Glider Collisions in Hybrid Cellular Automaton Rules

Fangyue Chen
Bo Chen
Junbiao Guan

Department of Mathematics, School of Science
Hangzhou Dianzi University
Hangzhou, Zhejiang, China

Weifeng Jin

School of Science, Shanghai University
Shanghai, China

Elementary cellular automaton (ECA) rules 9 and 74, members of Chua's Bernoulli shift rules and Wolfram's class 2, can generate a host of gliders and complicated glider collisions by introducing the hybrid mechanism. These gliders and collisions are more plentiful than those generated by ECA rule 110. This paper presents a discussion of the symbolic dynamics of the gliders in hybrid cellular automaton (HCA) rules 9 and 74. Moreover, with regard to HCA rules that are composed of more than a couple of ECA rules, three other HCA rules endowed with gliders and glider collisions are discussed.

1. Introduction

Cellular automata (CAs) are a class of spatially and temporally discrete dynamical systems characterized by local interactions [1]. As part of a significant renewal of interest in CAs, Stephen Wolfram introduced spatiotemporal representations of one-dimensional CAs and informally classified them into four classes by using dynamical concepts such as periodicity, stability, chaos, and complexity [2–4]. Based on previous work, L. O. Chua et al. provided a nonlinear dynamics perspective to Wolfram's empirical observations and grouped elementary cellular automata (ECAs) into six classes depending on the quantitative analysis of the orbits [5–9]. These six classes are established as period-1, period-2, period-3, Bernoulli σ_τ-shift, complex Bernoulli-shift, and hyper Bernoulli-shift rules. It is worth mentioning that some of their work is consistent with previous related studies. Specially, a kind of comparison of Wolfram's and Chua's classifications is explored in [10].

Among numerous local rules of CAs, the ones exhibiting plentiful gliders and glider guns have received special attention. They display

https://doi.org/10.25088/ComplexSystems.24.3.185

complex behaviors via the interactions of gliders from random initial conditions and have the potential to emulate universal Turing machines. Gliders in one-dimensional CAs are localized structures of nonquiescent and non-ether patterns (ether represents a periodic background) translating along the automaton's lattice. In 1991, Boccara et al. produced a list of gliders and discussed the existence of glider guns [11]. Subsequently, Hanson and Crutchfield applied finite-state machine-language representation to study defect dynamics in one-dimensional CAs and derived motion equations of filtered gliders [12]. Martin designed an algebraic group to represent collisions between basic gliders [13]. Shortly thereafter, Wolfram investigated glider collisions with long periods and derived several filters for detecting gliders and defects [14]. There are many more results concerning gliders; see [15–19] and references therein.

Notably, elementary cellular automaton (ECA) rule 110 in Wolfram's system of identification has captured special attention, due to the existence of a great variety of gliders in its evolution space. It is worth mentioning that Cook proved that ECA rule 110 is universal via simulating a cyclic tag system [20]. In their study of unconventional computation, Martínez et al. highlighted the dynamical characteristics of gliders in ECA rules 110 and 54 [21–24]. However, as gliders in ECA rule 54 are less complicated than those in ECA rule 110, so far no literature has proven that rule 54 is universal.

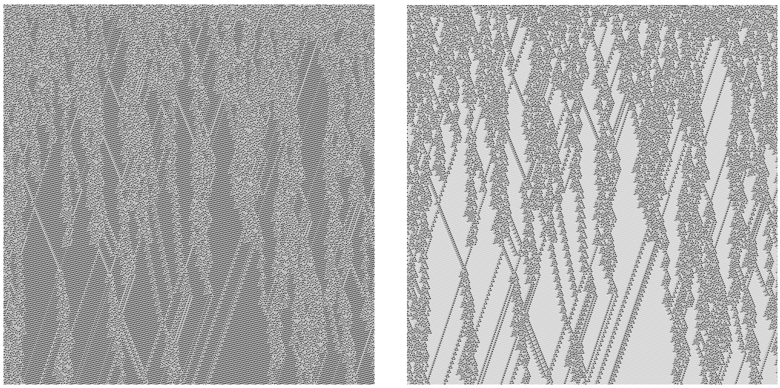
In the case of one-dimensional CAs, when the evolution of every single cell is only dependent on the unique global function, the CA is called uniform; otherwise, it is called a hybrid, that is, a hybrid cellular automaton (HCA). Denoted by HCA(N, M), the HCA rule is composed of ECA rules N and M. The HCA rule is specified to obey ECA rule N at odd sites of the cell array and rule M at even sites of the cell array [25, 26]. Although the simple rules of an HCA act on the same square tile structures, the evolution of an HCA may exhibit plentiful dynamical behaviors through local interactions. Several spatiotemporal patterns of HCA rules are presented in Appendix A. Based on a large amount of computer simulations and empirical observations, we find that HCA(9, 74) can generate rich gliders that are more complicated than those in ECA rules 110 or 54.

2. Glider Collisions in HCA(9, 74)

In order to gain further insights into the rich dynamics generated by HCA(9, 74), we present a systematic analysis of computational glider behaviors in this section. By designing a single filter (setting a different gray value of ether), the gliders are easy to distinguish from each

https://doi.org/10.25088/ComplexSystems.24.3.185

other. An example of a spatiotemporal pattern generated by HCA(9, 74) from disordered initial states is illustrated in Figure 1. It is noted that all patterns in this paper are studied under the periodic boundary condition.



(a) (b)

Figure 1. (a) Spatiotemporal pattern of HCA(9, 74), where white pixels are cells with state 0, and black pixels are cells with state 1. (b) The filter is applied to (a).

A pair of types—original glider and composite glider—are classified and coded depending on different shift velocities and volumes. Each glider is evolved under the uniform background of ether. As the background is a regular patchwork of the minimum component element, we call this element an *ether unit*. In HCA(9, 74), the ether unit is

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & & \end{pmatrix}.$$

For convenience, the shift feature of the ether unit is displayed in Table 1. There are only two cells "01" in the sixth row of the ether unit. In the process of ether matching, the remaining two cells are filled by the first row of the adjacent ether unit. The whole background can be obtained by splicing this ether unit repeatedly without any gap or overlap. Because of the hybrid mechanism, it is necessary to note that the first column of the ether unit begins with the odd sites.

https://doi.org/10.25088/ComplexSystems.24.3.185

OGs	Velocity	With	DCGs	Velocity	With
Ether Unit	−4/6	2–4			
a	−2/6	6	*ab*	−4/12	12–14
b	−2/6	8	*ac*	−4/12	12–16
c	−4/12	6–10	*ad*	−4/12	14–18
d	−4/12	8–14	*ae*	−4/12	14–18
e	−4/12	8–14	*ba*	−4/12	12–14
f	4/10	10–16	*bd*	−4/12	16–20
g	2/5	10–12	*be*	−4/12	16–20
g_2	2/5	12–14	*ca*	−4/12	14–16
g_3	2/5	8–10	*cb*	−4/12	16–18
h	−4/12	14–18	*da*	−4/12	18–20
i	4/21	10–20	*db*	−4/12	20–22
i_2	4/21	10–20	*dc*	−4/12	18–24
j	10/47	16–30	*de*	−4/12	20–24
k	2/16	18–24	*ea*	−4/12	16–18
k_2	2/16	24–32	*eb*	−4/12	18–20
l	2/49	8–22	*ec*	−4/12	18–22
m	10/102	18–42	*ed*	−4/12	20–26
n	0/55	20–40	*fg*	4/10	22–26
o	−2/17	14–22	*g2g*	2/5	24
p	2/27	16–24	*gf*	4/10	24–26
p_2	6/81	36–52	*hb*	−4/12	26
q	4/46	36–46	*hc*	−4/12	24–30

Table 1. Characterizations of OGs and DCGs.

2.1 Classification and Codings of Gliders

Besides the ether unit, the shift configurations without arbitrary combination of diverse gliders are called *original gliders* (OGs). The OGs are the independent gliders that cannot be decomposed into some smaller ones. However, if two different OGs have the same shift velocity, they will probably be assembled together, and there are no ether units between the two. These shift configurations are explicitly considered as new gliders—*double composite gliders* (DCGs). In a survey of spatiotemporal patterns of HCA(9, 74), 22 OGs and 22 DCGs that have occurred frequently are recorded as different codings. Table 1 presents their properties and Figure 2 illustrates their spatiotemporal patterns. For Table 1, columns 1 and 4 show the labels of gliders, columns 2 and 5 show the shift features of gliders, and columns 3 and 6 indicate maximal and minimal sizes of gliders. For any given glider, the velocity is calculated from its shift number divided by its period. The plus sign denotes that the glider shifts to the right and the minus sign denotes that the glider shifts to the left.

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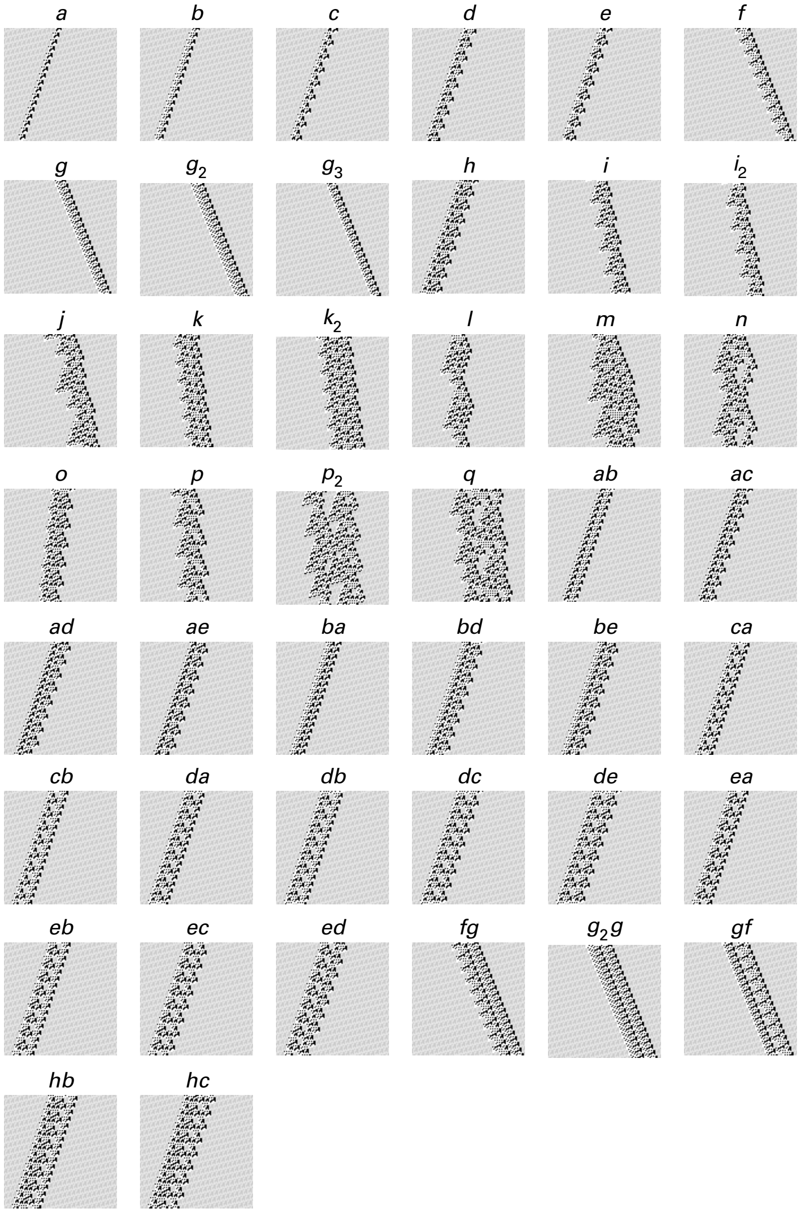

Figure 2. The spatiotemporal patterns of gliders.

However, if a glider is composed of two or more of the same OGs, it is not regarded as a new composite glider. As an example, the gliders *aa*, *bbb*, *cc*, *gg*, *ff*, *jj*, … are recognized as simple superposition of the original gliders *a*, *b*, *c*, *g*, *f*, *j*, …. Likewise, if OGs and DCGs have the same shift velocity, they also may be combined together without being divided by an ether unit. These shift configurations are regarded as new gliders—*multiple composite gliders* (MCGs). More specifically, the properties of several MCGs are listed in Table 2, where column 1 shows the labels of the gliders and column 2 shows the composition formulas of different gliders. Since the evolution of HCA(9, 74) is of high complexity and the collisions of different gliders can spark off new gliders, not all gliders are enumerated here.

MCGs	Composition Formulas
abb	$ab + bb$
aeb	$ae + eb$
aebbb	$aeb + bbb$
bae	$ba + ae$
bad	$ba + ad$
badb	$bad + db$
beb	$be + eb$
bhbb	$bh + hbb$
cae	$ca + ae$
daa	$da + aa$
eaa	$ea + aa$
eab	$ea + ab$
eaac	$eaa + ac$
hbb	$hb + bb$

Table 2. Characterizations of MCGs.

2.2 Collisions between Gliders

In this subsection, for the purpose of exploiting the mathematical definition of collisions between two OGs in HCA(9, 74), we perform an analysis of the collision formula via a one-dimensional string and introduce the *ether factor* and *glider factor*, which are denoted by E and $[\xi]$, respectively. Here, ξ is a concrete glider label. First, the ether factor is

$$E = (0, 0, 1, 1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1),$$

which means arranging the cell states of all the rows of the ether unit in an orderly fashion. The ether factor E is determined by the shift characteristic and width of the ether unit. Then, the glider factor $[\xi]$ is introduced, which stands for a row of cell states of the glider ξ under the background of the ether. If the glider ξ has the period M, it has M different glider factors. For example, as the glider a is endowed with the period 6 according to Table 1, there are six glider factors, and each one can produce the entire glider a after the evolution of six time steps.

For a pair of gliders ξ and η with different velocities, there are two scenarios: when particular kinds of glider factors $[\xi]$ and $[\eta]$ are chosen, the collision results may be diverse, as there are different numbers of ether factor E between $[\xi]$ and $[\eta]$. When different kinds of glider factors $[\xi]'$ and $[\eta]'$ are chosen for the same gliders, the collision results are possibly not the same as the first results.

For convenience, we only select a fixed glider factor for each OG (the strings are presented in Appendix B), and it has no effect on the observations of complex collisions concerning two OGs in this subsection. Thus, the collision results depend on the number of E between two OGs. Let Q denote this period's value. Each collision formula is introduced as $[\xi] \cup (QN+I)\ E \cup [\eta] \to \{\text{the result}\}$, where $QN+I$ is the number of ether factor E between two gliders, N is a natural number, and $I = 1, 2, \ldots Q$. For example, for collision $f \leftrightarrow e$, we have $Q = 2$, and the collision formulas are $[f] \cup (2N+1)E \cup [e] \to \{e, f\}$ and $[f] \cup (2N+2)E \cup [e] \to \{l\}$. Subsequently, the following five observations are obtained from the computer simulation.

2.2.1 Observations

1. When the gliders have colossal periods and widths, their collision results are intricate, such as OGs j, l, m, n, p_2, q.
2. All gliders can be generated by collisions.
3. For the collisions of two OGs, the phenomenon of "long reaction process" is discovered. Usually a host of gliders can be generated after a long reaction process.
4. In the collision courses of certain OGs, the phenomenon of "swerve" is discovered.
5. One gun and a series of solitons are discovered by collisions.

These observations are described in detail as follows:

1. For instance, the collision results concerning glider j and other OGs are quite intricate. Collision $j \leftrightarrow a$ has seven cases. Collision $j \leftrightarrow b$ has seven cases. Collision $j \leftrightarrow c$ has one case. Collision $j \leftrightarrow d$ has 10 cases. Collision $j \leftrightarrow e$ has 14 cases. Collision $j \leftrightarrow h$ has 14 cases. Collision $j \leftrightarrow i$ has one case. Collision $j \leftrightarrow i_2$ has one case. Collision $j \leftrightarrow k$ has three cases. Collision $j \leftrightarrow l$ has more than 15 cases. Collision $j \leftrightarrow m$ has more than 11 cases. Collision $j \leftrightarrow n$ has more than 11 cases. Collision $j \leftrightarrow o$ has 12 cases. Collision $j \leftrightarrow p$ has eight cases. Collision $j \leftrightarrow q$ has five cases.
2. The OGs can be generated through the collisions by themselves. As different collisions may produce the same or similar results, only one case is illustrated in Table 3. Since the DCGs and MCGs are assembled together by the OGs of the same velocity, it is conceivable that the collisions produce the whole gliders. To some extent, all gliders in HCA(9, 74) constitute a closed set in the sense of mutual collisions.

OGs	Collision Formulas
a	$[g_2] \bigcup (2N+2)E \bigcup [d] \to \{a\}$
b	$[k] \bigcup (N+1)E \bigcup [e] \to \{b\}$
c	$[g] \bigcup (2N+2)E \bigcup [e] \to \{c\}$
d	$[i_2] \bigcup (N+1)E \bigcup [l] \to \{d\}$
e	$[i_2] \bigcup (5N+2)E \bigcup [o] \to \{e\}$
f	$[g_2] \bigcup (2N+1)E \bigcup [p] \to \{f\}$
g	$[m] \bigcup (4N+4)E \bigcup [d] \to \{a, l, m, g\}$
g_2	$[m] \bigcup (4N+3)E \bigcup [c] \to \{e, a, p, g_2\}$
g_3	$[k] \bigcup (N+1)E \bigcup [a] \to \{a, a, d, g_3\}$
h	$[g] \bigcup (2N+2)E \bigcup [p] \to \{e, h, l, i\}$
i	$[g_2] \bigcup (N+1)E \bigcup [c] \to \{i\}$
i_2	$[i2] \bigcup (N+1)E \bigcup [e] \to \{k\}$
j	$[f] \bigcup (N+1)E \bigcup [c] \to \{j\}$
k	$[p] \bigcup (2N+2)E \bigcup [e] \to \{k\}$
k_2	$[g_3] \bigcup (N+1)E \bigcup [i] \to \{k_2\}$
l	$[g_3] \bigcup (N+1)E \bigcup [b] \to \{l\}$
m	$[l] \bigcup (5N+1)E \bigcup [b] \to \{m\}$
n	$[q] \bigcup (5N+1)E \bigcup [a] \to \{n, f\}$
o	$[m] \bigcup (2N+1)E \bigcup [b] \to \{a, o, g_3\}$
p	$[i] \bigcup (2N+2)E \bigcup [c] \to \{p\}$
p_2	$[j] \bigcup (8N+2)E \bigcup [p] \to \{p_2\}$
q	$[n] \bigcup (5N+3)E \bigcup [b] \to \{q\}$

Table 3. All OGs can be generated by collisions.

3. By systematically analyzing the spatiotemporal patterns of collisions, a majority of OGs are endowed with long reaction process. Nonetheless, the collisions lead to a regular bifurcation rather than evolve to an unordered state. $[f] \bigcup (7N+3)E \bigcup [m] \to \{e, b, jj, f, g, f, g2\}$ and $[f] \bigcup (7N+4)E \bigcup [m] \to \{d, a, b, b, b, b, f, f\}$ are illustrated in Figure 3.

4. The swerve means that some idiographic reaction configuration converges toward the left suddenly and forms the corresponding gliders. Roughly speaking, we believe the phenomenon occurs because of an interaction effect between the hybrid mechanism and the shift characteristic of the ether. For the visualization of swerve, the $[g] \bigcup (N+1)E \bigcup [a] \to \{a, e, hbb\}$ and $[g] \bigcup (N+1)E \bigcup [b] \to \{a, a, d\}$ are described in Figure 4.

5. Figure 5 presents the gun and solitons. In particular, the gun is obtained by the collisions: $[g_3] \bigcup (4N+4)E \bigcup [l] \to \{\text{gun}\}$, $[i] \bigcup (3N+1)E \bigcup [p_2] \to \{\text{gun}\}$, and so on. A series of solitons is obtained as $[f] \bigcup (2N+1)E \bigcup [e] \to \{e, f\}$, $[f] \bigcup (2N+1)E \bigcup [i] \to \{i, f\}$, $[l] \bigcup (10N+4)E \bigcup [e] \to \{e, l\}$, $[l] \bigcup (10N+2)E \bigcup [d] \to \{d, l\}$, $[l] \bigcup (10N+7)E \bigcup [d] \to \{d, l\}$, and $[p] \bigcup (2N+1)E \bigcup [e] \to \{e, p\}$.

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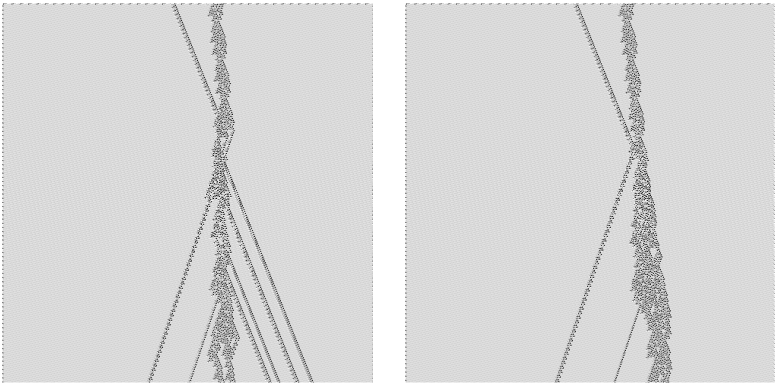
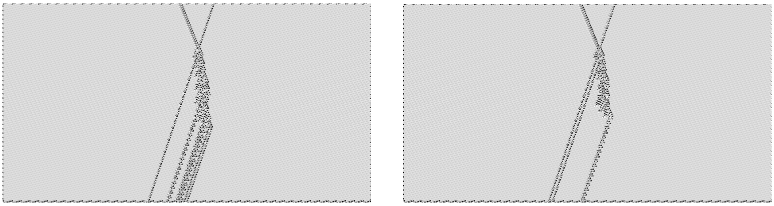


(a) (b)

Figure 3. The phenomenon of long reaction process. (a) Spatiotemporal pattern of collision $f \leftrightarrow m$ with $3E$ distance. (b) Spatiotemporal pattern of collision $f \leftrightarrow m$ with $4E$ distance.



(a) (b)

Figure 4. The phenomenon of swerve. (a) Spatiotemporal pattern of collision $g \leftrightarrow a$. (b) Spatiotemporal pattern of collision $g \leftrightarrow b$.

2.3 Symbolic Dynamics of Gliders

In the bi-infinite symbolic string space, some in-depth studies of the symbolic dynamics of gliders are carried out in the following. First and foremost, certain terminology and notations are the necessary prerequisite to the rigorous consideration of this subject.

The set of bi-infinite configurations is denoted by $S^{\mathbb{Z}} = \cdots S \times S \times S \cdots$ and a metric d on $S^{\mathbb{Z}}$ is defined as

$$d(x, \overline{x}) = \sum_{i=-\infty}^{+\infty} \frac{\tilde{d}(x_i, \overline{x}_i)}{2^{|i|}},$$

where $S = \{0, 1, \ldots, k-1\}$, $x, \overline{x} \in S^{\mathbb{Z}}$, and $\tilde{d}(\cdot, \cdot)$ is the metric on S defined as $\tilde{d}(x_i, \overline{x}_i) = 0$, if $x_i = \overline{x}_i$; otherwise, $\tilde{d}(x_i, \overline{x}_i) = 1$. For S, a word over S is a finite string $a = (\alpha_0, \ldots, \alpha_n)$ of elements of S. A set $X \subseteq S^{\mathbb{Z}}$ is F-invariant if $F(X) \subseteq X$ and strongly F-invariant if

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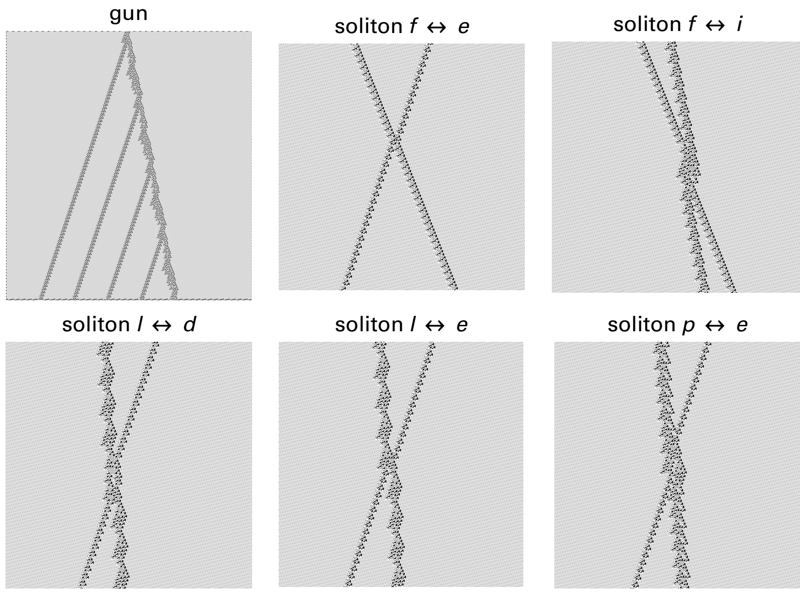

Figure 5. One gun and five solitons.

$F(X) = X$. If X is closed and F-invariant, then (X, F) or simply X is called a subsystem of F. For instance, let $\mathcal{A}$ denote a set of some finite strings over S, and $\Lambda = \Lambda_{\mathcal{A}}$ is the set that consists of the bi-infinite configurations made up of all the strings in $\mathcal{A}$. Then, $\Lambda_{\mathcal{A}}$ is a subsystem of $\left(S^Z, \sigma\right)$, where $\mathcal{A}$ is said to be the determinative block system of Λ. For a closed invariant subset $\Lambda \subseteq S^Z$, the subsystem (Λ, σ) or simply Λ is called a subshift of σ [27, 28].

The classical left-shift map $\sigma_L : S^Z \to S^Z$ is defined by $[\sigma_L(x)]_i = x_{i+1}$; the classical right-shift map $\sigma_R : S^Z \to S^Z$ is defined by $[\sigma_R(x)]_i = x_{i-1}$. A map $F : S^Z \to S^Z$ is a CA if and only if it is continuous and commutes with σ; that is, $\sigma \circ F = F \circ \sigma$, where σ is a left shift or right shift. Moreover, $\left(S^Z, F\right)$ is a compact dynamical system. By exploiting the mathematical definition of HCA rules, this section shows the symbolic dynamics of the gliders under the background of ether in the bi-infinite symbolic string space. For bi-infinite, one-dimensional CAs, there exist a radius $r \geq 0$ and a local rule $N : S^{2r+1} \to S$ such that $[F(x)]_i = N\left(x_{[i-r, i+r]}\right)$. For ECAs, $r = 1$ and $S = \{0, 1\}$. Each local rule of an ECA can be endowed with a Boolean function. For instance, the Boolean function of ECA rule 9 is expressed as

$$N_9\left(x_{[i-1,\, i+1]}\right) = \overline{x}_{i-1}\overline{x}_i\overline{x}_{i+1} \oplus \overline{x}_{i-1}x_i x_{i+1}, \qquad \forall\, i \in Z,$$

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where $x_i \in S$, $\cdot$, $\oplus$, and $-$ denote AND, XOR, and NOT logical operations, respectively. The Boolean function of ECA rule 74 is expressed as

$$N_{74}\left(x_{[i-1,\, i+1]}\right) = x_{i-1} x_i \overline{x}_{i+1} \oplus \overline{x}_{i-1} x_{i+1}, \qquad \forall\, i \in Z.$$

Consequently, the Boolean function of HCA$(9, 74)$ is induced as

$$[F(x)]_i = \begin{cases} N_9\left(x_{[i-1,\, i+1]}\right), & (i \bmod 2) \equiv 1 \\ N_{74}\left(x_{[i-1,\, i+1]}\right), & (i \bmod 2) \equiv 0 \end{cases}.$$

For each glider, a particular subsystem can be captured by enumeration and computer programming. Then the key question is whether the function is topologically mixing and possesses the positive topological entropy on its subsystems. The dynamical analysis of all gliders in this paper is carried out and the results are listed in Table 4. For clarity and convenience, one situation is discussed in detail in Proposition 1.

Type	Subsystem	Topological Entropy	Topological Mixing	Chaotic Property
a	$\Lambda_{\mathcal{A}_1}$	0.0990671	Yes	Li–Yorke and Devaney
b	$\Lambda_{\mathcal{A}_2}$	0.0944828	Yes	Li–Yorke and Devaney
c	$\Lambda_{\mathcal{A}_3}$	0.1039510	Yes	Li–Yorke and Devaney
d	$\Lambda_{\mathcal{A}_4}$	0.0915127	Yes	Li–Yorke and Devaney
e	$\Lambda_{\mathcal{A}_5}$	0.0987810	Yes	Li–Yorke and Devaney
f	$\Lambda_{\mathcal{A}_6}$	0.0871384	Yes	Li–Yorke and Devaney
g	$\Lambda_{\mathcal{A}_7}$	0.0831467	Yes	Li–Yorke and Devaney
g_2	$\Lambda_{\mathcal{A}_8}$	0.0813248	Yes	Li–Yorke and Devaney
g_3	$\Lambda_{\mathcal{A}_9}$	0.0876927	Yes	Li–Yorke and Devaney
h	$\Lambda_{\mathcal{A}_{10}}$	0.1376380	No	Li–Yorke
i	$\Lambda_{\mathcal{A}_{11}}$	0.0846730	Yes	Li–Yorke and Devaney
i_2	$\Lambda_{\mathcal{A}_{12}}$	0.0734324	Yes	Li–Yorke and Devaney
j	$\Lambda_{\mathcal{A}_{13}}$	0.0629498	Yes	Li–Yorke and Devaney
k	$\Lambda_{\mathcal{A}_{14}}$	0.0724487	Yes	Li–Yorke and Devaney
k_2	$\Lambda_{\mathcal{A}_{15}}$	0.0691436	Yes	Li–Yorke and Devaney
l	$\Lambda_{\mathcal{A}_{16}}$	0.0726517	Yes	Li–Yorke and Devaney
m	$\Lambda_{\mathcal{A}_{17}}$	0.0565970	No	Li–Yorke
n	$\Lambda_{\mathcal{A}_{18}}$	0.0578675	Yes	Li–Yorke and Devaney
o	$\Lambda_{\mathcal{A}_{19}}$	0.0773969	Yes	Li–Yorke and Devaney

Table 4. (*continues*).

Type	Subsystem	Topological Entropy	Topological Mixing	Chaotic Property
p	$\Lambda_{\mathcal{A}_{20}}$	0.0734324	Yes	Li–Yorke and Devaney
p_2	$\Lambda_{\mathcal{A}_{21}}$	0.0613904	Yes	Li–Yorke and Devaney
q	$\Lambda_{\mathcal{A}_{22}}$	0.0553908	Yes	Li–Yorke and Devaney
ab	$\Lambda_{\mathcal{A}_{23}}$	0.0832761	Yes	Li–Yorke and Devaney
ac	$\Lambda_{\mathcal{A}_{24}}$	0.0914153	Yes	Li–Yorke and Devaney
ad	$\Lambda_{\mathcal{A}_{25}}$	0.1414020	No	Li–Yorke
ae	$\Lambda_{\mathcal{A}_{26}}$	0.0846730	Yes	Li–Yorke and Devaney
ba	$\Lambda_{\mathcal{A}_{27}}$	0.0832761	Yes	Li–Yorke and Devaney
bd	$\Lambda_{\mathcal{A}_{28}}$	0.0773589	Yes	Li–Yorke and Devaney
be	$\Lambda_{\mathcal{A}_{29}}$	0.0815343	Yes	Li–Yorke and Devaney
ca	$\Lambda_{\mathcal{A}_{30}}$	0.0841022	Yes	Li–Yorke and Devaney
cb	$\Lambda_{\mathcal{A}_{31}}$	0.0775657	Yes	Li–Yorke and Devaney
da	$\Lambda_{\mathcal{A}_{32}}$	0.1337600	No	Li–Yorke
db	$\Lambda_{\mathcal{A}_{33}}$	0.0773589	Yes	Li–Yorke and Devaney
dc	$\Lambda_{\mathcal{A}_{34}}$	0.0750821	Yes	Li–Yorke and Devaney
de	$\Lambda_{\mathcal{A}_{35}}$	0.0940344	Yes	Li–Yorke and Devaney
ea	$\Lambda_{\mathcal{A}_{36}}$	0.0775657	Yes	Li–Yorke and Devaney
eb	$\Lambda_{\mathcal{A}_{37}}$	0.0750110	Yes	Li–Yorke and Devaney
ec	$\Lambda_{\mathcal{A}_{38}}$	0.1291430	No	Li–Yorke
ed	$\Lambda_{\mathcal{A}_{39}}$	0.0706930	Yes	Li–Yorke and Devaney
fg	$\Lambda_{\mathcal{A}_{40}}$	0.0692569	Yes	Li–Yorke and Devaney
g_2g	$\Lambda_{\mathcal{A}_{41}}$	0.0713481	Yes	Li–Yorke and Devaney
gf	$\Lambda_{\mathcal{A}_{42}}$	0.0760296	Yes	Li–Yorke and Devaney
hb	$\Lambda_{\mathcal{A}_{43}}$	0.0692569	Yes	Li–Yorke and Devaney
hc	$\Lambda_{\mathcal{A}_{44}}$	0.0682648	Yes	Li–Yorke and Devaney
ada	$\Lambda_{\mathcal{A}_{45}}$	0.0713481	Yes	Li–Yorke and Devaney
$aebbb$	$\Lambda_{\mathcal{A}_{46}}$	0.0803294	Yes	Li–Yorke and Devaney
bae	$\Lambda_{\mathcal{A}_{47}}$	0.0682119	Yes	Li–Yorke and Devaney
beb	$\Lambda_{\mathcal{A}_{48}}$	0.0704649	Yes	Li–Yorke and Devaney
daa	$\Lambda_{\mathcal{A}_{49}}$	0.0713481	Yes	Li–Yorke and Devaney
eaa	$\Lambda_{\mathcal{A}_{50}}$	0.0702522	Yes	Li–Yorke and Devaney
fgf	$\Lambda_{\mathcal{A}_{51}}$	0.0552894	Yes	Li–Yorke and Devaney
hbb	$\Lambda_{\mathcal{A}_{52}}$	0.0615264	Yes	Li–Yorke and Devaney

Table 4. Summary of quantitative properties of subsystem of several gliders along with their chaotic statements.

https://doi.org/10.25088/ComplexSystems.24.3.185

Proposition 1. For glider a, there exists $F^6(x)|_{\Lambda_{\mathcal{A}_1}} = \sigma_L^2(x)|_{\Lambda_{\mathcal{A}_1}}$, where $\Lambda_{\mathcal{A}_1} = \{x \in S^Z \mid x_{[i-11, i+10]} \in \mathcal{A}_1, \forall\, i \in Z\}$ is a subsystem of S^Z, and $\mathcal{A}_1 = \{$906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474115, 1313549, 1059895, 45277, 181109, 724436, 2897744, 3202368, 226562, 3474124, 1313587, 1060047, 45885, 183541, 734164, 2936656, 3358016, 849154, 3396619, 1003564, 4014259, 45942, 183769, 735076, 2940304, 3372608, 907522, 3630091, 1937452, 3555507, 1639117, 2362167$\}$. Thus, $\Lambda_{\mathcal{A}_1}$ is a subshift of finite type of (S^Z, σ_L).

Remark 1. Let $x_{[i-11, i+10]}$ denote the 22-bit string $(x_{i-11}, x_{i-10}, x_{i-9}, \ldots, x_{i+9}, x_{i+10})$ over $S = \{0, 1\}$. $\mathcal{A}_1$ is called the determinative system of $\Lambda_{\mathcal{A}_1}$, which is a 22-bit string set. For convenience, each $x_{[i-11, i+10]}$ in $\mathcal{A}_1$ is described by its decimal code form; that is, 906251 refers to the 22-bit string $(0, 0, 1, 1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 1)$.

In a nutshell, it is addressed that directed graph theory provides a powerful tool for studying such infinite strings. A fundamental method for constructing finite shifts starts with a finite directed graph and produces the collection of all bi-infinite walks (i.e., strings of edges) on the graph. A graph G consists of a finite set $V = V(G)$ of vertices (or states) together with a finite set $E = E(G)$ of edges. It is worth noting that $\Lambda_{\mathcal{A}_1}$ can be described by a finite directed graph $G_{\mathcal{A}_1} = \{\mathcal{A}_1, E\}$, where each vertex is a string in $\mathcal{A}_1$. Each edge $e \in E(G)$ starts at a string denoted by $a = (a_0, a_1, \ldots, a_{21}) \in V(G)$ and terminates at the string $b = (b_0, b_1, \ldots, b_{21}) \in V(G)$ if and only if $a_k = b_{k-2}$, $2 \le k \le 21$. A finite path $P = v_1 \to v_2 \to \cdots \to v_m$ on a graph G is a finite string of vertices v_i from G. The length of P is $|P| = m$. A cycle is a path that starts and terminates at the same vertex. A graph G is irreducible if for every ordered pair of vertices I and J there is a path in G starting at I and terminating at J. Each element of $\Lambda_{\mathcal{A}_1}$ has a certain path on the graph $G_{\mathcal{A}_1}$. The finite directed graph $G_{\mathcal{A}_1}$ is shown in Figure 6. All bi-infinite walks on the graph constitute the closed invariant subsystem $\Lambda_{\mathcal{A}_1}$.

A matrix D is positive if all of its entries are non-negative; irreducible if $\forall\, i, j$ there exist n such that $D_{ij}^n > 0$; and aperiodic if there exists N such that $D_{ij}^n > 0$, $n > N$, $\forall\, i, j$. If Λ is a two-order subshift of finite type, then it is topologically mixing if and only if D is irre-

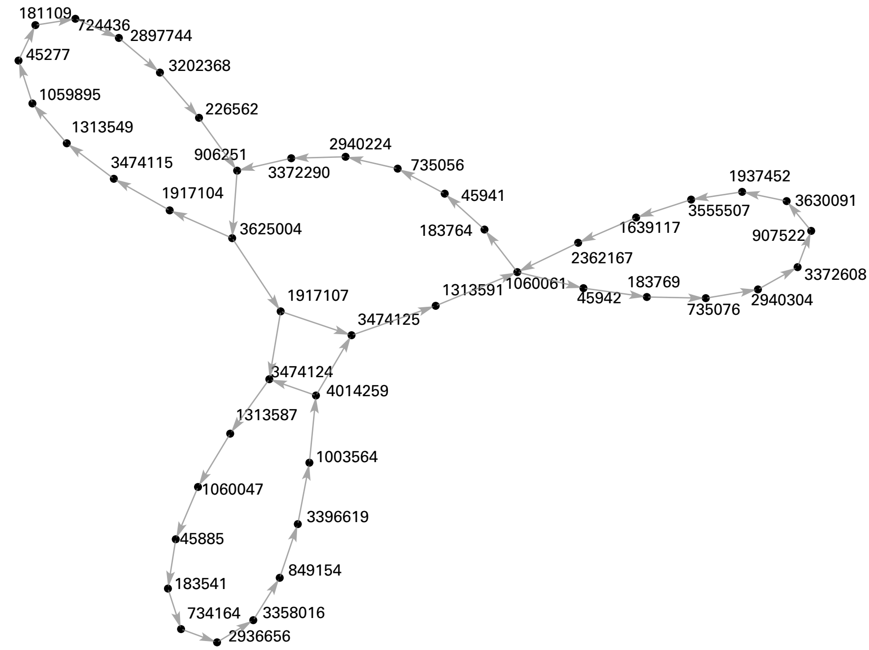

Figure 6. Graph representation of the subsystem $\Lambda_{\mathcal{A}_1}$ for glider a.

ducible and aperiodic, where D is its associated transition matrix with $D_{ij} = 1$, if $(i, j) \prec \Lambda$; otherwise, $D_{ij} = 0$. Then the topologically conjugate relationship between $\left(\Lambda_{\mathcal{A}_1}, \sigma_L\right)$ and a two-order subshift of finite type is established. The topological dynamics of F^6 on $\Lambda_{\mathcal{A}_1}$ is largely determined by the properties of its transition matrix. Let $\hat{S} = \{r_0, r_1, \ldots, r_{42}, r_{43}\}$ be a new symbolic set, where r_i, $i = 0, \ldots, 43$ stand for elements of $\mathcal{A}_1$, respectively. Then a new symbolic space $\hat{S}^Z$ can be constructed on $\hat{S}$. Denote by

$$\mathcal{B} = \left\{(r r') \,\middle|\, r = \left(b_0 b_1 \ldots b_{21}\right), r' = \left(b'_0 b'_1 \ldots b'_{21}\right) \in \hat{S}, \right.$$
$$\left. \forall\ 2 \leq j \leq 21 \text{ such that } b_j = b'_{j-2}\right\}.$$

Further, the two-order subshift $\Lambda_{\mathcal{B}}$ of σ_L is defined by

$$\Lambda_{\mathcal{B}} = \left\{r = (\ldots, r_{-1}, r_0^*, r_1, \ldots) \in \hat{S}^Z \,\middle|\, r_i \in \hat{S}, (r_i, r_{i+1}) \prec \mathcal{B}, \right.$$
$$\left. \forall\ i \in Z\right\}.$$

https://doi.org/10.25088/ComplexSystems.24.3.185

Therefore, it is easy to calculate the transition matrix D of the subshift $\Lambda_{\mathcal{B}}$ as follows:

$$\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

As a matter of fact, it is easy to verify that $D_{ij}^{n} > 0$, $n \geq 141$ for $1 \leq i, j \leq 44$, so D is aperiodic. Thus, the subshift of finite type $\left(\Lambda_{\mathcal{A}_1}, \sigma_L\right)$ is mixing, and $F^6(x)\,|_{\Lambda_{\mathcal{A}_1}}$ is also mixing. Furthermore, the topological entropy

$$\operatorname{ent}\left(F^6\,|_{\Lambda_{\mathcal{A}_1}}\right) = \operatorname{ent}\left(\sigma_L^2(x)\,|_{\Lambda_{\mathcal{A}_1}}\right),$$

and

$$\operatorname{ent}\left(\sigma_L^2(x)\,|_{\Lambda_{\mathcal{A}_1}}\right) = \log \lambda^* \doteq \log(1.10414) = 0.0990671,$$

where λ^* is the maximum positive real root of

$$\lambda^8\left(-1 + 3\lambda^{12} - 3\lambda^{24} - \lambda^{25} + \lambda^{36}\right) = 0,$$

the characteristic equation of D. Thus, the following proposition can be obtained promptly.

Proposition 2. (1) The map $F^6\,|_{\Lambda_{\mathcal{A}_1}}$ is topologically mixing. (2) The topological entropy of $F^6\,|_{\Lambda_{\mathcal{A}_1}}$ is positive.

It follows from [27, 28] that positive topological entropy implies chaos in the sense of Li–Yorke. Meanwhile, topological mixing is also an exceedingly complex property of dynamical systems. A system with the topologically mixing property has many chaotic properties in diverse senses. In particular, chaos in the sense of Li–Yorke can be deduced from positive topological entropy. Chaos in the sense of both Devaney and Li–Yorke can be deduced from topological mixing. In conclusion, the mathematical analysis presented provides the rigorous foundation for the following theorem.

Theorem 1. For glider a, $F^6|_{\Lambda_{\mathcal{A}_1}}$ is chaotic in the sense of both Li–Yorke and Devaney.

In order to describe the symbolic dynamics of all gliders in this paper in an unambiguous way, analogous results are listed in Table 4, where column 2 displays the particular subsystems. Their corresponding decimal code sets of determinative systems $\mathcal{A}_j$, $j = 1, \ldots 52$ are provided in Appendix C.

In addition, it is of interest that $\text{HCA}(9, 74)$ has practically unlimited chaotic subsystems, which is extrapolated for a pair of reasons. First, an increasing number of other gliders—the composite gliders especially—may be found via computer simulation. Second, different gliders of equal velocity can coexist independently in a spatiotemporal pattern. Then a multitude of corresponding subsystems will be captured for these combinations of gliders. Obviously, the dynamical behaviors of these subsystems are more complex than that of only a single glider.

3. Gliders in Other Hybrid Cellular Automaton Rules

As the coarse-grained preprocessing, when we treat two adjacent cells as the new smallest unit, $\text{HCA}(9, 74)$ is transformed to a certain one-dimensional uniform CA of four states and three neighbors. As the hybrid mechanism, it is necessary to note that the first cell of each unit shall begin with the odd sites. More precisely, let $\hat{S} = \{00, 01, 10, 11\}$ be a new symbolic set, and $\hat{S}^Z$ is the space of bi-infinite configurations over $\hat{S}$. The metric d^* on $\hat{S}^Z$ is

$$d^*(y, \overline{y}) = \sum_{i=-\infty}^{+\infty} \frac{\tilde{d}(x_{2i+1}, \overline{x}_{2i+1}) + 2\tilde{d}(x_{2i+2}, \overline{x}_{2i+2})}{4^{|i|}},$$

where $y, \overline{y} \in \hat{S}^Z$ and $\tilde{d}(\cdot, \cdot)$ is the metric on S. Then the block trans-

https://doi.org/10.25088/ComplexSystems.24.3.185

formation $B_{\langle 2\rangle}$ is defined as

$$B_{\langle 2\rangle} : S^Z \to \hat{S}^Z,\ y_i = \left[B_{\langle 2\rangle}(x)\right]_i = \begin{cases} 00 & \text{if } x_{2i+1} = 0, x_{2i+2} = 0 \\ 01 & \text{if } x_{2i+1} = 0, x_{2i+2} = 1 \\ 10 & \text{if } x_{2i+1} = 1, x_{2i+2} = 0 \\ 11 & \text{if } x_{2i+1} = 1, x_{2i+2} = 1 \end{cases}.$$

Let T be the new evolution function. It can easily be verified that block transformation $B_{\langle 2\rangle}$ is a homeomorphism map and $\left(\hat{S}^Z, T\right)$ is topologically conjugate with $\left(S^Z, F\right)$. Thereby, the dynamical properties of HCA$(9, 74)$ are consistent with the new uniform CA of four states and three neighbors. Meanwhile, 2^6 evolution results $\left[T(y)\right]_i$ can be obtained by all different input values (y_{i-1}, y_i, y_{i+1}). To name only a few, $\left[T(00, 00, 00)\right]_i = (10)$, $\left[T(00, 00, 01)\right]_i = (10)$, $\left[T(00, 00, 11)\right]_i = (11)$.

If the HCA rules are composed of t ECA rules $(t \geq 2)$, one interesting question is whether we can seek out some other HCA rules endowed with plentiful gliders. The HCA rules are denoted by HCA$(\theta_1, \theta_2, \ldots, \theta_{t-1}, \theta_t)$, where $\theta_1, \theta_2, \ldots, \theta_t$ imply the specific ECA rules. To this end, a generalization of the Boolean function F of the HCA rule is represented as

$$[F(x)]_i = \begin{cases} N_{\theta_1}\left(x_{[i-1,i+1]}\right), & (i \bmod t) \equiv 1 \\ N_{\theta_2}\left(x_{[i-1,i+1]}\right), & (i \bmod t) \equiv 2 \\ \cdots & \\ N_{\theta_{t-1}}\left(x_{[i-1,i+1]}\right), & (i \bmod t) \equiv t-1 \\ N_{\theta_t}\left(x_{[i-1,i+1]}\right), & (i \bmod t) \equiv 0 \end{cases}.$$

After introducing the extended space and distance, the block transformation $B_{\langle t\rangle}$ can be defined as

$$\left[B_{\langle t\rangle}(x)\right]_i = \begin{cases} 00\ldots0 & \text{if } x_{ti+1} = 0,\ x_{ti+2} = 0, \ldots, x_{ti+t} = 0 \\ 00\ldots1 & \text{if } x_{ti+1} = 0,\ x_{ti+2} = 0, \ldots, x_{ti+t} = 1 \\ \ldots & \\ 11\ldots1 & \text{if } x_{ti+1} = 1,\ x_{ti+2} = 1, \ldots, x_{ti+t} = 1 \end{cases}.$$

It is demonstrated that the new uniform CA of 2^t states and three neighbors is topologically conjugate with the original HCA. In what follows, three HCA rules endowed with gliders and glider collisions are found. Several spatiotemporal patterns of HCA(θ_1, θ_2, θ_3) are also presented in Appendix A.

3.1 Coding the Gliders of HCA(3, 28, 74)

Following Chua's classifications, ECA rule 28 belongs to the period-2 rules, and ECA rules 3 and 74 belong to the Bernoulli σ_τ-shift rules. Figure 7 illustrates the spatiotemporal pattern of $\text{HCA}(3, 28, 74)$. The Boolean function of $\text{HCA}(3, 28, 74)$ is presented as

$$[F(x)]_i = \begin{cases} N_3\left(x_{[i-1,i+1]}\right), & \left(i \bmod 3\right) \equiv 1 \\ N_{28}\left(x_{[i-1,i+1]}\right), & \left(i \bmod 3\right) \equiv 2 \\ N_{74}\left(x_{[i-1,i+1]}\right), & \left(i \bmod 3\right) \equiv 0 \end{cases}.$$

If each three-adjacent cell is regarded as a new smallest unit and the first cell of each unit begins with the sites of $\left(i \bmod 3\right) \equiv 1$ in Figure 7, $\text{HCA}(3, 28, 74)$ is topologically conjugate with a new uniform CA of eight states and three neighbors. Each glider is evolved under the same background with the ether unit

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

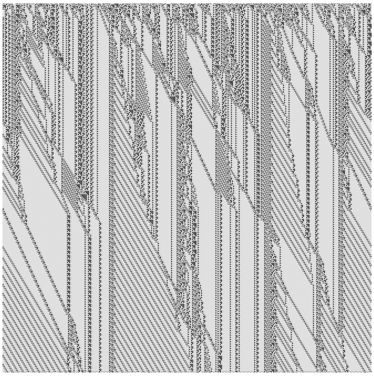


Figure 7. Spatiotemporal pattern of HCA(3, 28, 74).

We find 10 gliders and six guns, and their properties are listed in Table 5. Figure 8 illustrates the spatiotemporal patterns of the gliders and guns.

https://doi.org/10.25088/ComplexSystems.24.3.185

Gliders	Velocity	With
Ether Unit	−3/3	3
a	0/6	3−6
b	0/12	6−9
b_2	0/12	9−12
b_3	0/24	12−15
$\overline{b_3}$	0/24	15−18
b_4	0/24	15−18
c	0/24	12−15
d	3/5	6−9
e	3/8	6−9
f	6/13	6−12

Table 5. Characterizations of nine gliders.

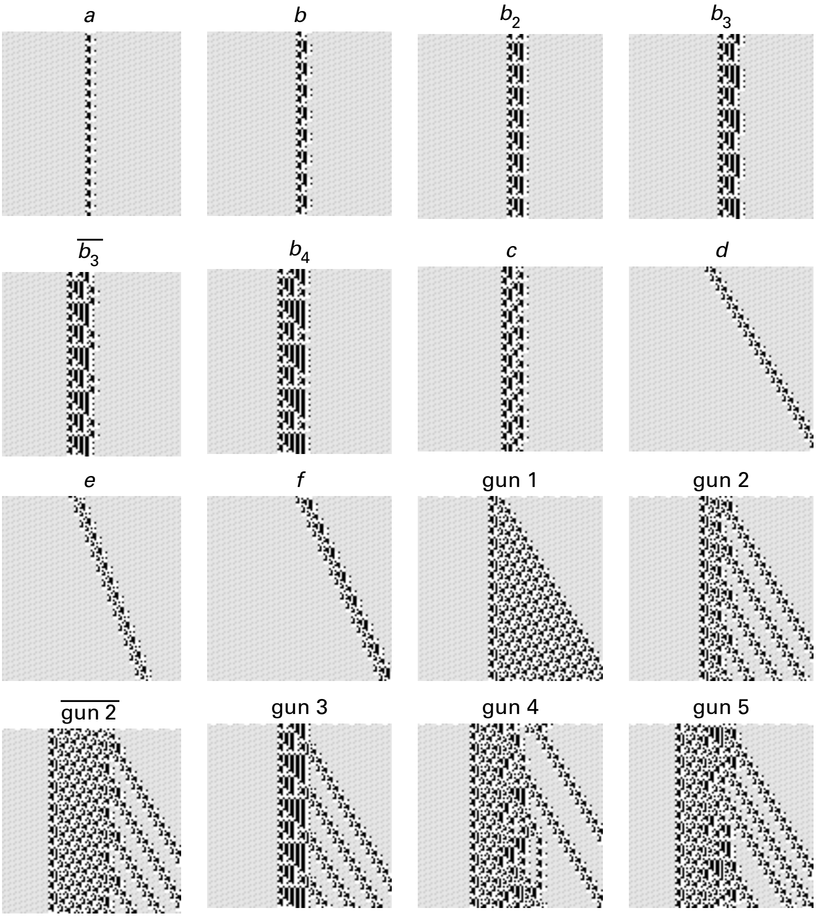


Figure 8. The spatiotemporal patterns of 10 gliders and six guns.

https://doi.org/10.25088/ComplexSystems.24.3.185

3.2 Coding the Gliders of HCA(163, 135, 105, 134)

Following Chua's classifications, ECA rules 134 and 163 belong to the Bernoulli σ_τ-shift rules, ECA rule 105 belongs to the complex Bernoulli-shift rules, and ECA rule 135 belongs the hyper Bernoulli-shift rules. Figure 9 provides the spatiotemporal pattern of HCA(163, 135, 105, 134). The Boolean function of HCA(163, 135, 105, 134) is induced as

$$[F(x)]_i = \begin{cases} N_{163}\left(x_{[i-1,i+1]}\right), & (i \bmod 4) \equiv 1 \\ N_{135}\left(x_{[i-1,i+1]}\right), & (i \bmod 4) \equiv 2 \\ N_{105}\left(x_{[i-1,i+1]}\right), & (i \bmod 4) \equiv 3 \\ N_{134}\left(x_{[i-1,i+1]}\right), & (i \bmod 4) \equiv 0 \end{cases}.$$

If each four-adjacent cell is regarded as a new smallest unit and the first cell of each unit begins with the sites of $(i \bmod 4) \equiv 1$ in Figure 9, HCA(163, 135, 105, 134) is topologically conjugate with a new uniform CA of 16 states and three neighbors. Its ether unit is

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

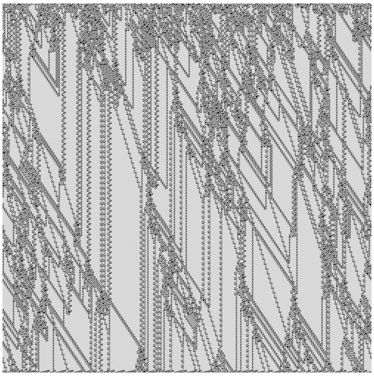


Figure 9. Spatiotemporal pattern of HCA(163, 135, 105, 134).

We find 14 types of gliders, whose detailed characterizations are listed in Table 6. Figure 10 illustrates spatiotemporal patterns of the gliders.

Gliders	Velocity	With
Ether Unit	−4/7	4
a	0/14	4–8
b	0/7	4
$\overline{b}$	0/8	8
c	0/14	4–12
$\overline{c}$	0/14	16–24
d	4/6	4–8
e	4/13	4–8
$\overline{e}$	4/13	12
f	4/6	8–12
$\overline{f}$	4/6	12–16
g	−4/15	8–12
ac	0/14	12–16
acb	0/14	20–24
bac	0/14	20–24

Table 6. Characterizations of 14 gliders.

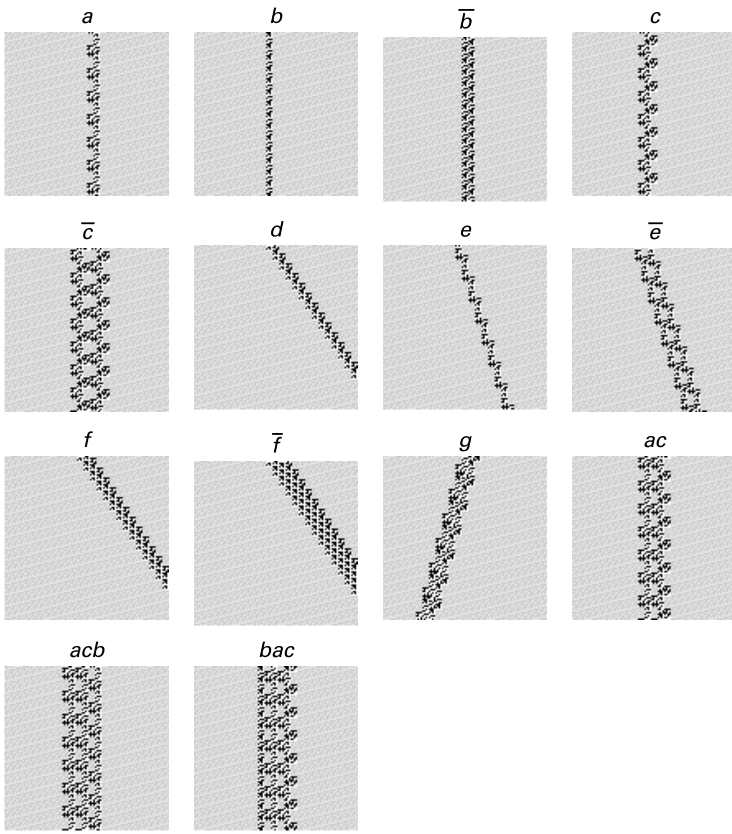


Figure 10. The spatiotemporal patterns of 14 gliders.

https://doi.org/10.25088/ComplexSystems.24.3.185

3.3 Coding the Gliders of HCA(90, 170, 74, 105, 129)

Following Chua's classifications, ECA rules 74 and 170 belong to the Bernoulli σ_τ-shift rules. ECA rules 90, 105, and 129 belong to the complex Bernoulli-shift rules. Figure 11 provides the spatiotemporal pattern of HCA$(90, 170, 74, 105, 129)$. The Boolean function of HCA$(90, 170, 74, 105, 129)$ is induced as

$$[F(x)]_i = \begin{cases} N_{90}\left(x_{[i-1,i+1]}\right), & (i \bmod 5) \equiv 1 \\ N_{170}\left(x_{[i-1,i+1]}\right), & (i \bmod 5) \equiv 2 \\ N_{74}\left(x_{[i-1,i+1]}\right), & (i \bmod 5) \equiv 3 \\ N_{105}\left(x_{[i-1,i+1]}\right), & (i \bmod 5) \equiv 4 \\ N_{129}\left(x_{[i-1,i+1]}\right), & (i \bmod 5) \equiv 0 \end{cases}.$$

If each five-adjacent cell is regarded as a new smallest unit and the first cell of each unit begins with the sites of $(i \bmod 5) \equiv 1$ in Figure 11, HCA$(90, 170, 74, 105, 129)$ is topologically conjugate with a new uniform CA of 32 states and three neighbors. Its ether unit is

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}.$$

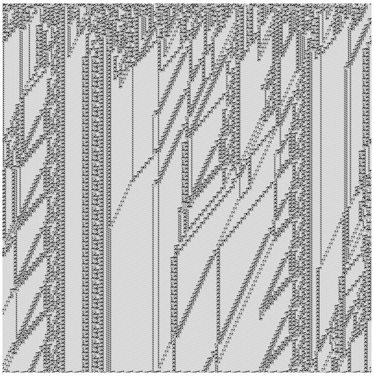


Figure 11. Spatiotemporal pattern of HCA(90, 170, 74, 105, 129).

Its characterizations of 10 gliders are listed in Table 7. Figure 12 illustrates spatiotemporal patterns of the gliders.

https://doi.org/10.25088/ComplexSystems.24.3.185

Gliders	Velocity	With
Ether Unit	−5/5	5
a	0/5	5
b	0/10	10
ba	0/10	15
$\overline{b}$	0/20	20
c	−5/11	5–10
d	−5/11	15–20
cd	−5/11	20–25
$\overline{d}$	−5/11	25–30
e	−10/12	5–15
$\overline{e}$	−10/12	20–25

Table 7. Characterizations of 10 gliders.

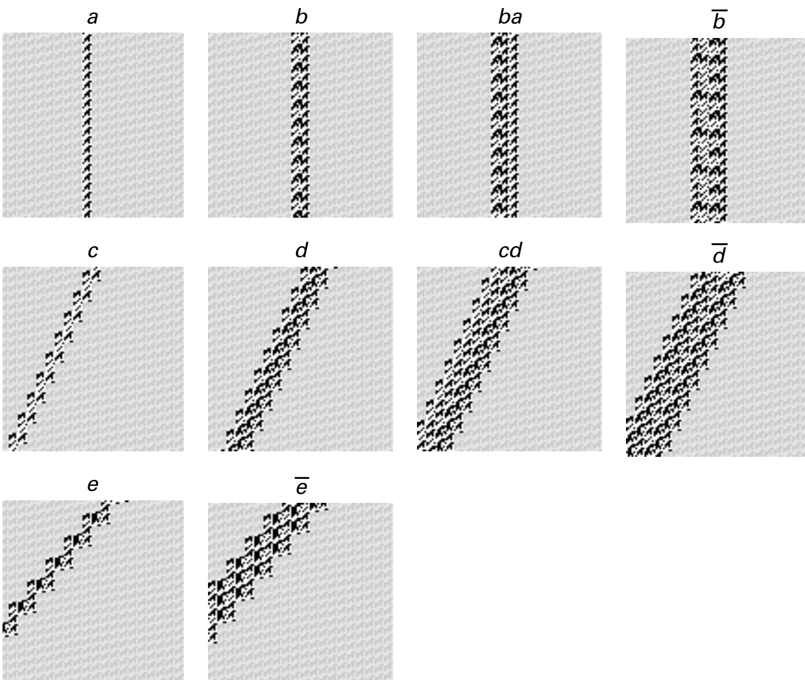


Figure 12. The spatiotemporal patterns of 10 gliders.

4. Concluding Remarks

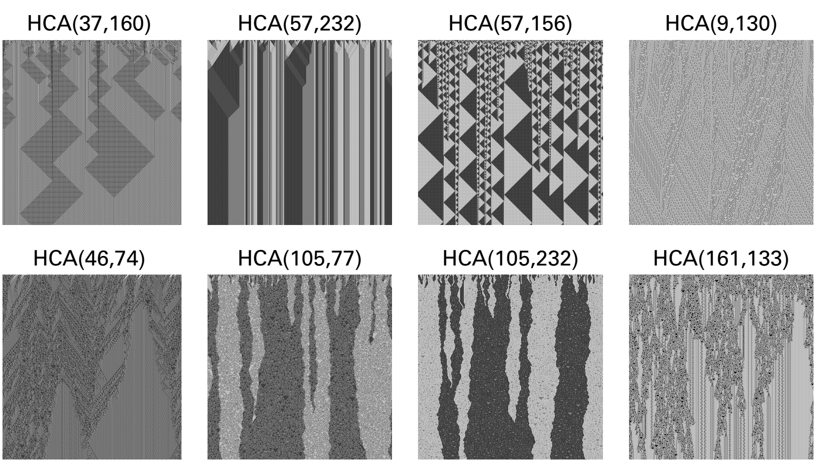
By classifying and coding the gliders that have occurred frequently in HCA(9, 74), we carry out the analysis of collisions between two original gliders (OGs) at a qualitative level. An interesting empirical observation is that the OGs can be generated from collisions by themselves. In addition, it is the hybrid mechanism that gives rise to the new phenomena of long-reaction process and swerve, which are not discovered in elementary cellular automaton (ECA) rules 110 and 54. Through exploiting the mathematical definition of hybrid cellular automaton (HCA) rules, we focus our attention on the discussion of the symbolic dynamics of all gliders in this paper. In particular, the gliders are identified with distinct subsystems. The rule of HCA$(9, 74)$ possesses positive topological entropy in all subsystems and is topologically mixing in most subsystems. Moreover, because of the newly found gliders and different combinations of extant gliders, it is inferred that the chaotic subsystems will be captured ad infinitum in HCA$(9, 74)$. On the other hand, when the hybrid cellular automata (HCAs) are composed of more than two ECA rules, we also present three other HCA rules of rich gliders.

Acknowledgments

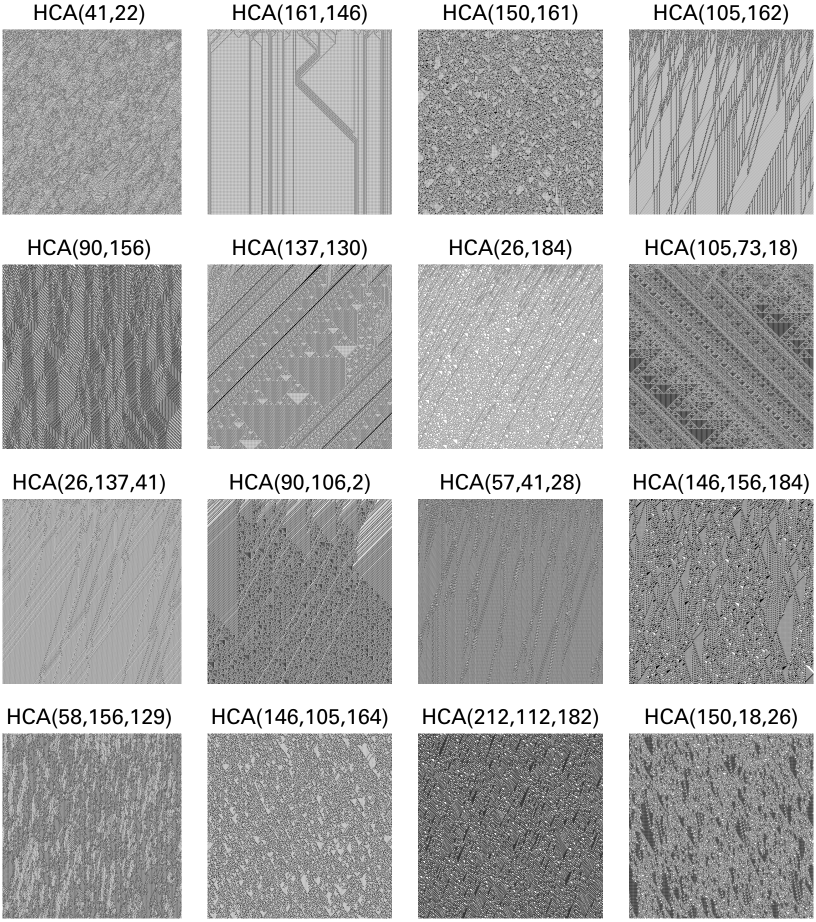
This research was supported by the Natural Science Foundation of Zhejiang Province (Grant No. LY13F030014) and the NSFC (Grant Nos. 11301122, 11171084, and 60872093).

Appendix

A. Several Spatiotemporal Patterns of Hybrid Cellular Automaton Rules



https://doi.org/10.25088/ComplexSystems.24.3.185



https://doi.org/10.25088/ComplexSystems.24.3.185

B. The Glider Factors of Original Gliders

If a glider has the period M, it has M different glider factors. For convenience, we only select a fixed glider factor for each glider. It is enough to obtain the five observations via the computer simulation in Section 2.2.

$$[a] = (0, 0, 0, 0, 1, 1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[b] = (0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[c] = (0, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[d] = (0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[e] = (0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[f] = (0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[g] = (0, 0, 1, 0, 1, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[g_2] = (0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[g_3] = (0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[h] = (0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[i] = (0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0,$$
$$1, 1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[i_2] = (0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[j] = (0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0,$$
$$1, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[k] = (0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 1,$$
$$0, 0, 1, 1, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[k_2] = (0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[l] = (0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 1, 1, 0, 0, 0,$$
$$0, 1, 1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[m] = (0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1,$$
$$1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 1, 0, 1, 1,$$
$$0, 0, 1, 1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[n] = (0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 0,$$
$$0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[o] = (0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0,$$
$$1, 1, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[p] = (0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 0, 1,$$
$$1, 0, 0, 1, 1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[p_2] = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 0, 0, 0,$$
$$0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1)$$
$$[q] = (0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 0, 0, 1, 0,$$
$$0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 1, 1, 0, 0, 1, 0, 1, 1)$$

https://doi.org/10.25088/ComplexSystems.24.3.185

C. The Decimal Code Sets of Determinative Systems

For glider *a*, the determinative system $\mathcal{A}_1 = \{906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474115, 1313549, 1059895, 45277, 181109, 724436, 2897744, 3202368, 226562, 3474124, 1313587, 1060047, 45885, 183541, 734164, 2936656, 3358016, 849154, 3396619, 1003564, 4014259, 45942, 183769, 735076, 2940304, 3372608, 907522, 3630091, 1937452, 3555507, 1639117, 2362167\}$.

For glider *b*, the determinative system $\mathcal{A}_2 = \{906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313539, 1059853, 45111, 180445, 721781, 2887124, 3159888, 56640, 226562, 3474124, 1313586, 1060043, 45871, 183485, 733941, 2935764, 3354448, 834880, 3339522, 775179, 3100716, 4014259, 45940, 183762, 735049, 2940196, 3372176, 905792, 3623170, 1909771, 3444780, 1196211, 590541, 2362167\}$.

For glider *c*, the determinative system $\mathcal{A}_3 = \{906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313538, 1059851, 45100, 180403, 721613, 2886455, 3157213, 3474124, 1313586, 1060043, 45868, 183475, 733901, 2935607, 3353821, 832373, 3329492, 45940, 183763, 735053, 2940215, 3372253, 906101, 3624404, 1914704, 3464512, 1275138\}$.

For glider *d*, the determinative system $\mathcal{A}_4 = \{906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313536, 1059841, 45060, 180240, 720960, 2883842, 3146763, 4140, 16563, 66253, 265015, 3474124, 1313586, 1060042, 45866, 183464, 733856, 2935426, 3353099, 829484, 3317939, 688845, 2755383, 2632925, 2143093, 45940, 183760, 735040, 2940162, 3372043, 905260, 3621043, 1901261, 3410743\}$.

For glider *e*, the determinative system $\mathcal{A}_5 = \{906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313536, 1059842, 45067, 180268, 721075, 2884301, 3148599, 11485, 3474124, 1313586, 1060042, 45867, 183468, 733875, 2935501, 3353399, 830685, 3322741, 708052, 2832208, 45940, 183760, 735043, 2940173, 3372087, 905437, 3621749, 1904084, 3422032, 1105216, 226562\}$.

For glider *f*, the determinative system $\mathcal{A}_6 = \{906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917106, 3474120, 1313568, 1059968, 45569, 182279, 729117, 2916469, 3277268, 526160, 2104640, 29954, 119819, 479276, 1313588, 1060050, 45898, 183594, 734377, 2937509, 3361428, 862800, 3451200, 1221890, 693259, 2773036, 2703539, 2425549, 1060060, 45937, 183748, 734992, 2939968, 3371264, 902144, 3608576, 1851394, 3211275, 262188, 1048755, 717, 2871, 11485\}$.

For glider *g*, the determinative system $\mathcal{A}_7 = \{906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917106, 3474123, 1313582, 1060025, 45796, 183184, 732736, 2930946, 3335179, 757804, 3031219, 3736269, 2362167, 1313588, 1060050, 45896, 183584, 734336, 2937346, 3360779, 860204, 3440819, 1180365, 527159, 2108637, 1060060, 45937, 183748, 734994, 2939978, 3371307, 902316, 3609267, 1854157, 3222327, 306397, 1225589, 708052, 2832208\}$.

For glider g_2, the determinative system $\mathcal{A}_8 = \{906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917106, 3474120, 1313568, 1059971, 45583, 182333, 729333, 2917332, 3280720, 539968, 2159874, 250891, 1003564, 4014259, 1313588, 1060050, 45899, 183598, 734393, 2937572, 3361680, 863808, 3455234, 1238027, 757804, 3031219, 3736269, 2362167, 1060060, 45937, 183748, 734994, 2939976, 3371296, 902272, 3609090, 1853451, 3219500, 295091, 1180365, 527159, 2108637\}$.

For glider g_3, the determinative system $\mathcal{A}_9 = \{906251$, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917106, 3474120, 1313568, 1059968, 45570, 182283, 729132, 2916531, 3277517, 527159, 2108637, 1313588, 1060050, 45898, 183595, 734380, 2937523, 3361485, 863031, 3452125, 1225589, 708052, 2832208, 1060060, 45937, 183748, 734992, 2939971, 3371277, 902199, 3608797, 1852277, 3214804, 276304, 1105216, $226562\}$.

For glider h, the determinative system $\mathcal{A}_{10} = \{906251$, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313538, 1059850, 45098, 180394, 721576, 2886304, 3156610, 43531, 174124, 696499, 2785997, 2755383, 2632925, 2143093, 3474124, 1313586, 1060043, 45868, 183472, 733888, 2935552, 3353602, 831499, 3325996, 721075, 2884301, 3148599, 11485, 45940, 183763, 735052, 2940210, 3372234, 906027, 3624108, 1913523, 3459789, 1256247, 830685, 3322741, 708052, $2832208\}$.

For glider i, the determinative system $\mathcal{A}_{11} = \{906251$, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917106, 3474120, 1313568, 1059968, 45568, 182275, 729100, 2916403, 3277005, 525111, 2100445, 13173, 52692, 210768, 843072, 1313588, 1060050, 45898, 183594, 734379, 2937519, 3361469, 862967, 3451869, 1224565, 703956, 2815824, 2874688, 3110146, 4051979, 1060060, 45937, 183748, 734992, 2939968, 3371266, 902153, 3608613, 1851541, 3211860, 264528, 1058112, 38146, 152587, 610348, 2441395, $1376973\}$.

For glider i_2, the determinative system $\mathcal{A}_{12} = \{906251$, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917106, 3474120, 1313568, 1059968, 45568, 182275, 729100, 2916400, 3276992, 525058, 2100235, 12332, 49331, 197325, 789303, 3157213, 1313588, 1060050, 45898, 183594, 734379, 2937519, 3361468, 862962, 3451851, 1224492, 703667, 2814669, 2870071, 3091677, 3978101, 3329492, 1060060, 45937, 183748, 734992, 2939968, 3371266, 902153, 3608613, 1851540, 3211859, 264525, 1058103, 38109, 152437, 609748, 2438992, 1367360, $1275138\}$.

For glider j, the determinative system $\mathcal{A}_{13} = \{906251$, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917106, 3474120, 1313568, 1059968, 45569, 182276, 729104, 2916416, 3277059, 525324, 2101296, 16578, 66313, 265252, 1061008, 49728, 198914, 795659, 3182636, 147635, 590541, 2362167, 1313588, 1060050, 45898, 183594, 734376, 2937504, 3361410, 862731, 3450927, 1220796, 688883, 2755532, 2633520, 2145472, 193282, 773131, 3092524, 3981491, 3343053, 789303, 3157213, 1060060, 45937, 183748, 734992, 2939968, 3371266, 902155, 3608620, 1851570, 3211977, 264997, 1059991, 45660, 182642, 730571, 2922284, 3300531, 619213, 2476855, 1518813, 1880949, $3329492\}$.

For glider k, the determinative system $\mathcal{A}_{14} = \{906251$, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313537, 1059845, 45077, 180308, 721235, 2884940, 3151155, 21711, 86845, 347381, 1389524, 1363792, 1260864, 849154, 3396619, 1003564, 4014259, 3474124, 1313586, 1060042, 45864, 183456, 733824, 2935297, 3352583, 827421, 3309686, 655833, 2623332, 2104720, 30272, 121090, 484363, 1937452, 3555507, 1639117, 2362167, 45940, 183760, 735042, 2940170, 3372074, 905385, 3621541, 1903253, 3418708, 1091920, 173376, 693506, 2774027, 2707500, 2441395, $1376973\}$.

For glider k_2, the determinative system $\mathcal{A}_{15} = \{906251$, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917106, 3474121, 1313572, 1059984, 45634, 182539, 730156, 2920624, 3293888, 592640, 2370560, 1093634, 180235, 720940, 2883763, 3146445, 2871, 11485, 1313588, 1060048, 45888, 183555, 734220, 2936883, 3358924, 852786, 3411146, 1061674, 52395, 209580, 838323, 3353293, 830263, 3321053, 701301, 2805204, 2832208, 1060060, 45936, 183746, 734987, 2939951, 3371197, 901879, 3607517, 1847156, 3194320, 194368, 777475, 3109901, 4050999, 3621085, 1901429, 3411412, 1062736, 56640, 226562}.

For glider *l*, the determinative system $\mathcal{A}_{16} = \{$906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313536, 1059843, 45068, 180274, 721099, 2884396, 3148976, 12995, 51981, 207927, 831709, 3326837, 724436, 2897744, 3202368, 226562, 3474124, 1313586, 1060042, 45867, 183471, 733885, 2935540, 3353555, 831308, 3325235, 718031, 2872125, 3099893, 4010964, 3460944, 1260864, 849154, 3396619, 1003564, 4014259, 45940, 183760, 735042, 2940169, 3372068, 905361, 3621447, 1902877, 3417206, 1085913, 149348, 597392, 2389568, 1169666, 484363, 1937452, 3555507, 1639117, 2362167}.

For glider *m*, the determinative system $\mathcal{A}_{17} = \{$906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917106, 3474120, 1313568, 1059968, 45569, 182276, 729104, 2916416, 3277057, 525319, 2101277, 16500, 66000, 264000, 1056000, 29698, 118793, 475172, 1900688, 3408450, 1050891, 9260, 37043, 148173, 592695, 2370781, 1094517, 1313588, 1060050, 45898, 183594, 734376, 2937504, 3361410, 862730, 3450921, 1220773, 688788, 2755152, 2632002, 2139402, 169003, 676012, 2704048, 2427584, 1321731, 1092620, 176179, 704717, 2818871, 2886877, 3158901, 52692, 210768, 843072, 1060060, 45937, 183748, 734992, 2939968, 3371266, 902155, 3608620, 1851568, 3211968, 264960, 1059840, 45059, 180236, 720944, 2883779, 3146508, 3122, 12491, 49967, 199869, 799479, 3197917, 208757, 835028, 3340112, 777536, 3110146, 4051979}.

For glider *n*, the determinative system $\mathcal{A}_{18} = \{$906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917106, 3474120, 1313568, 1059968, 45568, 182275, 729103, 2916413, 3277046, 525273, 2101092, 15760, 63040, 252162, 1008650, 4034602, 3555496, 1639072, 2361986, 1059339, 43052, 172211, 688845, 2755383, 2632925, 2143093, 1313588, 1060050, 45898, 183594, 734379, 2937518, 3361465, 862949, 3451796, 1224272, 702784, 2811138, 2855947, 3035180, 3752112, 2425536, 1313538, 1059851, 45100, 180403, 721613, 2886455, 3157213, 1060060, 45937, 183748, 734992, 2939968, 3371266, 902152, 3608608, 1851520, 3211776, 264194, 1056779, 32812, 131251, 525004, 2100018, 11467, 45868, 183475, 733901, 2935607, 3353821, 832373, 3329492}.

For glider *o*, the determinative system $\mathcal{A}_{19} = \{$906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313537, 1059845, 45076, 180304, 721219, 2884879, 3150909, 20725, 82900, 331600, 1326400, 1111298, 250891, 1003564, 4014259, 3474124, 1313586, 1060042, 45864, 183456, 733824, 2935299, 3352590, 827449, 3309796, 656272, 2625088, 2111746, 58379, 233516, 934067, 3736269, 2362167, 45940, 183760, 735042, 2940170, 3372075, 905390, 3621560, 1903328, 3419008, 1093122, 178187, 712748, 2850995, 3015373, 3672887, 2108637}.

For glider *p*, the determinative system $\mathcal{A}_{20} = \{$906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917106, 3474120, 1313568, 1059968, 45568, 182275, 729103, 2916412, 3277040, 525250, 2101003, 15404, 61619, 246477, 985911, 3943645, 3191669, 1313588, 1060050, 45898, 183594, 734379, 2937518, 3361465, 862948, 3451795, 1224268, 702771, 2811085, 2855735, 3034333, 3748725, 2411988, 1259344, 843072, 1060060, 45937, 183748, 734992, 2939968, 3371266, 902152, 3608608, 1851521, 3211783, 264221, 1056887, 33245, 132981, 531924, 2127696, 122176, 488706, 1954827}.

For glider p_2, the determinative system $\mathcal{A}_{21} = \{$906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313536, 1059840, 45056, 180226, 720907, 2883628, 3145904, 704, 2816, 11267, 45068, 180272, 721088, 2884352, 3148800, 12290, 49163, 196652, 786611, 3146445, 2871, 11485, 3474124, 1313586, 1060042, 45866, 183466, 733867, 2935468, 3353267, 830156, 3320626, 699594, 2798379, 2804911, 2831036, 2935538, 3353546, 831274, 3325099, 717484, 2869939, 3091149, 3975991, 3321053, 701301, 2805204, 2832208, 45940, 183760, 735040, 2940160, 3372035, 905229, 3620919, 1900765, 3408756, 1052112, 14146, 56585, 226341, 905364, 3621456, 1902912, 3417347, 1086477, 151607, 606429, 2425717, 1314260, 1062736, 56640, 226562}.

For glider *q*, the determinative system $\mathcal{A}_{22} = \{$906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917106, 3474120, 1313568, 1059968, 45568, 182275, 729103, 2916413, 3277046, 525273, 2101092, 15760, 63040, 252162, 1008650, 4034602, 3555498, 1639083, 2362031, 1059516, 43762, 175051, 700204, 2800819, 2814669, 2870071, 3091677, 3978101, 3329492, 1313588, 1060050, 45898, 183594, 734379, 2937518, 3361465, 862949, 3451796, 1224272, 702784, 2811138, 2855947, 3035180, 3752112, 2425536, 1313536, 1059842, 45065, 180261, 721044, 2884179, 3148109, 9527, 38109, 152437, 609748, 2438992, 1367360, 1275138, 1060060, 45937, 183748, 734992, 2939968, 3371266, 902152, 3608608, 1851520, 3211776, 264194, 1056779, 32812, 131251, 525004, 2100018, 11466, 45867, 183468, 733872, 2935488, 3353345, 830471, 3321885, 704629, 2818516, 2885456, 3153216, 29954, 119819, 479276}.

For glider *ab*, the determinative system $\mathcal{A}_{23} = \{$ 906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474115, 1313548, 1059890, 45259, 181039, 724157, 2896629, 3197908, 208720, 834880, 3339522, 775179, 3100716, 4014259, 3474124, 1313587, 1060047, 45885, 183540, 734162, 2936649, 3357988, 849040, 3396160, 1001730, 4006923, 3444780, 1196211, 590541, 2362167, 45942, 183769, 735076, 2940305, 3372612, 907536, 3630144, 1937666, 3556363, 1642540, 2375859, 1114829, 265015}.

For glider *ac*, the determinative system $\mathcal{A}_{24} = \{$906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474115, 1313548, 1059890, 45259, 181036, 724147, 2896589, 3197751, 208093, 832373, 3329492, 3474124, 1313587, 1060047, 45885, 183540, 734163, 2936653, 3358007, 849117, 3396469, 1002964, 4011856, 3464512, 1275138, 45942, 183769, 735076, 2940305, 3372615, 907549, 3630197, 1937876, 3557200, 1645888, 2389250, 1168395, 479276}.

For glider *ad*, the determinative system $\mathcal{A}_{25} = \{$906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 3372288, 906242, 3624971, 1916975, 3473597, 1311479, 1051613, 12149, 48596, 194384, 777536, 3110146, 4051979, 906250, 3625003, 1917100, 3474098, 1313481, 1059621, 44181, 176724, 706896, 2827584, 2921730, 3298315, 610348, 2441395, 1376973, 1917104, 3474115, 1313549, 1059892, 45264, 181056, 724224, 2896896, 3198978, 213003, 852012, 3408051, 1049293, 2871, 11485}.

For glider *ae*, the determinative system $\mathcal{A}_{26} = \{$906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474115, 1313548, 1059890, 45258, 181035, 724140, 2896563, 3197645, 207671, 830685, 3322741, 708052, 2832208, 3474124, 1313587, 1060047, 45885, 183540, 734160, 2936643, 3357965, 848951, 3395805, 1000309, 4001236, 3422032, 1105216, 226562, 45942, 183769, 735076, 2940304, 3372611, 907535, 3630141, 1937653, 3556308, 1642320, 2374976, 1111298, 250891, 1003564, 4014259}.

For glider *ba*, the determinative system $\mathcal{A}_{27} = \{$906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313539, 1059852, 45107, 180431, 721725, 2886901, 3158996, 53072, 212288, 849154, 3396619, 1003564, 4014259, 3474124, 1313586, 1060043, 45871, 183485, 733942, 2935769, 3354468, 834960, 3339840, 776450, 3105803, 4034604, 3555507, 1639117, 2362167, 45940, 183762, 735049, 2940197, 3372180, 905808, 3623232, 1910018, 3445771, 1200172, 606387, 2425549}.

For glider *bd*, the determinative system $\mathcal{A}_{28}$ = {906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313539, 1059852, 45106, 180426, 721706, 2886824, 3158688, 51842, 207371, 829484, 3317939, 688845, 2755383, 2632925, 2143093, 3474124, 1313586, 1060043, 45871, 183485, 733940, 2935760, 3354432, 834818, 3339275, 774188, 3096755, 3998413, 3410743, 45940, 183762, 735049, 2940196, 3372176, 905794, 3623179, 1909804, 3444915, 1196749, 592695, 2370781, 1094517}.

For glider *be*, the determinative system $\mathcal{A}_{29}$ = {906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313539, 1059852, 45106, 180426, 721707, 2886828, 3158707, 51917, 207671, 830685, 3322741, 708052, 2832208, 3474124, 1313586, 1060043, 45871, 183485, 733940, 2935760, 3354435, 834829, 3339319, 774365, 3097461, 4001236, 3422032, 1105216, 226562, 45940, 183762, 735049, 2940196, 3372176, 905795, 3623183, 1909821, 3444981, 1197012, 593744, 2374976, 1111298, 250891, 1003564, 4014259}.

For glider *ca*, the determinative system $\mathcal{A}_{30}$ = {906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 906250, 3625002, 1917096, 3474080, 1313410, 1059338, 43051, 172204, 688819, 2755277, 2632503, 2141405, 177013, 708052, 2832208, 1917104, 3474112, 1313538, 1059851, 45100, 180400, 721603, 2886413, 3157047, 45277, 181109, 724436, 2897744, 3202368, 226562, 3474124, 1313586, 1060043, 45868, 183475, 733900, 2935603, 3353807, 832317, 3329269, 734164, 2936656, 3358016, 849154, 3396619, 1003564, 4014259}.

For glider *cb*, the determinative system $\mathcal{A}_{31}$ = { 906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313538, 1059851, 45100, 180400, 721600, 2886403, 3157005, 45111, 180445, 721781, 2887124, 3159888, 56640, 226562, 3474124, 1313586, 1060043, 45868, 183475, 733900, 2935602, 3353803, 832303, 3329213, 733941, 2935764, 3354448, 834880, 3339522, 775179, 3100716, 4014259, 45940, 183763, 735053, 2940215, 3372253, 906100, 3624402, 1914697, 3464484, 1275024, 905792, 3623170, 1909771, 3444780, 1196211, 590541, 2362167 }.

For glider *da*, the determinative system $\mathcal{A}_{32}$ = {906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313536, 1059841, 45060, 180240, 720960, 2883840, 3146754, 4107, 16428, 65715, 262861, 1051447, 11485, 3474124, 1313586, 1060042, 45866, 183464, 733856, 2935426, 3353098, 829483, 3317932, 688819, 2755277, 2632503, 2141405, 177013, 708052, 2832208, 45940, 183760, 735040, 2940162, 3372043, 905260, 3621040, 1901251, 3410701, 1059895, 45277, 181109, 724436, 2897744, 3202368, 226562}.

For glider *db*, the determinative system $\mathcal{A}_{33}$ = {906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313536, 1059841, 45060, 180240, 720960, 2883840, 3146752, 4098, 16395, 65580, 262323, 1049293, 2871, 11485, 3474124, 1313586, 1060042, 45866, 183464, 733856, 2935426, 3353098, 829482, 3317931, 688812, 2755251, 2632397, 2140983, 175325, 701301, 2805204, 2832208, 45940, 183760, 735040, 2940162, 3372043, 905260, 3621040, 1901248, 3410691, 1059853, 45111, 180445, 721781, 2887124, 3159888, 56640, 226562}.

For glider *dc*, the determinative system $\mathcal{A}_{34}$ = {906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313536, 1059841, 45060, 180240, 720960, 2883840, 3146753, 4100,

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16400, 65600, 262402, 1049611, 4140, 16563, 66253, 265015, 3474124, 1313586, 1060042, 45866, 183464, 733856, 2935426, 3353098, 829482, 3317928, 688800, 2755202, 2632203, 2140204, 172211, 688845, 2755383, 2632925, 2143093, 45940, 183760, 735040, 2940162, 3372043, 905260, 3621040, 1901248, 3410690, 1059851, 45100, 180403, 721613, 2886455, 3157213}.

For glider *de*, the determinative system $\mathcal{A}_{35} =${906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313536, 1059841, 45060, 180240, 720960, 2883840, 3146752, 4097, 16388, 65552, 262208, 1048834, 1035, 4140, 16563, 66253, 265015, 3474124, 1313586, 1060042, 45866, 183464, 733856, 2935426, 3353098, 829482, 3317930, 688808, 2755232, 2632322, 2140683, 174124, 696499, 2785997, 2755383, 2632925, 2143093, 45940, 183760, 735040, 2940162, 3372043, 905260, 3621040, 1901248, 3410688, 1059842, 45067, 180268, 721075, 2884301, 3148599, 11485}.

For glider *ea*, the determinative system $\mathcal{A}_{36} =${ 906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313536, 1059842, 45067, 180268, 721072, 2884291, 3148557, 11319, 45277, 181109, 724436, 2897744, 3202368, 226562, 3474124, 1313586, 1060042, 45867, 183468, 733875, 2935500, 3353395, 830671, 3322685, 707829, 2831316, 2936656, 3358016, 849154, 3396619, 1003564, 4014259, 45940, 183760, 735043, 2940173, 3372087, 905437, 3621750, 1904089, 3422052, 1105296, 226880, 907522, 3630091, 1937452, 3555507, 1639117, 2362167}.

For glider *eb*, the determinative system $\mathcal{A}_{37} =${ 906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313536, 1059842, 45067, 180268, 721072, 2884288, 3148547, 11277, 45111, 180445, 721781, 2887124, 3159888, 56640, 226562, 3474124, 1313586, 1060042, 45867, 183468, 733875, 2935500, 3353394, 830667, 3322671, 707773, 2831093, 2935764, 3354448, 834880, 3339522, 775179, 3100716, 4014259, 45940, 183760, 735043, 2940173, 3372087, 905437, 3621748, 1904082, 3422025, 1105188, 226448, 905792, 3623170, 1909771, 3444780, 1196211, 590541, 2362167}.

For glider *ec*, the determinative system $\mathcal{A}_{38} =${ 906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313536, 1059842, 45067, 180268, 721072, 2884288, 3148546, 11275, 45100, 180403, 721613, 2886455, 3157213, 3474124, 1313586, 1060042, 45867, 183468, 733875, 2935500, 3353394, 830667, 3322668, 707763, 2831053, 2935607, 3353821, 832373, 3329492, 45940, 183760, 735043, 2940173, 3372087, 905437, 3621748, 1904083, 3422029, 1105207, 226525, 906101, 3624404, 1914704, 3464512, 1275138}.

For glider *ed*, the determinative system $\mathcal{A}_{39} =${ 906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313536, 1059842, 45067, 180268, 721074, 2884297, 3148581, 11413, 45652, 182608, 730432, 2921730, 3298315, 610348, 2441395, 1376973, 3474124, 1313586, 1060042, 45867, 183468, 733875, 2935501, 3353396, 830672, 3322688, 707840, 2831360, 2936834, 3358731, 852012, 3408051, 1049293, 2871, 11485, 45940, 183760, 735043, 2940173, 3372087, 905436, 3621744, 1904066, 3421962, 1104938, 225451, 901804, 3607219, 1845965, 3189559, 175325, 701301, 2805204, 2832208}.

For glider *fg*, the determinative system $\mathcal{A}_{40} =${ 906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917106, 3474120, 1313568, 1059968, 45569, 182279, 729116, 2916465, 3277252, 526098, 2104394, 28971, 115884, 463539, 1854157, 3222327, 306397, 1225589, 708052, 2832208, 1313588, 1060050, 45898, 183594, 734377, 2937509, 3361428, 862800, 3451201, 1221892, 693264, 2773059, 2703629, 2425911, 1315037, 1065845,

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69076, 276304, 1105216, 226562, 1060060, 45937, 183748, 734992, 2939968, 3371264, 902144, 3608576, 1851394, 3211272, 262176, 1048707, 527, 2109, 8437, 33748, 134992, 539968, 2159874, 250891, 1003564, 4014259}.

For glider g_2g, the determinative system $\mathcal{A}_{41} = \{$ 906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917106, 3474120, 1313568, 1059971, 45583, 182332, 729329, 2917316, 3280658, 539722, 2158891, 246956, 987827, 3951309, 3222327, 306397, 1225589, 708052, 2832208, 1313588, 1060050, 45899, 183598, 734393, 2937572, 3361680, 863809, 3455236, 1238032, 757827, 3031309, 3736631, 2363613, 1065845, 69076, 276304, 1105216, 226562, 1060060, 45937, 183748, 734994, 2939976, 3371296, 902272, 3609090, 1853448, 3219488, 295043, 1180175, 526397, 2105589, 33748, 134992, 539968, 2159874, 250891, 1003564, 4014259}.

For glider gf, the determinative system $\mathcal{A}_{42} = \{$ 906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917106, 3474120, 1313568, 1059968, 45570, 182283, 729134, 2916537, 3277541, 527252, 2109011, 47437, 189751, 759005, 3036021, 3755476, 2438992, 1367360, 1275138, 1313588, 1060050, 45898, 183595, 734380, 2937522, 3361480, 863008, 3452032, 1225217, 706567, 2826269, 2916469, 3277268, 526160, 2104640, 29954, 119819, 479276, 1060060, 45937, 183748, 734992, 2939971, 3371277, 902196, 3608786, 1852234, 3214634, 275625, 1102501, 215700, 862800, 3451200, 1221890, 693259, 2773036, 2703539, 2425549}.

For glider hb, the determinative system $\mathcal{A}_{43} = \{$ 906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313538, 1059850, 45098, 180394, 721576, 2886304, 3156610, 43530, 174122, 696491, 2785964, 2755251, 2632397, 2140983, 175325, 701301, 2805204, 2832208, 3474124, 1313586, 1060043, 45868, 183472, 733888, 2935552, 3353602, 831499, 3325996, 721072, 2884288, 3148547, 11277, 45111, 180445, 721781, 2887124, 3159888, 56640, 226562, 45940, 183763, 735052, 2940210, 3372234, 906027, 3624108, 1913523, 3459788, 1256242, 830667, 3322671, 707773, 2831093, 2935764, 3354448, 834880, 3339522, 775179, 3100716, 4014259}.

For glider hc, the determinative system $\mathcal{A}_{44} = \{$ 906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474114, 1313546, 1059883, 45230, 180920, 723680, 2894720, 3190274, 178186, 712744, 2850976, 3015296, 3672578, 2107403, 41004, 164019, 656077, 2624311, 2108637, 3474124, 1313587, 1060044, 45872, 183490, 733960, 2935842, 3354762, 836139, 3344556, 795312, 3181250, 142090, 568363, 2273452, 705203, 2820813, 2894647, 3189981, 177013, 708052, 2832208, 45943, 183772, 735091, 2940364, 3372851, 908492, 3633968, 1952963, 3617548, 1887283, 3354828, 836400, 3345603, 799501, 3198007, 209117, 836469, 3345876, 800592, 3202368, 226562}.

For glider ada, the determinative system $\mathcal{A}_{45} = \{$ 906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474115, 1313549, 1059892, 45264, 181056, 724224, 2896896, 3198978, 213002, 852011, 3408044, 1049267, 2765, 11063, 44253, 177013, 708052, 2832208, 3474124, 1313587, 1060047, 45884, 183536, 734146, 2936586, 3357738, 848043, 3392172, 985776, 3943107, 3189517, 175159, 700637, 2802549, 2821588, 2897744, 3202368, 226562, 45942, 183769, 735076, 2940307, 3372620, 907568, 3630272, 1938179, 3558412, 1650739, 2408655, 1246013, 789749, 3158996, 53072, 212288, 849154, 3396619, 1003564, 4014259}.

For glider $aebbb$, the determinative system $\mathcal{A}_{46} = \{$ 906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474115, 1313549, 1059892, 45267, 181068, 724272, 2897091, 3199756, 216114, 864459, 3457839, 1248445, 799476, 3197906, 208713, 834852, 3339409, 774724, 3098896, 4006976, 3444994, 1197067, 593964, 2375859, 1114829, 265015, 3474124, 1313587, 1060047, 45884, 183537, 734151, 2936604, 3357811, 848335, 3393341, 990452, 3961810, 3264329, 474404, 1897617, 3396164, 1001744, 3444992, 1197056, 593922, 2375691, 1114156, 262323, 1049293, 2871, 11485, 45942, 183769, 735076, 2940305, 3372613, 907541, 3630166, 1937753, 3556708, 1643921, 2381380, 1136912, 353344, 1413376, 1459200, 1642498, 2375690, 1114154, 262315, 1049260, 2739, 10957, 43831, 175325, 701301, 2805204, 2832208 }.

For glider *bae*, the determinative system $\mathcal{A}_{47} = \{$ 906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313539, 1059852, 45107, 180431, 721724, 2886897, 3158983, 53020, 212083, 848335, 3393341, 990453, 3961812, 3264336, 474432, 1897730, 3396619, 1003564, 4014259, 3474124, 1313586, 1060043, 45871, 183485, 733942, 2935769, 3354468, 834961, 3339845, 776469, 3105878, 4034905, 3556708, 1643920, 2381376, 1136898, 353291, 1413164, 1458355, 1639117, 2362167, 45940, 183762, 735049, 2940197, 3372180, 905808, 3623232, 1910016, 3445760, 1200129, 606212, 2424848, 1310784, 1048834, 1035, 4140, 16563, 66253, 265015}.

For glider *beb*, the determinative system $\mathcal{A}_{48} = \{$ 906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 735057, 2940228, 3372304, 906304, 3625216, 1917952, 3477505, 1327108, 1114128, 262208, 1048832, 1024, 4098, 16395, 65580, 262323, 1049293, 2871, 11485, 3372288, 906240, 3624962, 1916938, 3473450, 1310890, 1049256, 2720, 10882, 43530, 174122, 696491, 2785964, 2755251, 2632397, 2140983, 175325, 701301, 2805204, 2832208, 906250, 3625002, 1917099, 3474092, 1313456, 1059520, 43776, 175106, 700427, 2801708, 2818224, 2884288, 3148547, 11277, 45111, 180445, 721781, 2887124, 3159888, 56640, 226562}.

For glider *daa*, the determinative system $\mathcal{A}_{49} = \{$ 906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313536, 1059841, 45060, 180240, 720960, 2883840, 3146754, 4106, 16427, 65708, 262835, 1051341, 11063, 44253, 177013, 708052, 2832208, 3474124, 1313586, 1060042, 45866, 183464, 733856, 2935426, 3353098, 829483, 3317932, 688816, 2755267, 2632461, 2141239, 176349, 705397, 2821588, 2897744, 3202368, 226562, 45940, 183760, 735040, 2940162, 3372043, 905260, 3621040, 1901251, 3410700, 1059891, 45263, 181053, 724213, 2896852, 3198800, 212288, 849154, 3396619, 1003564, 4014259}.

For glider *eaa*, the determinative system $\mathcal{A}_{50} = \{$ 906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917104, 3474112, 1313536, 1059842, 45067, 180268, 721072, 2884291, 3148556, 11315, 45263, 181053, 724213, 2896852, 3198800, 212288, 849154, 3396619, 1003564, 4014259, 3474124, 1313586, 1060042, 45867, 183468, 733875, 2935500, 3353395, 830671, 3322685, 707830, 2831321, 2936676, 3358096, 849472, 3397890, 1008651, 4034604, 3555507, 1639117, 2362167, 45940, 183760, 735043, 2940173, 3372087, 905437, 3621750, 1904089, 3422053, 1105300, 226896, 907584, 3630338, 1938443, 3559468, 1654963, 2425549}.

For glider *fgf*, the determinative system $\mathcal{A}_{51} = \{$ 906251, 3625004, 1917107, 3474125, 1313591, 1060061, 45941, 183764, 735056, 2940224, 3372290, 1917106, 3474120, 1313568, 1059968, 45569, 182279, 729116, 2916465, 3277252, 526098, 2104394, 28971, 115884, 463539, 1854156, 3222322, 306378, 1225515, 707759, 2831037, 2935541, 3353556, 831312, 3325248, 718082, 2872331, 3100716, 4014259, 1313588, 1060050, 45898, 183594, 734377, 2937509, 3361428, 862800, 3451201, 1221892, 693264, 2773059, 2703629, 2425911, 1315037, 1065844, 69072, 276290, 1105161, 226340, 905360, 3621440, 1902850, 3417099, 1085484, 147635, 590541, 2362167, 1060060, 45937, 183748, 734992, 2939968, 3371264, 902144, 3608576, 1851394, 3211272, 262176, 1048707, 527, 2109, 8437, 33748,

https://doi.org/10.25088/ComplexSystems.24.3.185

134992, 539971, 2159884, 250928, 1003712, 4014850, 3476491, 1323052, 1097907, 197325, 789303, 3157213}.

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