction. Moreover, this interaction has the curious property that it cannot be harnessed for the transmission of information from one measurement site to the other—though this curiosity is just what one would expect were there not in fact an interaction, but rather just a manifestation of a preestablished correlation.

6. Multiple worlds or quantum collapse?

The multiplicity of worlds in the multiple-worlds interpretation of quantum mechanics stems from what we might call the *contagion* of a particle's superposition when its state is observed by another particle—the other not only assumes a superposition of states, but it assumes a correlated superposition. The resulting quantum state therefore cannot be expressed simply as the product of two independent superpositions, but must instead designate superpositions of configurations of both particles. And as observations of observations cascade, arbitrarily many particles may join the correlated superposition, effectively splitting the universe into separate versions, at least as far as the participating particles are concerned. An elegant formalism for this process, explored herein as an analog of Everett's formulation, is to represent the quantum universe in terms of weights on total classical states of the universe. These weights flow deterministically through configuration space, and a "split" occurs when an observation causes weights on already distinct classical states that are already separated along some configuration-space dimension to separate along another dimension as well.

The Copenhagen interpretation is almost identical. In particular, the contagion of superposition when particles interact is also present in the Copenhagen formalism, which is, indeed, indentical to the Everett formalism. Contagion of superposition is what explains the quantum hide-and-seek game, providing a correlation between the observer and the observed, and a concomitant inherent complementarity between that correlation and quantum interference.

But the Copenhagen interpretation diverges from the formalism by postulating an extra event, the collapse of the superposition into just one of its superposed states, which contradicts the formalism. According to the Copenhagen interpretation, this collapse occurs at some unspecified point along the cascade of microscopic observations so that, at least by the time the observations culminate in a conscious observation by a human being (or perhaps by the time they culminate in a macroscopic observation by, say, a laboratory instrument), the superposition has vanished.

The original motivation for postulating the collapse was straightforward: following a quantum experiment, the formalism predicts a continuing superposition of states. But the experimenter clearly observes only one state from that superposition; therefore, the superposition has collapsed into a unique state, contrary to the formalism. Everett's central contribution was to demonstrate that the formalism already accounts for the seemingly unique outcome since, although the formalism describes a continuing superposition of states, it also describes a corresponding superposition of mutually isolated observers, each of whom will therefore see only one outcome.

Thus, contrary to how the interpretational debate is typically framed, the difference between the Copenhagen and Everett interpretations is not a dispute between a single universe or multiple universes. The multiplicity of universes, in the sense of the contagion of superposition when particles interact, is a property of the formalism shared by both interpretations. The actual difference is whether or not to postulate an extra kind of event, namely the collapse of the superposition.

In view of Everett's explanation, the seemingly unique outcome of a quantum observation does not provide evidence for a collapse. Nor does any other such evidence exist. Postulating the collapse thus becomes a gratuitous complication and contradiction of the massively confirmed formalism. Moreover, the collapse renders quantum theory incomplete and ambiguous:

- The theory becomes incomplete because it cannot describe a quantum state of some portion of the universe, except relative to some other portion that embodies an observer. The theory cannot in principle describe the quantum state of the universe as a whole and give laws for the evolution of that state. The Everett formulation can and does.

- The theory is ambiguous as to what sort of physical interaction constitutes a superposition-collapsing observation. Yet the theory makes different predictions depending on whether such an observation has occurred. In particular, if the observation is later "reversed," reconverging the superposed states (as in figure 14 in section 5.3), interference occurs if the superposition is intact, but cannot occur if the superposition had collapsed, leaving nothing to interfere with. (But a collapse is only postulated when reversal is prohibitively unlikely, so that the distinguishing experiment is prohibitively impractical.)

Not only is the postulated collapse gratuitous, and incompletely and ambiguously specified, but furthermore all of the problematic features of quantum physics—the apparent non-objectivity of the state of the universe, apparent nonlocality of the effects of a measurement, and apparent nondeterminism—result from postulating the collapse.

7. Conclusion

Quantish physics, while not identical to actual quantum physics, shows by example *how it could be* that local, deterministic laws produce a quantum-like interference-observation duality. Everett's formulation does the same for a more complicated example—the real world.

Everett's interpretation of quantum mechanics accounts for actual quantum phenomena in terms of an elegant formalism—one so basic to the phenomena that even the Copenhagen interpretation invokes that formalism, together with a gratuitous complication, the quantum collapse. Everett's model *explains* quantum observation—its nature can be deduced from the model—instead of requiring an ad hoc, imprecisely specified distinction between observation interactions and all other physical interactions.

The quantish-physics model is much simpler than, but deeply similar to, Everett's formulation. Quantish physics faithfully exhibits not only the fundamental quantum interference-observation duality, but also (with respect to definite states) Heisenberg's impossibility of eliminating interfering superposition without thereby introducing some complementary superposition. The quantish model captures the fundamental issues that the interpretational debate appeals to, and captures them in a precise formalism, but without appeal to the training required of physicists. By substituting trivial Fredkin-gate mechanics for real-world wave mechanics, quantish physics allows one to devote full attention to what is special and perplexing about quantum uncertainty.

Quantish physics may be helpful for introducing quantum mechanics to undergraduates (perhaps even to many high school students[8]), and for explaining quantum uncertainty to the technically oriented segment of the general population. I think doing so would be important for more than the usual reasons of scientific literacy. The difference between a mechanistic and nonmechanistic universe is as profound a philosophical matter as humanity has ever grappled with. To the extent that this dispute focuses on quantum mechanics, a simplified model such as quantish physics may provide a common ground on which laypersons, philosophers who are not physicists, and physicists who are not philosophers can communicate with precision.

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[8] Aside from many-dimensional space and elementary probability, no mathematical concepts beyond a high-school level appear in the exposition. Even the use of complex numbers could be avoided by substituting two-dimensional vectors. Discussion of the EPR experiment can also be omitted.

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