# The Effect of Network Structure on Individual Behavior

#### **James Hay**

Department of Chemical and Biochemical Engineering Western University, London, ON, Canada jmhcontact@cogeco.ca

#### **David Flynn**

King's University College, London, ON

Using computer networks built from cellular automata fundamentals, this paper shows how the behavior of individuals at the nodes or vertices in those networks is affected by the network structure, analogous to the way the behavior of individuals in a social network may be affected by social structure. In this paper, we use global cellular automata (GCAs) arranged in global cellular automata networks (GCANs). The networks studied are random, small worlds, cycle, wheel, star, and hierarchical, and we compare our results with those generally expected.

# 1. Introduction

# 1.1 Cellular Automata, Global Cellular Automata, and Global Cellular Automata Networks

There are innumerable cellular automata (CAs), but this global cellular automaton (GCA) study is based on simple, one-dimensional CAs. Their behavior is based on one of 256 Boolean rules that determine how a given two-valued cell changes its value based on its current value and that of its immediate neighbors on its left and right. The cellular automaton (CA) begins with a row of these cells, the initial condition, and a rule that determines each cell value for the next time step. All cells change simultaneously at each time step. The pattern developed after a number of these time steps reveals the behavioral state of the CA. Wolfram [1] presented a detailed examination of these CAs and many other discrete systems, that is, systems made up of discrete individuals where change happens in discrete steps.

A GCA is a one-dimensional CA that contains two or more rules rather than just one rule, along with a method of determining which rule applies at each time step, a design suggested by Wolfram [2]. A global cellular automaton network (GCAN) is a network connecting many GCAs, a structure developed by Chandler [3].

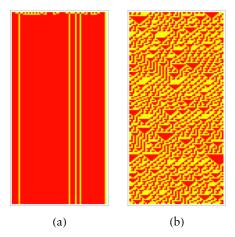
At each time step, a GCA selects which rule to use, based on global information from the GCAN. This is done using a technique devel-

oped in [3] and described in detail by the authors in [4]. Essentially, each GCA looks at the middle cell of each GCA connected to it and uses this set of ones and zeros as the initial condition for what we call a processing CA operating under a certain rule, in our case, rule 30. This CA is then run a sufficient number of time steps to involve all the cells. The value of the first cell in the final time step is then used to decide which of the two rules will be used. The effect of the rule choice is small and is discussed in [4, p. 228].

Each GCA can be thought of as an individual in the network that is the GCAN. In the present paper, we have examined the effect on the state of the GCAs in that network based on the type of network of the GCAN. Each GCA may be affected by the behavior of the other GCAs to which it is connected and the manner in which they are connected in the network. We examine the state of all the GCAs in the system over a number of time steps to find out the proportion of ordered GCAs. This does not directly measure whether the state of the total GCAN system has changed, but it does indicate how particular networks change the state of individual GCAs.

# ■ 1.2 Global Cellular Automaton Behavior

In his massive study of discrete systems including CAs and many others, Wolfram determined he could classify essentially all behavioral outcomes into four classes, namely, fixed order, repetitive order, complexity, and chaos [1, p. 231]. In general, the class of any CA could be determined by examining the CA after a sufficient number of time steps. For this study, we label both types of order as ordered and both complexity and chaos as chaotic. We apply this classification system to the GCAs in the GCAN. Figure 1 shows typical behavior.



**Figure 1.** Typical GCAs: (a) is an ordered GCA and (b) is a chaotic GCA.

In our book, [5, p. 20] we extended this classification to social systems, arguing that since social systems are also discrete systems, where individual behavior varies from moment to moment according to interactions with other individuals, then the state of a social system also can be classified into one of these same four classes.

# 2. Network Structures

For this study, we examine networks with 300 vertices, which gives a significant sampling from the set of all possible pairs of the 256 simple CAs. Each vertex is a GCA controlled by one of two rules, where the pair of rules is randomly selected from the set of the 256 rules of simple CAs [2]. At each time step, the GCA selects which one of these two rules to apply to that time step, based upon input received from those GCAs to which it is connected in the network [2]. We run all GCAs for 150 time steps and then classify each GCA as ordered or chaotic. Chaotic includes chaotic and complex states. We report the fraction of GCA sites that are ordered.

For a detailed discussion of networks (including graph theory, the mathematical analysis of networks), see Newman [6]. The structure of particular networks that we studied is illustrated and discussed in this section, along with our expectations about how chaotic or ordered their states might be.

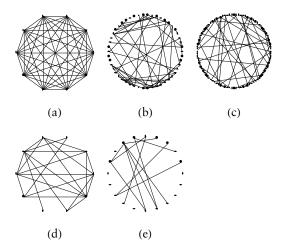
# **■ 2.1 Random Networks**

Figure 2 shows five possible random networks, with the first being a complete graph where all vertices are connected to each other. The remaining four incomplete graphs vary by the number of vertices and the probability of one edge being connected to another, that is, its edge probability (EP).

In our study of random networks, 300 vertices were used, with various EP values. Each vertex has the same specified EP, and the connections to individual vertices are randomly selected based on this probability. In Figure 2, for clarity, the number of vertices shown is varied.

In many social systems, such as a university class or a political party, people are not totally randomly connected, but they do tend to come into contact with each other more or less randomly.

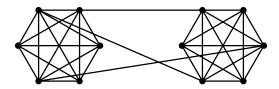
In such randomly connected systems, we would expect behavior to be related to the EP. At low probability with few connections, the individual is unlikely to change, so it would tend toward order. As EP increases, the system moves toward the complete network. Since in a complete network any new outside information will be transmitted throughout the system, we would expect more chaos. As EP approaches one, where each vertex is connected to all other vertices, we would expect the behavior to move toward chaos and unpredictability [5, p. 34]. This would approximate a social system where everyone is connected, such as a sports team, family dinner, or a school class discussion.



**Figure 2**. Examples of random networks: (a) Complete graph with 10 vertices and 1.0 EP, (b) 50 vertices and 0.05 EP, (c) 100 vertices and 0.005 EP, (d) 10 vertices and 0.5 EP, (e) 20 vertices and 0.05 EP.

# ■ 2.2 Small-World Networks

The model of a small-world network is the famous Watts-Strogatz solution to the problem of why information can be transmitted to everyone in a social system within a relatively short series of links [7, 8]. A group of people where everyone knows each other, combined with a few members who have links to other groups, is the ideal compromise between the chaos of a completely connected network and the order of a random network with few connections. So we would expect more order than most random networks but less order than the more structured networks described in the next sections.



**Figure 3.** Small-world network of two six-vertex groups with three connections between the groups.

# ■ 2.3 Cycle, Wheel, and Star Networks

The three networks of cycle, wheel, and star are similar and related. They were used in the classic studies of small groups that grew out of Moreno's early experiments [9, p. 10]. Those studies looked at the effects of different structures on individual morale and group solidarity, not order versus chaos as we are doing here.



Figure 4. Cycle, wheel, and star networks.

Cycle Network. While a cycle network is not often fully realized in social systems, it is approximated in a spy network, where information (sometimes false) is passed in secret from one person to another and may eventually get back to the spy who started it. Another example is the party game where a message (a rumor) is whispered from player to player sitting in a circle, to see what changes are introduced from the initial message after it is circulated around the network.

One would expect, as the examples above illustrate, that there is some unpredictability in such networks, a tendency toward chaos, although less so than in a random network. It is difficult to control future patterns since there is no central control, so we would expect more chaotic behavior than in either a wheel or star network.

Wheel Network. An example of a wheel graph would be some manufacturing systems such as computers, where the product of a single company is moved from site to site or from company to company as it is being built, accompanied by overall coordination from a central manager. Or it could represent an organization with one overall leader and all others connected to two others and to the leader.

The wheel is like the star, except that some members can communicate directly with each other. This introduces some unpredictability, so the wheel should be midway between the more chaotic behavior of the cycle and the more ordered behavior of the star.

Star Network. An example is a licensing system where a central government controller regulates competitors. An example in the field of telecommunications is the Canadian Radio-television Telecommunications Commission. Another example is a small organization with no communication except through the leader.

Because the leader has such control over results, we would expect more predictability and order from the star network arrangement, compared to the cycle or wheel.

# ■ 2.4 Hierarchical Network

A hierarchy (called a *k*-ary tree in graph theory) has several levels, each of which has supervisors to whom members report. Ideally, this arrangement is to guarantee predictability and order; hence, it is the network used in bureaucracies or large manufacturing firms, where the goal is repetitive output with few errors.

One way to categorize different hierarchies is by the span of control (SOC), the number of workers supervised at each managerial level. SOC has been studied extensively since the 1920s to determine its effect on organization performance. The size of the supervisory bureaucracy is reduced by increasing the SOC. As the SOC is increased, at some point efficiency is reduced. This number depends on many factors, such as the actual business of the organization, training of employees, managers' abilities, and so on [10, 11]. We wish to determine if the stability—the order—is affected by the SOC variable itself.

We began with an SOC of five, but then varied the span to see what effect that had on the class of order.



**Figure 5.** Hierarchy network (*k*-ary tree) with 30 vertices and an SOC of five, and thus almost three full levels.

# 3. The Computer Experiments

We began with 300 GCAs in each network, each GCA with two rules randomly selected from the set of 256 simple CA rules. The 300 GCAs, each with a different rule pair, are a representative sample of all possible GCA pairs of the 256 simple CAs. For hierarchical networks, some additional sizes were studied, as explained later.

In his studies, Wolfram classified the resulting behavior of each rule as ordered (two types of order), chaotic, or complex when run as a CA from particular initial conditions.

A previous paper [4, p. 225] showed that there was no simple relationship between the class of the individual CAs in a given pair in a GCA and the behavior of that GCA. This behavior is the result of a complicated, deterministic process, and chaotic or ordered behavior can arise from all types of pairings in the two rule sets.

In the computer experiments discussed here, we examined the class of each of the 300 GCAs in the GCAN after running 150 time steps of the system and then determined the fraction of ordered GCAs. Our objective was to determine how the fraction of ordered GCAs varied among different network structures.

By using different networks with other conditions constant, we can discover whether network structure itself changes the fraction of the GCAs that are ordered.

We also tested other variables within each type of network. For random networks, we altered the EP from one (a complete network) to 0.05 to test the hypothesis that lower EP produces more order. We also repeated this experiment with another initial condition (arrangement of the first row of cells) and with another pair of rules to see if the relationship between EP and order was retained.

For small-world networks, we tried the experiment with different numbers of groups and different numbers of vertices in each group, from networks with four, five, six, and 10 groups with 75, 60, 50, 30 vertices each, still retaining a total of 300 vertices. We would assume that altering the number of groups and vertices would not affect the proportion of ordered GCAs, but we wanted to test that assumption. We also chose two different initial conditions and two sets of rule pairs, to see if this affected the results for small-world networks.

For hierarchical networks, in addition to varying the SOC we also repeated this test with a different total number of vertices. This was because fixing the total number of vertices affected the number of levels as we varied SOC, since the SOC is built from the top down. When the SOC is large, few vertices are left for the lower levels of the hierarchy. Again, we assume the main cause of variation in ordered GCAs is the SOC.

In all cases, the results are presented as the fraction of ordered GCAs in the GCAN, where each vertex is a GCA.

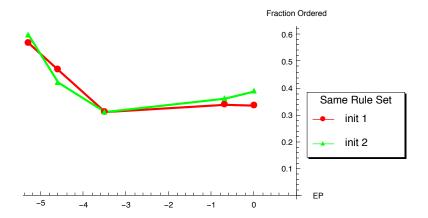
# 4. Results

We present the results of our computer simulations of various networks. In addition to noting the proportion of ordered classes produced by each network, we compare the effects of varying other factors, as discussed in Section 3.

#### ■ 4.1 Random Networks

Figure 6 shows how the fraction of ordered GCAs in a random network is affected by the number of connections in the network plotted as the log of the EP (0 is a complete graph).

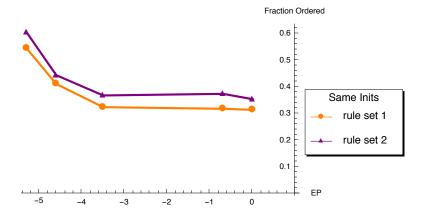
As shown in Figure 6, order decreases as we increase EP (move to the right) until we reach an EP of about 0.0247 ( $\ln = -3.7$ ), an average of about 7.5 connections each for 300 vertices. Above that EP, the fraction ordered remains constant at about 0.35.



**Figure 6.** The effect of increasing EP on fraction ordered for GCANs with two initial conditions but the same rule set.

Comparing two sets of initial conditions, we see little change in the results.

We studied whether different rule sets cause random networks to be more or less ordered with the same initial conditions as shown in Figure 7. The relationship to order remains similar to the graphs in Figure 6 for each rule set; that is, the proportion of order falls from a high of about 0.6 for low EP to about 0.35 for a complete graph.



**Figure 7.** The effect of increasing connections at vertices on the ordered fraction of a GCAN for two different rule sets with the same initial condition.

Each rule set contains its own set of GCAs (although some rules may appear in both sets), and each GCA in turn contains two of the simple rules. Since elementary rules used in each rule set are different, since neither is a perfect representation of the total set of rules, the two representative collections of GCAs do not give identical graphs in Figure 7.

Since for any initial condition there will always be some two-rule GCAs that will generate order and some that will generate chaos under all network conditions, this would explain why the ordered fraction does not fall below 0.3 or rise above 0.6.

# **■ 4.2 Small-World Networks**

We can also compare the proportion of ordered GCAs for small-world networks as we use varying numbers of groups and varying connections between groups. The number of groups in each network is 4, 5, 6, or 10 groups, with each group having 75, 60, 50, or 30 vertices, respectively. Again, we also examine the results for two rule sets and for two initial conditions to see how well the results hold up. The results are shown in Table 1.

With the same initial condition and two different rule sets						
Rule Set 2 Rule Set 1						
4 Groups	0.363	0.283				
5 Groups	0.340	0.323				
6 Groups	0.357	0.323				
10 Groups	0.360	0.387				
With one rule and two different initial conditions						
	Init 2	Init 1				
4 Groups	0.370	0.330				
5 Groups	0.327	0.327				
6 Groups	0.370	0.320				
10 Groups 0.380		0.313				

**Table 1.** Small-world networks showing the fraction of ordered GCAs.

In all cases, small-world networks have about the same relatively low order. In all cases, the proportion of ordered GCAs is about one-third—about the same as more highly connected random networks. The effect of varying rule sets, initial conditions, or group size is small.

# ■ 4.3 Cycle, Wheel, and Star Networks

The results for these networks are shown in Table 2, again with two different initial conditions and two different rule sets. These three networks are more ordered than most random or small-world forms.

The star network is actually a simple form of a hierarchical organization. As one would expect, then, a star organization has a higher or-

dered fraction than the other two. However, the wheel has a lower ordered fraction than a cycle, which seems counterintuitive, although the differences are not large. This suggests that the wheel organization, although it has a risk of going to chaotic behavior, is the structured organization that is most likely to lead to creativity in the complex state.

Our results indicate that in most cases, order is quite high, as we predicted—higher than small-world networks and higher than random networks, except those with a low EP. Variations are probably a function of the particular rules in that set.

With constant initial conditions and two rule sets						
Rule Set 2		Rule Set 1				
Cycle	0.553	0.420				
Wheel	0.526	0.333				
Star	0.780	0.660				
With one constant rule set and two initial conditions						
	Init 2	Init 1				
Cycle	0.467	0.493				
Wheel	0.347	0.343				
Star	0.677	0.653				

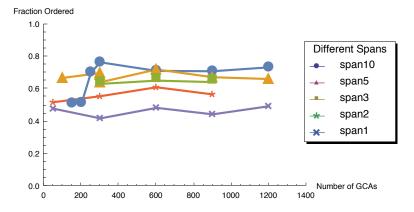
**Table 2.** Cycle, wheel, and star networks showing the fraction of ordered GCAs.

#### ■ 4.4 Hierarchical Networks

As expected, hierarchical networks have more order, in general, than the networks discussed so far. The variables involved in establishing an appropriate SOC are discussed in [12]. However, in the literature there appears to be no sense that increasing SOC itself leads to increased order.

We studied hierarchical networks by looking at hierarchies that have a different SOC, which is the same at all levels of the system. Figure 8 shows that order increases as SOC increases. For a constant population of 300 GCAs, the number of levels depends on the SOC, so we also examine the effect of varying the total number of GCAs.

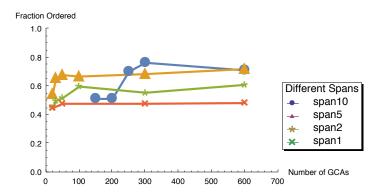
The values for each SOC value vary slightly as the total population is increased from about 200 to 1200 GCAs, which might be attributed to differences in the rule set composition as we increase the population. The fraction ordered does increase systematically, however, with increasing SOC. The fraction ordered for an SOC of 10 approaches that of the star, which has an SOC equal to the population minus one.



**Figure 8.** Hierarchical network—fraction ordered as a function of number of GCAs for different SOC values.

The population of 300 gives a statistically relevant sample of all the possible rule permutations (factorial 256). For smaller populations, the rule set is not as good as a sample.

Figure 9 does use these smaller populations to see the effect of SOC for smaller populations where the number of levels is reduced for each SOC.



**Figure 9.** The fraction ordered at populations less than 300 for various SOC values.

Despite more variability, as we would expect using these smaller populations, it is apparent that all SOC values tend to converge to a fraction ordered of 0.5 as population decreases.

The SOC of 10 falls off much more quickly than the others. This seems likely to be related to the number of levels in the system. For an SOC of 10, the levels used for populations of 10, 100, and 1000 are 1, 10, and 100.

Thus for an SOC of 10, a population of 250 covers only 14 percent of the fourth level, whereas for an SOC of five, (1:5:25:125:625) covers about 25 percent of the fifth level, and even at a population of 150 almost the whole of level four is covered.

Thus as SOC is reduced in large populations, the fraction ordered decreases, but as system size is reduced, the effect of SOC becomes less and less significant.

Overall, though, the effect of SOC itself is not considered in the organizational literature, and the increase in order with increasing SOC is surprising [10].

In Section 5, we discuss the implications of these results.

# 5. Discussion of Results

It is generally accepted that network structure has an effect on the behavior of social systems [9–14]. Sociologists use terms such as group cohesion and individual morale to describe these effects, but none of these terms are easily translated into Wolfram's four classes. Hence this paper is, as far as we know, the first abstract study on the effect of network structure on the behavior of individuals in that network in terms of behavior range from chaos to order.

In [5] the authors describe how network structure, or more generally, connectedness, affects the state of a social system. We begin by showing how the state of a social system depends on the effect of the external environment on the system, which we call centrality (c), along with the internal structure of the system, which we define as the system's ability to deal with its environment, labeled as differentiation (d). We summarize the effect of these two variables as the d/c ratio. With increased value of this ratio, that is, higher differentiation and lower centrality, we predict more order.

Borrowing from Page's analysis of social structure [14], the internal variable of differentiation is broken down into four parameters: connectedness, diversity, interdependence, and adaptability. In [5] the authors show how three of the parameters—diversity, interdependence, and adaptability—are directly correlated with differentiation, hence with order, but we were unable to find a clear relationship between differentiation and connectedness, a term we equate to network structure. We speculate how variations in network structure might be expected to influence differentiation and therefore the state of a given system, but come to no clear conclusions.

In a previous paper [4] the authors demonstrated with virtual models the effect of another Page parameter, increasing diversity, and found that it does, indeed, produce more order.

In this paper, we have examined the effect of various network structures on the behavior of individual GCAs at the vertices of a GCAN network. Changes in this behavior of individual GCAs, the changes in the ordered fraction, do not in themselves indicate a change in the state of the system. However, it is reasonable to assume that as the fraction of ordered individuals increases in the network, the entire GCAN would be more likely to be ordered.

Table 3 presents a summary of these networks, their expected degree of order, and the results of our experiments.

	Expected State	Results Fraction Ordered	Average Degree	Average Path Length	Clustering Coefficient
Random (complete)	Chaotic	-0.32	n	1	Low
Random	Order	0.6–0.32 function of EP	≪ n	< n	Low
Small-World	Complex on the edge of chaos	0.32-0.36	High but < n	short but > 1	High
Cycle	Order	0.43-0.55	2	n/4	Low
Wheel	Order but possible complex	0.33-0.53	3 +	2	Low
Star	Order	0.65-0.78	1 +	2	Low
Hierarchical	Order	0.41–0.79 function of <i>s</i>	~ s	6 + 1/2 (#Levels)	Low +

**Table 3.** Summary of network structures and results, n = number of vertices; s = span of control.

The column titled "Expected State" indicates the system state that this network supports. As discussed, the actual system state depends on many other factors in addition to its network structure.

Results are the fraction of ordered GCAs, where higher fractions are indicators of the ordered state for a particular network structure produced in our experiments. The actual state of any network system depends on many other factors.

In the remaining three columns, we list other ways of distinguishing one network from another. This is our attempt to find some underlying general characteristics of all networks that influence the state of a system. The characteristics used to distinguish networks include degree (the number of edges connected to a vertex), path length (the lowest number of steps between two vertices), and clustering (defined as the probability that two vertices are connected to each other if they have a third vertex in common [6, p. 286]. Usually the degree, path length, and clustering coefficient are system averages.

In the discussion that follows, we examine each type of network in the light of these characteristics and the experimental results.

# **■** 5.1 Random Networks

A complete or fully connected random network has all vertices connected to each other with degree equal to the number of vertices in the system and a path length of one. In such a network, small changes rapidly penetrate the whole network, and the network is very sensitive to change. This network would predispose a system to a chaotic state.

In this paper, we have shown that the fraction of ordered individuals is low for complete random graphs: 0.35. This is as expected. The interesting result is that the proportion of ordered GCAs remains low until the EP decreases to about 0.0247 ( $\ln = -3.7$ ), corresponding to a relatively low degree of about seven for a 300-vertex system. At still lower degrees, the fraction of ordered GCAs increases, and therefore, it appears that systems with relatively few links are more ordered.

Thus, most random networks have a predisposition to chaotic behavior. Not until EP falls to a very low level does order increase.

# **■** 5.2 Small-World Networks

All the small-world systems behave the same, independent of the size and number of groups.

As small worlds consist of a number of complete random networks with a few connections among them, their EP is quite high, much higher than 0.0247. Their average path length, while somewhat longer than a complete network, is shorter than most random networks. Thus, we would expect the fraction of ordered GCAs to be about the same as random networks with EP above 0.0247, the value above which the fraction of ordered GCAs stays low and constant. That is, small worlds should tend to fall somewhere between chaos and order.

Our results show that their ordered fraction is, indeed, higher than most random networks but lower than cycle, wheel, and star. Thus small-world networks fall between order and chaos, which Wolfram classifies as complexity. As suggested in [5, p. 36], the implications of the small-world model systems is that most social systems fall into the region of complexity.

Almost by definition, small-world networks have high clustering coefficients.

# ■ 5.3 Cycle, Wheel, and Star Networks

These networks have much lower average degree and much longer average path lengths than complete random networks or small-world

networks, so we would expect them to have higher ordered fractions. The experimental results support this view.

The star organization has a higher ordered fraction than the other two. However, the wheel has an average lower fraction of ordered GCAs than a cycle, which seems counterintuitive, although there is some overlay, and the differences are not large.

This suggests, though, that of these three networks, the wheel is the structured organization that is most likely to lead to creativity in the complex state.

#### ■ 5.4 Hierarchical Networks

As discussed earlier, the star network is the simplest hierarchical network. For this network, there is one vertex with degree equal to the total of connected vertices for that one, and all others are one. Similarly, path length is two for all but one of the vertices. As we increase SOC, the average degree increases, as does average path length.

One would expect that increasing degree would increase the fraction of ordered GCAs, as is the case in GCANs above a certain size, where a larger SOC increases the ordered fraction. However, for each SOC, the ordered fraction decreases below this certain size, about 300 for an SOC of 10.

This limiting size decreases with decreasing SOC. It seems likely that this is related to the number of levels for a given SOC. Levels increase with decreasing SOC for a given system size. Moreover, all SOC values have a fraction ordered of about 0.5 at the smallest system size.

The assumption in the management literature is that a manager with too many people reporting to him or her cannot handle all the demands, and the system loses order [12]. The interesting result from our experiments is that the effect of SOC on the fraction ordered and thus on the state of the system is dependent on system size. For a large organization, such as the military, our results say that a large SOC may ensure ordered behavior. The relationship between SOC and order is not as clear for smaller organizations.

In addition to the overall effect of SOC in real organizations, managers must consider the informal cliques that can form in such organizations, which will make the hierarchy closer to a wheel or even a small world, hence more complex and creative.

#### 6. Conclusion

This work demonstrates that network structure directly affects the state of individuals in a network system, and this implies that this structure can affect the behavior of the entire network system itself.

Moreover, the differences among the different network structures is demonstrated. Thus the structure, that is, the connectedness of a system, is an important variable determining the behavior of a system.

Because social systems, like the virtual systems modeled by Wolfram that we are using here, are also discrete, we believe we can generalize these results to apply to *social* systems [5, p. 20]. Hence, the organization of a social system should be carefully selected based on the output state desired.

If a more chaotic state is desired, say to generate wild, improvisational music or other art forms, connect a group of people in a random, highly connected fashion—the more connections the merrier!

For creative endeavors, such as problem-solving task forces, a small-world, cycle, or wheel network is preferred, where the state will be more complex. We have also demonstrated that hierarchies with fewer members may also be more complex.

For the repetitive work of assembly lines or hazardous environments where predictability and order are important, the star network is more appropriate or, with larger organizations, a hierarchy. Our results show that a larger span of control (SOC) generates more order in larger organizations.

Much more work with virtual systems is required. It would be useful to demonstrate a correlation between the state of individual vertices and the overall state of the entire network.

There remain two Page parameters of differentiation to be tested using virtual systems: interdependence and adaptability [5, pp. 39–41], and it would be useful if those parameters could be modeled with global cellular automata networks (GCANs).

Finally, more work needs to be done on the underlying characteristics that distinguish one network from another, in such a way that those characteristics could be correlated with system states.

We look forward to more discussion on how network structure affects individual and system behavior.

# References

- [1] S. Wolfram, A New Kind of Science, Champaign, IL: Wolfram Media, Inc., 2002.
- [2] S. Wolfram. "Cellular Automata with Global Control" from *The NKS Forum—A Wolfram Web Resource*. (Nov. 4, 2014) http://forum.wolframscience.com.

- [3] S. Chandler, "GCANs: Global Cellular Automaton Networks," presentation given at NKS 2004 Conference, Boston. (Nov. 4, 2014) http://www.wolframscience.com/conference/2004/presentations/ HTMLLinks/index\_18.html.
- [4] J. Hay and D. Flynn, "Changing the States of Abstract Discrete Systems," *Complex Systems*, 19(3) 2011 pp. 211–233. http://www.complex-systems.com/pdf/19-3-2.pdf.
- [5] D. Flynn and J. Hay, *Making Social Change*, Sarnia, Canada: Grafiks Marketing and Communications, 2011.
- [6] M. Newman, A.-L. Barabási, and D. Watts (eds.), *The Structure and Dynamics of Networks*, Princeton: Princeton University Press, 2006.
- [7] D. J. Watts, Small Worlds: The Dynamics of Networks between Order and Randomness, Princeton: Princeton University Press, 1999.
- [8] D. J. Watts, Six Degrees: The Science of a Connected Age, New York: Norton, 2003.
- [9] S. Wasserman and K. Faust, Social Network Analysis: Methods and Applications, New York: Cambridge University Press, 1994.
- [10] Wikipedia. "Span of Control." (Nov 4, 2014) http://en.wikipedia.org/wiki/Span\_of\_control.
- [11] The Economist. "Idea: Span of Control." (Nov 4, 2014) http://www.economist.com/node/14301444.
- [12] A. Gupta. "Organization's Size and Span of Control." Practical Management. (Nov 4, 2014) http://practical-management.com/Organization-Development/ Organization-s-size-and-span-of-control.html.
- [13] R. A. Eve, S. Horsfall, and M. E. Lee (eds.), *Chaos, Complexity, and Sociology*, Thousand Oaks, CA: Sage Publications, 1997.
- [14] S. E. Page. "Understanding Complexity." The Teaching Company. (Nov 4, 2014) http://www.thegreatcourses.com/courses/understanding-complexity.html.