Complex Systems **6** (1992) 433–441

A New Candidate Rule for the Game of Three-Dimensional Life

Carter Bays
Department of Computer Science,
University of South Carolina, Columbia, SC 29208 USA

The two-dimensional cellular automaton "Life," originally described by Conway, has been expanded to three dimensions, with three different defining rules being noted (see [1, 2, 5]). Each rule can be written in the form $E_bE_hF_bF_h$, where E_b is the minimum number of live neighbor cells that must touch a currently living cell **C** in order to guarantee that **C** remain alive the next generation; F_b is the minimum number touching a currently dead cell **D** in order that **D** come to life next generation; and E_h and F_h are the corresponding upper limits. These rules are called the "environment" and "fertility" rules. According to this notation, Conway's Life would be written "Life 2333."

Previously discovered three-dimensional life rules are Life 4555, Life 5766, and Life 5655. Each of these rules exhibits distinct characteristics. Life 5766 was shown to contain Conway's two-dimensional Life 2333; Life 4555 sports many period-four oscillators; and Life 5655, until now the most recently discovered rule, barely qualifies as a game of life, as its glider (translating oscillator) is quite rare in terms of its ability to condense out of "primordial soup" (that is, random initial configurations).

Utilizing the methods explained in [4], yet another rule has been discovered. This rule, 6855, appears to satisfy both criteria for a game of life; these criteria, described in [1], are repeated below.

1. Primordial soup experiments must exhibit bounded growth. This means that an initial random "blob" must eventually stabilize and cannot grow without limit.
2. A glider must occur "naturally"—that is, it must be discoverable by repeatedly performing primordial soup experiments.

The rule 6855 (hereafter called Life 6855) appears to satisfy these criteria. When primordial soup experiments are performed, all finite blobs tested eventually shrink and stabilize or disappear. This point was emphasized by checking primordial universes that were wrapped in two out of three dimensions, thereby giving the effect of an infinite blob with a boundary. In every case tested the blob shrank along the direction of the non-wrapped axis and stability eventually resulted. It should be noted, however, that when wrapped in all three dimensions, the universe (depending upon the initial density of the "soup") can remain in constant turmoil forever. In this respect, Life 6855 is similar to the "dense packed spheres" rule, Life 3333 (see [3]). Nevertheless, since finite random blobs eventually stabilize, Life 6855 satisfies the first criterion.

The second criterion is beautifully fulfilled, for Life 6855 sports three gliders—two with a period of two and the other with a period of three (see figure 1). The larger of the period-two gliders, the "double glider," is essentially composed of two of the smaller period-two gliders moving in tandem, spewing out debris as they go. The signatures have been given at appropriate phases (see [3]).

In terms of the relative frequency of its gliders, Life 6855 shows much more promise than Life 5655 and, with a small caveat, may be almost as important as Life 4555. The caveat is as follows. In purely random soup configurations, the 6855 gliders are rather rare objects—much rarer than the already scarce 4555 and 5766 gliders, which, given a small random starting configuration, appear about once every 1000–6000 experiments (depending upon the initial conditions). But when symmetry is imposed upon the initial random configuration, the picture changes drastically, with 4555, 6855, and 5766 gliders then appearing quite frequently.

We experimented with an initially random $10 \times 10 \times 10$ (roughly) blob of about 10–40% density, which was contained in a much larger ($45 \times 45 \times 45$) and otherwise empty universe. We detected the appearance of gliders as objects that struck the boundary of the larger universe (see [4, 5] for details). The period-two 6855 glider appears frequently if one employs symmetric soup of the type used to discover the 5655 glider (see [5]). Hence a series of experiments was performed where the initial random blob was bilaterally symmetric with respect to a plane perpendicular to a coordinate axis. The purpose was to find the optimal starting densities for smallish initial blobs, in order to maximize the frequency of appearance for the 4555 glider and the small period-two 6855 glider. For comparison, experiments were also run for the small 5766 glider. The results are summarized in table 1. (Each entry contains averages for at least 5000 experiments. The "generations until stability" columns exclude experiments where gliders were discovered.)

From table 1 we observe that the period-two 6855 glider, under optimal conditions using bilaterally symmetric random soup, appears more frequently than the 5766 glider, but not as often as the 4555 glider. Without imposed symmetry, the 4555 glider appears roughly once every 4000 experiments, the 5766 glider once every 800 or so, and the 6855 glider once in about 10^6 experiments. Interestingly, symmetry seems to enhance the appearance of the 5766 glider by a factor of two or three only, whereas the 6855 glider is enhanced by a factor of 10^4.

One implication of this ability to turn gliders into "common" objects is that it may be possible to utilize such symmetry in the construction of

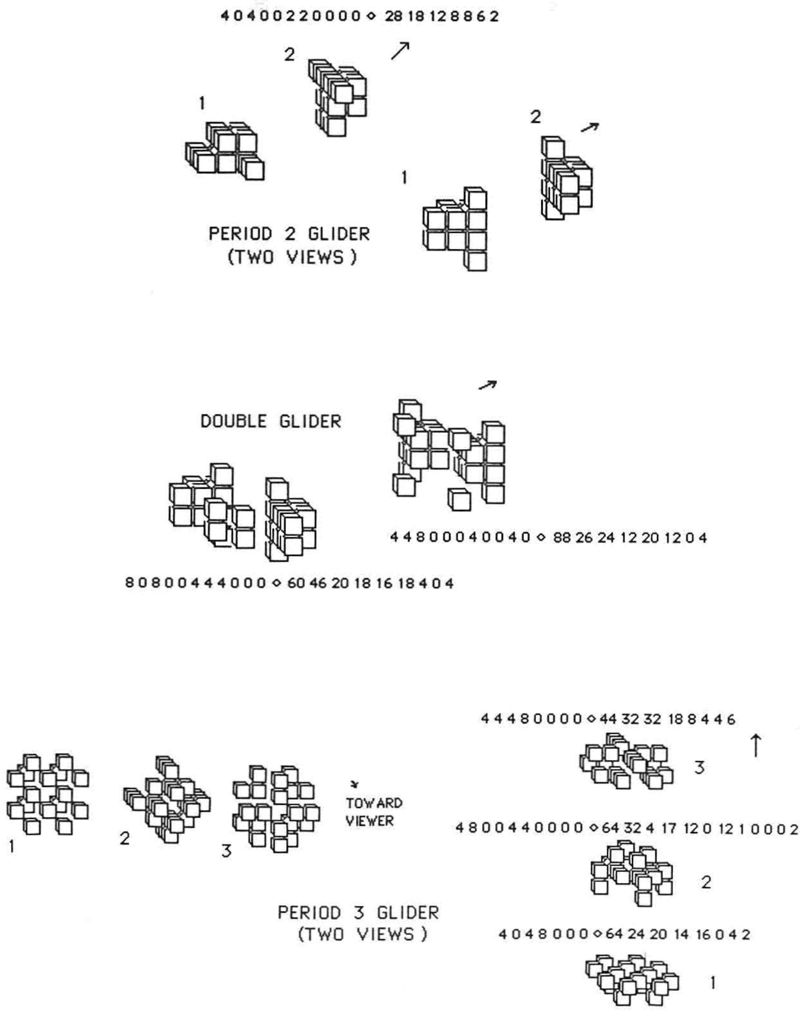

Figure 1: The Life 6855 gliders have periods of two and three. After one cycle, the period-two gliders have moved diagonally one unit in the direction shown. The double glider can be characterized essentially as two of the smaller period-two gliders moving in tandem, spewing out debris. The period-three glider moves in the direction of a coordinate axis, one unit per cycle. The signatures (see [3]) are given at the appropriate phases.

Dimensions of starting blob	Percent filled with live cells	Number of generations until stability			Number of experiments to find glider		
		4555	6855	5766	4555	6855	5766
$6 \times 9 \times 9$	12	11	6	6	31	330	360
	16	15	9	10	31	204	253
	40	12	14	12	30	125	445
$8 \times 11 \times 11$	12	16	8	10	30	241	352
	15	18	12	12	35	168	300
	18	20	15	15	32	118	250
	25	23	19	18	34	101	371
	30	21	20	17	51	105	310

Table 1: Optimal starting densities to find gliders.

a "glider gun" (a device that spews out an endless supply of gliders). A common object is much more likely than a rare object to settle from debris—whether the debris is random or man-made.

Another interesting aspect of Life 6855 is that most of the smaller oscillators appear to have a period of two (see figure 2). Note also that some of the smallest oscillators are identical to those for Life 5655, but not those for Life 4555. Oscillators with period greater than two appear to be rare—at least for primordial soup experiments (see figure 3). The elegant period-three oscillator at the top right of figure 3 exists for both Life 6855 and Life 5655. To find some of these oscillators, random soup of various symmetries was employed. The type of symmetric soup utilized is usually made apparent by observing the oscillator in question. Note that small experimentally-produced stable objects are rather rare. The bottom of figure 3 illustrates a few of the larger forms that can be condensed from symmetric soup.

Time-space barriers for Life 6855

When the first two games of life (4555 and 5766) were investigated, it was noted that we could construct "time-space barriers" for Life 5766 (hereafter called simply barriers). These barriers are stable planar arrays of live cells, stabilized by non-planar configurations at their borders, which prohibit any growth in the adjacent plane (see [1]). For example, the Life 5766 barrier is composed of a plane of live cells in which each cell has seven live neighbors and is therefore stable. Each cell in the immediately neighboring plane thus has (at least) eight live neighbors—this plane must therefore remain a "dead zone." It is the Life 5766 barrier configuration that allows us to build any (or all) two-dimensional Conway (Life 2333) constructs. Indeed, Life 5766 was shown thus to contain Conway's game. It is also possible to construct barriers for Life 6855 (see figure 4); the effect is somewhat different, however.

For a three-dimensional game to have a two-dimensional analog, live cells must exist in adjacent pairs, with each pair corresponding to a single cell

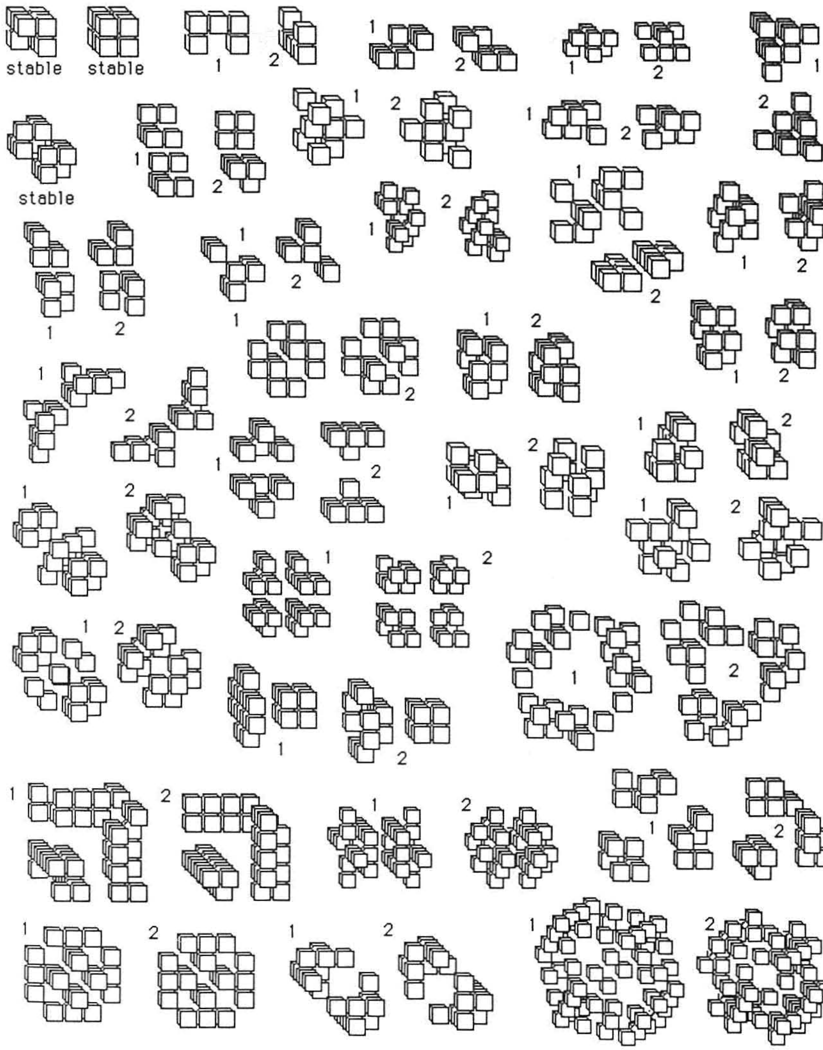

Figure 2: Illustrated here are most of the smallest stable and period-two oscillating Life 6855 forms, along with a few larger forms that require symmetric soup to find. Many of the shapes at the top are also forms for Life 5655. None are valid in Life 4555. The shape at the top left (a cube with a corner missing) is also stable for Life 5766 and Life 5655.

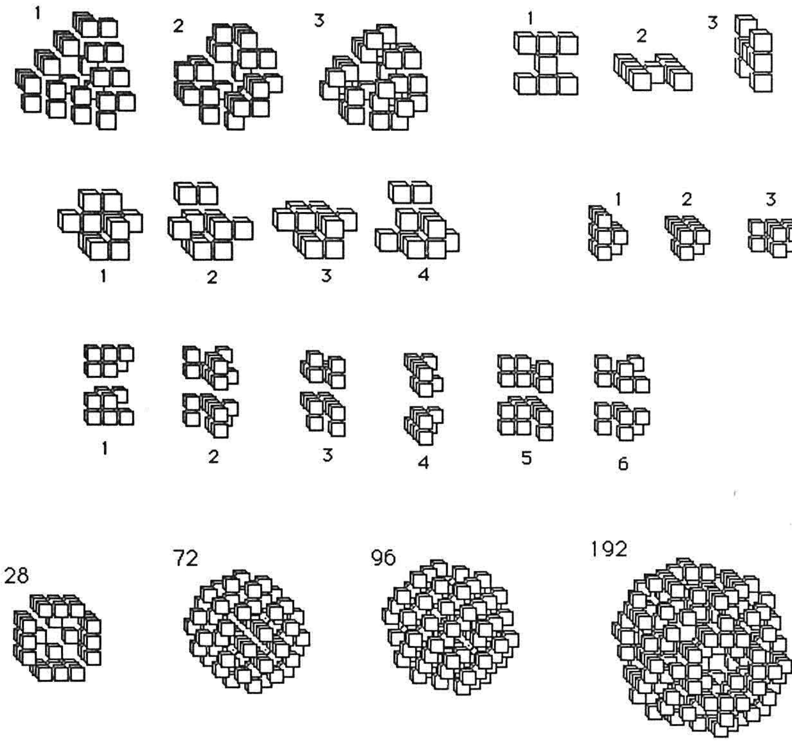

Figure 3: Stable forms and oscillators with a period greater than two appear to be rather rare. The "flipping H" shape at the upper right is one of the few naturally occurring oscillators with a period higher than two. It also appears in Life 5655. The stable forms at the bottom are accompanied by the number of live cells in each. To be discovered, many of the objects illustrated here required "symmetric soup."

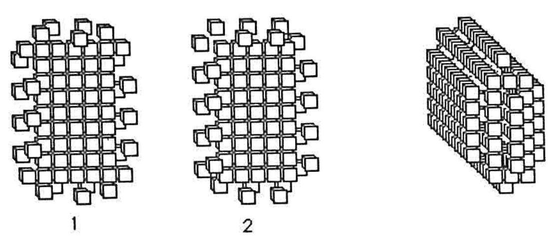

Figure 4: Barriers for Life 6855 can be constructed in many ways. The barrier at the left is stabilized by a period-two oscillating border. The more mundane object at the right can of course be stretched in any direction parallel to a coordinate axis.

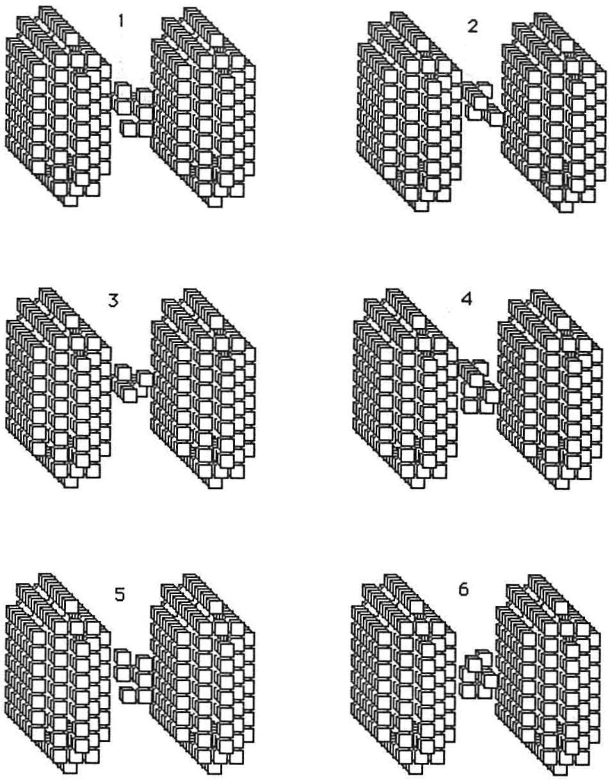

Figure 5: This period-six oscillator would not exist without barriers.

in the two-dimensional analog. All life is confined to the two "liveable" planes between the barriers. Note that any non-living cell in these two planes would thus touch either 0, 2, 4, 6... live cells. Hence, Life 6855 cannot have a two-dimensional analog—for that matter, no such analog can exist whenever the fertility rule of the two-dimensional game is strictly odd (see [1]). Nevertheless, when we position barriers exactly four cells apart and then place random configurations into the two live planes between, an entirely new universe unfolds. A myriad of small period-two oscillators—different from those illustrated in figure 2—can be discovered easily. Much more interesting, however, are the many higher-period small oscillators, which are typified by those illustrated in figures 5 and 6, and which exist nowhere but in this barrier game. Note that for any such "barrier oscillator" we can place mirror images of the oscillator one plane apart to create a "dead zone" similar to that imposed by a barrier. This effect is true for any life game that has only odd fertility rules, as cells in the plane between the mirrored oscillators are always bounded by an even number of live cells and thus can never become alive. The period-six oscillator in the middle of figure 3 is

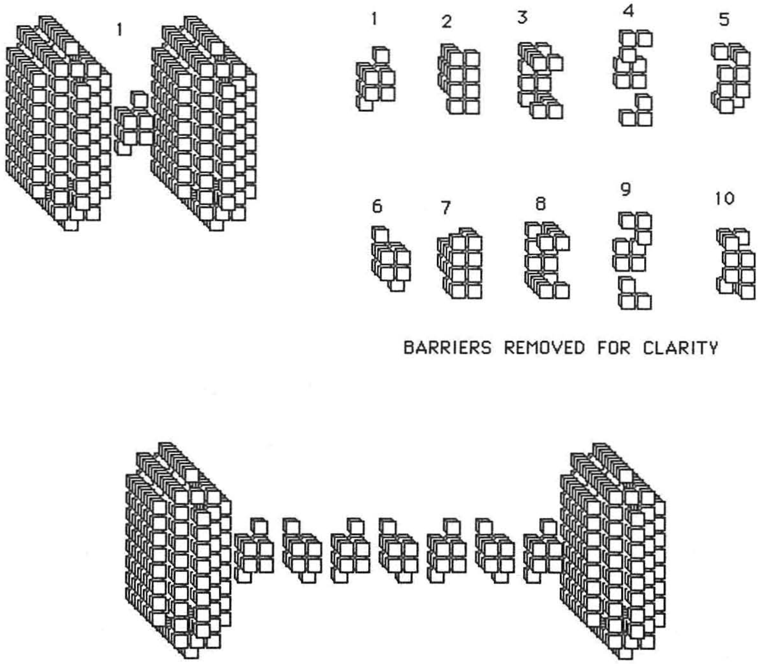

Figure 6: Typical of the many higher-period "barrier oscillators," this period-ten oscillator is shown without the barriers for clarity. We can create a string composed of as many of these as we wish by placing mirror images one plane apart. Of course we must "tie" the ends of the string with barriers. When one barrier is removed, the object behaves like a "fuse"—one oscillator is burned away every five generations.

somewhat similar to a barrier oscillator; it only works when the two halves are placed as mirror images one plane apart. If we remove one half, the other would tend to grow in the direction of the dead zone, and the object would self-destruct.

A search has also been made for a translating barrier oscillator. So far no such "pseudoglider" has been discovered. (Such a glider is referred to as "pseudo" because it would eventually attempt to emerge from the barriers and, hence, self-destruct.)

In conclusion, it is apparent that Life 6855 holds at least as much promise as the original games, Life 4555 and Life 5766. Life 6855 is the first rule where three gliders have been discovered; and although there is no two-dimensional analog, the unusual "barrier life" construction certainly deserves further investigation.

References

[1] C. Bays, "Candidates for the Game of Life in Three Dimensions," *Complex Systems*, **1** (1987) 373–400.

[2] A. K. Dewdney, "The Game Life Acquires Some Successors in Three Dimensions," *Scientific American*, **224**(2) (1986) 112–118.

[3] C. Bays, "Patterns for Simple Cellular Automata in a Universe of Dense Packed Spheres," *Complex Systems*, **1** (1987) 853–875.

[4] C. Bays, "The Discovery of a New Glider for the Game of Three-Dimensional Life," *Complex Systems*, **4** (1990) 599–602.

[5] C. Bays, "A New Game of Three-Dimensional Life," *Complex Systems*, **5** (1991) 15–18.