Complexity Classes in the Two-dimensional Life Cellular Automata Subspace

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This paper presents results from a systematic exploration of a two-dimensional cellular automata subspace based on Conway's "Game of Life." A qualitative study of its dynamics has led to a refinement of Wolfram's classification. Evidence that rules supporting gliders are located in the vicinity of a phase transition between order and chaos are found.

1. Introduction

Cellular automata (CA) are a class of discrete dynamical models which represent an approximation to physical systems composed of a large number of simple and locally interacting elements [18]. They have caught the interest of many researchers because of their rich and diverse phenomenology [16]. In this framework, Wolfram has shown that CA exhibit the full spectrum of dynamical behaviors [19] and proposed a qualitative classification of their dynamics into four groups [20].

Class I is associated to limit points in the phase space; for almost all configurations, cellular elements have the same value after a relatively short transient period.

Class II is associated to limit cycles; almost all configurations lead to an homogeneous state, except for some stable and oscillating patterns.

Class III is associated to chaotic behaviors which refer to apparently unpredictable space-time behaviors.

Class IV is associated to complex behaviors characterized by long transients and the existence of propagating patterns. Also, some Class IV CA are suspected to support universal computation.

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In 1991 Langton proposed the "Edge of Chaos" hypothesis which claims the existence of a phase transition between ordered and chaotic behaviors for one-dimensional CA and locates complex dynamics in the vicinity of this transition [11]. He also defined the λ parameter, which could be described as the fraction of a rule's input configuration that leads to its quiescent value (for binary CA, 0 is the quiescent value) [12]. According to Langton, this parameter allows travelling continuously across the space of rules and, as λ is incremented from 0 to $1 - 1/k$ (where k is the number of cell states), the average behavior of rules passes through the following regimes:

$$\text{Class I} \rightarrow \text{Class II} \rightarrow \text{Class IV} \rightarrow \text{Class III}.$$

Therefore, CA with computational capabilities are likely to be found in the vicinity of the phase transition between ordered (Class I & II) and chaotic behaviors (Class III). Evidence of this location has also been given in [9, 15].

However, Langton's hypothesis has been recently criticized by several authors [6, 14]. Critics mainly discuss the validity of the λ parameter. Langton himself [11] reported that for binary CA, the λ parameter does not fit very well and that another parameter could be better [21]. Also, while robust and general, the Wolfram classification is not precise enough for describing the richness and diversity of CA dynamical behaviors.

This paper is divided into two parts. In the first part, a systematic exploration of a subspace of two-dimensional CA based on Conway's Game of Life is performed. This allows us to identify various dynamical behaviors and to create a behavioral map revealing the structure of the Life subspace. Given these results, the validity of the λ parameter and the Edge of Chaos hypothesis are reviewed.

In the second part of this paper, a systematic search for propagating patterns in the Life subspace is presented. This search is motivated by the fact that "gliders" represent good signs for detecting interesting Class IV CA, the ones that are suspected to support universal computation. Then, results of this search are used in order to check the consistency of Langton's theory in the Life subspace, that is, the location of complex rules in the vicinity of a phase transition between ordered and chaotic behaviors. Finally, these results are discussed and the paper concludes on possible future works.

2. The two-dimensional Life cellular automata subspace

The 2D CA Game of Life, originally described in [8] has been presented in a large number of publications. It shows complex dynamics [16] and has been proven capable of supporting universal computation [5]. Few variants have been mentioned, and according to several authors, Life seems to be an exception [13, 16]. However, recently a new candidate rule exhibiting the same complex dynamics but with shorter transients has been discovered [10]. Since the set of all possible two-dimensional CA represents a huge space of transition rules and not all rules are interesting, it has been decided to focus this study on a subspace of two-dimensional CA based on a generalization of Life.

There are many ways for generalizing the rule originally designed by Conway. The most recent variation on this theme has been a series of articles by Bays [1–4]. Bays has extended Life into three dimensions and proposed a general definition for the set of possible rules, which will be referred to as the *Bays space*. Each rule in the Bays space can be written in the form $E_b E_h F_b F_h$ where E_b is the minimum number of living neighbor cells that must touch a currently "living" cell in order to guarantee that it will remain alive in the next generation. F_b is the minimum number touching a currently "dead" cell in order for it to come to life in the next generation, and E_h and F_h are the corresponding upper limits. These rules are called the *environment* and *fertility* rules. According to this notation, Conway's Life would be written Life 2333, that is, $E_b = 2$, $E_h = 3$, $F_b = 3$, and $F_h = 3$.

More formally, let S_t define the state of a cell and N_t be the number of living cells in the neighborhood of the considered cell at time t. The transition rule for the Bays space is then:

$$\text{if } (S_t = 0 \;\&\; N_t \leq F_b \;\&\; N_t \geq F_h) | (S_t = 1 \;\&\; N_t \leq E_b \;\&\; N_t \geq E_h)$$
$$S_t + 1 = 1,$$
$$\text{else}$$
$$S_t + 1 = 0.$$

In a two-dimensional square grid, a cell is surrounded at most by eight neighbors. The environment and fertility intervals are also bound between 1 and 8, since 0 is prohibited for quiescent reasons. So intervals can be any among (1,1) (1,2) ... (1,8) (2,2) (2,3) ... (8,8). Thus there are $(8 + 7 + \cdots + 1) = 36$ possible intervals. Environment and fertility being independent, the Bays space contains $36 \times 36 = 1296$ rules.

3. Qualitative study of the Life subspace

3.1 Experimental environment

The systematic exploration of the Life CA subspace was performed using a 64-cell square grid with boundary conditions, that is, on a toroïdal universe. For each rule several runs were observed for the following living cells densities: 0.1, 0.2, 0.5, and 0.8. For each run, all cells were randomly initialized with the probability of being alive corresponding to the average density. The duration of the observation

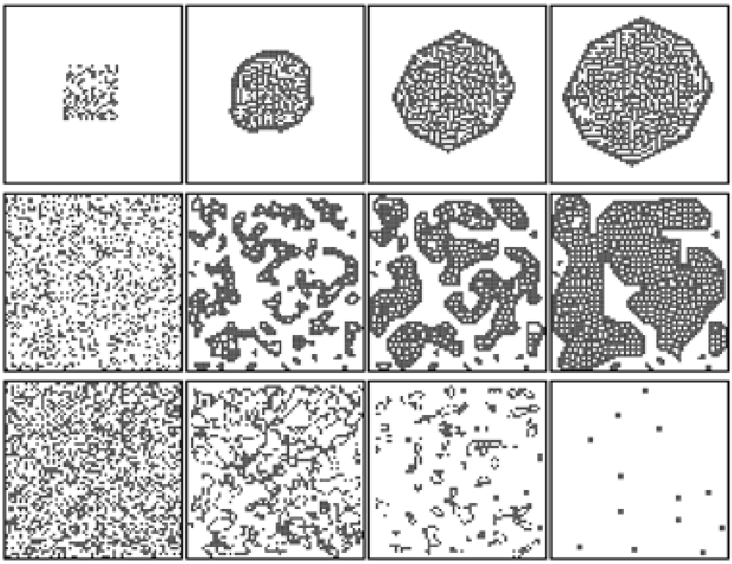

Figure 1. Top view shows infinite growth for rule 3634 at generations 1, 10, 20, and 30. Middle view shows bounded growth for rule 2746 at generations 1, 4, 9, and 20. Bottom view shows falling behavior for rule 3445 at generations 1, 3, 6, and 15.

varied, depending on the transient time of each run. The *Life32* software by John Bontes[1] was used, and since it does not yet allow boundary conditions, a dedicated Java applet was also developed.

3.2 Overview of dynamical behaviors

This section presents typical dynamics found during the systematic exploration of the Life subspace. As expected, the qualitative observation has shown a wide diversity of dynamics. This diversity can be expressed in terms of *growth* and *global periodicity*.

3.2.1 Growth

Observed growth can be classified into three basic types: *infinite*, *bounded*, and *falling* (*cf*. Figure 1). First, we define the *outline polygon*, as the polygon surrounding all living cells in a given configuration.

For almost all initial configurations, an *infinite growth* shows a continuously increasing surface of the outline polygon, until it completely fills the entire universe. Since density remains nearly the same inside the polygon, the number of living cells increases with the polygon surface. If the initial density is low, some CA can also exhibit a falling behavior.

[1] *Life32* can be freely dowloaded at http://soup.math.wisc.edu/Life32.html.

Different infinite growths can be distinguished by the shape of the outline polygon which can be roughly described by its shape (e.g., square, hexagon, octagon, circle), and its growth speed.

Bounded growth is characterized by a constant outline polygon. This type of growth can be compared to water drops on a window. In the beginning, some cells die, and the universe is composed of several drops of living cells. Drops "wriggle" and close drops merge together, forming new bigger drops, increasing the number of living cells. After a transient time, drops stop to evolve and the final density depends on the initial density value. With a sufficient value, all of the outline polygon is likely to be filled and all living cells form a unique drop. Also, there is a density threshold below which CA exhibit no growth, or even have a falling behavior.

The last type of observed growth is *falling behavior*. Whatever the initial configuration, all cells die after a varying transient time, leaving the universe empty with the exception of some rare stable or oscillating patterns.

3.2.2 Global periodicity

This section describes typical dynamics in terms of *global periodicity*. Observed behaviors can be divided into three main classes: *chaotic*, *cyclic*, and *stable*. Despite the close vocabulary used for naming classes in this section, we must underline that this classification is not the one stated by Wolfram. The main difference is that the membership of a given rule to one of these three classes depends on the global behavior of the lattice; that is, observed on a majority of cells, not necessarily all cells. Thus, some small parts of a configuration could exhibit different local dynamics.

Chaotic dynamics refers to the unpredictability of the CA behavior. It should be pointed out that on a finite universe, the number of configurations is finite. So, the CA behavior must be cyclic. However, for chaotic dynamics, the period is exponential with the size of the pattern while for cyclic dynamics the period is constant, typically short.

Almost all observed *cyclic dynamics* exhibit a period of two. Typically, after a varying transient time, the majority of cells seems to be inverted at each generation. This is not so surprising since Life is a binary CA. Of course, some local patterns show longer periods, which are generally even.

Finally, other CA exhibit *stable dynamics*, meaning that after a varying transient period, all cells remain in the same state at each generation. See Figure 2 for examples of each.

3.3 A new complexity classification

The qualitative observation of these behaviors leads to a refinement of Wolfram classification for the Life subspace. This new classification

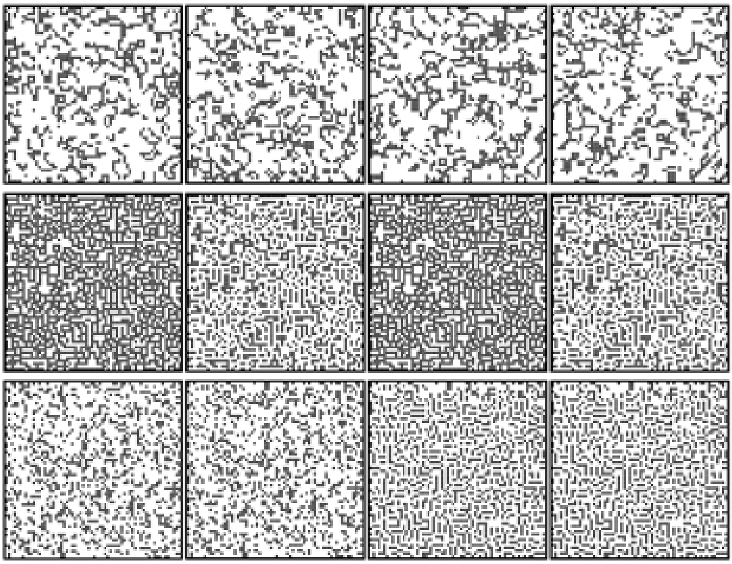

Figure 2. Top view shows a typical example of a chaotic rule (1112) at generations 20, 21, 22, and 23. Middle view shows a typical example of a cyclic rule (7717) at generations 10, 11, 12, and 13. Bottom view shows a typical example of a stable rule (1311) at generations 1, 10, 11, and 12.

is based on the two attributes *growth* and *global periodicity*. Since each of these attributes has three values, one could expect the resulting classification to contain nine classes that represent all combinations of growth and periodicity. However the classification is not so simple. First, all falling dynamics have been grouped into one class since there is no different global periodicity when a majority of cells are dead. In addition, chaotic dynamics with infinite growth were divided into four different classes. Thus the resulting classification includes 10 different classes (*cf*. Figures 3, 4, and 5).

Class 1 corresponds to falling dynamics (*cf*. Figure 3). After a transient time, all configurations lead to an empty universe, except for some local cyclic or stable patterns. Conway's Life (rule 2333) belongs to this class with a typical transient time of 500 generations.

The next three classes are characterized by a limited growth, which recalls some kind of "water drop" behavior (*cf*. Figure 4).

Class 2 typically exhibit stable dynamics with bounded growth while *Class 3* is also characterized by limited growth but with cyclic dynamics. Drop periods are short and in most cases even. *Class 4* exhibits chaotic behavior with bounded growth. This class differs from Class 3 by the fact that drops have a period which is an exponential function of their size.

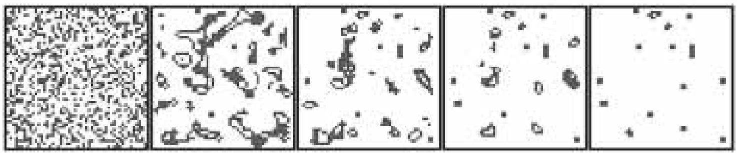

Figure 3. A typical example of a Class 1 CA is rule 3546, shown here at generations 1, 5, 10, 15, and 25.

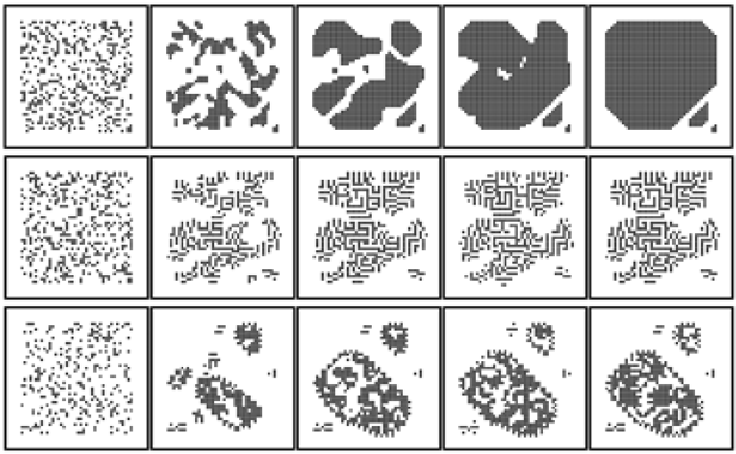

Figure 4. Water drop growth behavior. Top view shows a typical example of a Class 2 rule (2847) at generations 1, 3, 10, 20, and 70. Middle view shows a typical example of a Class 3 rule (3668) at generations 1, 5, 15, 16, and 17. Bottom view shows a typical example of a Class 4 rule (4638) at generations 1, 20, 60, 61, and 62.

The six remaining classes are characterized by an infinite growth (*cf.* Figure 5). *Class 5* corresponds to an infinite growth with stable dynamics in the outline polygon. *Class 6* is close to Class 5 but with a cyclic periodicity instead of stable dynamics. Figure 5 shows a typical example of this class where cells reverse their state at each generation.

All of the following classes are characterized by chaotic dynamics but differ regarding their type of chaotic behavior. *Class 7* exhibits some rare stable patterns that survive only a few generations and then are destroyed by the surrounding chaos. Figure 5 shows some configurations using rule 4711 with some stable patterns. In four generations, most of these patterns are almost destroyed. *Class 8* is cyclic with a global period of 2, but some moving areas are changing so that the universe has a period exponential with its size. *Class 9* recalls the surface of the sun. The universe is spotted by different sized spots of dead cells. With subsequent generations, each spot size decreases. When a spot disappears,

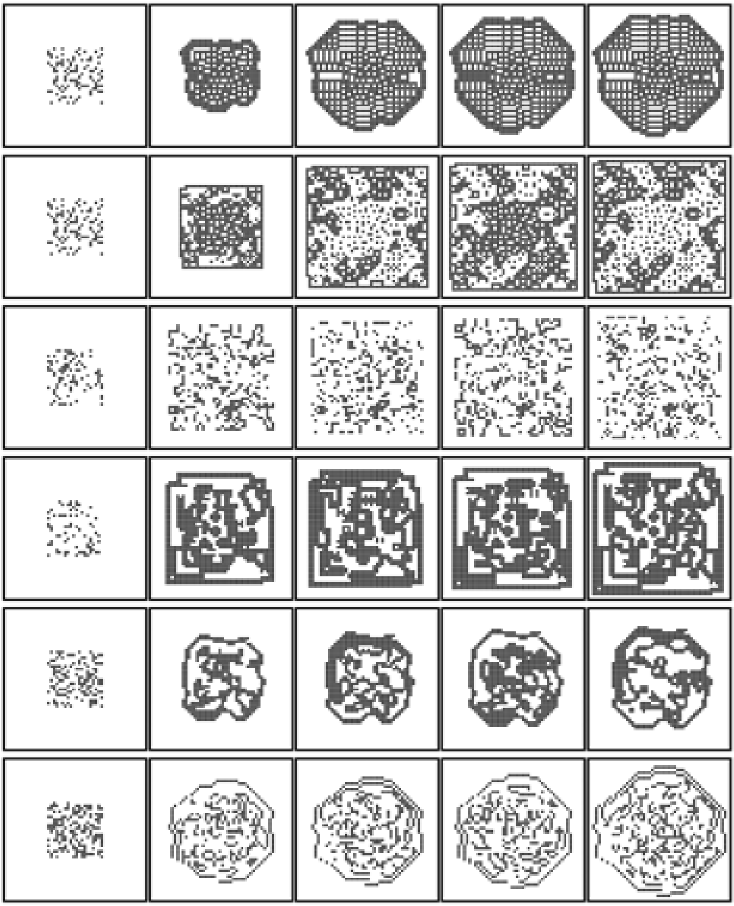

Figure 5. From top to bottom, typical examples of Classes 5 through 10 are shown. Class 5 is represented by rule 2725 at generations 1, 6, 13, 14, and 15. Class 6 is represented by rule 3318 at generations 1, 6, 13, 14, and 15. Class 7 is represented by rule 4711 at generations 1, 10, 11, 12, and 13. Class 8 is represented by rule 1616 at generations 1, 11, 12, 13, and 15. Class 9 is represented by rule 3526 at generations 1, 7, 8, 9, and 10. Class 10 is represented by rule 1111 at generations 1, 10, 11, 12, and 15.

it is generally replaced by a new spot at the next generation. Figure 5 gives an example of such behavior using rule 3526. It shows a white spot on the right which is reducing until it totally disappears at generation 10. At generation 7, the center is occupied by a dark spot which disappears at the next generation, giving place to a new white spot. *Class 10* exhibits fully chaotic dynamics. The outline polygon increases its size and seems to be randomly initialized at each generation.

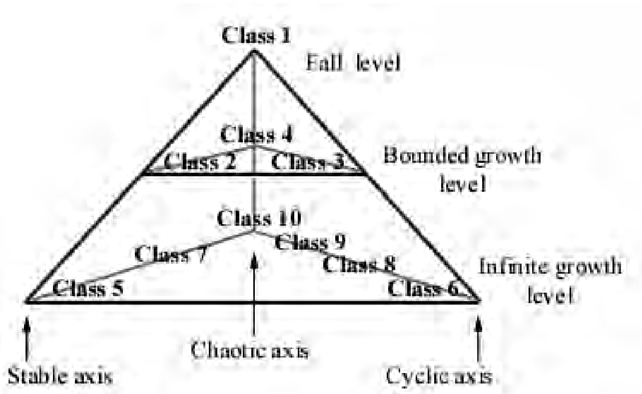

Figure 6. A qualitative distance between classes can be shown on a three-dimensional pyramid graph.

3.4 Structure of the Life subspace

The fact that this classification is mainly based on two attributes allows us to define a qualitative notion of distance between classes. Two classes are considered to be close together if they differ only in one attribute, either growth or global periodicity. For instance, Class 3 and Class 4 are close because they have the same growth attribute. Therefore, chaotic, cyclic, and stable dynamics can be considered as different axes in a three-dimensional graph, while growth can be ordered from falling to infinite growth. These features are best shown on a pyramid graph (*cf.* Figure 6). Note that Class 7 describes behaviors which can be qualitatively placed between Class 5 and Class 10, and Classes 8 and 9 describe behaviors between Classes 6 and 10.

Besides a qualitative distance between classes, a topological notion of distance between rules is needed to describe the structure of the Life subspace. In the following sections, two rules will be considered close together if they differ by one and only one parameter. For instance, rule 2637 has eight neighbors that are 1637, 3637, 2537, 2737, 2627, 2647, 2636, and 2638.

From a topological perspective, since each rule is described by four parameters, the Life subspace is a four-dimensional space. Figures 7, 8, and 9 show some partial two-dimensional and three-dimensional projections of this four-dimensional space which is too large and too complex to be represented on a single graph. On these projections, topologically close rules are shown as adjacent cubes. We also refer to *areas*, such as the xx2x area, which represents a three-dimensional projection of the behavioral map where all rules are characterized by $F_b = 2$. Since the xx3x area is adjacent to the xx2x area, rules at the same location in their respective figures are also topologically close. Thus each

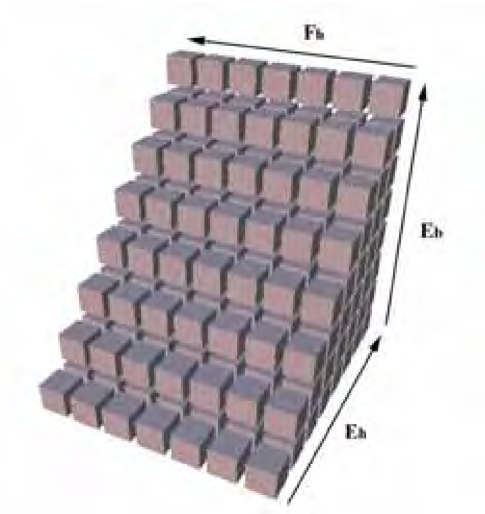

Figure 7. Given an F_b value, a single area xxF_bx of the four-dimensional Life subspace can be represented as a three-dimensional stairway composed of adjacent rules. Each rule is represented as a cube.

area can be represented as a three-dimensional stairway composed of cubes (*cf*. Figure 7).

An important feature that appears in Figure 8 is that topologically close rules belong to close classes when not of the same class. Behaviorally close rules are adjacent, forming three-dimensional patterns with clear outlines. These outlines are the evidence of a phase transition in the Life subspace from a behavioral perspective. As an example, Figure 8 presents evidence of a phase transition between chaotic and more ordered rules. These results partially confirm the existence of a phase transition between order and chaos as described by Langton. Figure 9 gives evidence of a similar phase transition between infinite and bounded growth, that was not expressed in Langton's hypothesis.

However, this behavioral landscape is not so simple and homogenous. The Life subspace is very complicated and shows a variety of transitions between classes. As an example, Figure 10 shows different kinds of transitions. The xx23 area exhibits a transition between Class 10 and Class 5 rules, which are close classes. Moreover, in the vicinity of this transition are located some Class 7 rules. Class 7 is located between Classes 5 and 10, so such a transition could be qualified as a smooth transition. The xx18 area also exhibits a smooth transition since rules in this area belong to close classes. Nevertheless, this area is more complex because classes are not simply spatially distributed. As an example of a sharp transition, the xx35 area presents a transition between Class 1

Figure 8. From left to right, three-dimensional projections of areas xx2x, xx3x, and xx4x. Each cube represents a rule with chaotic global periodicity while other rules (cyclic and stable) are not displayed.

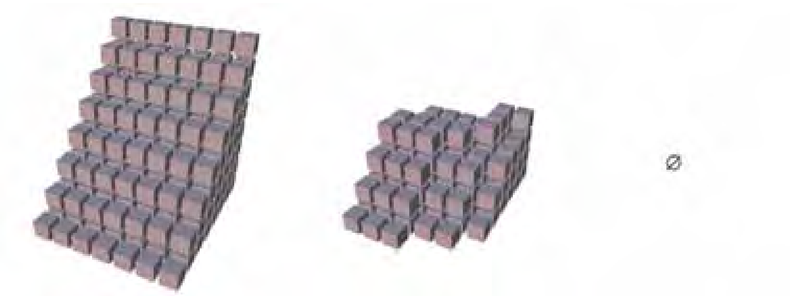

Figure 9. From left to right, three-dimensional projections of areas xx2x, xx3x, and xx4x. Each cube represents a rule characterized by an infinite growth. All rules belonging to the xx2x area exhibit infinite growth, while the opposite is true for the xx4x area (no cubes).

and Class 10; that is, between falling dynamics and chaotic infinite growth.

3.5 Lambda parameter

According to the Edge of Chaos hypothesis, as λ is incremented from 0 to 0.5 (for binary CA), the average CA dynamics passes from Wolfram Class I/II to Class III with a phase transition where Class IV CA are located. There must also be a symmetry in this picture as λ is incremented from 0.5 to 1. Therefore, as λ becomes close to 0.5, a majority of CA should belong to chaotic classes. Unfortunately, the left part of Figure 11 shows that it is not true for the Life subspace. Chaotic CA are not likely to be found for λ close to 0.5. As a conclusion, λ is not a good parameter to predict the class membership of a rule in the Life subspace. However, it seems that λ is better for predicting CA growth.

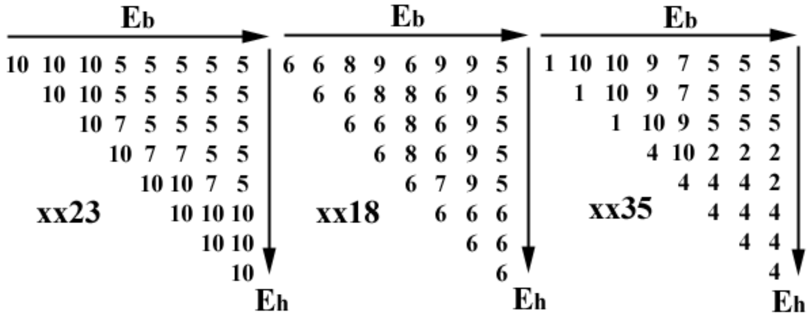

Figure 10. Some two-dimensional projections of the four-dimensional Life subspace exhibiting different transitions between classes. From left to right, the xx23 area shows a smooth and simple transition, the xx18 area shows a smooth and complex transition, and the xx35 area shows a sharp and complex transition.

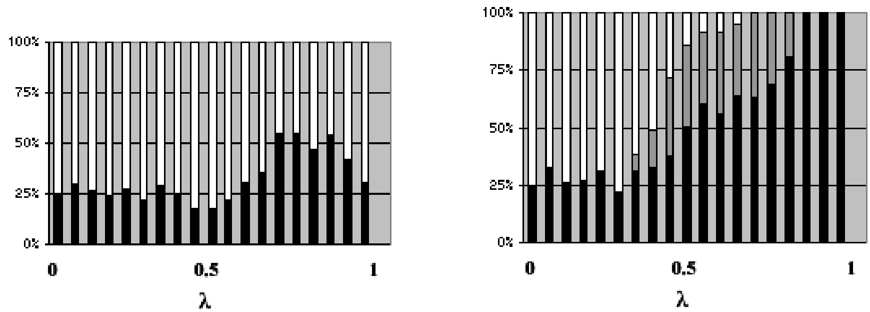

Figure 11. Left view shows percentage of chaotic rules in the Life subspace function of λ. Right view shows: percentage of infinite growth rules in black, limited growth rules in gray, and falling rules in white.

The right part of Figure 11 shows that infinite growth is likely to be found as λ becomes high.

4. Locating propagating patterns in the Life subspace

4.1 Gliders as signs of universality

As far as we know, Conway's Life is the only CA that has been proven capable of universal computation. This proof mainly relies on the existence of gliders and the analogy between glider flows with binary pulses in logical gates. A glider is a propagating pattern, that is, a periodic configuration of living cells which is translated across the lattice with generations. The existence of gliders alone is not a sufficient condition to establish universality, because other patterns such as "glider guns" are also needed [7]. However, CA admitting gliders deserve attention because they represent good candidates to support universal computation. Thus, in the following sections, a systematic search for gliders in the Life subspace is reported. This allows verification that candidate rules for universal computation are located near a phase transition between ordered and chaotic dynamics.

4.2 Searching for gliders

The existence of gliders in the Life subspace was addressed in two different ways. The first approach was based on the Bays test [1]. In this test, a centered square of 20×20 cells is randomly initialized in a 100×100 empty grid. Then, transition rules are applied for a fixed number of generations unless an object hits the edge of the universe. For each rule, 200 tests were performed for three different initial densities. Hitting objects may be gliders but may also be spurious patterns. However, they can be easily distinguished by a simple visual observation.

The second approach for finding gliders was a systematic test of all possible patterns contained in a 4×5 rectangle. Each pattern was evolved during 10 generations. Then, the resulting pattern was considered a glider if one of its phases occurred again with a translation from its initial location.

A set of 18 different gliders were found by applying these two tests. The Bays test found only a few of them, while the exhaustive test exhibited all gliders. Gliders differ by their period, their speed, their orientation, and, of course, their transition rule(s). Figure 12 shows the 18 gliders with their related rules. Their period and their speed can be easily deduced from the figure.

4.3 Between order and chaos

Rules admitting gliders can be divided into two groups. The first group is located in the xx2x area, while the second is located in the xx3x area. As shown in Figure 8, all rules in the xx2x area exhibit an infinite growth. In this area, even a small pattern grows forever, so stable and oscillating patterns are very rare and do not occur naturally. Since oscillating and stable patterns are also required for a universality proof, rules located in this area do not represent good candidates, despite the existence of gliders.

In contrast, most of the rules admitting gliders in the xx3x area exhibit infinite growth, but with a density threshold below which rules exhibit falling dynamics. Under these conditions stable and oscillating patterns are likely to emerge. Note that Conway's Life (rule 2333) is

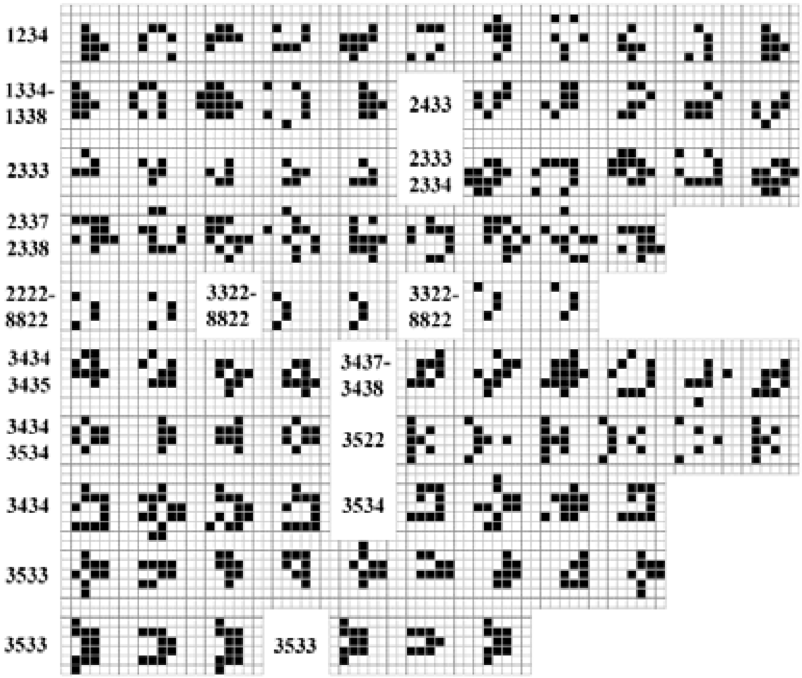

Figure 12. The 18 gliders found in the Life subspace. xxxx-yyyy means that the related glider can be found in rules xxxx through yyyy.

located in this area, showing long transients in a global density falling behavior.

These observations suggest that rules admitting gliders in the xx3x area are possible candidates for supporting universal computation, while rules in the xx2x area are discarded. Figure 13 shows the location of rules admitting gliders in the xx3x area. Since the Life subspace is a four-dimensional space, all neighbor rules are not represented but rules admitting gliders are clearly located in the vicinity of an ordered and chaotic phase transition, thus partially confirming Langton's hypothesis. Moreover, these rules are also located in the vicinity of the phase transition between infinite and bounded growth.

5. Discussion

This paper proposed a classification of dynamical behaviors for the Life subspace. While Wolfram's classification is wide-purpose and based on the observation of one-dimensional CA, Life's classification comes from the systematic qualitative observation of the Life subspace which represents an interesting but reduced set of two-dimensional binary CA. Thus there does not exist a one-to-one correspondence between

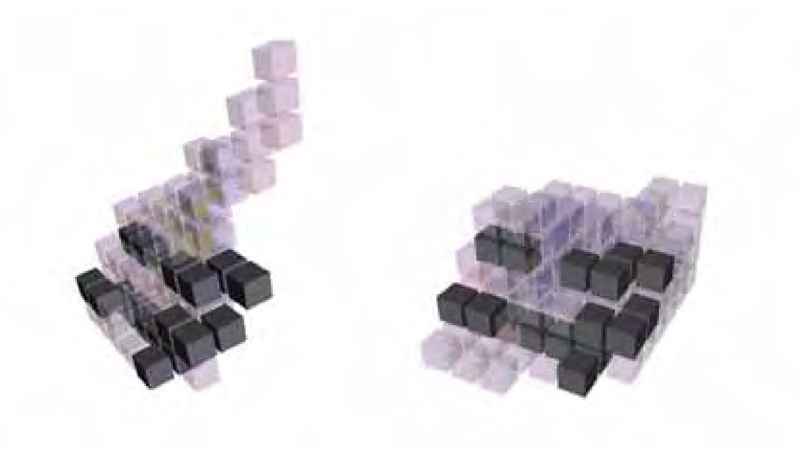

Figure 13. Location of rules admitting gliders in the xx3x area. The left diagram shows chaotic rules as translucent cubes and rules characterized by gliders in dark gray. The right diagram shows rules with infinite growth as translucent cubes and rules characterized by gliders in dark gray.

these two classifications. However, they are close enough to allow a relationship between the different classes, except for Wolfram's Class IV which has no counterpart here. In the proposed classification, "complex CA" are not grouped in a dedicated class, but rather defined as candidate rules for supporting universal computation. Since this classification is based on an observation of qualitative behaviors, it is not always easy to determine exactly the class of a rule. As an example, rule 3737 has required a careful evaluation of its proportion of stable and periodic patterns in order to decide its class. Therefore, a quantitative analysis is needed to confirm this work.

In the second part of this paper, results from a systematic search for propagating patterns were presented. Due to the size of the space of this search, it has been limited to gliders contained in a 4×5 cell rectangle, and having a period less than 10 generations. Thus, some gliders may have been missed. However, as emphasized, it is not gliders themselves that are interesting but rather the sign of universality that they embody. For Conway's Life universality proof, various glider-based configurations such as collisions and glider guns were used [5]. The probability of natural gliders, as well as the existence of useful glider collisions and glider guns, decreases with glider size. In this context, we have focused the search for relatively small propagating patterns.

Bays has performed quite a similar search for Life "worthy of the name" in the set of three-dimensional binary CA [1]. He defined two criteria that have to be respected. First, gliders must occur naturally by the application of the rule on random initial configurations. The second criteria is that all random initial configurations must exhibit bounded growth. Since the Bays test of searching for gliders missed most of them, we have used another test which does not respect these criteria. These two criteria seem too strong. If gliders are sufficiently small, then interesting configurations such as glider guns and glider collisions are more likely to emerge. In addition, a bounded growth is not necessary if glider collisions do not exhibit infinite growth. There are some possibilities that rules with infinite growth, but having a density threshold below which falling dynamics occur, may support universality. This hypothesis must be verified by a careful analysis of each candidate rule. By the way, these results conform to Bays's theorems [1], since this study has not revealed any rules admitting a glider with $F_b \geq 4$, and all rules having $F_b \leq 2$ exhibit an infinite growth.

As expected, the behavioral structure of the Life subspace is complex, composed of homogeneous areas separated by smooth or sharp transitions. The existence of these phase transitions between ordered and chaotic dynamics, as well as the location of complex CA in the vicinity of one of these transitions partially confirms the Edge of Chaos hypothesis. In addition, similar results were found between infinite and limited growth. Nevertheless, Langton's λ parameter was not accurate for predicting the class of a given rule in the Life subspace. Rather, it can be used more accurately as an estimation for CA growth.

Since Life is only a small subset of the whole space of two-dimensional binary CA, one can ask questions about its relevance. As an example, this study could have been done on another generalization of Conway's Life where the fertility and environmental rules are defined as discontinuous sets of values, which correspond to Wolfram's "outer totalistic" rule type [17]. This Life subspace is then composed of 2^{16} rules instead of 1296, and includes Heudin's Life which exhibits a dynamical behavior close to the one of Conway [10]. A complete answer to the complexity structure of the two-dimensional binary CA space requires constructing a map of all two-dimensional binary CA. Since such a space is huge, it cannot be done using a qualitative observation of dynamical behaviors, but rather with an automatic classification based on quantitative measurements.

6. Conclusion

This paper has presented results from a systematic qualitative exploration of the dynamics of a two-dimensional binary CA subspace based on Conway's "Game of Life." The main goal was to check the consistency of Langton's "Edge of Chaos" hypothesis on this subspace, which claims the existence of a phase transition between ordered and chaotic dynamics and locates complex behaviors in the vicinity of this transition.

The exploration of the Life subspace led to a refinement of Wolfram's classification and evidence that rules admitting gliders were located in the vicinity of order and chaos were found, thus partially confirming Langton's hypothesis. A transition between infinite and bounded growth was also found. In addition, 18 propagating patterns, called *gliders*, were found and located in the Life subspace. Future works include a quantitative validation of these results and the extension of the study on a larger Life subspace.

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