Self-regulation in a Simple Model of Hierarchically Organized Markets

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We study a model for a market which has a tree form structure. Each branch of the tree is composed of identical firms. The branch of the first level is formed by the firms producing raw material, and the branches of the last level are the retail outlets. The branching points (tree nodes) are micromarkets for firms forming branches connected with a given node. The prices and the production rate are controlled by the balance in supply and demand. Due to the material conservation law the network nodes in such a market function perfectly: the prices are specified by the production expense only, whereas demand determines the production rate.

We construct an efficiency functional with an extremal that gives the governing equations for the market. It turns out that this ideal market is degenerated with respect to its structure. It is shown that such a market functions ideally: the prices are determined by the costs of producing the goods, and the level of production of goods of any kind is defined only by demand.

We propose a simple model of trade interaction among firms which are able to join together in order to decrease total cost. It is shown that self-organization, due to optimization of each firm's profit, leads to hierarchical organization of the trade network.

1. Introduction

The characteristic properties of commodity market evolution have been intensively studied during recent years. However, at the present time, theoretical descriptions of the self-organization phenomena in microeconomic markets are still practically unresolved and pose challenging problems. The reason is that the commodity market is a radically open system that exists while and if it exchanges substance by energy; and, what is not of less importance, by information with the surrounding world (i.e., social and physical environment). This is the main difference between market systems and physical systems. Really, for physical systems it is possible to indicate a state of a thermodynamic equilibrium, where all streams are equal to zero. Such an equilibrium state is usually described by a small number of macroscopic parameters, that is, parameters of "order." But economic systems exist only while they have the streams of commodity and money. Besides, the number of parameters which define their state, at first glance, seems to be actually endless.

Nevertheless, we can consider dissipative systems that are far from equilibrium found in physics and chemistry as some rough analogs of such economic systems from the viewpoint of common features of evolution. During the past 20 years these dissipative systems were profoundly investigated (e.g., [1–7]). It has been found that their processes of dissipation can produce self-organization in various types of dissipative structures.

It should be noted, that a physical or chemical system at the moment of the origin of dissipative structures transforms into a state with an intermediate degree of chaos [4]. The entropy of this state is less than maximum, corresponding to thermodynamic equilibrium. At the same time it is possible to describe the state with a set of macroscopic (or mesoscopic) parameters of order. Due to the slow change of these parameters the evolution of such systems is usually realized as the process of transition through the sequence of nonequilibrium quasistationary states, distinguished from each other by their dissipative structures.

In economic systems, complex dissipative structures also appear as stipulated by the will of people. These structures arise from individual interaction between agents and spontaneous formation of some order in their relationships (see [8, 9] for reviews of this problem from the viewpoint of synergetics).

As an example of such self-organization phenomena one can consider the creation and function of trade networks in the systems with distributed auctions, as investigated in [10–14]. It has been supposed that the number of trade agents is given beforehand and does not vary in time. It should be noted that self-organization and evolution of these structures is similar to nonequilibrium phase transitions under the changes of external parameters [14].

In the first stage of the development of self-organization theory in microeconomics it is necessary to choose some adequate zero approximation, which actually is a detailed description of the market ideal state, or its "norm." Such a norm is well known. It is an equilibrium market, where the interests of buyers and sellers completely coincide with each other in the way, that, within the given price, the value of supply is equal to the value of demand. However, for the analysis of self-organization processes a more detailed description is required, for example, to pair interactions of sellers and buyers.

Apparently, such a rather detailed description of economics as a whole does not make much sense, because the markets of radically different products would be only slightly associated. For example, the market of meat items and the market of furniture can interact between themselves mainly through changes in financial states of the whole collection of consumers. It should be noted that such a situation is similar to the situation realized in living organisms. Indeed, if a living organism is not in extreme conditions, each of its organs is supplied with an amount of blood completely satisfying its needs, regardless of the other organ's function. It is supplied by the system of large arteries. These arteries form the infinite tank of blood for systems of separate organs, and the heart replies to some aggregated information about the state of the organism as a whole [15, 18].

Therefore it would be worthwhile to restrict our consideration to any commodity market, related from the viewpoint of possibility of their mutual substitution. Producers, dealers, and consumers of these goods will form a common connected network, carrying on the goods from the producers of raw material up to the buyers which purchase these goods in retail shops.

Nevertheless, the set of products made for a given market forms only a small part of the whole public market. The state of the rest of the economic system has to be taken into account in some set of governing parameters, which do not influence local changes in the given market and reflect aggregated information about society as a whole.

In other words, for the analysis of self-organization processes it seems reasonable to restrict ourselves to research some small markets of goods, which nevertheless form a common connected system between consumers and all producers of these goods. Such a mesoscopic market (mesomarket) is a subject of research in the present paper. Concerning the common requirements applied to the possible model of its norm, we single out the following items.

- The model should be of an intermediate degree of chaos. In other words, on the one hand, individual behavior of the participants in an ideal market is not prescribed from any center. More likely, it is determined by their own goals, based on a small part of information about the market state. On the other hand, the ideal market contains structures, generated by individual interaction of the participants as a result of the establishment of spontaneous order in their ratios. These structures form the streams of goods and corresponding flow of money in the opposite direction.

  These streams of money bear some kind of information self-processing, which provides a way for each participant to react adequately on various changes of the market state, based only on a small share of information, which aggregates information about the market state as a whole (e.g., [16–18]).

  It should be noted that a strictly administrative (completely ordered) system, based on the ideal overall plan, as well as formally equivalent to

Walras's "auctioneer" (i.e., completely unregulated system, where everyone can contact each other through some process such as "tatonment") cannot function in reality, because for proper functioning these systems require physically infinite time to collect the information [19, 20].

Summing up this item we can formulate that a model of the norm has to contain individual interaction of the participants, described with classical laws of demand and supply. And the realization of their contacts has to be given by the micromarket network structure, where a physically finite number of participants comes into contact.

- The prices, arising as the result of individual trade relations, normally have to be determined only by real material costs on commodity production. It should be noted, however, that this correspondence should not be an outcome of price control, but a result of the self-organization processes.

- Spontaneously arising structures should be organized hierarchically. This is due to the fact of the existence of huge amounts of various goods in the market in contrast to the small amount of raw materials. Therefore, before the final goods are complete, the material of which they are made will be processed during the stages of their production many times. Their sequence and content are determined by the sort of product made and vary for different types of products. A simple example of such a hierarchical structure is the tree form. The root of this tree is formed by firms obtaining raw materials and the branches of the last level are the retail shops.

- The ideal market should be characterized by separability concerning the production of the commodity. In other words, let $\{\alpha\}$ be a collection of the final types of goods of the given market, $\boldsymbol{p} = \{p_\alpha\}$ is their price in retail shops, and $\boldsymbol{S}(\boldsymbol{p},\phi) = \{S_\alpha(\boldsymbol{p},\phi)\}$ is their demand function, that is, a consumption level at a given price. The demand function depends also on some internal parameters ϕ, describing the state of consumers, for example, their average yearly income and so forth. Then the change in the demand for the goods of sort α (i.e., change of the function $S_\alpha(\boldsymbol{p},\phi)$, for example, as a result of variations in ϕ) should not affect prices or production level for other types of goods $\alpha' \neq \alpha$ under their invariable demand (i.e., in the absence of changes in the function $S_{\alpha'}(\boldsymbol{p},\phi)$).

  Really, in an ideal case the consumption level of the goods of sort α and their price p_α are the result of the balance of total costs and utility of their production. Therefore, if the given balance is disturbed only because of a change in demand for goods of another sort $\alpha' \neq \alpha$ without technology change, then it points to an imperfection of the market and can result in chaos in the commodity production. The same is true for one sort of goods. If customers compose various groups, separated from each other in space, they are supplied by various market elements.

The given requirement of separability of the ideal market is a typical condition imposed on the process of commodity production and on properties of separate producers interacting with one another. This requirement does not impose any restrictions on possible immediate interrelations in demand of the consumers for goods of various sorts. Such interrelations can arise, for example, because one individual is a consumer of different types of goods.

It should be noted that the problem of how a "living" system that is complex in structure can respond perfectly to local changes in the environment has been investigated in [16–18]. It was shown that such a system possesses a cooperative mechanism of self-regulation by which the system as a whole can react perfectly. The mechanism implemented in natural systems is based on an individual response of each element to the corresponding small piece of information on the state of the system. The conservation of flux through the supplying network gives rise to a certain processing of information and the self-consistent behavior of the elements, leading to the perfect self-regulation.

In the present paper we have formulated a simple model of the commodity market, which functions ideally.

2. Model of the market

Let us consider the market of some goods, made of one sort of raw material. We shall present the collection of the consumers $\mathcal{M}$ of these goods as a union

$$\mathcal{M} = \bigcup_{\alpha} \boldsymbol{m}_{\alpha} \tag{2.1}$$

of various groups $\boldsymbol{m}_{\alpha}$. The difference of these groups is defined in such a way that they consume different sorts of goods, or that they acquire formally the common sort of goods, but are supplied by different branches of the market structure.

It should be noted that in the first case, groups $\boldsymbol{m}_{\alpha}$ and $\boldsymbol{m}_{\alpha'}$ (for $\alpha \neq \alpha'$) can be physically composed of the same collection of people. In the second case different consumers are separated either in space, or belong to various strata of society.

Let us suppose that the structure of the given market $\mathcal{N}$, formed with an industrial and commercial network, has the form of a tree (Figure 1). The branch i of this tree is a collection of n_i independent firms, bringing out the products of sort i, which buy the products from firms belonging to a lower level of the network hierarchy $\mathcal{N}$, and sell the results of their activity to the firms on a higher level. Firms forming the common branch i are considered identical from the viewpoint of technological processes and their trade relations. The root of the tree $\mathcal{N}$ (the branch of the first level) is formed by the firms obtaining and processing raw material. The tree branches of the last level are the points of retail trade, supplying only one group of consumers. We also suppose that for any

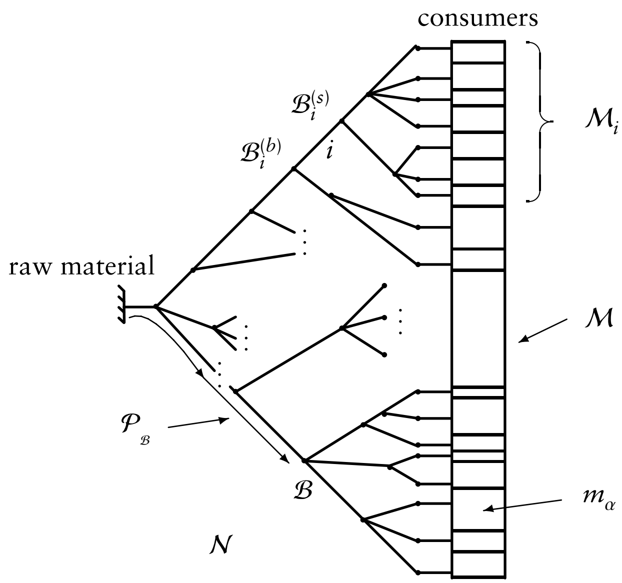

Figure 1. Structure of the mesomarket under consideration.

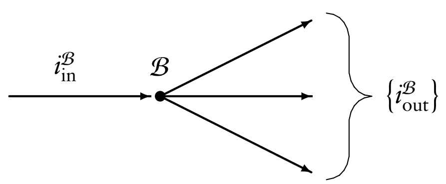

Figure 2. Structure of an elementary micromarket.

branch i the number of firms n_i belonging to this branch is a large value

$$n_i \gg 1 \qquad \forall i. \tag{2.2}$$

This equation allows us to consider the values $\{n_i\}$ as a continuous variable $(n_i + 1 \approx n_i)$.

Each network node of $\mathcal{N}$; for example, node $\mathcal{B}$, is a micromarket in which only those firms belonging to the branch $\{i_{\text{in}}^{\mathcal{B}}\}$ participate, going in a node $\mathcal{B}$, and firms belonging to branches $\{i_{\text{out}}^{\mathcal{B}}\}$, going out of the given node (sellers and buyers in this market, respectively, Figure 2). All products on the given branch are sold under one price $p_{\mathcal{B}}$. Firms which belong to the root of a tree $\mathcal{N}$ extract raw material and the firms of the last level of a hierarchy sell the goods directly to the consumers.

The network $\mathcal{N}$ actually determines the economic ratios between the market participants and sets interrelations of product flows. We shall consider the situation when the market of the considered goods exists, that is, all the streams should be greater than zero.

Let us measure the level of the firm's production which belongs to the branch i in units of the initial raw material flow x_i, "flowing through" the given firm. It should be noted that in this system of units the value x having dimensionality (*material*)/(*time*) and index i really determines the sort of production of the considered firm. The full product stream of X_i can be represented as

$$X_i = n_i x_i. \tag{2.3}$$

Due to the material conservation law, in network nodes $\mathcal{N}$ relations

$$X_{i^{\mathcal{B}}_{\text{in}}} = \sum_{j \in \{i^{\mathcal{B}}_{\text{out}}\}} X_j \tag{2.4}$$

are fulfilled for any node $\mathcal{B}$.

The whole production of the last level of firms is acquired by the consumers. In doing so, for a branch i_α supplying a group of consumers $\boldsymbol{m}_\alpha$, we can write the following:

$$X_{i_\alpha} = X_\alpha, \tag{2.5}$$

where X_α is the consumption level of the goods by group $\boldsymbol{m}_\alpha$, expressed in raw material flow units.

It should be noted that goods produced in this mesomarket can include not only the raw material but also other additional materials. The latter, however, are acquired by firms individually and their costs are included with the cost of production. The raw material of the considered model is like a binding of the different producers in the single network that is expressed in the conservation laws of equation (2.4).

The trade interactions of the micromarket $\mathcal{B}$ result in the origin of interchanged production prices. Taking this into account, we shall measure the interchanged production by the effective price $p_{\mathcal{B}}$ of the raw material unit. Then, in the result of its activity, each firm which belongs to branch i obtains the unit time profit π_i, which is equal to

$$\pi_i = (p_i^{(s)} - p_i^{(b)})x_i - t_i(x_i). \tag{2.6}$$

Here $p_i^{(s)}$ and $p_i^{(b)}$ are prices on the micromarkets where firms i represent themselves as the seller and buyer, respectively and $t_i(x)$ is the total cost for production at level x_i. Following the standard view we suppose that $t_i(x)$ is an increasing convex function, growing faster than $x^{1+\epsilon}$ (where ϵ is some positive constant). We also consider that

$$t_i(0) > 0 \qquad \forall i. \tag{2.7}$$

It should be noted that this equation is a condition imposed on the specific production costs of one firm, considered as indivisible. Total costs T_i, connected with the activity of all firms which form branch i, are equal to $T_i = n_i t(x_i)$ and depend on two arguments: n_i and x_i. If the value x_i is considered as the given parameter of production, then by virtue of equation (2.3) we can express $T_i = X_i t_i(x_i)/x_i$ and therefore $T_i \to 0$ at $X_i \to 0$, since equation (2.2) allows considering n_i as a continuous variable.

For firms obtaining raw material we have the expression

$$p_i^{(b)}\Big|_{i\in\text{root}} = 0. \tag{2.8}$$

We suppose that the individual goal of each firm is to reach the maximum profit. In this case we can describe the production of firms at level x_i as satisfying the following condition

$$\frac{\partial \pi_i(x_i)}{\partial x_i} = 0. \tag{2.9}$$

The change in demand, in particular, causes change in the production level, and therefore, results in the value of gained profit. The latter, in its turn, stimulates or suppresses activity of the firms. In the same case this stipulates firms arising or vanishing in the market. If a competition is perfect (i.e., when there are no barriers for the entrance of new firms into the existing market) at the equilibrium status, the profit received by the firm, taking into account all costs, should be equal to zero (e.g., [21, 22]). Assuming this condition is met, the value x_i should satisfy the conditions

$$\pi_i(x_i) = 0. \tag{2.10}$$

The consumers, purchasing the goods at the points of retail trade, try to maximize their utility. Taking into account their utility and budget constraints we shall describe the behavior of the consumer group $\boldsymbol{m}_\alpha$ by a positive definite demand function $S_\alpha(\boldsymbol{p}, \phi)$

$$X_\alpha = S_\alpha(\boldsymbol{p}, \phi), \tag{2.11}$$

where $\boldsymbol{p} = \{p_\alpha\}$ is the collection of prices of all final goods in the shops of retail trade, and the parameter ϕ reflects total expenditure. Equations (2.3) through (2.5) and (2.8) through (2.11) describe the functioning of the given market and, in particular, determine the number of firms $\{n_i\}$ participating in production and trade of commodities.

We now analyze characteristics of the formulated model.

3. Mechanism of ideal self-regulation

With the given structure of a micromarket the unknown variables are: the level of each firm's production $\{x_i\}$, their number in different branches $\{n_i\}$, and prices on the micromarkets $\{p_{\mathcal{B}}\}$.

Equation (2.6) for the profit π_i of firm i contains only two independent variables, the difference $(p_i^{(s)} - p_i^{(b)})$ and the level of production x_i. Therefore, on the accepted assumptions about the type of functions $t_i(x)$, the system of equations (2.9) and (2.10) has a single solution x_i^*, being the root of the equation

$$\left.\frac{d\ln t_i(x)}{d\ln x}\right|_{x=x_i^*} = 1. \tag{3.1}$$

The rate of production x_i^* corresponds to a difference in prices:

$$\Delta p_i \stackrel{\text{def}}{=} (p_i^{(s)} - p_i^{(b)}) = t_i'(x_i^*). \tag{3.2}$$

This equation allows immediately finding the price $p_{\mathcal{B}}$, established on the micromarket (node) $\mathcal{B}$. Really, considering equation (2.8) and summing up equation (3.2) along the path $\mathcal{P}_B$, leading from the root of the tree $\mathcal{N}$ to the given node $\mathcal{B}$, (Figure 1) we have the following expression for the price:

$$p_{\mathcal{B}} = \sum_{i\in\mathcal{P}_B} t_i'(x_i^*). \tag{3.3}$$

Selecting, in particular, the micromarket of the last level α; that is, the node linking the firms of the last level and the group of consumers $\boldsymbol{m}_\alpha$, we find the prices for goods of sort α:

$$f_\alpha \equiv p_\alpha = \sum_{i\in\mathcal{P}_\alpha} t_i'(x_i^*). \tag{3.4}$$

Thus we arrive at the following conclusion.

Suggestion 1. In the given model with the fixed structure $\mathcal{N}$ the prices $\{p_{\mathcal{B}}\}$, established on the micromarkets $\mathcal{N}$, are determined only by technological processes, and not by demand of the consumers.

This is a direct consequence of the perfect competition. In other words, with perfect competition prices reflect only the cost of the production of goods in spite of the fact that physically they are established as the result of supply and demand balance. It should be noted that this result is similar to that obtained in Leontjev models.

The full stream of production X_i, "flowing" through the branch i, reasonably depends on demand. Thereby, in the given model just the number of firms $\{n_i\}$ forming the branches $\{i\}$ of the market structure $\mathcal{N}$ are controlled by demand of consumers $\mathcal{M}$. In other words, the change of values $\{n_i\}$ is a manifestation of self-regulation of this type of market.

As is seen from equations (2.5) and (2.11), the total stream of goods $X_\alpha = X_{i_\alpha}$, made by firms of the last level, which form, for example, the branch α, is determined by

$$X_\alpha = S_\alpha(\boldsymbol{f}, \phi), \tag{3.5}$$

where $\boldsymbol{f} = \{f_\alpha\}$.

The conservation law of equation (2.4) of the material streams on the micromarkets for an arbitrary branch i allow us to write an expression:

$$X_i = \sum_{\alpha \in \mathcal{M}_i} X_\alpha \tag{3.6}$$

where $\mathcal{M}_i$ is a collection of groups of consumers, connected directly with the given branch i through the higher level branches (Figure 1)

$$n_i = \frac{1}{x_i^*} \sum_{\alpha \in \mathcal{M}_i} S_\alpha(f, \phi). \tag{3.7}$$

In conclusion to this part of the paper, it should be noted that equation (3.5) testifies to the separability of the given model of the market from the viewpoint of the absence of mutual influence of consumers on different types of commodity, as stated in Suggestion 2.

Suggestion 2. Any kind of change in demands of one group of consumers $\boldsymbol{m}_\alpha$ for the goods of sort α in no way affects the price and production of the other sort of goods $\alpha' \neq \alpha$.

4. Variational principle for the ideal market model

The set of values $\{x_i\}$, $\{n_i\}$, and $\{p_{\mathcal{B}}\}$ actually give all main conditions of the production and trade for the given commodity market. In other words, these are the parameters of order. The strategy of firms and the individual behavior of consumers (equations (2.9) through (2.11)) determine concrete values of these parameters. Nonetheless, the formulated model of the market contains an implicit image of one more parameter of order, namely, structure of the local micromarkets, which defines the order of trade interactions by firms. Really, with organization of the commodity market of a given type, the multitude of consumers $\mathcal{M}$ and the source of raw materials are initially fixed. Realization of technological processes and commercial networks is a concrete solution for the problem of supplying consumers with the required goods. Solution of the given problem can be ambiguous, and in this case, the market self-organization process has to select this realization of the network $\mathcal{N}$.

The next section is devoted to the analysis of the ideal market model from this point of view.

4.1 Global function of consumption utility

To begin we shall construct some function $\mathcal{U}(\boldsymbol{X})$ from $\boldsymbol{X} = \{X_\alpha\}$, which are extremum properties that define the function of demand of the consumers $\boldsymbol{S}(f, \phi) = \{S_\alpha(f, \phi)\}$. Let us suppose that the collection of consumers $\mathcal{M}$ consists of $\{\mathcal{M}_k\}$ individuals or various groups, considered as separated individuals

$$\mathcal{M} = \bigcup_k \mathcal{M}_k. \tag{4.1}$$

A full set of goods, necessary for the life activity of each of them, is $\{\gamma\} = \{\alpha\} \cup \{\beta\}$, where, as used earlier, $\{\alpha\}$ are goods delivered in the considered market and $\{\beta\}$ are goods made in other markets. In this part of the paper we shall suppose that appropriate prices $\boldsymbol{c} = (\boldsymbol{p}, \boldsymbol{q})$ for the goods $\{\gamma\}$ are given. Here, following the previous sections, we use the notations $\boldsymbol{c} = \{c_\gamma\}$, $\boldsymbol{p} = \{p_\alpha\}$, and $\boldsymbol{q} = \{q_\beta\}$.

Individuals $\{\mathcal{M}_k\}$ are independent. Each individual k, on gaining a set of goods $\boldsymbol{Z}^{(k)} = (\boldsymbol{X}^{(k)}, \boldsymbol{Y}^{(k)})$ (where $\boldsymbol{Z}^{(k)} = \{Z_\gamma^{(k)}\}$, $\boldsymbol{X}^{(k)} = \{X_\alpha^{(k)}\}$, and $\boldsymbol{Y}^{(k)} = \{Y_\beta^{(k)}\}$) maximizes their utility function $U_k(\boldsymbol{Z}^{(k)}) = U_k(\boldsymbol{X}^{(k)}, \boldsymbol{Y}^{(k)})$ proceeding from the budget constraint ϕ_k. In other words, each individual solves the problem

$$\max_{(\boldsymbol{X}^{(k)}, \boldsymbol{Y}^{(k)})} U_k(\boldsymbol{X}^{(k)}, \boldsymbol{Y}^{(k)}) \tag{4.2}$$

if $(\boldsymbol{X}^{(k)} \cdot \boldsymbol{p} + \boldsymbol{Y}^{(k)} \cdot \boldsymbol{q}) = \phi_k$.

The whole consumption level for goods of type α is given by

$$X_\alpha = \sum_k X_\alpha^{(k)}. \tag{4.3}$$

Equation (4.2) is reduced to a solution of the system of equations

$$\frac{\partial U_k(\boldsymbol{Z}^{(k)})}{\partial Z_\gamma^{(k)}} = \Lambda_k c_\gamma, \tag{4.4}$$

where $\Lambda_k > 0$ are the Lagrange multipliers and the index γ runs through the whole set $\{\gamma\}$. The solution of the given system of equations determines the demand function of consumer k

$$\boldsymbol{Z}^{(k)} = \boldsymbol{S}^{(k)}(\boldsymbol{c}\Lambda_k) = (\{S_\alpha^{(k)}(\boldsymbol{c}\Lambda_k)\}, \{S_\beta^{(k)}(\boldsymbol{c}\Lambda_k)\}), \tag{4.5}$$

which depends on both the price $\boldsymbol{c}$, and the value Λ_k. The latter itself becomes the function of the price $\boldsymbol{c}$ and of the budget constraint ϕ_k and satisfies the equation

$$\boldsymbol{S}^{(k)}[\boldsymbol{c}\Lambda_k(\boldsymbol{c}, \phi_k)] \cdot \boldsymbol{c} = \phi_k. \tag{4.6}$$

The utility function $U_k(\boldsymbol{Z}^{(k)})$ is defined with the accuracy of monotone transformation $U_k \to \mathcal{G}(U_k)$, that conjugates also with the transformation $\Lambda_k \to \Lambda_k/\mathcal{G}'(U_k)$. The final type of the function $S^k(c, \phi_k)$ remains constant. Equation (4.5) as the argument contains the product

$\boldsymbol{c}\Lambda_k(\boldsymbol{c}, \phi_k)$, where the second multiplier (in its turn, possibly, multiplied by the function of type $1/\mathcal{G}'(U_k)$) actually aggregates in itself information about the state of the market of commodities γ as a whole and of budget constraints of the consumer k. Evident dependence in this type of function $S_\gamma^{(k)}$ on the price $c_{\gamma'}$ with $\gamma \neq \gamma'$ reflects mutual influence of the goods of sorts γ and γ' for the process of their choice. In a considered model a set of goods α in its own right complies with some requirement of the person, while other goods $\{\beta\}$ correspond to other independent requirements. Therefore, it is natural to expect, that a choice of the set X^k will be defined only by prices $\{p\}$ for these goods and by common budget constraints, aggregated in some uniform multiplicand depending already on all prices $\boldsymbol{c}$ and constraints ϕ_k. Note that the given assumption is similar to the aggregated description of the utility function of various commodity groups [23–25], however it has differences (see section 4.2). Such behavior of consumers will be realized, if using some transformation $\mathcal{G}$, the utility function U_k can be reduced to such a type that for any α and β

$$\frac{\partial^2 U_k(\boldsymbol{X}^{(k)}, \boldsymbol{Y}^{(k)})}{\partial X_\alpha^{(k)} \partial Y_\beta^{(k)}} = 0. \tag{4.7}$$

Taking this equation into account we shall accept the following assumption.

Suggestion 3. We suppose, that for the goods $\{\alpha\}$ of the ideal mesoscopic market for any consumer k, there is a utility function $U_k(\boldsymbol{Z}^{(k)})$ representable as

$$U_k(\boldsymbol{X}^{(k)}, \boldsymbol{Y}^{(k)}) = u_k(\boldsymbol{X}^{(k)}) + v_k(\boldsymbol{Y}^{(k)}). \tag{4.8}$$

Then, by virtue of equations (4.4) through (4.6) the demand function for the goods $\boldsymbol{Z}^{(k)} = (\boldsymbol{X}^{(k)}, \boldsymbol{Y}^{(k)})$ can be rewritten as

$$\boldsymbol{S}^{(k)}(\boldsymbol{c}, \phi_k) = (\boldsymbol{S}_x^{(k)}(\boldsymbol{p}\Lambda_k), \boldsymbol{S}_y^{(k)}(\boldsymbol{q}\Lambda_k)), \tag{4.9}$$

where the functions $\boldsymbol{S}_x^{(k)}(\boldsymbol{p}\Lambda_k) = \{S_\alpha^{(k)}(\boldsymbol{p}\Lambda_k)\}$ and $\boldsymbol{S}_y^{(k)}(\boldsymbol{q}\Lambda_k) = \{S_\beta^{(k)}(\boldsymbol{q}\Lambda_k)\}$ satisfy the conditions

$$\left.\frac{\partial u_k(\boldsymbol{X}^{(k)})}{\partial X_\alpha^{(k)}}\right|_{\boldsymbol{X}^{(k)}=\boldsymbol{S}_x^{(k)}} = \Lambda_k p_\alpha, \tag{4.10a}$$

$$\left.\frac{\partial v_k(\boldsymbol{Y}^{(k)})}{\partial Y_\beta^{(k)}}\right|_{\boldsymbol{Y}^{(k)}=\boldsymbol{S}_y^{(k)}} = \Lambda_k q_\beta. \tag{4.10b}$$

And the function $\Lambda_k(\boldsymbol{p}, \boldsymbol{q}, \phi_k)$ is found from the equation

$$\boldsymbol{S}_x^{(k)}(\boldsymbol{p}\Lambda_k) \cdot \boldsymbol{p} + \boldsymbol{S}_y^{(k)}(\boldsymbol{q}\Lambda_k) \cdot \boldsymbol{q} = \phi_k. \tag{4.11}$$

The value of the Λ_k parameter is controlled by the state of the whole economic system. Therefore local changes of the price $\boldsymbol{p}$ on the mesoscopic market of goods $\{\alpha\}$ should vary the value Λ_k. Let us demonstrate this with an example of small fluctuations of the prices $\delta \boldsymbol{c} = (\delta \boldsymbol{p}, \delta \boldsymbol{q})$. Varying equation (4.11) under condition $\phi_k = \text{const}$, we get

$$\sum_\alpha (1 - \epsilon_\alpha^{(k)}) S_\alpha^{(k)} \delta p_\alpha + \sum_\beta (1 - \epsilon_\beta^{(k)}) S_\beta^{(k)} \delta q_\beta = \left(\sum_\alpha \epsilon_\alpha^{(k)} S_\alpha^{(k)} p_\alpha + \sum_\beta \epsilon_\beta^{(k)} S_\beta^{(k)} p_\beta \right) \frac{\delta \Lambda_k}{\Lambda_k}, \tag{4.12}$$

where

$$\epsilon_\gamma = - \left. \frac{\partial S_\gamma^{(k)}}{\partial c_\gamma} \right|_{\Lambda_k = \text{const.}} \tag{4.13}$$

From this it immediately follows that, when the number N of independent mesomarkets is rather great and the fluctuation of the prices is not so significant that they do not change the state of the consumer, the mean value $\langle \delta\Lambda_k / \Lambda_k \rangle \to 0$, at $N \to \infty$ for $1/\sqrt{N}$. This leads us to the following.

Suggestion 4. At local changes in the market of goods $\{\alpha\}$ the value Λ_k can be considered as some constant macroscopic variable, describing the common state of the consumer k.

Taking into account equation (4.10a), from this assertion we also receive the following.

Suggestion 5. At local changes in the market of the goods $\{\alpha\}$ the strategy of the consumer k can be described as finding a maxima of the function

$$\max_{\boldsymbol{X}^{(k)}} \left[\frac{1}{\Lambda_k} u^{(k)}(\boldsymbol{X}^{(k)}) - \boldsymbol{p} \cdot \boldsymbol{X}^{(k)} \right].$$

In this case the function

$$\frac{1}{\Lambda_k} u^{(k)}(\boldsymbol{X}^{(k)})$$

can be considered as a utility function of the good's required $\boldsymbol{X}^{(k)}$, depending also on some macroscopic parameters. This utility function not only defines the preference of a choice, but also measures this preference in monetary units.

Let us now consider the result of collective behavior of individuals $\{\mathcal{M}_k\}$. A full consumption level X_α of the goods of type α by virtue of equation (4.3) is

$$X_\alpha = \sum_k S_\alpha^{(k)}(p\Lambda_k). \tag{4.14}$$

Let us define the function

$$\mathcal{U}(p, \{\Lambda_k\}) \stackrel{\text{def}}{=} \sum_k \frac{1}{\Lambda_k} u_k(S_x^{(k)}(p\Lambda_k)). \tag{4.15}$$

If in equations (4.14) and (4.15) we consider values of p as some formal set of parameters, then equation (4.14) can be inverted, resulting in some dependence and specifying the vector p as the function of a vector X. The latter allows us to consider equation (4.15) as the function $\mathcal{U}(X|\{\Lambda_k\})$ of arguments X. By virtue of equations (4.10a), (4.10b), (4.14), and (4.15) we get

$$\begin{aligned}
\frac{\partial \mathcal{U}}{\partial X_\alpha} &= \sum \frac{\partial \mathcal{U}}{\partial p_{\alpha'}} \frac{\partial p_{\alpha'}}{\partial X_\alpha} = \sum_{k,\alpha',\alpha''} \frac{\partial \mathcal{U}}{\Lambda_k \partial S_{\alpha''}} \frac{\partial S_{\alpha''}}{\partial p_{\alpha'}} \frac{\partial p_{\alpha'}}{\partial X_\alpha} \\
&= \sum_{k,\alpha',\alpha''} p_{\alpha''} \frac{\partial S_{\alpha''}}{\partial p_{\alpha'}} \frac{\partial p_{\alpha'}}{\partial X_\alpha} = \sum_{\alpha',\alpha''} p_{\alpha''} \frac{\partial p_{\alpha'}}{\partial X_\alpha} \frac{\partial}{\partial p_{\alpha'}} \left(\sum_k S_{\alpha''}^{(k)} \right) \\
&= \sum_{\alpha',\alpha''} p_{\alpha''} \frac{\partial p_{\alpha'}}{\partial X_\alpha} \frac{\partial X_{\alpha''}}{\partial p_{\alpha'}} = \sum_{\alpha''} p_{\alpha''} \delta_{\alpha,\alpha''} = p_\alpha.
\end{aligned}$$

Thus, collective behavior of the consumers in the market of goods $\{\alpha\}$ can be presented as optimization of the global utility function of consumption $\mathcal{U}(p, \{\Lambda_k\})$, which depends also on some macroscopic parameters $\{\Lambda_k\}$ and is measured in monetary units, as stated in Suggestion 6.

Suggestion 6. At the local changes in the market of goods $\{\alpha\}$ the strategy of cooperative behavior of the consumers can be described as finding a maximum of the following function:

$$\max_X \left[\mathcal{U}(X|\{\Lambda\}) - p \cdot X \right]. \tag{4.16}$$

The common function of demand $S(p|\{\Lambda\})$ of the goods $\{\alpha\}$ satisfies the equation

$$\left. \frac{\partial \mathcal{U}(X|\{\Lambda\})}{\partial X_\alpha} \right|_{X=S} = p_\alpha. \tag{4.17}$$

Measurability of the global utility function in monetary units allows comparing it with production costs of the goods in the given market and to describe its operation as a whole in terms of the variational problem. Section 4.2 is devoted to consideration of this problem.

4.2 Functional of the market efficiency

We now use the known description of the market system in terms of the global efficiency criterion. Let us set a functional of efficiency $\mathcal{D}$ of the market goods $\{\alpha\}$ to be:

$$\mathcal{D} = \mathcal{U}(\{X_\alpha\}|\{\Lambda\}) - \sum_i n_i t_i(x_i). \tag{4.18}$$

Here indexes α and i run over all units of sets $\mathcal{M}$ and $\mathcal{N}$, respectively. The arguments of this functional are $\{n_i\}$, $\{x_i\}$, and X_α. The structure of the network of supply $\mathcal{N}$ gives the conservation laws of material streams, and, hence, the interrelation between values $\{n_i\}$, $\{x_i\}$, and X_α. The first term in the right part of equation (4.18) is a global utility of the good's consumption at level X_α, and the second term is the production costs.

Suggestion 7. The laws of functioning of the ideal mesomarket can be rewritten as equations for the extremals of the functional (equation 4.18)), considering the network $\mathcal{N}$, as given.

Indeed, variables $\{n_i\}$, $\{x_i\}$, and X_α are not independent, but connected by equations (2.4) and (2.5). Let us take advantage of the Lagrange method, which in this case will correspond to the prices $p_{\mathcal{B}}$ on the micromarket at node $\mathcal{B}$

$$\mathcal{D}^P = \mathcal{D} + \sum_\alpha p_\alpha (n_{i_\alpha} x_{i_\alpha} - X_\alpha) + \sum_{\mathcal{B}} p_{\mathcal{B}} \left(n_j x_j \Big|_{j=i^{\mathcal{B}}_{\text{in}}} - \sum_{j \in \{i^{\mathcal{B}}_{\text{out}}\}} n_j x_j \right). \tag{4.19}$$

Here index $\mathcal{B}$ runs over all of the nodes (micromarkets) of network $\mathcal{N}$ except nodes that immediately connect firms and consumers, and $\{p_\alpha\}$ and $\{p_{\mathcal{B}}\}$ are Lagrange multipliers (prices) attributed by the network $\mathcal{N}$. For equation (4.19), the arguments $\{n_i\}$, $\{x_i\}$, $\{X_\alpha\}$, $\{p_\alpha\}$, and $\{p_{\mathcal{B}}\}$ are supposed to be independent. It is well known that in this case the extremals of equations (4.18) and (4.19) coincide with each other. Equation (4.19) can also be rewritten as:

$$\mathcal{D}^P = \mathcal{U}(\{X_\alpha\}|\{\Lambda\}) - \sum_\alpha p_\alpha X_\alpha + \sum_i n_i \pi_i (x_i\ ,\ p_i^{(s)} - p_i^{(b)}), \tag{4.20}$$

where the function $\pi_i(\dots\ ,\ \dots)$ is determined by equation (2.6). Then differentiating equation (4.19) with respect to variables $\{p_\alpha\}$ and $\{p_{\mathcal{B}}\}$, and equation (4.20) with respect to variables $\{n_i\}$, $\{x_i\}$, and $\{X_\alpha\}$ we get the equations of the model formulated in section 2 where the Lagrange multipliers act as prices, and the demand function $\boldsymbol{X} = \boldsymbol{S}(\boldsymbol{p}|\{\Lambda\})$ satisfies equation (4.17).

We now consider the condition of extremality of a functional (equation (4.19) or (4.20)) concerning all the parameters of order, including the parameters describing the production and trade system. As follows from the result of section 2, the expression of the functional $\mathcal{D}_*^P$, received after maximization over all arguments, except the structure of network $\mathcal{N}$, is determined by

$$\mathcal{D}_*^P = \mathcal{U}(\{X_\alpha\}|\{\Lambda\}) - \sum_\alpha f_\alpha X_\alpha. \tag{4.21}$$

Values of $\{f_\alpha\}$, and therefore values of $\{X_\alpha\}$, are given by the production processes and geometry of network $\mathcal{N}$. If their changes are such that variations $\{\delta f_\alpha\}$ and $\{\delta X_\alpha\}$ are small, then, by virtue of equations (4.17) and (4.21) the variation of functional $\mathcal{D}_*^P$ is

$$\delta\mathcal{D}_*^P \approx -\sum_\alpha X_\alpha \delta f_\alpha. \tag{4.22}$$

From this equation the comparison criterion of the function efficiency of ideal markets follows, identical for their nomenclature of the goods of consumption $\{\alpha\}$, which differ by organization of the goods production as stated in Suggestion 8.

Suggestion 8. Increasing the functional efficiency value of an ideal mesomarket, owing to the changes of its production and trade structure, corresponds to decreasing, on average, retail prices for the goods of consumption.

The given result allows, at least on an intuitive level, to hypothesize that for an ideal mesomarket the functional of efficiency will have the extremum including market structures. Really, it seems to be quite reasonable, that due to the self-organization process alone, those structures will survive which offer the goods with lower prices to the consumers.

The second result, following from equation (4.22), is that for the ideal market plenty of realizations exist for production and trade structure. These are identical from the viewpoint of extremality of functional efficiency as stated in Suggestion 9.

Suggestion 9. The ideal market is degenerated concerning the network of supply $\mathcal{N}$.

Really, according to equation (3.4) the number of parameters specifying the value of $\{f_\alpha\}$ far exceeds the number of these values in consequence of hierarchical organization of the network $\mathcal{N}$. That is why the set of values $\{f_\alpha\}$ can be obtained by a large number of methods. In particular, two realizations of the network $\mathcal{N}$ will result in the single value $\mathcal{D}_*^P$, if they contain the same number of hierarchy levels. Firms belonging to one level are characterized by the same function of the costs $t_i(x_i)$, and networks differ only by organization of intermediate

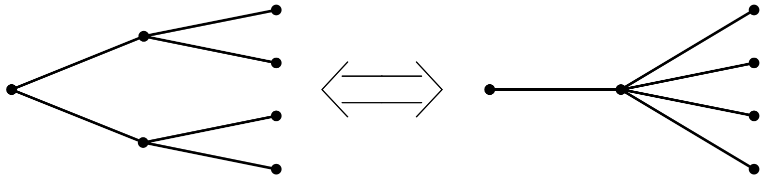

Figure 3. Equivalent fragments of the production-trade network.

bifurcations (Figure 3). In other words, market structures with identical technology, which have various organizations of the micromarket network, are equivalent in the ideal market.

5. Self-organization of the structure in a simple market model

In the previous sections we analyzed characteristics of a market system that are caused by interaction between a great number of firms in the case that the structure of the market is given beforehand. So, we have considered that the time of network evolution is a significant parameter in comparing the process of trade relations between firms involved in selling and buying interactions. Of course, this raises the following questions. What principles are responsible for network organization? Why do the firms organize their market network in a form similar to the tree form? What cues are available for firms to use in organizing a market network? Practically all of these questions are still without answer. In these cases the assumption accepted in the article can give rise to some objections. For example, accepting a tree form of the market structure seems to be hard-wired and is not proved in the framework of this paper. In order to reject such an objection and justify the proposed model a little, let us consider the small market community, the properties of their agents can change during the interaction process.

In the framework of this approach we can show that hierarchical organization of the firms can lead to substantial decreases in the prices at the retail outlets. In this case let us confine our consideration to the regular market system presented in Figure 4(a). This system consists of $(N_x * N_y)$ identical firms (the domains have a square form and the squares represent the size of each firm) connected to each other in the vertical direction by material flow (cursors indicate material flow in the system). In this model, firms are interacting on the horizontal boundaries and each horizontal line corresponds to some given price p_i. The profit of each firm; for example, of firm k at level i, can be represented as

$$\pi_i^k = (p_i - p_{i-1})x_i - t_i^k(x_i), \tag{5.1}$$

where i corresponds to the hierarchy level and runs from 1 to N_k. (In

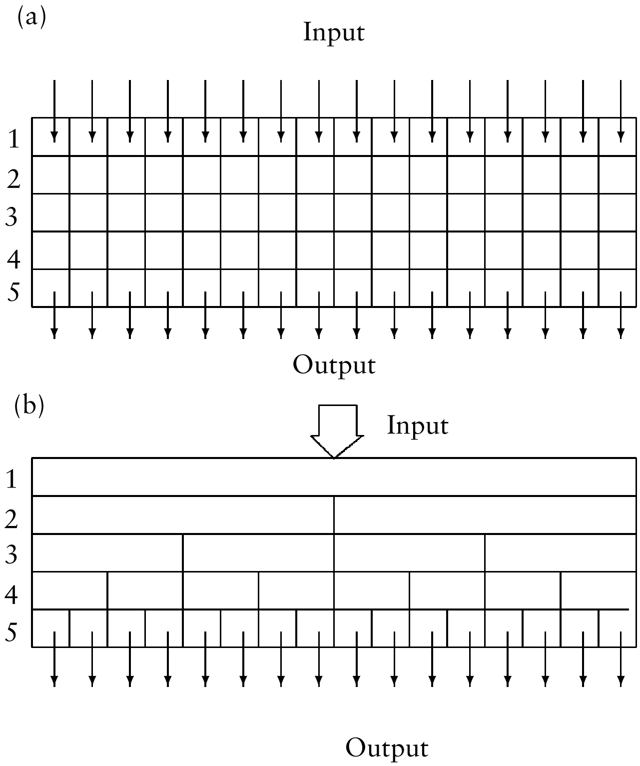

Figure 4. Representation of the sizes of firms interacting with each other by selling and buying processes at the intial stage (a) and after union of their efforts into more powerful firms (b).

Figure 4(a) $N_k = 5$.) In the initial stages there are no interactions in the lateral direction and each firm optimizes its own profit:

$$\Delta p_i = (p_i - p_{i-1}) = \frac{dt_i^k(x_i)}{dx_i}. \tag{5.2}$$

So, according to equations (5.1) and (5.2) we get

$$\pi_i^k = \left. \frac{dt_i(x_i)}{d\ln x_i} - t_i^k(x_i) \right|_{x=x_i^*}, \tag{5.3}$$

the solution of which will be $x_i = x_i^*$.

Let us suppose that the firms taking part in the production process have the possibility of joining their efforts in the production activity (the size of the firms will increase after incorporating several firms n) and, depending on the hierarchy level, the total production cost will decrease strongly if the firm is located closer to the input of the system. Such a property will lead to hierarchical organization of the production network and to decreasing the prices of retail trade.

Let us consider a simple example. For the sake of simplicity we assume that the cost of the firm on level i has the form

$$t_i^k(x) = f^k(n(i), i) x_i^{1+\epsilon} = \frac{1}{2^{2N_y - 2i} + 1} \left[\frac{2^{2N_y - 2i}}{n(i)} + n(i) \right] x_i^{1+\epsilon} \tag{5.4}$$

and $x_i^* = 1$. Here $n(i)$ is the number of firms that can join together to make up a single firm. Each firm of level i tries to optimize its own profit and in this case equations (5.2) and (5.3) should be fulfilled:

$$p_i - p_{i-1} = (1 + \epsilon) f(n(i), j). \tag{5.5}$$

The optimization of firm size

$$\frac{\partial f^k(n(i), i)}{\partial n(i)} = 0 \tag{5.6}$$

gives the solution $n_i = 2^{N_y - i}$ and then we get the result

$$p_i - p_{i-1} = (1 + \epsilon) \frac{2^{N_y + 1 - i}}{2^{2(N_y - i)} + 1}$$

where, as done previously, we can consider that $p_0 = 0$.

The typical example of such a structure is presented in Figure 4(b) for $N_x = 16$, $N_y = 5$, and $i = 1 \ldots 5$. The recurrent relationship (equation (5.5)) determines the prices during the selling and buying process between firms of different hierarchy levels. The distribution of prices along the axis directed with material flow is presented in Figure 5. In this case

$$p(i) = \sum_{m=1}^{i} p_m.$$

As already noted, the prices in such a model are determined by the total costs and not by the demand of the consumers. From this point of view such reorganization of the market structure will survive and this consideration gives indirect confirmation of the fact that the real market structure should be hierarchically organized.

Suggestion 10. The hierarchical organization of the market should survive due to providing goods for the ultimate consumers at lower prices.

If we suppose that the market structure presented in Figure 4 is three-dimensional and in the third direction we can denote the number of firms taking part in the process, it is easy to fulfill any demand according to

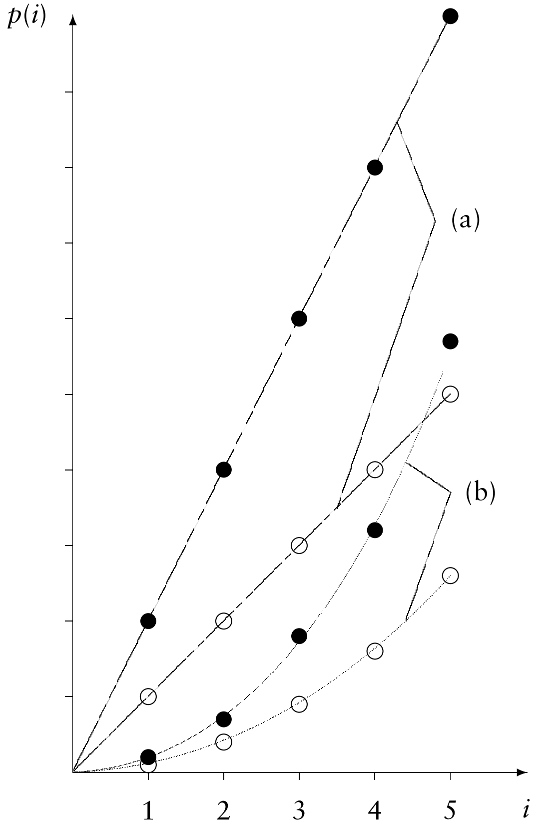

Figure 5. Plot of the prices p_i *versus* level i before self-organization (a) and after it (b).

the demand function $X_\alpha = S_\alpha(f, \phi)$. The number of firms participating in the production of goods α should be determined by a formula similar to equation (3.7).

6. Remarks

In the present paper we tried to demonstrate that the problem of self-regulation of market systems can be solved through the appearance of production and trade hierarchical structures. The classical economic theory and Walras's auctioneer suggestion assumes the availability of complete information about the economic state of a market. However, it is physically impossible due to the large number of participants in the market system. This limitation can be removed, if the market participants restrict their contacts to a small number of other participants, to form a connected network of micromarkets. In doing so, however, another problem occurs.

Each market participant, having contact with a limited number of other participants, receives a small share of information, on which to orient their planning and select the strategy for their activity. At first glance such limited information (usually as the prices for some production) cannot immediately tell them what and how much to produce. Really, with the availability of the developed system of micromarkets, the result of their work is usually an intermediate product (semiproducts). And the market participant is involved in a long production chain, connecting the production of raw materials and the final goods offered to the consumers. And the prices are established during the process of mutual convention among a small number of people.

The formulated model of the ideal market with hierarchical structures of the micromarkets demonstrated the existence of self-regulation processes in such systems. It allows the market participants to react adequately to changes in consumer demand based only on a value of the prices in the two appropriate micromarkets, connected together by the participants. The conservation laws of material flows and streams of money on the micromarkets are the basis for this self-regulation. Owing to the fact that the stream of money "flows" in a backward direction (from the consumers to the firms obtaining raw materials) in contrast to the stream of materials, the smaller-sized streams of money are all going into larger-sized streams. Integration in the money stream provides that the prices aggregate information about the state and demand of consumers on larger and larger scales. The latter provides the self-processing of information, latent in the relation of prices for semiproducts of different hierarchical levels.

In summary we shall note the following. First, in an accepted model it was considered that production and trade network $\mathcal{N}$ has the form of a tree. For a market with perfect competition such an assumption is quite justified. Really, as follows from equations (2.9) and (2.10), the conclusion about the independence between the Δp_i difference in prices and demand is not connected with the geometry of network $\mathcal{N}$ and is the sequence of perfect competition. Let us assume now that to some micromarket (node of the network $\mathcal{N}$) there are two incoming branches (technological ways), connecting it with firms obtaining raw material (root of the network $\mathcal{N}$). Then the prices for production, proposed on the given micromarket for these two routes of production, should be different (except by accidental agreement). Therefore one of these routes of production has to disappear. And the network $\mathcal{N}$ has to become of a tree form (graph without cycles).

Second, for the ideal market the prices, the number of firms participating in production and trade, and also the streams of goods appear to be defined values. The market deviation from ideality; for example, when the competition is not perfect, and the firms accomplishing the same production are not identical, will produce changes in these values. However, if these deviations are not too large, it is possible to expect, that the changes in the prices and in the commodity production level will also be insignificant. The situation varies significantly concerning market structures. Owing to a degeneracy of the market norm over production and trade structures, small deviations from perfection can result in self-organization "surviving" the structures of various forms. Upon it, small variations of parameters can stipulate significant changes in the market structures. It is likely that this effect was observed in [14]. It should be noted that plenty of realizations of production and trade structures, which are identical from the viewpoint of the ideal market, point to the possibility of existence of any macroscopic parameters of order, characterizing an architecture of these structures in aggregated form.

In summary to the given part of the paper we note that the assumption about the possibilities of reducing the common utility function (equation (4.7)) is similar to an aggregated description of utility functions for the goods belonging to different groups [23–25]. The main difference is, that in the proposed approach, budget constraints are not imposed individually on the goods of each mesomarket and are considered as common constraints for all sorts of goods. As a result, in the given approach, individual indexes of the prices are absent. The description of local behavior of consumers in a mesomarket (equation (4.16)) in its form is different from the global optimality condition (equation (4.2)). Besides, the utility function in the form of equation (4.7) is uniquely defined, and thus does not allow beforehand the homogeneity of the first degree.

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References

[1] G. Nikolis and I. Prigogin, *Self-organization in Nonequilibrum Systems: From Dissipative Structures to Order through Fluctuations* (J. Wiley and Sons, New York, 1977).

[2] H. Haken, *Synergetics: An Introduction*, *Springer Series in Synergetics, Volume 1* (Springer-Verlag, 1988).

[3] H. Haken, *Information and Self-organizing: A Macroscopic Approach to Complex Systems* (Springer-Verlag, 1988).

[4] Yu. L. Klimontovich, *Statistical Theory of Open Systems* (Janus, Moscow, 1995).

[5] B. S. Kerner and V. V. Osipov, *Autosolitons: A New Approach to the Problem of Self-organization and Turbulence* (Kluwer, 1994).

[6] I. A. Lubashevsky and V. V. Gafiychuk, "The Projection Dynamics of Highly Dissipative Systems," *Physical Review E*, **50** (1) (1994) 171–181.

[7] V. V. Gafiychuk and I. A. Lubashevsky, "Variational Representation of the Projection Dynamics and Random Motion of Highly Dissipative Systems," *Journal of Mathematical Physics*, **36** (10) (1995) 5735–5752.

[8] F. A. Hayek, *The Fatal Conceit: The Errors of Socialism* (The University of Chicago Press, 1988).

[9] W. Weidlich, "Physics and Social Science: The Approach of Synergetics," *Phys. Rep.*, **204** (1) (1991) 1–163.

[10] R. C. Baesemann, "The Formation of Small Market Places in a Competitive Economic Process: The Dynamics of Agglomeration," *Econometrica*, **45** (2) (1977) 361–376.

[11] A. Rubinstein and A. Wolinsky, "Equilibrium in a Market with Sequential Bargaining," *Econometrica*, **53** (5) (1985) 1133–1150.

[12] A. Wolinsky, "Information Revelation in a Market with Pairwise Meetings," *Econometrica*, **58** (1) 1990 1–24.

[13] C. L. van Raalte and R. P. Gilles, *Endogeneous Formation of Trade Center: An Evolutionary Approach* (Department of Economics and Center, Tilburg University, 1994).

[14] S. Guriev and M. Shahova, "Model of Self-organizing of Commercial Networks in Economy with Incomplete Infrastructure," in *Mathematical Models of the Analysis of Data and Management in Microeconomics* (Computer Center of Academy of Sciences of Russian Federation, Moscow, 1995).

[15] G. Mchedeshvili, *Microcirculation of Blood: Common Features of Regulation and Violations* (Science, Leningrad, 1989).

[16] I. A. Lubashevsky and V. V. Gafiychuk, "A Simple Model of Self-regulation in Large Natural Systems," *Journal of Environmental Systems*, **23** (3) (1995) 281–289.

[17] V. V. Gafiychuk and I. A. Lubashevsky, "On Hierarchical Structures Arising Spontaneously in Markets with Perfect Competition," *Journal of Environmental Systems*, **25** (2) (1996–1997) 159–166.

[18] I. A. Lubashevsky and V. V. Gafiychuk "Cooperative Mechanism of Self-regulation in Hierarchical Living Systems," *SIAM J. Appl. Math.*, **60** (2) (1999) 633–663; e-print, http://xxx.lanl.gov/abs/adap-org/9808003 (1998).

[19] I. G. Pospelov, "Dynamic Model of the Market," *Economy and Math. Methods*, **24** (3) (1988).

[20] I. G. Pospelov, "Optimal Strategy of Behaviour in the Dynamic Market Models," *Automation and Telemechanics*, **2** (1989) 113–123.

[21] B. R. Binger and E. Hoffman, *Microeconomics with Calculus* (Harper Collins Publishers, 1988).

[22] F. M. Scherer and D. Ross, *Industrial Market Structure and Economic Performance* (Houghton Mifflin Company, Boston, 1990).

[23] A. A. Shananin, "A Condition of Integrability in the Problem of Finite Goods," *Rep. AS USSR*, **294** (3) (1987) 553–555.

[24] S. D. Vratenkov and A. A. Shananin, *Analysis of a Structure of Consumer Demand with Economic Indexes* (Computer Center AS USSR, Moscow, 1991).

[25] A. A. Shananin, "A Nonparametric Method of the Analysis of the Consumer's Demand Structure," *Math. Simulation*, **5** (9) (1993) 3–17.