Pinning Control in a System of Mobile Chaotic Oscillators

Reza Dariani*

Strasbourg University, France

Arturo Buscarino†
Luigi Fortuna‡
Mattia Frasca§
University of Catania, Dipartimento di Ingegneria Elettrica, Elettronica e Informatica
Viale A. Doria 6, 95125 Catania, Italy
and
Laboratorio sui Sistemi Complessi, Scuola Superiore di Catania, University of Catania
Via Valdisavoia 9, 95123 Catania, Italy

**reza.dariani@gmail.com*
†arturo.buscarino@dieei.unict.it
‡lfortuna@diees.unict.it
§mfrasca@diees.unict.it

In this paper, pinning control in a system of moving agents (each one associated with a chaotic dynamical system) was investigated. In particular, we studied and compared two different strategies for pinning control and discussed the nontrivial relation between synchronization and chaotic agent control. Our results show how system parameters such as agent density are critical in order to reach synchronous agent behavior as well as to reach global control of the system by pinning a reduced set of agents.

1. Introduction

Complex networks are the subject of intense research in fields including mathematics, chemistry, neuroscience, physics, biology, electrical engineering, and social science [1]. A classic example of a complex network is the human society: the nodes are human beings while the links are social relationships between them. Other paradigmatic examples of complex networks are the World Wide Web and internet, in which the nodes are computers and the links are cables or wireless connections between them. Usually, complex networks are considered to be *static* networks, that is, networks with links that do not evolve over time. However, to account for the most general case of links that may change during time, network models with time-varying links

https://doi.org/10.25088/ComplexSystems.20.3.279

should be considered. A class of such networks is represented by interaction networks that arise in a system of mobile agents that interact only with neighbors. Such a class of systems has been studied under several points of view in [2–6]. In this paper, we investigate pinning control in such a system.

Each agent of the system is associated with a nonlinear (in particular, chaotic) dynamical system. In static networks, when the nodes are assumed to be nonlinear dynamical systems and the links are channels for exchanging information about their state, collective behavior such as synchronization may emerge [7, 8]. This phenomenon (particularly interesting when the nodes are chaotic systems) has been widely investigated; different phenomena have been discovered and different methodologies for their investigation have been introduced [9]. In some cases, it is necessary not only to achieve synchronization but also to control the entire network. However, controlling each node of a complex network composed of many units may be difficult and unnecessary.

Some strategies to regulate and control the dynamical behavior of networks have been proposed in the literature; among them, pinning control attracted much attention. The general idea behind pinning control is to apply a feedback loop only to a small fraction of nodes that propagate the control effect to the rest of the network through the existing links [10]. In the case of static networks, two main strategies have been proposed in the literature for pinning control: random pinning and selective pinning. Random pinning refers to a random selection of the nodes to be pinned, while in selective pinning the most important nodes are pinned. Depending on the way in which the importance of a node is defined, different selective pinning strategies arise, for instance those based on the node degree (the nodes with higher degree are selected), on betweenness, or on other topological features of the network.

In the case study investigated in this paper, time-varying networks generated by random walkers are considered. In the case of selective pinning, it is nontrivial to define what (if any) are the most important nodes. In the system under examination, in fact, due to the motion characteristics, the degree distribution is very peaked around the average node degree, so that we cannot choose the nodes to pin according to their degree. Similar considerations hold for other pinning criteria. Obviously, this does not apply if the motion of agents is ruled by different laws, for instance by a Levy flight, and selective pinning strategies can be thus defined. For this reason, in this paper we focus only on random pinning. However, since the network topology is time-varying, we also proposed letting the selection of the pinning nodes be variable in time. Therefore, in the following two different pinning strategies have been considered: random and fixed-in-time (RF) pinning control and random and time-dependent pinning (RV) control.

The rest of the paper is organized as follows. The time-varying interaction network model based on mobile agents is first introduced.

https://doi.org/10.25088/ComplexSystems.20.3.279

Then, simulation results showing the effects of pinning control on a network of moving chaotic agents are reported. Finally, conclusions are drawn.

2. The Model

Let us consider N mobile agents moving in a two-dimensional space of size L with density $\rho = N / L^2$. The agents in our model are random walkers: each agent moves with velocity $v(t)$, constant in modulus, and with a direction of motion $\theta_i(t)$ that is updated stochastically at each time step. Hence, the position and orientation of our agents in space are updated according to

$$\begin{cases} y_i(t + \Delta t) = y_i(t) + v_i(t)\,\Delta\mathrm{t} \\ \theta_i(t + \Delta t) = \eta_i(t + \Delta\mathrm{t}) \end{cases}, \tag{1}$$

where $y_i(t)$ is the position of the i^{th} agent in the plane at time t and $\eta_i(t)$ are N independent random variables chosen at each time with uniform probability in the interval $[-\pi, \pi]$. Moreover, each agent is also characterized by internal state variables $x^i(t) \in R^3$, which evolve according to the following equations describing a Rössler chaotic oscillator [11]:

$$\begin{cases} \dot{x}_1^i = -x_2^i - x_3^i \\ \dot{x}_2^i = x_1^i + a\,x_2^i \\ \dot{x}_3^i = b + x_3^i\left(x_1^i - c\right) \end{cases} \tag{2}$$

with $x^i(t) = \left[x_1^i(t)\,x_2^i(t)\,x_3^i(t)\right]^T$. Parameter values have been chosen as $a = 0.2$, $b = 0.2$, and $c = 7$ in order to ensure that each oscillator exhibits a chaotic behavior. Δt represents both the motion and the dynamics integration step size. Although it is possible to consider different values for the motion and the dynamics integration step, in this paper we restricted our investigation to the case in which they have the same value (in our simulations, $\Delta t = 0.001$).

As mentioned in Section 1, each agent is able to connect only to its *neighbors*, that is, those agents that are within its interaction radius r at time t. When two agents interact, the state equations of each agent are changed to include a diffusive coupling term with the neighbor agent, acting on the state variable x_1^i. Based on this interaction between neighbors, the state dynamics of each agent is described by

$$\dot{x}^i = f\left(x^i\right) + K \sum_{j=1,\, j\neq i}^{N} a_{i\,j}(t)\,\Gamma\left(x^j - x^i\right), \tag{3}$$

https://doi.org/10.25088/ComplexSystems.20.3.279

where $a_{ij}(t)$ is the generic element of the adjacency matrix $A(t) = \{a_{ij}(t)\} \in R^{N\times N}$, that is, $a_{ij}(t) = a_{ji}(t) = 1$ if the two agents are neighbors at time t, otherwise $a_{ij}(t) = a_{ji}(t) = 0$. K is the coupling strength, and $\Gamma \in R^{3\times 3}$ is a constant $0-1$ matrix indicating the coupled variables and defined, in our simulations, as

$$\Gamma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Considering the Laplacian of the network ($l_{ij}(t) = -1$ if $i \neq j$ and a link between i and j exists at time t; $l_{ii}(t) = k_i(t)$, where $k_i(t)$ is the node degree at time t), the model can be rewritten in a simpler form:

$$\dot{x}^i = f(x^i) - K \sum_{j=1}^{N} l_{ij}(t)\, \Gamma\, x^j. \tag{4}$$

Furthermore, according to the idea of pinning control, in some nodes a control input has been added with the aim of stabilizing the whole network onto the homogeneous stationary state: $x^1 = x^2 = \cdots = x^N = \overline{x}$, with $f(\overline{x}) = 0$. The subset of network nodes $\mathbb{N}_l$ on which the control acts is defined as the set of *pinned nodes*. Control is therefore introduced only on l nodes. It consists of a linear feedback controller [12, 13] described by

$$u^i = -K\, d^i\, \Gamma_c (x^i - \overline{x}), \qquad i \in \mathrm{N}_l, \tag{5}$$

where $d^i > 0$ is the positive feedback control parameter and Γ_c is the identity matrix (in all the simulations, $d^i = 100\ \forall\ i = 1, \ldots, N$). Considering the introduction of pinning control, the dynamics of the oscillator's ensemble can be described as

$$\begin{cases} \dot{x}^i = f(x^i) - K \sum_{j=1}^{N} l_{ij}(t)\, \Gamma\, x^j + u^i, & i \in \mathrm{N}_l \\ \dot{x}^i = f(x^i) - K \sum_{j=1}^{N} l_{ij}(t)\, \Gamma\, x^j, & \text{otherwise} \end{cases}. \tag{6}$$

The system under investigation is schematically represented in Figure 1.

https://doi.org/10.25088/ComplexSystems.20.3.279

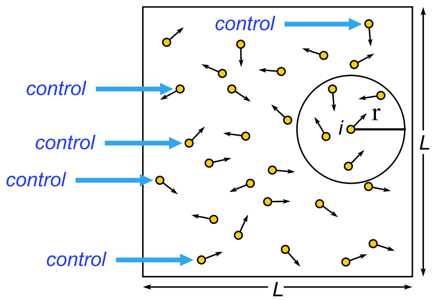

Figure 1. Schematic representation of the system of pinned chaotic agents.

3. Results

In this section, we discuss the emergence of synchronization and control in equation (6) according to two different pinning strategies: RF and RV. In the first case, the set $\mathbb{N}_l$ is fixed in time; the pinned nodes are therefore randomly chosen at the beginning of the simulation and then pinned for the entire duration. In the second case, the set $\mathbb{N}_l$ is not fixed in time, that is, $\mathbb{N}_l = \mathbb{N}_l(t)$; each Δt_{sw} simulation step's new nodes (randomly chosen) are pinned and control is applied to them. Δt_{sw} thus represents how often the pinned nodes are changed (in simulation steps).

We first note that synchronization and control are related in a nontrivial way. When the nodes are controlled, they converge to the same equilibrium point $\overline{x}$ because control is a form of synchronization. Therefore, control implies synchronization, but we verified that synchronization in general does not imply control, that is, there exist regimes in which the unpinned nodes synchronize following a chaotic orbit. This means that control is not a direct consequence of a topology able to support synchronization.

We first discuss the RF strategy and notice that when pinning control is not applied [4], agent density is a critical parameter for the onset of synchronization. Increasing the density has an effect similar to increasing the coupling strength in static networks, leading the system first to synchronization from a disordered condition and then again through a second bifurcation to an unsynchronized status.

When pinning control is applied to the moving chaotic agent network, different parameters have to be taken into account to investigate the onset of synchronization and the control efficacy. To this aim, two parameters representing average errors have been defined: $\langle\delta_s\rangle$, which gives indications on network synchronization, and $\langle\delta_c\rangle$, which monitors the effectiveness of the control strategy. In particular,

https://doi.org/10.25088/ComplexSystems.20.3.279

the synchronization error has been defined as

$$\delta_s(t) = \sum_{i=l+1}^{N-1} \left(\left| x_1^i - x_1^N \right| + \left| x_2^i - x_2^N \right| + \left| x_3^i - x_3^N \right| \right) / \left(3 \left(N - l - 1 \right) \right) \tag{7}$$

and $\langle \delta_s \rangle$ as the mean of $\delta_s(t)$ over the last 100 000 integration steps. The control error has been defined as

$$\delta_c(t) = \sum_{i=l+1}^{N} \left(\left| x_1^i - \overline{x}_1 \right| + \left| x_2^i - \overline{x}_2 \right| + \left| x_3^i - \overline{x}_3 \right| \right) / \left(3 \left(N - l - 1 \right) \right) \tag{8}$$

and $\langle \delta_c \rangle$ as the mean of $\delta_c(t)$ over the last 100 000 integration steps. Both the parameters are calculated on nodes with nontrivial dynamics. In fact, we assume that the pinned nodes are the first l nodes. Therefore, their state converges toward the equilibrium point $\overline{x}$.

Because of the nontrivial relation between synchronization and control, the two parameters $\langle \delta_s \rangle$ and $\langle \delta_c \rangle$ have to be examined together to derive the system behavior.

Figure 2 shows the parameters $\langle \delta_s \rangle$ and $\langle \delta_c \rangle$ with respect to different densities. It can be observed that three different regimes exist. For instance, consider $l = 10\,\%$: for $\rho < 0.06$, there is neither control or synchronization; for $0.06 < \rho < 0.08$, the unpinned nodes are synchronized ($\langle \delta_s \rangle = 0$ and $\langle \delta_c \rangle \neq 0$); and for $\rho > 0.08$, all the nodes are controlled (and so synchronized).

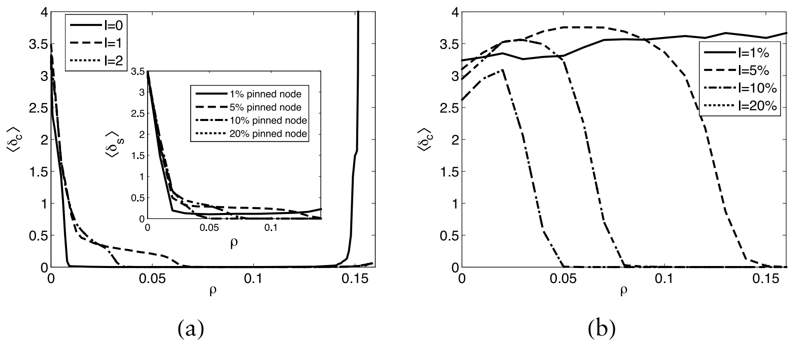

Figure 2. (a) Synchronization index $\langle \delta_s \rangle$ versus density ρ for $N = 10$ agents for different values of l. The coupling strength is fixed to $K = 10$. The other parameters have been chosen as follows: $v = 1$, $r = 1$. Results are averaged over 10 realizations. Inset: synchronization index $\langle \delta_s \rangle$ versus density ρ for $N = 100$ agents. (b) Control index $\langle \delta_c \rangle$ versus density ρ for $N = 100$ agents.

Moreover, it has also been observed that, when pinning is introduced, the synchronization range changes. In fact, as shown in Figure 2, the synchronization threshold of the system increases, hence the presence of pinned nodes (when the system is not fully controlled) acts as a source of perturbation.

https://doi.org/10.25088/ComplexSystems.20.3.279

We now discuss the comparison between the RF and RV strategies. They have been compared in terms of the number of nodes it is possible to control by pinning l nodes. In fact, when l nodes are pinned, other nodes may become virtually controlled and, obviously, the larger the number of controlled nodes, the better the control strategy. Figure 3 shows the results for different values of the parameter Δt_{sw}; different values of the density have also been considered. As can be observed, the RV strategy is very efficient, especially at low density values. Figure 3(a) refers to $\rho = 0.001$ and shows how, while the RF strategy is able to control only the pinned nodes, the number of controlled nodes in the RV strategy is larger. It is important to note that RF is efficient even when the RF fails, that is, when RF is able to control only the l pinned nodes. Furthermore, the smaller Δt_{sw} is, the larger the number of nodes it is possible to control by pinning only l nodes. In the case of $\rho = 0.01$, shown in Figure 3(b), as soon as the RV strategy becomes efficient (i.e., when enough nodes are pinned), it outperforms the RF. When the density is high ($\rho = 0.1$, in Figure 3(c)), RF and RV show similar performances.

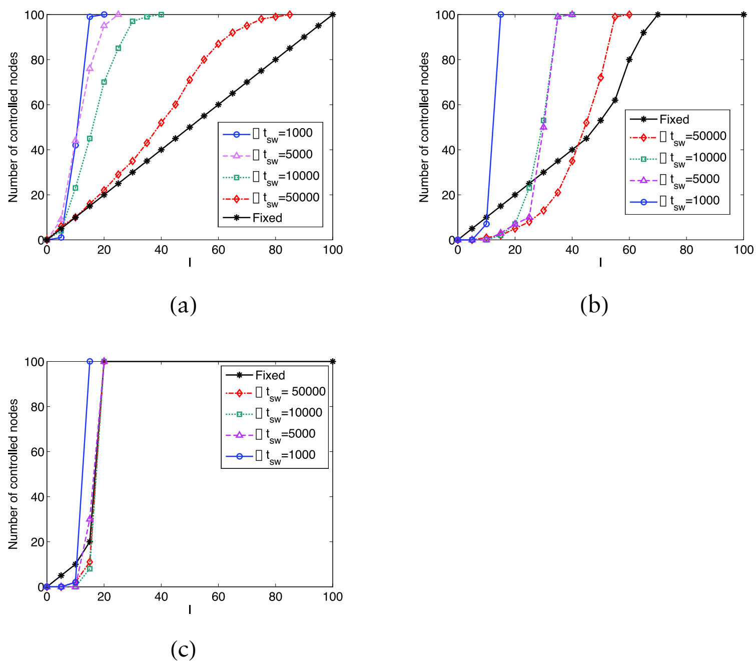

Figure 3. Number of controlled nodes versus number of pinned nodes l for different pinning control strategies and at different values of the density: (a) $\rho = 0.001$; (b) $\rho = 0.01$; and (c) $\rho = 0.1$.

https://doi.org/10.25088/ComplexSystems.20.3.279

4. Conclusions

This paper focused on pinning control of a dynamical time-variant complex network representing the interaction among a group of chaotic agents. The possibility of obtaining control of the network by pinning only a fraction of the network nodes was investigated. It has been shown that global synchronization and stabilization can be achieved by a random (and fixed in time) choice of pinned nodes as well as by a random (and variable in time) pinning strategy. The latter strategy in general performs better.

We noticed that synchronization and control are related but in a nontrivial way. Control implies synchronization; in fact, when the network is globally controlled, the nodes are also synchronized (they evolve toward the same trajectory, in this case an equilibrium point). However, the opposite is not true; there exists a region of parameter values in which the system is synchronized but not controlled. By increasing the number of pinned nodes, the density ranges in which the network is controlled tend to be those where all the agents are synchronized.

References

[1] S. Boccaletti, V. Latora, Y. Moreno, et al., "Complex Networks: Structure and Dynamics," *Physics Reports*, **424**(4–5), 2006 pp. 175–308. doi:10.1016/j.physrep.2005.10.009.

[2] M. Frasca, A. Buscarino, A. Rizzo, et al., "Dynamical Network Model of Infective Mobile Agents," *Physical Review E*, **74**(3), 2006 p. 036110. doi:10.1103/PhysRevE.74.036110.

[3] A. Buscarino, L. Fortuna, M. Frasca, et al., "Disease Spreading in Populations of Moving Agents," *Europhysics Letters*, **82**(3), 2008 p. 38002. doi:10.1209/0295-5075/82/38002.

[4] M. Frasca, A. Buscarino, A. Rizzo, et al., "Synchronization of Moving Chaotic Agents," *Physical Review Letters*, **100**(4), 2008 p. 044102. doi:10.1103/PhysRevLett.100.044102.

[5] S. Meloni, A. Buscarino, L. Fortuna, et al., "Effects of Mobility in a Population of Prisoner's Dilemma Players," *Physical Review E*, **79**(6), 2009 p. 067101. doi:10.1103/PhysRevE.79.067101.

[6] S. Scellato, L. Fortuna, M. Frasca, et al., "Traffic Optimization in Transport Networks Based on Local Routing," *The European Physical Journal B*, **73**(2), 2010 pp. 303–308. doi:10.1140/epjb/e2009-00438-2.

[7] S. Boccaletti, V. Latora, and Y. Moreno (eds.), *Handbook on Biological Networks: World Scientific Lecture Notes in Complex Systems*, Vol. 10, Singapore: World Scientific, 2009.

[8] I. D. Couzin, J. Krause, N. R. Franks, et al., "Effective Leadership and Decision-Making in Animal Groups on the Move," *Nature*, **433**, 2005 pp. 513–516. doi:10.1038/nature03236.

[9] S. Boccaletti, *The Synchronized Dynamics of Complex Systems*, Amsterdam: Elsevier Science, 2008.

[10] X. Li, X. F. Wang, and G. R. Chen, "Pinning a Complex Dynamical Network to Its Equilibrium," *IEEE Transactions on Circuits and Systems I: Regular Papers*, **51**(10), 2004 pp. 2074–2087. doi:10.1109/TCSI.2004.835655.

[11] O. E. Rössler, "An Equation for Continuous Chaos," *Physics Letters A*, **57**(5), 1976 pp. 397–398. doi:10.1016/0375-9601(76)90101-8.

[12] H. Weiming, S. Hongfei, and H. Wei, "Pinning Control of Complex Networks with General Topology," in *8th IEEE International Conference on Control and Automation (ICCA10)*, Xiamen, China, Piscataway, NJ: IEEE, 2010 pp. 360–364. doi:10.1109/ICCA.2010.5524222.

[13] Q. Jiang, G. P. He, J. P. Zhang, et al., "A General Adaptive Pinning Control for Climate Complex System," in *2nd International Conference on Environmental Science and Information Application Technology (ESIAT10)*, Wuhan, China (Q. Luo, ed.), Piscataway, NJ: IEEE, 2010 pp. 135–138. doi:10.1109/ESIAT.2010.5568457.