# How External Environment and Internal Structure Change the Behavior of Discrete Systems

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This paper continues our computer studies using virtual systems to examine the behavior of subsystems in a system as a model of the behavior of social systems made up of individuals. The subsystems in our virtual system are global cellular automata (GCAs) as suggested by Wolfram [1], placed at the vertices of a GCA network (GCAN) as developed by Chandler [2]. The behavioral results are based on the four classes of cellular automata output patterns as identified by Wolfram [3] and are measured by the fraction of ordered GCAs in a GCAN.

Our objective has been to show how our theory of social dynamics explains this behavior. That theory states that the behavior of a social system and of our virtual systems model depends upon the external environment of the system defined as centrality and the internal structure of the system defined as the four parameters of differentiation, namely, diversity, connectedness, interdependence, and adaptability of the subsystems in the system. In previous papers we have shown the effect of diversity and connectedness. In this paper we show that behavior becomes more ordered and focused as interdependence and adaptability increase and as centrality decreases.

## 1. Introduction

This paper continues our study of the variables that change the behavior of virtual discrete systems. Our studies are motivated by the fact that social systems are also discrete systems—systems made up of individual subsystems that change incrementally (not continuously) as a result of their interaction with each other. Therefore, studying virtual discrete systems (models with individual cells) can lead to insights into the causes of change in real social systems.

We continue to publish the application of this theory to real social systems, including corporations [4, pp. 108–165], indigenous social systems [5], the evolution of art [4, pp. 165–203], and the history of


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American jazz [6]. We have also published two papers on virtual systems, which test part of our theory [7, 8]. We will discuss in this paper the implications of all our experiments using virtual systems to test our theory.

## 2. The Differentiation/Centrality Theory of System Change

The ability to study virtual discrete systems depends fundamentally on the work of Wolfram that showed that the behavior of such systems falls into four and only four classes—chaos, complexity, and two kinds of order [3]. In our theory, we use the word *focus* to describe the continuum from chaos to order. As output patterns of virtual systems become more fixed and predictable, that is, more ordered, we see them as more focused. Similarly, the behavior of social systems becomes more focused as they move from chaos to order.

We have developed the d/c theory to explain why systems become more focused. The theory incorporates two social science variables: differentiation (d) and centrality (c). Differentiation stands for the internal structure of the system. Centrality describes the external environment of the system. Our theory is that greater focus, that is, the change from chaos through complexity to order, is determined by the ratio of these two variables: differentiation/centrality, or d/c [4, p. 51]. As differentiation increases and centrality decreases, the system becomes more focused.

Differentiation, the system's internal ability to cope with its external environment, can be broken down into four parameters, originally applied by Page [9] as parameters for describing complex systems, but used by us to measure differentiation. The four parameters are diversity, connectedness (networks), interdependence, and adaptability among the individuals in the system. The d/c ratio predicts that the system should move toward order, become more focused, as each of the differentiation parameters increases, provided that centrality is held constant.

Centrality is the measure of the external environment of the system and can be thought of as the variety of information presented to the system. Increasing centrality makes the system *less* focused and moves the system back from order toward chaos—as long as differentiation does not change. The system becomes overwhelmed by the increase in external information.

Combining the effects of the two variables of differentiation and centrality, we can say that the behavioral state of a system depends upon d/c.

In an earlier paper we used virtual systems to model the effect on focus of increases in diversity, the first differentiation parameter, and


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confirmed that there is a positive relationship between diversity and focus [7].

In a second paper derived from our experiments with virtual systems, we investigated the relationship of connectedness to focus. Since there are many different network structures, the relationship between connectedness and focus is more complicated. For so-called random networks, order increases when the number of connections for each node *decreases*. In general, however, we found that order increases as hierarchical structure increases [8].

In each case, centrality was kept constant as the two parameters of diversity and connectedness were altered. The two papers confirmed that our virtual systems moved toward order, with increasing differentiation as the system becomes more and more capable of handling outside information.

In this paper we use virtual discrete models to continue our study of what happens to focus when the other two parameters of differentiation—interdependence and adaptability—increase. In addition, we examine the effect on focus of increasing centrality.

We use global cellular automata networks (GCANs) made up of interacting global cellular automata (GCAs), as described in detail below.

Based on the d/c theory, we expect that order will increase with increasing interdependence and ability to adapt but will decrease as we increase centrality.

In the next section we outline our methodology.

## 3. Using Global Cellular Automata Networks to Examine Adaptability, Centrality, and Interdependence

Our studies are based fundamentally on cellular automata (CAs). A simple one-dimensional CA uses one of the 256 rules that determine how a given two-valued (two colors) cell changes its color based on its current color and that of its immediate neighbors on its left and right. The CA begins with a row of these cells, and each cell determines its color for the next time step using the specified rule. For example, one rule for a black cell might be that if the two adjoining cells are black, the cell remains black on the next step. All cells change simultaneously at each time step. The pattern developed after a number of these time steps reveals the behavioral state of the CA and can be observed to vary in focus from chaos to complexity to the two types of order. Wolfram presents a detailed examination of these CAs and the resulting four classes generated [3, p. 231].

There are many other more complicated CAs, including multivalued (colored) cells with large numbers of rules and CAs whose rules may involve more than just the two nearest neighbors. For all of these, Wolfram found the same four classes.

In order to test the effect on focus when adaptability, centrality, and interdependence are increased, we used GCANs, a network consisting of many GCAs. A GCA is a one-dimensional CA that contains two or more rules rather than just one rule and has a method of determining which rule applies at each time step [1].

In a GCAN as developed by Chandler [2], the GCA selects which rule to use based on information from the other GCAs to which it is connected in the GCAN. Each GCA can be thought of as an individual subsystem in the GCAN network.

In our experiments on the differentiation parameters of adaptability and interdependence, we used GCAs in which individual cells can be more than two colors (representing wider adaptability, as explained below), and in which each individual cell in the GCA can be affected by more than just the cell's nearest neighbors (increasing interdependence).

As the number of colors or the number of neighbors increases, the number of rules increases dramatically. If $k$ is the number of colors and $r$ is the number of neighbors on each side, the number of rules is $k^{k^{(2r+1)}}$. Thus, for simple CAs with two colors, dependent on only nearest neighbors, the number of rules is 256, but for three colors there are 7 625 597 484 987 possible rules!

If we do not consider the location of the cells relative to each other and just consider the average color of the cells under consideration, the so-called totalistic approach [3 p. 60], we greatly reduce the number of rules to $k^{(1+(k-1)(2r+1))}$. If $r$, the number of neighbors on each side, is 1, then for two colors there are 16 totalistic rules, for three colors there are 2187, and for four colors there are 1 048 576. In all the GCA work presented here, we use the totalistic approach.

The GCAN we used was made up of 300 GCAs connected in a complete network, that is, a network in which every GCA is connected to all other GCAs. Of all network types, this network is the most likely to produce chaos [8].

Each GCA has two rules randomly selected from a set of all possible totalistic rules for the given number of colors and the number of neighbors. In the study of adaptability and centrality, we use only nearest neighbors $(r = 1)$. For each time step, the GCA selects which rule to use based on information from all the GCAs connected to it, in this case, all of the GCAs in the complete network. The procedure involves the repeated use of rule 30, the so-called random number generating rule, with the initial condition being the midpoint of all the connected GCAs [2, 7].


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In the next sections we explain how the GCAN is modified to test for the effects of changing interdependence, adaptability, and centrality.

## 3.1 Adaptability

We are unable to dynamically model adaptation itself, so we propose that increasing the number of alternative value choices for individual cells within a GCA is the same as giving it more ability to adapt. Hence, we modify the experiment by comparing results for GCAs with varying numbers of colors to show the effect of adaptability on changes in behavioral focus.

In each experimental run for a certain number of colors, we randomly select the two rules to be used by each GCA from a fraction of the maximum number of possible rules. The maximum number of rules for a given number $k$ of colors for totalistic GCAs is given by the formula $k^{[1+(k-1)(2r+1)]}$ for $r$ neighbors on each side. This population can still get very large as the number of colors increases. For three colors and $r$ equal to one, it is 2187, but for four colors it is 1 048 576. We randomly select the rules for each GCA from sets of various sizes taken from the entire pool of available rules.

We then run our GCAN so that each GCA takes 150 time steps and examine the output of each GCA to determine if it is ordered or chaotic. That is, we combine both Wolfram classes of order, and combine complex and chaotic classes. We then determine the fraction of ordered GCAs among the 300 GCAs in the GCAN. We wish to see what effect increasing the number of colors (adaptability) has on the fraction ordered.

As adaptability (number of colors) increases, we expect order to increase.

## 3.2 Centrality

For each set of GCA colors, we are able to vary the size of the pool of numbers from which rules are chosen for each GCA up to the maximum. Since this pool of rules is the environment of the GCAN, it represents centrality, which is defined as the information presented to a system. In selecting pools of increasing size, we are presenting the system, each GCA, with increasing centrality for a fixed number of colors (which in the preceding section we used as a measure of adaptability).

As centrality goes up for a given number of colors, with more available rules, the resulting focus should decrease from a more ordered state toward more chaos.


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### 3.3 Interdependence

To study interdependence, we do the same experiment except for a constant number of colors (adaptability is kept constant) and for a constant pool of rules (fixed centrality). We vary the number of neighbors on each side that are used to determine the color of each cell at each time step of the GCAs in the GCAN. As the number of neighbors connected to each cell increases, we are increasing the interdependence among them. Increasing the interdependence among the cells, while keeping adaptability, centrality, connectedness, and diversity all constant, should increase order.

In all of these GCAN experiments, we examine the state of all the GCAs in the GCAN after 150 time steps to find out the fraction of ordered GCAs. This does not directly measure whether the state of the total GCAN system has changed, but it does indicate the effect of changing the number of colors (adaptability), the number of available rules (centrality), and the number of neighbors (interdependence) on the prevalence of the ordered or chaotic state of individual GCAs.

In summary, in these experiments we can independently demonstrate the effects on focus of changes in adaptability, interdependence (differentiation), and centrality.

We report the following results of these experiments in two separate sections, one dealing with increasing adaptability and centrality, the second with the effects of changes in interdependence.

## 4. Results

### 4.1 The Effect on System Focus of Increasing Adaptability (Differentiation) and Centrality

Since we are using the same GCAN to test for the effects of higher adaptability and higher centrality, we will combine the results. Increasing the number of rules in the rules pool—higher centrality—for a fixed number of colors—constant adaptability (differentiation)—should *decrease* order, while increasing the number of colors (adaptability) for constant centrality should increase order.

Figure 1 is an example of what the resulting patterns look like for a three-color GCAN. The chaotic result on the left is the result of drawing from a large rule pool; the ordered pattern on the right comes from a much smaller rule pool.

Figure 2 combines the results of the GCAN experiments on changes in adaptability and centrality. We can see that for a fixed population set of rules, increasing the number of colors increases the fraction ordered, as we would expect, since the colors represent adaptability, one of the indicators of differentiation.


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Second, for a fixed number of colors, increasing the population size from which the rules are selected *decreases* the fraction ordered. Note the size of the rule set is given as a logarithm to base $e$.

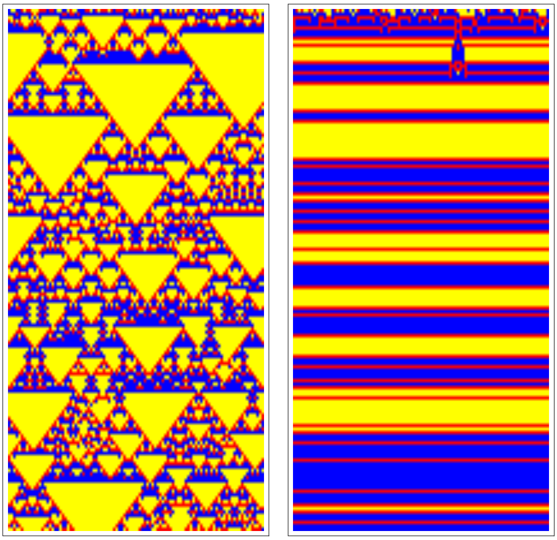

**Figure 1.** For a three-color GCA the example on the left is chaotic—large rule pool, and on the right, ordered—smaller rule pool.

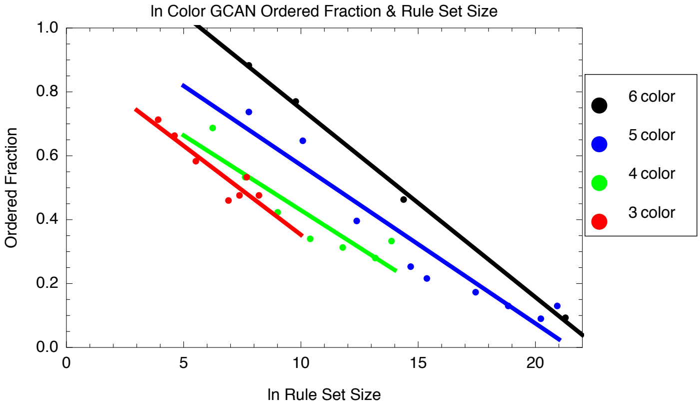


**Figure 2.** The effect of the number of rules (centrality) and the number of colors (ability to adapt) on the fraction of ordered GCAs.


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### 4.2 The Effect of Increasing Interdependence (Differentiation) on System Focus

Figures 3 and 4 show the effect of increasing the number of neighbors (interdependence) for a given number of colors (fixed adaptability) and for two differently sized rule pools (fixed centrality).

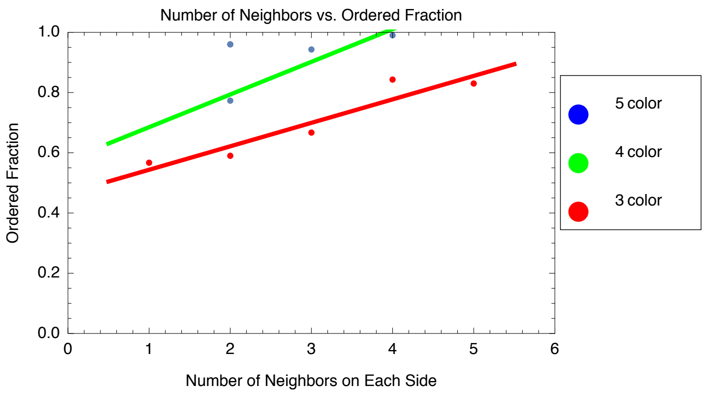


**Figure 3.** The effect of the number of neighbors (interdependence) on the fraction of ordered GCAs for a given number of colors (adaptability). The pool of numbers (centrality) from which the rules are drawn is 2186.

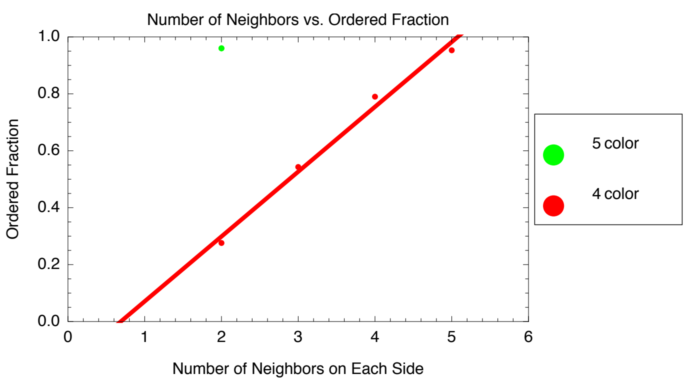


**Figure 4.** The effect of the number of neighbors (interdependence) on the fraction of ordered GCAs for a given number of colors (adaptability). The pool of numbers (centrality) from which the rules are drawn is 1 048 576.

In each case, increasing interdependence (more neighbors) adds more order, when adaptability and centrality are held constant.

## 5. Discussion

In the first set of experiments, the results demonstrated that the fraction ordered increased as adaptability (the number of colors) increased. A greater number of colors increases the number of alternate values for the cells in the GCA, which, in turn, increases the ability of that GCA to adapt. Thus, as we increase the ability to adapt, we increase order, as we have outlined previously would be the case [4, pp. 40–41].

The second experiments showed that when interdependence (number of neighbors used) increased, so did order. The number of neighboring cells considered by each GCA cell at each step involved more cells in each decision. Using more neighboring cells is equivalent to individuals in a social system consulting more friends before making a decision [4, pp. 39–40].

Combined with our earlier papers on virtual models of differentiation, these results confirm that as the four parameters of differentiation—diversity, connectedness, adaptability, and interdependence—increase, so does system focusing, as long as centrality does not vary.

It is also apparent from the first experiments that for any given number of colors (differentiation), as we increase the size of the pool of rules from which we select the rules for each GCA—which we argue represents increasing centrality—the fraction ordered *decreases*. The system becomes more chaotic with higher centrality, as long as the differentiation parameters are held constant. This effect is also apparent when examining the differences between Figures 3 and 4. This is as we expect, because when centrality *increases*, the d/c ratio goes down, representing a loss of focusing.

## 6. Conclusion

This paper and our two previous papers use global cellular automata (GCAs) and global cellular automata networks (GCANs) to model the d/c theory of social focusing.

Our first paper [7] used GCANs to demonstrate that the increase in the fraction ordered was the result of increasing diversity, often considered the major factor in keeping systems strong [10]. The implications for social systems are obvious: having a greater division of labor in a social system helps keep it more organized.

In our second paper [8], again using GCANs, we studied a wide range of networks and demonstrated how different networks affect the fraction of ordered GCAs. With the exception of random networks, which are essentially unstructured and chaotic—a good example is equal participation small groups—increasing the number of con-


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nections in organized structures produces more order. In general, more hierarchical networks tend to be more ordered. The ultimate hierarchy is a dictatorship.

In the present paper, we have shown that increasing the remaining two parameters of differentiation—interdependence and the ability to adapt—increases the fraction ordered overall. Thus, with all four experiments on the four parameters of differentiation, we have demonstrated that differentiation is more than simple diversity. Simply having a variety of individuals in a system does not guarantee focus. Focus also depends upon the other Page parameters of interdependence and adaptability, as well as the type of network connecting individuals.

Finally, in this paper, we have shown that increasing centrality decreases the fraction ordered. Together, these results support our theory that social focus can be predicted by the d/c ratio.

There are deeper issues of how much computer models can model social behavior. As with any model, economic or virtual, they require assumptions, and those assumptions can be challenged.

On the other hand, the big advantage of virtual models is that we can easily vary one variable at a time, while keeping other variables constant, to isolate the relationship between independent and dependent variables. Of course, social scientists have long used regression analysis to isolate relationships statistically, but regression models require a lot of data, and the resulting statistics are only an indirect way to draw out relationships. Studying relationships directly may be more convincing.

Taken together, our papers support the d/c theory, using four indicators for differentiation and one for centrality, along with the dependent variable of focusing type.

The implications for social systems, be they small work groups or large organization such as corporations or, indeed, entire countries, is quite profound. In most social systems, it is possible to change some of the factors we have examined and therefore change the behavioral output of that social system, whether we wish to increase order or increase creativity by moving from order into a complex state.

We look forward to other tests of our theory, both with real social systems and with computer models.

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