The “Two’s Company, Three’s a Crowd” Game

Philippe Collard
Université Côte d'Azur, CNRS, I3S, France
UNS, Parc Valrose, 06108 Nice, France
Philippe.Collard@unice.fr

The so-called Two’s Company, Three’s a Crowd game is a tiny artificial world populated by individuals, each with their own behavior, which is expressed by the way they move around the world; when they move, individuals meet others and establish social links with some of them. This model allows carrying out experiments in silico; its goal is not truly to model the real world but rather to suggest that a system of individuals, moving through an artificial world and reacting together, is adequate to account for the formation of some patterns comparable to those resulting from animal and human behavior. First, according to the density of individuals and the distribution of mobility behaviors, we study the properties of the resulting relational network. Then, assuming that in turn, proximity links may influence the behaviors, we study the impact of the feedback loop on both spatial distribution and social patterns. Such dynamics lead to various kinds of homophilous groups where links between separate groups are weak, while links within a group are strong. Although the emergent social networks could be seen as the result of individual strategies striving for uniformity, seclusion, gregarious instinct or the need to live as a couple or in a narrow group, it is suggested that the explanation does not require a reductionist theory.

1. Introduction

The general context of this paper was formulated by Schelling [1], in these words: “The […] subject that occupied me in the seventies was the ways that individual behavioral choices could aggregate into social phenomena that were unintended or unexpected.” Here we consider a tiny artificial world composed of individuals who have their own behavior and who can establish some links with other individuals. Although simple rules of interaction are considered, the aim is to show that the emergence of some meaningful and recognizable structures akin to those encountered in animal or human social organization can be observed. Nevertheless, we know from the arguments of Gilbert and Troitzsch [2] that social reality is not that reductionist: microscopic interactions take place within a larger, emergent and self-

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organizing pattern. That is why we assume there are reciprocal influences from local behaviors onto macroscopic regularities and also from the top downward onto the microscopic interactions. However, our approach is not to conduct a social simulation about a specific social problematic and therefore it is not intended to be linked to empirical data. The present work takes inspiration from the contributions of Luhmann, Silverman and Bryde on systems sociology [3, 4]: "By viewing society from its earliest beginnings, prior to the existence of any societally-defined modes of interaction and communication, the systems sociology approach hopes to develop a theoretical understanding of the fundamental behavioral characteristics which lead to the formation of social order." In this way, our approach is reminiscent of the Alife approach to modeling "life-as-it-could-be"; the so-called Two's Company, Three's a Crowd model perspective leads us to examine *socialization-process-as-it-could-be*.

We propose a minimalist model in which both individual behaviors and social patterns mutually interact and influence each other. Granovetter claims that "linkage of micro and macro levels is no luxury but of central importance to the development of sociological theory"; furthermore, he outlines that taking mobility into consideration is of "special importance in developing micro-macro linkage with the help of the network analysis" [5]. All other things remaining equal, in the same vein as the Game of Life by Conway [6], this model can be seen from the angle of a psychosocial game; so, the Two's Company, Three's a Crowd game is a tiny model where (*i*) individual behavior is the way to move around the world; (*ii*) social links are induced by proximity contacts; and (*iii*) a self-evident psychological rule affects the behaviors between neighbors in the relational network. (Of course, neither the Game of Life nor the Two's Company, Three's a Crowd game are games in the ordinary sense, as the outcome is determined as soon as an initial configuration is chosen [7].) The game needs both a model of mobility and a model of influence. The former is based on the observation that in real life each person periodically comes back to a given place; although the way we implement this assumption is as simple as possible, it is much more realistic than the very common "random wanders." The model of influence implements the Two's Company, Three's a Crowd rule of individual behavior; the key idea is that people are more likely to have intimate feelings in company with only one other person [8]; more precisely, at a given time, an individual is influenced if there is one and only one other individual in their vicinity at that moment; in such a circumstance, the individual changes their own mobility behavior by imitating that of their unique neighbor.

This work is composed of three main parts. In Section 2, the model of mobility is presented and we introduce the proximity network built by contact relationships between individuals. Section 3 defines the Two's Company, Three's a Crowd game, in which mobility behaviors and social patterns mutually interact and influence each other. In Section 4, considering an artificial society composed of agents, we conduct computer simulations and study the impacts of interactions between the micro level of individual behaviors and the macro level of the structural population. Finally, we discuss results and offer our conclusions and hints for future research.

2. Mobility versus Proximity Network

The main component parts of the Two's Company, Three's a Crowd game (hereinafter TC game) are a model of mobility and a model of influence. While the first determines how individuals behave and how the proximity network is built, the second explains how individual behaviors can be affected over time by neighbors in the network. According to our main objective, what will guide our choices of these models concerns both their simplicity and their connection with the day-to-day reality. In this section we present the model of mobility and the consequential proximity network.

2.1 Mobility Model

As an individual's behavior is only expressed by the way they move around the world, it is crucial to define what we mean by mobility. This is all the more necessary because the term can be ambiguous; for instance, there is a kind of mobility in the Game of Life where a persistent pattern of cells seems to move (e.g., the glider translating oscillator). In the TC game, each agent really changes its position at each time step. First we will characterize the elementary moves; then we will be able to deduce the shape and the length of a trajectory; finally, we will specify what happens when individuals cross each other.

Many models of mobility were proposed in the literature, and the interested reader will usefully consult [9–11] on this topic. Beyond realism, what will guide us is to choose a model that is as simple as possible; this means that we would like: (*i*) to tune mobility with a single parameter; and (*ii*) to build trajectories one step at a time by means of deterministic calculations. Therefore, we exclude models based on random motion [12, 13], even if they are among the most used. As there are few direct studies on human mobility, it is useful to look at research on related subjects to obtain additional insight into the way people move. Research on time diaries provides indirect indications on mobility and shows that periodicity at different timescales is a major phenomenon [14]. Time geography integrates space and time in geographical phenomena [15]; this paradigm explicitly models the concept of space and formulates basic rules: such as, for instance, people can only be in one place at a time, activities take time, space is limited, mobility in space takes time, and so forth [16, 17]. As human mobility has structural patterns due to geographic and social constraints, spatial social networks can provide pointers to model mobility [18]; for example, using cell phone location data and data from online location-based social networks, Cho et al. [19] find that humans experience a combination of periodic movement and seemingly random jumps; the authors show that social relationships can explain about 10% to 30% of all human movement, while periodic behavior explains 50% to 70%.

2.1.1 Motivations

While the concept of mobility has multiple meanings, we consider here mobility as the motion of living animal or human entities. To combine simplicity and realism, we start from the observations that every form of life is highly constrained by the cycles of nature, like circadian rhythms and seasons, or by the basic need to survive; for example, opposition between diurnal and nocturnal animals or migratory birds exhibiting some large-scale seasonal movements. Obviously, rhythms also concern human life in all its individual, social and cultural dimensions. It is the reality of everyday life that many people commute on a daily basis between the place of residence, the place of work (school, office, etc.) and some local services (city hall, post office, pharmacy, medical clinic, general hospital, grocery store, etc.) [20, 21]; some only walk around their city block, others do not go outside their neighborhood, while others walk throughout the city.

2.1.2 Piecewise Polygonal Trajectory

We assume that time is discretized and at each time step t, each individual is characterized by position P_t in the two-dimensional space and heading h_t. The heading indicates the direction the individual is facing; this is a number greater than or equal to 0 and less than 2π; 0 is east, north $\pi / 2$, and so on.

An elementary move is a combination of only two basic actions: *go forward* and *turn*. More precisely, each individual moves one step forward according to their heading and then rotates through an angle θ that is a fraction of the full turn length that would be needed to repeatedly come back to their initial position: $\theta = 2\pi / fTL$ radians; we will say that fTL is the move parameter. Let us note that in such circumstances the moving speed is constant. More formally, an *elementary move* of parameter fTL is defined as follows.

Definition 1. Let $P_t = x_t + i.y_t$ be the position of the individual in the complex plane and h_t their heading at time t; then we define an elementary move by:

$$P_t = P_{t-1} + \cos(h_{t-1}) + i\,.\,\sin(h_{t-1})$$
$$h_t = h_{t-1} + \frac{2\,\pi}{f\,T\,L},\ \text{with}\, fTL \in \mathbb{N}\backslash\{0,\,1\}. \tag{1}$$

As the fTL value may change over time, we have to consider fTL as a function of t.

Proposition 1. Let P_t be the position of the individual in the complex plane after t time steps, and P_0 its starting point; then

$$P_t = P_0 + \sum_{n=1}^{t} \cos\left(h_0 + \sum_{k=1}^{n} \theta_k\right) + i\,.\,\sum_{n=1}^{t} \sin\left(h_0 + \sum_{k=1}^{n} \theta_k\right), \tag{2}$$

where $\theta_k = 2\pi \big/ fTL(k)$.

Proof. It results from Definition 1 that

$$P_t = P_{t-1} + \cos\left(h_0 + \sum_{k=1}^{t} \theta_k\right) + i\,.\,\sin\left(h_0 + \sum_{k=1}^{t-1} \theta_k\right), \tag{3}$$

where $\theta_k = 2\pi \big/ fTL(k)$. □

Definition 2. A trajectory is the sequence of points $(P_0, \ldots, P_t, \ldots)$ where P_0 is the starting point and P_t the point reached at time step t.

As a consequence, a trajectory is a succession of pieces of a polygon.

Definition 3. A trajectory $(P_0, \ldots, P_t, \ldots)$ is p-periodic if and only if there exists $p \in \mathbb{N}$ such that $P_{n+p} = P_n$ for all values of n.

A trajectory is eventually periodic if and only if it can be made periodic by dropping some finite number of terms from the beginning.

2.1.3 Polygonal Trajectory

If the move parameter fTL is time invariant, each trajectory is a periodic regular polygon with one vertex at each time step. This corresponds to the eternal-return (ER) model of mobility first proposed in [8]; it implements the idea that each individual comes back to a given place. Walking all the way around on their own polygon, one individual makes one full turn; fTL is the length of the path—full turn length—an individual has to follow to come back to a given position. So the fTL period is both the number of time steps needed to make one full turn and the number of sides of the polygon (note that as soon as the fTL is large enough, a trajectory looks like a circle).

Proposition 2. Let $P_t = x_t + i \,.\, y_t$ be the position in the complex plane of the individual after t time steps; then

$$P_t = P_0 + \sum_{n=1}^{t} \cos(h_0 + n\theta) + i \,.\, \sum_{n=1}^{t} \sin(h_0 + n\theta), \tag{4}$$

where $\theta = 2\pi / fTL$.

Proof. Assuming that θ is time invariant, this results directly from Proposition 1. □

Proposition 3. In the ER mobility model, $P_{fTL} = P_0$.

Proof. With no loss of generality, we can assume that the initial heading h_0 is null.

As

$$\sum_{n=1}^{t} \cos(n\theta) = \sin\frac{(t-1)\theta}{2} \,.\, \frac{\cos\frac{t\theta}{2}}{\sin\frac{\theta}{2}} - 1 \tag{5}$$

and

$$\frac{\sum_{n=1}^{t} \sin(n\theta) = \sin\frac{t\theta}{2} \,.\, \sin\frac{(t-1)\theta}{2}}{\sin\frac{\theta}{2}} \tag{6}$$

and

$$t = fTL \left(\text{i.e., } t = 2\frac{\pi}{\theta}\right), \tag{7}$$

then

$$\begin{aligned} \sin\left(\frac{(t-1)\theta}{2}\right) &= \sin\left(\frac{\theta}{2}\right) \\ \cos\left(\frac{t\theta}{2}\right) &= 1 \\ \sin\left(\frac{t\theta}{2}\right) &= 0, \end{aligned} \tag{8}$$

so $P_{fTL} = P_0$.

Let us remark that as $fTL > 2$, $\sin(\theta / 2) \neq 0$. □

Let us note that if two individuals meet at one time, then they will meet periodically. More precisely, if individuals a_i and a_j meet for the first time at time t, then they meet again at $t + \mathrm{lcm}(fTL_i, fTL_j) \times k$, where $\mathrm{lcm}(n, m)$ is the least common multiple between the integers n and m.

We will say that individuals with low fTL are *sedentary* individuals while those with high fTL are *travelers*. Although the model is freely inspired and very restrictive, it is sufficient to express the facts that individuals commute and some individuals go across large spaces, while others are confined in small areas.

2.1.4 Periodic Piecewise Polygonal Trajectory

There are trajectories built from elementary moves (Definition 1) that are periodic even though they are not polygonal; to obtain those, it will be necessary to assume that the fTL parameter changes periodically over time. For instance, a simple case is the *12-periodic* trajectory $(A, B, A, B, C, B, C, B, A, B, A, B, \ldots)$ with A, B and C the vertex of a regular triangle where, during a period, the fTL adopts the values $(\pi, \pi, 2\pi/3, \pi, \pi, 2\pi/3, \pi, \pi, 2\pi/3, \pi, \pi)$. In the following, we will see more examples with a p-period of 8, 9, 32 and 555, respectively (Figure 5).

2.2 From Mobility to Proximity Network

As individuals move, at any time, each can meet some others and take the opportunity to establish some links with them: for example, this may be the case with a *hand-to-hand* contact or *word-of-mouth* relation between two people. Taken together, these contacts constitute the so-called *proximity network*. To formalize the dynamics growth of this network, we have to define the notion of proximity bubble and describe the process to build it.

Definition 4. For each individual a, the proximity bubble is defined by

$$\mathrm{proxiBubble}(a) = \{x \in A - \{a\} \mid d(a, x) \leq \mathrm{pr}\}, \tag{9}$$

where A is the set of individuals, d is the Euclidean distance and pr is the proximity radius.

A network is an ordered pair (A, E) comprising a set A of nodes together with a set E of edges, which are two-element subsets of A [22].

Definition 5. The proximity network is the network where: (*i*) nodes are individuals; and (*ii*) two individuals a and b are connected by an edge if there is at least one time with $b \in \mathrm{proxiBubble}(a)$.

Let us note that such a network is dynamic, as new contacts may potentially occur at any time during the moves; in addition, if mobility is defined by the ER model, each individual polygonal trajectory is deterministic and periodic, and so the relational network will reach a fixed point in a finite amount of time; it therefore makes sense to study its characteristics.

In real life, if a contact is established, a persistent social link is or is not created. Let us note that persistence is most often a necessary precondition for a link to become a social link. In the ER mobility model, provided that one contact is established, it will be established from time to time and so it becomes persistent (Proposition 2). Despite that, the proximity network does not show particular social structures and, as a consequence, we cannot see it as a social network.

3. The Two's Company, Three's a Crowd Game

In Section 2, we showed how mobility can induce a proximity network; now in addition we consider the conditions under which this network can impact an individual's behavior. As each individual's behavior is characterized only by how they move, the act made under the influence of the proximity network needs to affect mobility; consequently, each individual trajectory may change over time, depending on the encounters.

3.1 A Model of Influence

To give thought to how the proximity network can impact an individual's behaviors, we have to consider the questions: (*i*) under what conditions can an individual be under the influence of someone other than themself?; and (*ii*) what is the nature of the influence? Let us remember what will guide us are simplicity and connection with the day-to-day reality.

3.1.1 "When and with Whom"

Considering the first point, our proposition is guided by the work of the sociologist Simmel [23, 24]. He provides an illustration of the emergence of qualitative changes when he elaborates on the distinction between two individuals and three. It is a fact of life that a pair of living beings make something different than three individuals together. The expression "two's company, three's a crowd" sums up well this kind of behavior. We have all seen evidence of the following scenario, which exemplifies this rule: if you meet an unknown person along a mountain path you say hello, whereas you might ignore that person on a crowded sidewalk. In the research domain on group communication and collaboration, it is established that two people will cooperate intuitively but groups of people need more. For instance, in [25] Colman et al. say: "Married couples or pairs of business partners may be able to rely on this type of intuitive cooperation, to an extent, but larger groups need explicit communication and planning. Mechanisms need to be put in place to facilitate it. Intuitive cooperation is really a case of two's company, but three's a crowd." In social ecology, Chase et al. performed experiments for testing whether several basic aspects of dominance were the same in isolated pairs as in pairs within a social group of three or four; they found that the social context, whether a pair was isolated or within a group, strongly affected the basic properties of dominance relationships [26]. In molecular biology, Khan and Molloy in a recent paper entitled "Self-Organization: Two's Company, Three's a Crowd" provided insight into the physical rules governing self-organization in complex living systems [27].

3.1.2 "Do As I Do"

The most straightforward way for an individual to be under the influence of another person is by imitation. Imitation is recognized as being a basic behavior in both the animal and human world [28]. For example: (*i*) Akins and Zentall [29] show that birds imitate foraging behaviors; (*ii*) Hayes and Hayes [30] used the "do-as-I-do" procedure to demonstrate the imitative abilities of their trained chimpanzee; and (*iii*) data collected by Metzoff [31] suggests that infants are prewired to imitate the behavior of conspecifics.

3.1.3 The Two's Company, Three's a Crowd Rule

All these things lead us to define the Two's Company, Three's a Crowd rule (hereinafter TC rule) as an implementation of the model of influence.

Definition 6. At a given time, an individual is influenced if and only if there is one and only one other individual in their vicinity at that moment; in such a circumstance, the individual changes their own mobility behavior by imitating (i.e., adopting) that of a unique neighbor.

3.2 From Physical Distance to Social Distance and Vice Versa

All that we have introduced earlier leads us to define the so-called Two's Company, Three's a Crowd game as the result of the feedback loop between individual mobilities and the proximity network.

Definition 7. The TC game is a combination of both the mobility model based on piecewise polygonal trajectories and the feedback loop defined by the TC rule.

During the game, at each time step, each individual runs in parallel the TC rule (Definition 6) and then conducts an elementary move according to its fTL (Definition 1). Let us note that: (*i*) imitation concerns only the way to move; (*ii*) as neither the heading nor the direction changes during the imitation process, the two trajectories do not become identical; and (*iii*) the imitation process is not symmetrical because one individual may have only one neighbor in their proximity bubble, while at the same time that neighbor may have many other neighbors.

The resulting gap between micro and macro levels is due to dynamic neighborhoods: an individual that moves according to their own behavior can affect not only the neighborhood they leave and the one they arrive in, but also, in the long run, everyone. Over time, movements change the behaviors through a chain reaction until an equilibrium may be reached. The dynamics start with a transitional phase then end possibly on a fixed point; what we mean by fixed point is a spatial configuration in which no individual changes their own trajectory over time. Let us note that during the transitional phase we leave the ER model of mobility because individual trajectories are no longer polygonal. The questions arise, therefore, whether: (*i*) there are stable social groups under this dynamics; (*ii*) such groups can result from the dynamics; and (*iii*) as soon as a fixed point is reached, trajectories are polygonal again. The first question is examined in the following, and we will address the remaining issues in Section 4.

3.3 Cellular Automaton and the Two's Company, Three's a Crowd Game

A cellular automaton (hereinafter CA) is a regular lattice of cells where each cell has its own state with a finite number of values and is updated in discrete time steps according to a rule that depends on the state values in some neighborhood around it [32]. As the Game of Life is a typical example of a two-dimensional CA (2D-CA), it may be tempting to draw a parallel with the TC game; however, according to Definition 7, the TC model is not a true 2D-CA, mainly because individuals really move. A way to make the link between the two models would be to consider one individual as a cell with its mobility state (fTL) but with a neighborhood that changes over time, depending on the position in the 2D space. Nonetheless, such a delineated view appears not to provide a helpful approach, and we focus rather on methodology as presented in the seminal book *A New Kind of Science* [33], where Wolfram argues that simple programs are enough to produce complex behavior like what we see in nature, and that simple computer experiments may reveal a vast world of diverse and surprising phenomena. Keeping this in mind, we can also highlight such common notions as survivability/robustness or networks generated from competing agents [33, p. 280].

3.4 How to Preserve Mobility Behavior?

The simpler condition for a trajectory to become periodic is when all the individual mobilities are time invariant; so, assuming that, we will examine various ways for an individual to preserve their own mobility behavior. The following propositions can be seen either as potential fixed-point configurations for the dynamics or strategies to preserve mobility; thus, stable configurations are presented here without presuming anything about their true occurrence at the end of a simulation.

3.4.1 Striving for Uniformity

Uniformity is the most trivial situation, where all the individuals have the same mobility; of course, in such a case mobility never changes, because if one individual meets other individuals, mobility values may be exchanged, but in the end, mobility does not vary. A uniform population can result either from a centralized decision—thus far from our approach—or from a dynamic where one group sharing the same mobility becomes a hegemonic group; in this case the loss of diversity, due to a kind of preferential attachment, leads to a strong pressure toward uniformity.

3.4.2 Seclusion

Seclusion is the act of keeping apart from others; it can concern an individual, a couple or a larger group. Seclusion can be chosen (e.g., a hermit) or imposed by the social environment (e.g., a homeless person [34, 35]). In the TC game, seclusion is an extreme and trivial case also: if one individual never meets another, they will never be influenced, and then their behavior will never change. Let us note that such a circumstance depends on the trajectories of the other individuals: the greater the density and the mobility, the less likely is seclusion.

3.4.3 Gregarious Behavior

Gregarious individuals tend to move in or form a group with others of the same kind. There are many answers to the origin of such behavior, ranging from individual gregarious instinct [36] to more complex social interaction between individual and collective levels [37]. In the TC game, gregarious behavior is a combination of both uniformity and seclusion. While seclusion is applied to a single individual, gregarious instinct concerns a group that shares the same mobility. This allows such a group to concentrate in a confined area. Let us note that this kind of configuration can be broken if the containment area is crossed by one individual with another mobility.

3.4.4 Hand-in-Hand Group

While all the previous strategies are context dependent, there is one particular strategy that allows mobility to be preserved whatever the crowd does. For one individual, this is done by establishing a permanent link with at least one other individual with the same mobility and by moving forward hand in hand; this is possible if over time each member of the group remains in the proximity bubble of at least one other member of the group. We will call groups with such a cluster *hand-in-hand*. A hand-in-hand group appears as an inseparable unit able to travel all around the world without any outside disturbance. The likelihood of spontaneous emergence of such a group decreases as both the mobility and the size of the group increase and as the proximity radius decreases. As the smallest hand-in-hand group is a pair, we can really ask the question whether a dyad, triad or larger group commonly results from simulations.

3.5 Homophilous Group

As all the individuals within a group share the same behavior, according to the size of the group, each of the four previous situations displays some degree of homophily [38]: one individual for seclusion (extreme case); dyad or triad for hand-in-hand group; larger group for gregarious instinct and the whole population for uniformity. Let us remember that such spatial structures, homogeneous in terms of mobility behavior, should result from the process induced by both mobility and the TC rule. Because a large proportion of links are deleted during this process, and because the process plays out over multiple “generations,” we can speculate that relatively small spatial homophilous clusters will tend to be amplified over time via a cumulative advantage-like process [39], thereby producing striking patterns—analogous to so-called tipping models of residential segregation [40]. As the TC rule is a psychosocial rule and the resulting network may show particular social-like structures, from now on we will consider the relational network as a *social network*.

4. Simulation and Results

An agent-based model (hereinafter ABM) allows us to model a social system based on individual-level interaction. The aim of ABMs is to produce interesting features from physical, social or biological structure. Therefore, emphasis is not on the social structure, nor on isolated individuals, but rather on their interaction [41]. In the context of this paper, one central point is the claim by Axelrod in his seminal work *The Complexity of Cooperation*: “As such, these models do not necessarily … aim to provide an accurate representation of a particular empirical application. Instead, the goal of agent-based modelling is to enrich our understanding of fundamental processes that may appear in a variety of applications [..] making these models more realistic might add complexity that could undermine their usefulness as tools for theoretical research" [42]. Following the Alife vein, we use ABMs to perform experiments that explore plausible processes that may underlie observed social patterns.

Agent-based simulations are conducted with the NetLogo [43] multi-agent programmable modeling environment—the source code is available [44]—and we use the Gephi [45] tool to analyze networks. Assuming that trajectories are polygonal and mobility is time invariant, the relational network converges toward a fixed point in a finite amount of time. If mobility changes over time under the TC rule, there is a primary time phase that may or may not end in a fixed-point configuration. For the purpose of the simulation, the world is put on an $L\times L$ grid where L is set to 100 (unless otherwise specified). The grid is a toroid where the top and bottom edges, as well the left and right edges, are connected to each other. Each agent is initially located on the grid (i.e., initial coordinates are random integers), but as soon as agents move, positions are floating points. The density δ is the proportion of agents regarding the number of nodes on the grid; that is $\#\text{agents} = \delta\times L^2$. In this section, we simulate the TC game, first without feedback from the proximity network to the individual behaviors, then we will consider the full game.

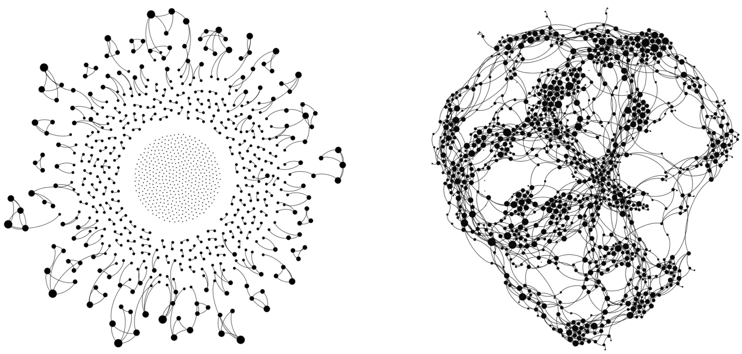
4.1 Time-Invariant Mobility

Here, mobilities do not change over time and all trajectories are polygonal. According to the mobility distribution over the population, we are going to consider two extreme situations.

4.1.1 The Same Mobility for All Agents

Assuming that mobility is the same for all agents, the issues are about correlation between the mobility value and the number of components in the network and the percolation threshold. These points are of interest as soon as we look at information flow from agent to agent when transmission requires proximity contact [46]. In the following, we look at the influence of density and mobility (i.e., fTL) once the networks have ended up converging.

For instance, considering that there are 1000 agents and the proximity radius is one, Figure 1 shows the proximity network according to two values for mobility; as expected, we can see that the number of components decreases as mobility increases.



(a) (b)

Figure 1. Proximity network for two mobility values (node size reflects its degree): $\delta = 0.1$, pr $= 1$. (a) $fTL \in \{3\}$, #components = 670. (b) $fTL \in \{24\}$, #components = 18.

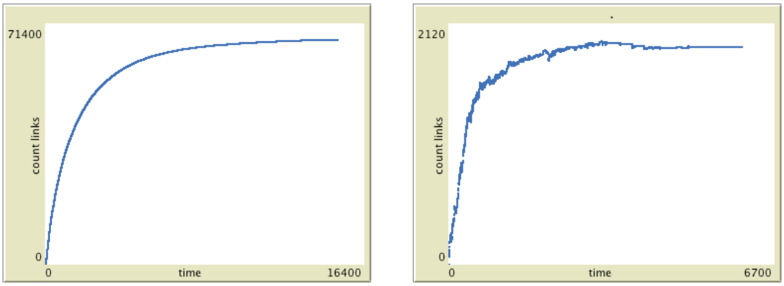
The percolation threshold is a particular value for density such that below this value a giant connected component does not exist, while above it, there exists a giant component of the order of system size. In [8] it was established that the percolation threshold follows an approximate power-law decrease according to mobility.

4.1.2 Uniform Random Mobilities

Assuming a uniform random mobility in the range $[3; 300]$, the population is composed of agents from sedentary to travelers. The degree of an agent node in the proximity network is the number of connections the node has. We look at the total number of links and the degree distribution when the network has converged. The following results from some representative runs.

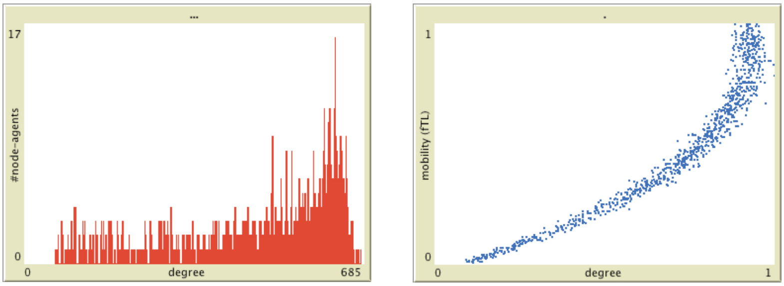
Figure 2(a) is the time evolution of the number of links during the transition period leading to the formation of the ultimate network; as expected, this shows that the number of links increases monotonically toward a limit value (approximately 71 000).

In Figure 3(a) we can see that the degree distribution for the network exhibits a peak for very high degree; this means that strongly connected agents are in large numbers, while for other degrees there are fewer agents. The issue now will be correlation for an agent between mobility and degree. The scatter plot of mobility versus degree (Figure 3(b)) shows a strong correlation: low degree values correspond to low mobility values, and high degree values correspond to high mobility values. As expected, this reveals that sedentary agents tend to have few proximity contacts, while travelers have the highest number of links. Let us note that the scatter plot shows a saturation effect when mobility is above a certain value (say $fTL > 200$). All these results will be used as the baseline for comparison in the following investigations with the feedback loop.



(a) (b)

Figure 2. Time evolution of the number of links: $\delta = 0.05$, pr $= 2$, initial $fTL \in [3; 300]$. (a) Time-invariant mobility. (b) Dynamic mobility.



(a) (b)

Figure 3. Time-invariant mobility: proximity network $\delta = 0.10$, pr $= 2$, initial $fTL \in [3; 300]$ at $t = 10\,000$. (a) Degree distribution. (b) Mobility versus degree (normalized).

4.2 Dynamic Mobility

Here we simulate the TC game in its entirety. We assume that for each agent: (*i*) the initial mobility is chosen at random; (*ii*) trajectories are piecewise polygonal; and (*iii*) mobility may change over time according to the TC rule. We will see that many runs end on a fixed-point network, despite there being no theoretical guarantee that such configurations emerge.

Considering the TC game with many agents and many mobilities, the process of social construction is far too cumbersome to analyze; this is why we first analyze two particular cases with two values for the fTL only.

4.2.1 What Happens with Only Two Agents?

First we consider the case with only two agents (hereinafter agent_1 and agent_2). The aim is to show that although it is the very simplest situation, the dynamics of motion can be complicated, even unexpected. Let us remember that the overall process is deterministic and, in this particular case, as soon as there is a contact between the agents, the TC rule applies; so during a run, the fTL of each agent can change depending on their reciprocal contacts. We ask the following questions: (*i*) does the system always reach a fixed-point configuration (i.e., a configuration for which trajectories are time independent)?; and (*ii*) does a time-independent trajectory necessarily correspond to a polygonal trajectory?

For these experiments, the world is put on a 20×20 grid. The agents are initially positioned at the center and directions are set to 1. For agent_1, initially the fTL is 6 and the heading is 0. For agent_2, the initial fTL is 3 and we conduct a set of experiments where its initial heading (hereinafter h_2) will vary between 1 and 360°. Figure 4 is a scatter plot where for each value of h_2 in abscissa the set of points visited by agent_2 between time steps 5000 and 7000 is plotted; more exactly, h_2 being fixed, each point (x, y) on the trajectory of agent_2 corresponds to a dot with abscissa h_2 and ordinate $(x+y)/2$. We can observe that according to the initial heading, some trajectories become periodic, while others tend to occupy the entire space.

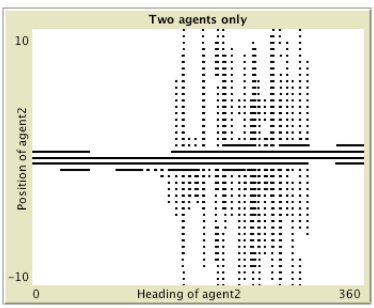
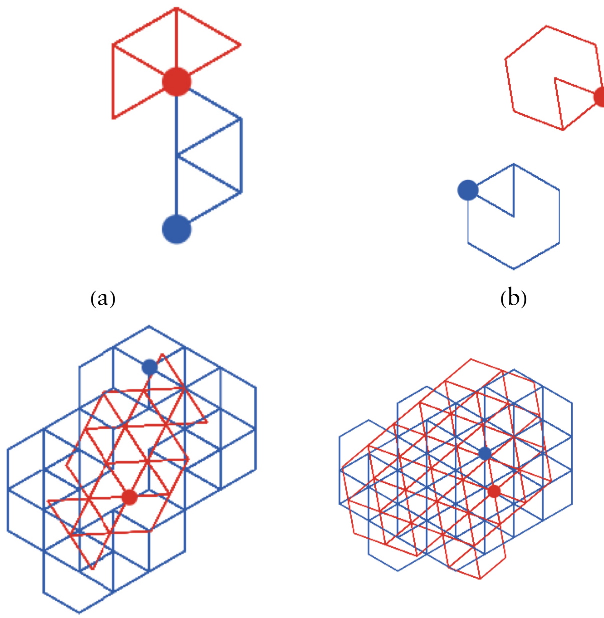


Figure 4. All the spatial positions of one agent versus its initial heading. Two agents, pr $= 2$, $fTL \in \{3, 6\}$.

Periodic patterns. Figure 5 shows the trajectories of the two agents at convergence for four particular h_2 values; we can observe periodic trajectories with or without contact. The key point here is that trajectories are no longer regular polygons: this shows that the dynamic may lead to the emergence of a fixed-point configuration even though individuals periodically (ex)change their own mobility. However, in all the simulations performed with a significant density of agents, such a pattern (if any) is destroyed as soon as its components come into contact with other agents, and then trajectories become regular polygons again.

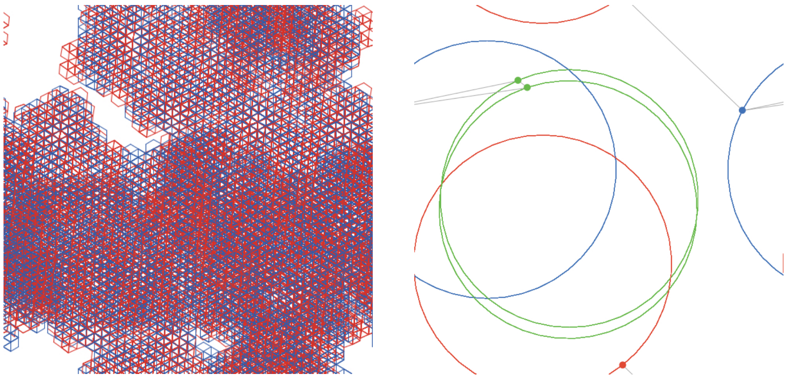


(a) (b)

(c) (d)

Figure 5. Examples of periodic trajectories between time 5000 and time 7000. Two agents, pr = 2, initially $fTL_1 = 6$, $h_1 = 0$, $fTL_2 = 3$. (a) $h_2 = 240$, Period = 8. (b) $h_2 = 291$, Period = 9 (no contact). (c) $h_2 = 147$, Period = 132. (d) $h_2 = 170$, Period = 555.

Chaotic dynamics. Figure 6(a) presents chaotic dynamics where one trajectory tends to occupy the entire space, with unpredictable outcomes. This provides a counterexample to the affirmation that all systems reach a fixed-point configuration. However, we might ask whether such a case really exists in a more general situation. For instance, we can raise the question of knowing the potential influence of a hand-in-hand dyad on the preceding dynamics. To do so, we initially add (by hand) a dyad revolving around the two previous agents. Let us remember that a dyad is a hand-in-hand group able to travel the world without any outside disturbance. Compared to Figure 6(a), Figure 6(b) corresponds to the same initial configuration but with the two agents initially inside the dyad's zone. As soon as one of the two agents meets one component of the dyad, its mobility changes and, in doing so, step by step, the agents build new polygonal trajectories. This simple scenario shows that a hand-in-hand dyad allows chaotic dynamics to be avoided and incidentally favors the exploration of the space.



(a) (b)

Figure 6. Influence of a hand-in-hand dyad on chaotic dynamics: trajectories. Two agents, $\mathrm{pr} = 2$, initially $fTL_1 = 6$, $h_1 = 0$, $fTL_2 = 3$, $h_2 = 186$. (a) Two agents only ($t = 100\,000$). (b) Two agents + one dyad ($100\,000 < t < 102\,000$).

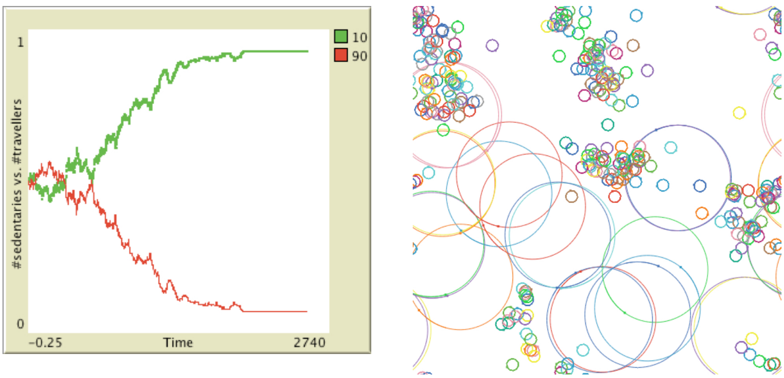
4.2.2 Many Agents with Two Mobility Values Only

Given that there are only two types of agents: sedentary agents and travelers, the aim is to answer the following basic questions: (*i*) what would be the long-term outcome of sedentary agents?; (*ii*) what would be the long-term outcome of travelers?; (*iii*) if any, how do large communities emerge?; and (*iv*) if any, how do hand-in-hand groups emerge?

There are 150 sedentary agents (respectively 150 travelers) with fTL set to 10 (respectively 90), and we look at the time evolution of the two populations. Figure 7(a) shows that the number of travelers drastically decreases to finally occupy only 4% of the population. At the end of the process, when there are no changes in trajectory, travelers go hand in hand, most often by dyad or triad, and sedentary agents are in large groups (Figure 7(b)).

Vulnerability versus robustness. To explain this phenomenon, we need to think in terms of *vulnerability/robustness* regarding the capacity of an agent to preserve their own mobility behavior. At the early stage of the process, all the agents, whatever their mobility is, are vulnerable in the sense that they can easily change their mobility by close contact: a traveler becoming a sedentary agent and vice versa.

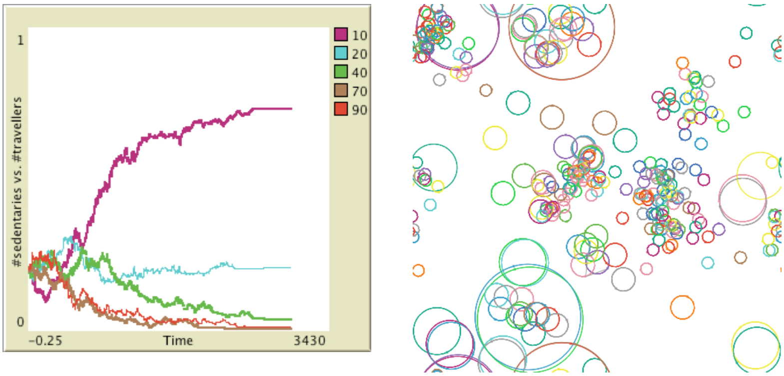
- A *traveler2sedentary* transition can occur when one traveler is in the vicinity of one sedentary agent, hence the traveler becomes sedentary; we get two sedentary agents with a high probability to stay in close proximity and thus, according to the TC rule, protecting each other from future potential changes.
- A *sedentary2traveler* transition can occur when one sedentary agent is in the vicinity of one traveler, hence the sedentary agent becomes a traveler, but with a very low probability to go hand in hand with its traveler contact and, thus, a high probability to remain vulnerable.



(a) (b)

Figure 7. Two fTL values only: $\delta = 0.03$, $\text{pr} = 1.5$, $fTL \in \{10, 90\}$. (a) Time evolution of agents with the same fTL. (b) Fixed-point configuration (tick = 2500).

Along the way this causes a rapid decrease in the number of travelers and thus, in the longer term, a decrease in the number of *traveler2sedentary* transitions. All this explains why the system converges toward a configuration with many sedentary agents and some hand-in-hand dyads or triads of travelers. It can therefore be assumed that *the less we move, the more we will be robust and inversely, the more we move, the more we will be vulnerable*. However, there is one exception related to hand-in-hand travelers for which high mobility results in strong robustness; in fact, this does not invalidate the hypothesis because the latter is related to individual behaviors, while the exception considers the case of a particular social structure. To reinforce this hypothesis, we conduct a complementary experiment with 300 agents and five values, from 10 to 90, for the initial fTL. We choose the same initial proportion for the five subpopulations and we look at their evolution over time. Figure 8(a) shows that globally, populations grow for sedentary agents and decline for travelers.



(a) (b)

Figure 8. Five fTL values only: $\delta = 0.03$, $\mathrm{pr} = 1.5$, $\mathrm{fTL} \in \{10, 20, 40, 70, 90\}$. (a) Time evolution of agents with the same fTL. (b) Fixed-point configuration ($t = 3000$).

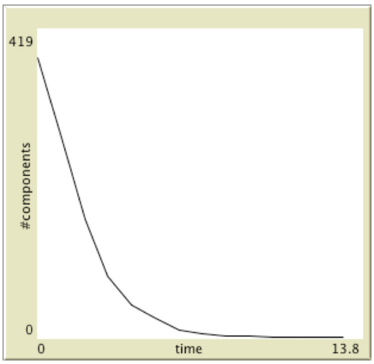
As the previous results suggest that the fTL distribution evolves from a uniform to a decreasing distribution, a question can be asked about the nature of the decrease; for example, exponential or power law? As, in addition, it seems intuitive that a node's degree in the social network is low for sedentary agents and high for travelers; it may be assumed that there is a positive correlation between mobility and degree. Both these points will need to be addressed in the general case with many agents and many mobility behaviors.

4.2.3 General Case

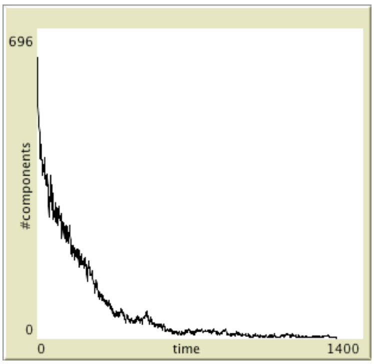
Now we can consider the general case for the TC game with a lot of agents and many values for mobility. Paying close attention to the social network, we are more specifically interested in the number of links, the number of components and the degree distribution.

Number of links. Figure 2(b) is a significant example of the time evolution of the number of links during the transition period leading to the formation of the ultimate social network. The first observation is that the growth is not continued: while the number of links globally increases it may, from time to time, slightly decrease. Then we can observe two phases: first a rapid growth, then a relative stagnation; furthermore, the entire phenomenon is much faster than in the case of time-invariant mobility (Figure 2(a)). As the number of existing links (approx. 2000) is finally far from saturating the total number of possible links, the network is characterized by the sparsity of links (compared to 71 000 for time-invariant mobility).

Number of components. Figure 9 represents two significant examples of the time evolution of the number of components during the transition period. In both cases, static and dynamic mobility, the network evolves toward one giant component. With dynamic mobility, the decrease is not continued (Figure 9(b)): while the number of components globally decreases, it may, from time to time, slightly increase; this can be explained by the fact that when a new link is created, some other links are broken. In the case of time-invariant mobility, there is a very rapid monotonous decrease of the number of components (Figure 9(a)) while, for dynamic mobility, the overall loss is very slow.



(a)

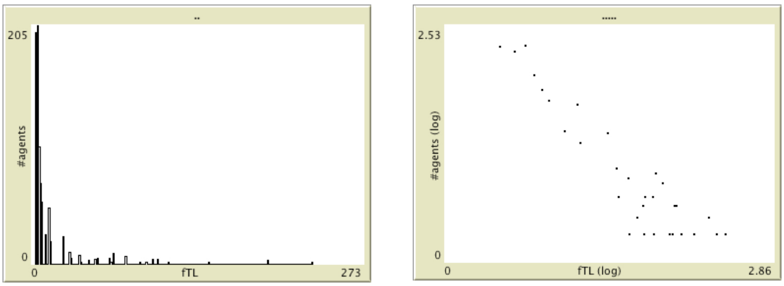


(b)

Figure 9. Time evolution of the number of components in the relational network. $\delta = 0.10$, $\text{pr} = 2$, initial $fTL \in [3; 300]$. (a) Time-invariant mobility: $t = 15$. (b) Dynamic mobility: $t = 1300$.

Mobility and degree distribution. As mobility changes according to the TC rule, the mobility distribution also evolves. The question is, what can we know about mobility and degree when many clusters and hand-in-hand groups emerge? In the following we consider a typical fixed-point configuration obtained from a population of 1000 agents with a uniform initial distribution of mobility from 3 to 300 for the fTL.

Mobility distribution. The mobility distribution (Figure 10(a)) exhibits a tail indicating the presence of a few agents with a much higher mobility than others. The corresponding log-log plot (Figure 10(b)) shows a reasonably consistent linear cloud; this enables us to assume that the distribution follows approximately a *power law*; that is, the number of agents having a certain mobility is found to decrease as a power of mobility; it would look similar to an exponential decay, but the tail does not decay as quickly, leaving large mobility still possible: there are large numbers of sedentary agents and nevertheless some travelers sufficiently numerous to make it possible to communicate between large groups of sedentary agents. We are aware that the proper fitting of power-law distributions is made difficult by the fluctuations that occur in the tail [47]; however, a prerequisite for using a statistical method, like the one proposed by Clauset et al. [48], would be to have a larger sample population.

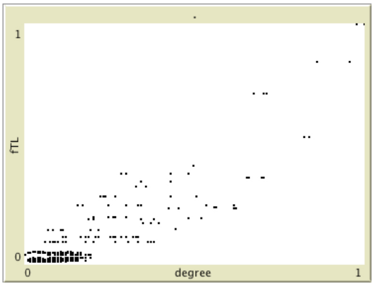


(a) (b)

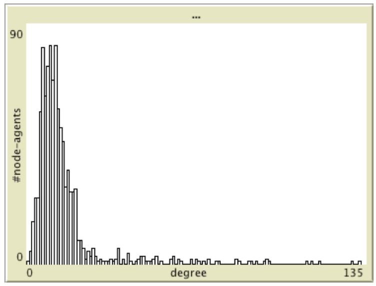
Figure 10. Mobility distribution: $\delta = 0.10$, pr $= 2$, initial $fTL \in [3; 300]$, $t = 5000$. (a) Linear scale. (b) Log-log scale.

The mobility distribution also reveals that the dynamic process leads to a loss of the initial behavioral diversity. Such organization has turned out to be fairly robust to random death although very vulnerable to targeted attacks [49]; the key point to be emphasized is that following the random disappearance of one agent (most likely to be a sedentary agent), the system is able to reconfigure itself in order to reach a fixed-point configuration again.

Degree distribution. As expected, the scatter plot of mobility versus degree (Figure 11(a)) always shows a strong positive correlation: sedentary agents tend to have few proximity contacts, while travelers have the highest number of links. Nevertheless, the respective distributions are quite different; indeed, the degree distribution (Figure 11(b)) shows a peak for low degrees (around a value of 10). This reveals that: (*i*) agents with very low degree are scarce; (*ii*) relatively low-connected agents are in large numbers; and (*iii*) for high degree (approximately from 25 to 133) there are few agents. The first case corresponds to isolated sedentary agents or very small groups of such agents, the second case to large groups of sedentary agents, and the last case to hand-in-hand groups of travelers. Let us note that compared to static mobility (Figure 3(a)), this is a reversal situation.



(a)



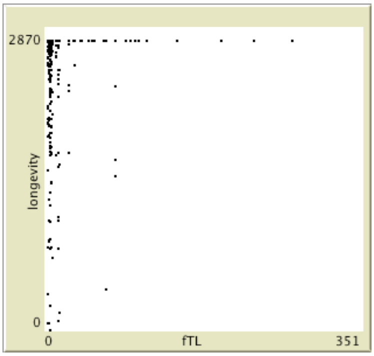
(b)

Figure 11. Social network: $\delta = 0.10$, pr $= 2$, initial $fTL \in [3; 300]$, $t = 5000$. (a) Mobility versus degree (normalized). (b) Degree distribution.

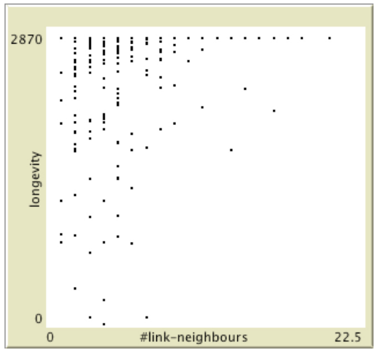
Stages of group formation. We have previously observed that the "demographic" changes over time according to mobility behavior: the number of sedentary agents increases while the number of travelers decreases. But that does not provide any evidence about the different stages of group formation. For instance, taking a closer look at the dates the fTL are finally set would allow us to know if some agents fix their behavior at the beginning or at the end of the process. For each agent, we define *longevity* as the elapsed time since the last change in its fTL.

Figure 12(a) is a scatter plot corresponding to a fixed-point spatial configuration where, for each agent plot, the abscissa is its fTL and the ordinate is its *longevity*. First, this confirms that in the end there are many sedentary agents and very few travelers, but much more, this shows that traveler behaviors are fixed in the early stages of the process, while new sedentary behaviors emerge throughout the process.

As sedentary agents are grouped together into large groups, we can raise the question of knowing if there is a correlation between *longevity* and spatial position within a group. Figure 12(b) is a scatter plot corresponding to a fixed-point spatial configuration where, for each sedentary agent, the abscissa is the number of link neighbors with the same fTL and the ordinate is the *longevity*. It can be observed that the more surrounded an agent is, the earlier the behavior is set. This indicates that a group of sedentary agents grows by aggregation of new elements on its periphery. Thus, there are agents near the border that actually change their own behavior: some leaving the group, while some others are absorbed by the group. Incidentally, this also shows that agents on the frontier provide protection to those who are on the inside.



(a)



(b)

Figure 12. Longevity correlation: $\delta = 0.10$, pr $= 2$, initial $fTL \in [3; 300]$, $t = 3000$. (a) Longevity versus mobility. (b) Longevity versus number of link neighbors (sedentary agents only).

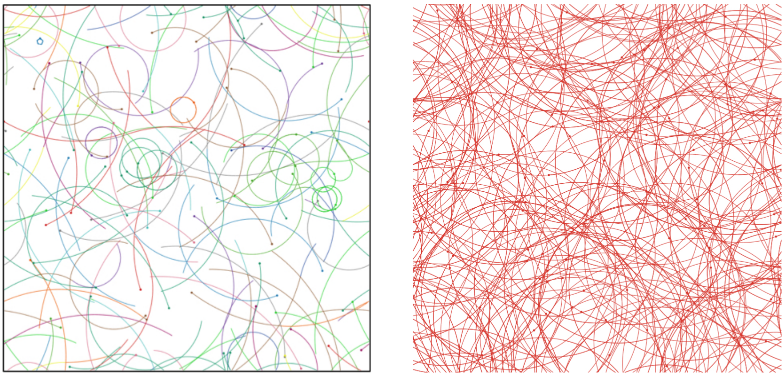
4.2.4 Emergence of Self-Organizing Patterns

Spatial and social patterns are closely tied because of the mutual interaction between these two dimensions; a structured pattern is no longer a given macro state but is coming from complex dynamics.

Let us note that for a given set of global parameter values (i.e., density and proximity radius), as the initial state of each agent (i.e., spatial position, heading, mobility and direction) is random, the outcome of the dynamics may be qualitatively completely different from one run to another. So, given a set of parameters, it makes no sense to compute averaged results; as a consequence, we do not sweep through the parameter space and we look rather for typical runs to show the various structural forms achievable by the dynamics.

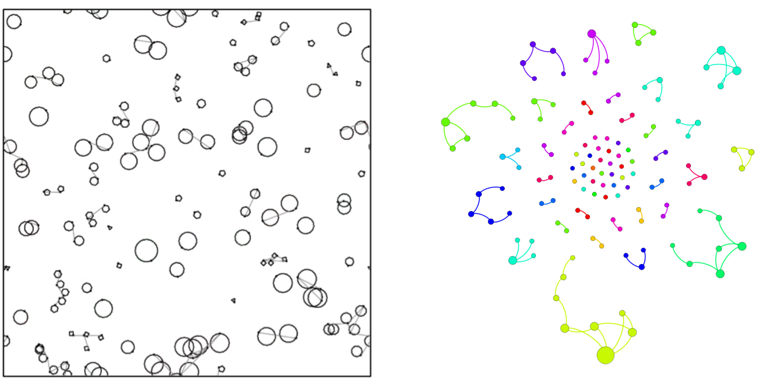
Uniformity. To illustrate uniformity from simulations, we choose an initial population of 100 travelers ($\delta = 0.01$) with uniform random mobility in the range $[250; 300]$ and proximity radius set to 1. Then, after 2000 time steps the system reaches a fixed point where all the agents have the same mobility ($fTL = 174$) (Figure 13).

Seclusion and gregarious instinct. We address seclusion and gregarious instinct together because they often happen together. To illustrate these phenomena, we choose an initial population of 90 sedentary agents with random uniform mobility in the range $[3; 20]$ and the proximity radius is set to 3. Then, after 1000 steps, the system reaches a fixed-point configuration with many groups, each concentrated in a confined area (Figure 14(a)), and where all the agents "living" in a group have the same mobility. Figure 14(b) is a view on the corresponding relational network: individuals with the same color have been influenced by the same mobility behavior, and groups with only one agent (see center) correspond to seclusion.



(a) (b)

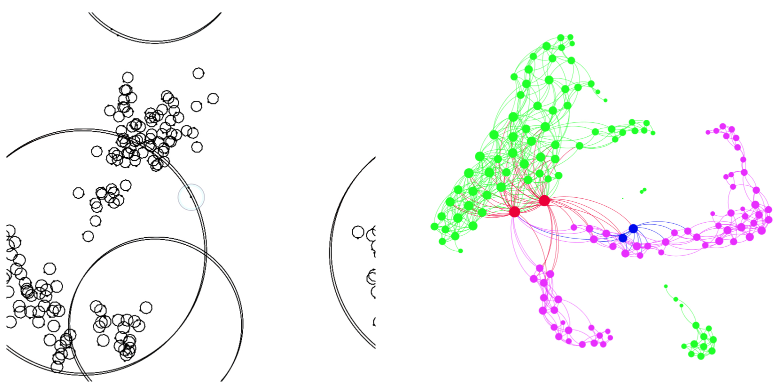
Figure 13. Evolution of the trajectories toward uniformity during one particular run. $\delta = 0.01$, $\text{pr} = 1$. (a) $t = 40$, $fTL \in 250; 300$. (b) $t = 2000$, $fTL \in \{174\}$.



(a) (b)

Figure 14. Fixed-point configuration: seclusion and gregarious instinct. $\delta = 0.009$, $\text{pr} = 3$, initial $fTL \in [3; 20]$, $t = 1000$. (a) Trajectories and social links. (b) Social network (node size reflects the degree).

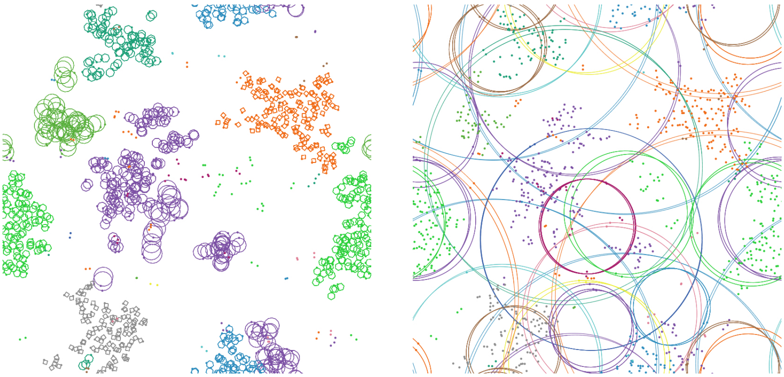
Pairwise matching. The pairwise matching strategy leads to a form of organization quite different because: (*i*) it concerns the travelers; (*ii*) it does not lead to a partition of the space in isolated areas; and (*iii*) for a pair of travelers, the way to move forward hand in hand is context independent. This last point means that a dyad can cross a cluster of linked agents without interfering with its members and without losing its own mobility. To establish a pairwise matching, the two agents need to have the same mobility and furthermore, in order to remain always in the same proximity bubble, the two trajectories need to be very close to each other. As soon as two agents go hand in hand, whenever they met, they form an indestructible pair. Such a pair presents inner "solidarity" coupled with exclusiveness toward the outside. Although such matching appears unlikely, the phenomenon happens frequently during simulations. Figure 15(a) shows a fixed-point configuration with 150 agents where we can see four secluded clusters of sedentary agents and two pairs of travelers can cross any cluster without a hitch. Looking at the network (Figure 15(b)), we see that a pair of travelers can connect clusters of sedentary agents far apart from each other; in a way, a pair seems to be invoked as a go-between between agents (and groups) over distance. A dyad looks like a local bridge connecting two communities, to the extent that it is the only alternative to communicate between each community. So, as a dyad tends to reduce the number of components in the network, the pairwise matching phenomenon is of importance as soon as we look at information flow when transmission requires a direct contact between individuals.



(a) (b)

Figure 15. Fixed-point configuration: gregarious instinct and pairwise matching. $\delta = 0.015$, pr = 3, initial $fTL \in [3; 300]$, $t = 5000$. (a) Trajectories. (b) Social network (node size reflects the degree).

Hand-in-hand group. The previous case with only two agents is the minimal case for a hand-in-hand group. To establish such a group, several agents (say a dyad, a triad or a quaternion) need to have the same mobility and their trajectories need to remain very close to each other. Figure 16 represents a more complex fixed-point configuration obtained after 15 000 time steps with 1000 agents. In order to better distinguish between big clusters and hand-in-hand groups, trajectories of sedentary agents and travelers are shown in two separate figures (Figure 16(a) and 16(b)).



(a) (b)

Figure 16. Fixed-point configuration: (a) gregarious instinct and (b) hand-in-hand groups. $\delta = 0.10$, pr $= 2$, initial $fTL \in [3; 300]$, $t = 15\,000$. (a) Trajectories of sedentary agents only ($fTL < 20$). (b) Trajectories of travelers only ($fTL > 60$).

Now it is worth posing the question, is the TC rule really necessary? To look at this issue, we conducted experiments—not reported in this paper due to space—with a variety of alternatives for the TC rule (Definition 6) by relaxing the uniqueness constraint. It is a fact that in no cases did we observe the emergence of agents in a pair (or more) able to cross a cluster. This lends support to the hypothesis that the TC rule is at the heart of the pairwise matching process, but, of course, does not prove it.

4.2.5 The Strength of Weak Ties

The TC game can be interpreted in the light of Granovetter's strength of weak ties (hereinafter SWT) [50]. The author proposed a model "for linkage of small-scale levels with one another and with larger, more amorphous ones; where emphasis has been placed more on weak ties than on strong. Weak ties are more likely to link members of different small groups than are strong ones, which tend to be concentrated within particular groups." For Granovetter, a given tie is strong, weak or absent; he defines the strength of a tie as "a combination of the amount of time, the emotional intensity, the intimacy, and the reciprocal services which characterize the tie"; he deliberately ignores other aspects, such as the "relation between strength and degree of specialization of ties, or between strength and the hierarchical structure."

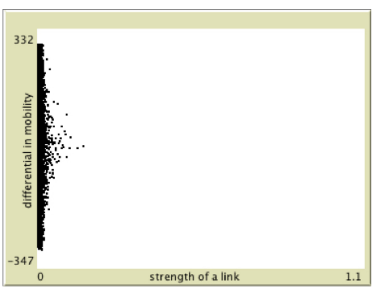
In the TC game, we define the strength as the amount of time, that is, a real number in the range $]0; 1]$ representing the (normalized) amount of time that one end spends with the other end. In the TC game, no contact between two individuals corresponds to an "absent tie."

Definition 8. The strength of a link connecting two agents is the mean frequency of proximity contacts between them.

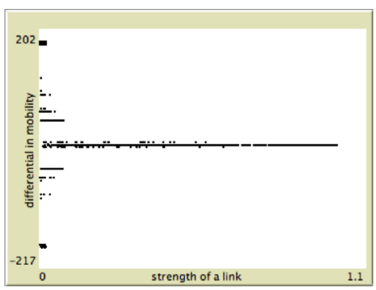
Proposition 4. The strength of a link connecting the two members of a hand-in-hand dyad is equal to one.

Proof. For a link, the strength is equal to one if and only if the two ends are in continuous contact. □

Figure 17(b) represents for each link its strength versus the difference of mobility between its two ends; the scatter plot shows that: (*i*) a strong link connects two individuals with the same mobility; and (*ii*) two agents with different mobilities are connected by a weak link. For instance, the strength of the link between two agents linked inside a hand-in-hand group is very strong because they stay in permanent touch with one another; at the opposite extreme, the link between a traveler inside a hand-in-hand group and a sedentary agent in a community is weak because a contact between such agents is rare. Let us note that if mobility is time invariant (Section 4.1.2), the same type of scatter plot shows that the proximity network is made with weak links only (Figure 17(a)).



(a)



(b)

Figure 17. Link's strength versus the difference of mobility between the ends of a link: $\delta = 0.050$, pr $= 2$, initial $fTL \in [3; 300]$. (a) Time-invariant mobility, $t = 25\,000$. (b) Dynamic mobility, $t = 2500$.

Considering any two arbitrarily selected agents and the set S of all agents with ties to either or both of them, Granovetter formulates the hypothesis that the stronger the tie between the agents, the larger the proportion p_{both} of individuals in S to whom they will both be tied. Figure 18 confirms this property for the TC game: each dot shows one link's strength versus its p_{both} proportion, that is, the degree of overlap of two individuals' friendship networks versus the strength of their link to one another. The scatter plot indicates that the stronger the strength, the larger the proportion; the continuous line gives the mean proportion for adjacent values of the strength by increments of 0.10.

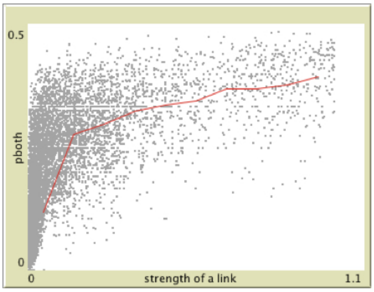


Figure 18. Link's strength versus the p_{both} proportion: $\delta = 0.10$, pr $= 2$, initial $fTL \in [3; 300]$, $t = 1000$.

The SWT principle helps us understand how information flows through a social network. In the study of diffusion, bridges play an important role because a "bridge is a line in a network which provides the only path between two points" [51]. Granovetter asserts that all bridges in a relational network are weak ties, and so more people can be reached through weak ties. In a similar way, in the TC game the scale-free distribution is very efficient for a communications network and favors the spreading of information through weak links provided by the hand-in-hand groups.

5. Conclusion

In this paper we have proposed a scale game of psychosocial interactions between individual behaviors and social structures. We have translated the Simmel hypothesis about the role of the number of people in the neighborhood in human relations into a computer simulation and explored the consequences of this hypothesis on the emergence of homophilous groups. Obviously, animal or human behaviors are far more complicated than this artificial game; however, despite its intrinsic simplicity, it shows that a feedback loop between micro and macro levels allows some patterns to emerge as the ones resulting in individual strategies such as striving for uniformity, seclusion, gregarious instinct or living with a partner.

Starting from a uniform distribution of mobilities, the system converges to a fixed spatial configuration with many homophilous groups of various sizes that coexist despite the movement of agents. In particular, there are many sedentary agents, with few proximity contacts, gathered in confined areas and some travelers, with many contacts, moving in hand-in-hand groups. The decrease in the number of travelers and the correlative increase of sedentary agents can be explained by the vulnerability of the former and the robustness of the latter regarding their capacity to preserve their behavior. These observations allow us to formulate the general hypothesis: the less we move, the more we will be robust and, inversely, the more we move, the more we will be vulnerable, with, however, one notable exception related to hand-in-hand travelers for which high mobility results in strong robustness. That said, we are aware that such a claim needs strengthening by a more measurable, or quantitative, approach [52].

Considering the strength of a relational link as the amount of time that one end spends with the other end allows us to compare the emergent network with real social networks such as those highlighted by Granovetter with his principle about the strength of weak ties (SWT); as a consequence, the degree of overlap of two individuals varies with the strength of their link to one another. Strong links ensure the robustness of the system and weak links ensure the ability to communicate between the components. A weak link, for instance between a traveler and a sedentary agent, allows effective communication on the condition that the information that flows through the agents has a long lifetime; this would favor the forwarding of perennial information rather than labile information such as a virus.

We have established that the behaviors of travelers are fixed in the early stages of the process, while sedentary behaviors emerge throughout the process. Furthermore, a sedentary group grows by aggregation of new elements on its periphery; thus, there are agents near the boarder that actually change their behavior, and agents on the frontier provide protection to those who are on the inside.

The Two's Company, Three's a Crowd game (TC game) dynamics are characterized by: (*i*) no creation of new behaviors: the repertoire of behaviors is available since the beginning; (*ii*) no reward or no punishment for some local behaviors; (*iii*) no positive selection based on fitness criteria: no individual is better (or fitter) than anyone else; (*iv*) negative selection: mobility of individuals under influence is deleted; and (*v*) reproduction by imitation of the behavior for individuals influencing others. All this shows that while this process can "succeed" (i.e., converge) by trial-and-error, it is far from the evolutionist loop where mutation creates new solutions and selection is based on the "fittest."

In the future, we intend to further the present work in the following ways: (*i*) study different models of mobility: variants of the piecewise polygonal model or random walk models or even a mix between the two approaches; (*ii*) study the influence of global parameters on such emergent structures as the population density or the proximity radius; (*iii*) further investigate the study of fixed trajectories with only a few agents; (*iv*) further investigate the internal structure of hand-in-hand groups; (*v*) relax the "Two's Company, Three's a Crowd" rule and the strict imitation process to know whether, or to what extent, adjustments in local behavior influence the emergence of social-like structures; (*vi*) exploit fully the concept of tipping point—for instance, study the "life expectancy" of a group according to its characteristics: mobility, size, location, shape, … ; (*vii*) strengthen our claim about correlation between mobility and robustness by a more measurable approach; and (*viii*) establish whether or not there is a connection between the exploitation/exploration concepts and the sedentary/traveler dichotomy.

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