

# Influence of Excess $1/f$ Noise on Channel Capacity

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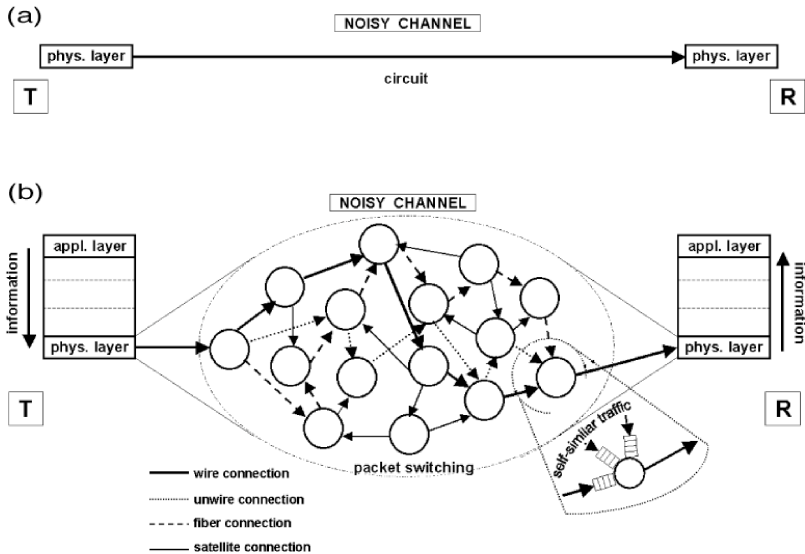
This paper deals with new analytical and experimental aspects of a channel's capacity in the presence of excess nonwhite Gaussian noise with long-range dependencies described by the Hurst parameter  $H$ . Shannon's theory of information based on the assumptions given by the Boltzmann-Gibbs extensive thermodynamical basis, does not allow description of many different phenomena directly connected with the ideas of a complex systems approach. This theory is also a basis of many considerations in communication, but a new approach to transmission channels is needed. The transmission channels are no longer simple systems built with only one wire connection, but consist of many different transmission media. For each type of partial connection in such channels there are many various interferences that influence some parts of the channel in different ways. We suggest that in many cases the real capacity of the whole channel can no longer be determined by Shannon's equation without taking into account the problem of excess  $1/f$  noise, which appears as an intrinsic feature of dynamically packet switched networks. The ideas presented in this paper show how the complex system approach can provide a good perspective for analyzing the whole transmission channel.

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## 1. Introduction

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The system notion, in a wide range of disciplines from ecology to physics, social sciences, and informatics, has received significant attention in recent years. Generally, the notion *system* can be interpreted as a structure consisting of a large number of interdependent elements. To understand the behavior of a complex system, in contradiction to a simple system approach, requires understanding not only the behavior of the individual elements but also how they act together [1]. This approach should be used in many cases, and it seems that it may be appropriate to describe the problem of information transmission through a noisy channel, which is a very dynamic process.



**Figure 1.** (a) The channel as a simple system based on circuit switching where the information flow is “laminar” and (due to the homogeneity) the problems with queueing can be described by Kendall’s approach. (b) The channel as a complex system, where information from the transmitter to the receiver can follow different, dynamically switched paths that consist of different types of transmission media (heterogeneity), which are connected in nodes, but with queueing systems that manage self-similar traffic.

More precisely, the classical approach seems to be based on tacit assumptions that the channel is a connection that once established (by a circuit switching) does not change during the transmission (Figure 1(a)). According to Shannon [2, 3], the channel is “merely the medium used to transmit the signal from the transmitting to the receiving point.” This suggests that Shannon probably did not pay much attention to the channel as a thing (system). He used the expression “merely the medium”, which can have many interpretations and, for example, can be understood as a “thing” that is not as important as it now seems to be. Of course, in the next sentence he wrote “It may be a pair of wires, a coaxial cable, a band of radio frequencies, a beam of light, etc.” But this is an explanation of what a channel can be and not the assumption that a channel can be a system (particularly a complex one) that itself can influence the transmission (or the channel capacity) by the existence of excess noise or distortion.

We suggest that the channel capacity analysis proposed by Shannon considers the case when the channel has a spatial homogeneous structure, and that possible interactions “inside” the channel are governed by short-range dependencies. That is, from the thermodynamical point of view, the whole system stays in equilibrium and the information flow is laminar. But the assumption that the channel is only a

medium with the properties mentioned seems to fail. The situation nowadays is diametrically different because information from the transmitter to the receiver can follow different dynamically switched paths (packet switching), which can be disturbed by different types and levels of noise (Figure 1(b)). These paths are heterogeneous, that is, a channel is a hybrid consisting of wires and wireless technologies and the contact between them is usually built by a queue, which influences the time that packets need to stay and get out of an actual part (path) of the channel. This heterogeneity and queueing lead to the generation of long-range dependent processes and nonequilibrium states. The information flow becomes turbulent and excess  $1/f$  noise appears. Because circuit switching was replaced by packet switching (which itself is a process that so far cannot be fully understood) information can follow many different paths to “create” the channel rather as a (complex) system built from many different parts. Thus, from the complex systems approach, a channel seems to be a thing that emerges as a system consisting of many heterogeneous parts, which in addition interact (e.g., by queueing problems, congestions, different faults, etc.). The whole problem of describing transmission needs an approach other than information theory.

It should also be noted that during the transmission at each node, the primary route can be dynamically changed. A few questions immediately arise: Why does this change occur? Is this change always a desirable process? Does it lead to self-organization? The resulting routes can have different properties that can be disturbed by various levels of noise or have smaller buffers in their queue. This additionally increases the degree of complexity of such a system. But this complex process can be analyzed by the problem, which is well known in the literature.

Each switched path can be compared to the generation-recombination (g-r) process with a power law that determines the trap’s lifetime, which, for example, can be governed by long-range dependencies and directly leads to excess  $1/f$  noise [4]. This similarity is due to the fact that each dynamically switched path can be a kind of “trap” for packets and its lifetime can vary (e.g., due to the problem of queue overload at the end of a path, see Figure 1(b)). This variety can be governed by a power law and the dependencies that appear can have the long-term property.

As already mentioned, each packet switching is done in the network, which is not a homogenous system, and can be a very dynamic and complex process, whose description requires a wider view based on problems that concern real network topology (e.g., as a “small world”) and a proper description of the network as a queueing system with the dispersion of delays due to different buffer lengths or queue regulations. This process itself can lead to excess noise.

We think that the change from circuit switching (where an established path is disturbed by a level of noise that does not undergo dynamic changes due to path switching and the queueing problems can be described by Kendall's approach [5]) to packet switching (where the levels of noise for different parts of a channel can dynamically change and the queueing problems cannot be described by Kendall's approach due to the existence of self-similarity in network traffic [6, 7]) suggests that total channel capacity can also undergo dynamic changes and even more, can be less than that given by Shannon's theory of information [2, 3]. The whole problem can be understood only when a complex systems approach is taken because in many cases the transmission channels should be considered systems that are no longer simple.

It is known that the capacity of a noisy channel is described by the famous Shannon-Hartley theorem of channel capacity that states [2]:

$$C = W \log \left( 1 + \frac{S}{N} \right), \quad (1)$$

where  $W$  denotes the frequency band,  $S$  is the source power, and  $N$  is the total noise power.

However, is it possible to understand precisely what kinds of processes exist and influence channel capacity in reality by taking into account only equation (1)? This formula cannot say what will happen when the problem of long-range dependencies is taken into account. This paper addresses the possible influence of excess  $1/f$  noise on channel capacity. To understand the problems under consideration, a brief introduction to noise theory is given in Section 2, where some interesting properties of long-range dependencies in noises via spectrum analysis are described. The fundamentals of Shannon's communication theory are given in Section 3, which covers the basic assumptions of Shannon's theory of information and the channel capacity theorem. It also presents a possible influence of the excess  $1/f$  noise on channel capacity and the results of our investigations. Section 4 offers conclusions.

## 2. Noise

Noise in electronic systems is the effect of electrical fluctuations in a structure of elements or electrons carrying the current as they are jolted around by thermal energy. In each circuit many different types of noise can exist, especially thermal, flicker, and generation-recombination (g-r). The statistical properties of each noise can be analyzed in the time domain by its autocorrelation function  $R(t)$ . By the Wiener-Khinchin theorem it is known that for each stationary noise there is a

transformation from the time domain to the frequency domain that is usually called the power spectrum  $S(f)$ :

$$S(f) = \int_{-\infty}^{\infty} R(t) e^{j2\pi f t} dt. \quad (2)$$

A thermal noise in the frequency domain is sometimes called “white noise”, because its spectrum is flat, that is, independent of frequency  $f$ . This noise in the time domain has an autocorrelation function  $R(t)$  that quickly goes to 0. In other words, this noise does not have any long-range dependencies [8, 9]. But when the power spectrum is not flat and behaves like

$$S(f) \approx \frac{1}{f^\alpha}, \quad (3)$$

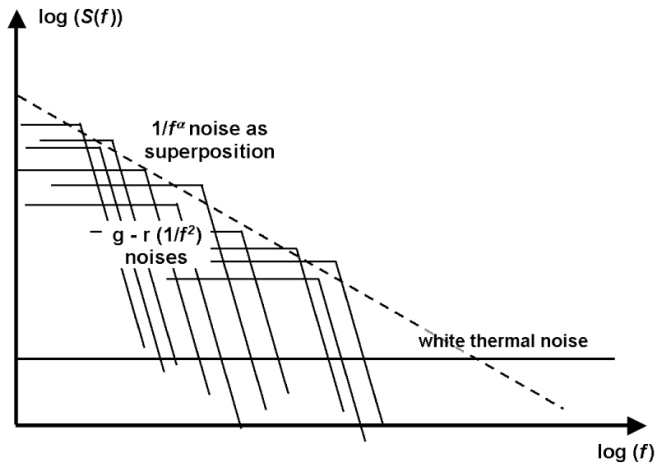
it can be said that such a noise is governed in the time domain by long-range dependencies [8].

In addition, if the probability density function of such a noise is Gaussian or has different densities with a finite value of variance, its fractal properties can be described by the Hurst parameter  $H$ , which is directly connected with  $\alpha$  by the relation [8]:

$$\alpha = 2H - 1 \quad \text{for } \alpha < 1. \quad (4)$$

Such a noise is sometimes called the fractional Gaussian noise or simply  $1/f^\alpha$  (1 over  $f$ ). If  $\alpha = 1$ , it is a “flicker noise” or a “pink noise”. Another important property of  $1/f$  noise is that it is a low-frequency phenomenon that in the higher frequencies is overshadowed by a white thermal noise.

It can be shown that in semiconductors (e.g., in MOS transistors [4])  $1/f$  noise can occur as a superposition of g-r processes that has a flat (white) spectrum to some frequency  $f'$  and then vanishes like  $1/f^2$  [10]. The graphical illustration of equation (3) in Figure 2 shows where each type of noise occurs.



**Figure 2.** Superposition of long-range dependencies in g-r processes gives the type of excess  $1/f^\alpha$  noise that exists in each time scale [10].

In the case of packet switching it is possible to proceed similarly to the problem of the superposition of  $1/f^2$  noises, because each packet or a set of packets can be “trapped” in a path with different noise levels (i.e., different noise power values with the flat spectrum  $S(f)$ ). The flat part of the spectrum of a g-r process will represent a possible level of noise in a given part of a channel, and the  $1/f^2$  part of the characteristics will represent the state of the packet switching (i.e., the situation when a new path is obtained for successive packets). Two possible situations can be assumed. The first situation is when the process of packet trapping is not governed by long-range dependencies, that is, the resultant spectrum of this process will be flat. The second is when the process is governed by long-range dependencies and the dispersion of trap lifetimes is governed by the power law. If the second case appears, the resultant spectrum will show the existence of excess  $1/f$  noise.

### 3. Channel Capacity in the Presence of Excess $1/f$ Noise

Many statements in the literature say that information is the same as entropy. The reason given to Tribus [11] is that Shannon did not know what to call his measure so he asked von Neumann, who said: “You should call it entropy (...) [since] no one knows what entropy really is, so in a debate you will always have the advantage.” Despite this doubt, researchers maintain Shannon’s definition of information

as an entropy for a discrete source where each symbol is some kind of random variable [2].

If source  $S$  sends a symbol from the finite alphabet  $\Psi = \{s_0, s_1, \dots, s_{K-1}\}$  in each time step, then each symbol will occur with probability  $P(S = s_k) = p_k$ ,  $k = 0, 1, \dots, K-1$ . The function  $I$  that describes the amount of information that is connected to each symbol  $s_k$  should be [2]:

$$I(s_k) = \log \left( \frac{1}{p_k} \right). \quad (5)$$

For a discrete information source,  $I(s_k)$  is a discrete random variable that has the values  $I(s_1), I(s_2), \dots, I(s_{K-1})$  with probabilities  $p_0, p_1, \dots, p_{K-1}$ . The average value of such a variable is:

$$H(\Psi) = E[I(s_k)] = \sum_{k=0}^{K-1} p_k I(s_k) = \sum_{k=0}^{K-1} p_k \log \left( \frac{1}{p_k} \right). \quad (6)$$

To analyze the entropy for a continuous random variable instead of a discrete variable, a definition of the differential entropy similar to equation (6) is needed [2, 12]:

$$h(X) = \int_{-\infty}^{\infty} f_X(x) \log \left( \frac{1}{f_X(x)} \right) dx. \quad (7)$$

Shannon showed that when a random variable has a Gaussian distribution, its differential entropy equals [2]:

$$h(X) = \frac{1}{2} \log(2\pi e \sigma^2), \quad (8)$$

and its value is the largest of all probability densities with a finite variance.

When any information is transmitted through some noisy channel, a few entropies can be calculated: the entropy of the source  $H(x)$ , the entropy of the signal received  $H(y)$ , the conditional entropies  $H_y(x)$  and  $H_x(y)$ , and a joint entropy  $H(x, y)$ . If  $H(x) = H(y)$  there is no noise in the channel. The transmission rate  $R$  cannot exceed [2]:

$$R = H(x) - H_y(x), \quad (9)$$

thus the capacity  $C$  of a channel is the maximal transmission rate  $R$  and equals equation (1) [2, 12], where  $N$  is the power of interferences that have a white noise power spectrum. Shannon showed in [3] that white noise is the worst case, because the channel disturbed by such a noise has the smallest total capacity. The power  $P$  of an analyzed

signal can be obtained from the power spectrum of the analyzed noise:

$$P = \int_0^\infty S(f) df. \quad (10)$$

Because the spectrum of white noise has a constant power  $N_0$  for each frequency in band  $W$ , the power spectrum is equal to

$$P_{WN} = \int_0^W N_0 df = N_0 f \Big|_0^W = N_0 W. \quad (11)$$

Taking this into account, equation (1) can be rewritten (assuming that  $S = \text{const}$ ) in the following form:

$$C = W \log \left( 1 + \frac{S}{N_0 W} \right). \quad (12)$$

In the case of  $1/f$  noise, assuming that  $\alpha \neq 1$ , a similar method can be used to obtain (recalling equation (4)):

$$P_{1/f} = \int_0^W \frac{N_{01/f}}{f^\alpha} df = N_{01/f} \frac{f^{1-\alpha}}{1-\alpha} \Big|_0^W = N_{01/f} \frac{W^{2-2H}}{2-2H}. \quad (13)$$

Assuming that the  $1/f^\alpha$  noise and the white noise should have the same power in band  $W$ , the value for  $N_{01/f}$  can be computed from:

$$N_{01/f} = \frac{N_0 W (2-2H)}{W^{2-2H}} = N_0 \frac{2-2H}{W^{1-2H}}. \quad (14)$$

In [12] it was shown that Shannon's considerations are agreeable only if important assumptions are made: the total power of noise having the flat spectrum ( $S_{WN}(f)$ ) and the  $1/f$  spectrum ( $S_{FN}(f)$ ) should be equal for each frequency  $f$  in a band limited to  $W_{\max}$ , that is:

$$\int_0^{W_{\max}} S_{WN}(f) df = \int_0^{W_{\max}} S_{FN}(f) df. \quad (15)$$

This comes from Shannon's observation that if Gaussian noise has a spectrum  $S(f)$  that is different from white noise, then its entropy power  $N_1$  equals [2, 3]:

$$N_1 = W \exp \frac{1}{W} \int_0^W \log S(f) df. \quad (16)$$



For the white noise it will be:

$$\begin{aligned} N_1 &= W \exp \frac{1}{W} \int_0^W \log N_0 df = \\ &W \exp \frac{1}{W} \log N_0 W = WN_0 = N, \end{aligned} \quad (17)$$

thus if  $S(f) = \text{const}$ , then  $N_1 = N$ .

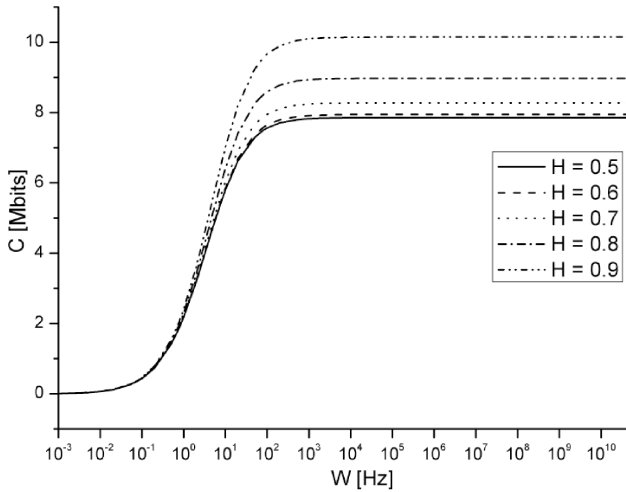
For the  $1/f$  noise it will be:

$$\begin{aligned} N_1 &= W \exp \frac{1}{W} \int_0^W \log \left( \frac{N_{0_{1/f}}}{f^\alpha} \right) df = \\ &W \exp \frac{1}{W} f \left( \log(N_{0_{1/f}} f^{-\alpha}) + \alpha \right) \Big|_0^W = \\ &W \exp \left( \log(N_{0_{1/f}} W^{-\alpha}) + \alpha \right) = N_{0_{1/f}} W^{1-\alpha} e^\alpha. \end{aligned} \quad (18)$$

The entropy power, in the case of  $1/f^\alpha$  noise taking into account equations (14) and (18), equals [12]:

$$N_1 = e^{2H-1} W N_0 (2 - 2H), \quad (19)$$

where  $N_0$  is the entropy power for white noise. From equation (19) it can be seen that if  $H = 0.5$  (the white noise case), then  $N_1 = N$ ; in other cases  $N_1 < N$  (except the special case when  $H = 1$ ). The white noise case is the worst one, giving the smallest channel capacity  $C$  (see Figure 3).



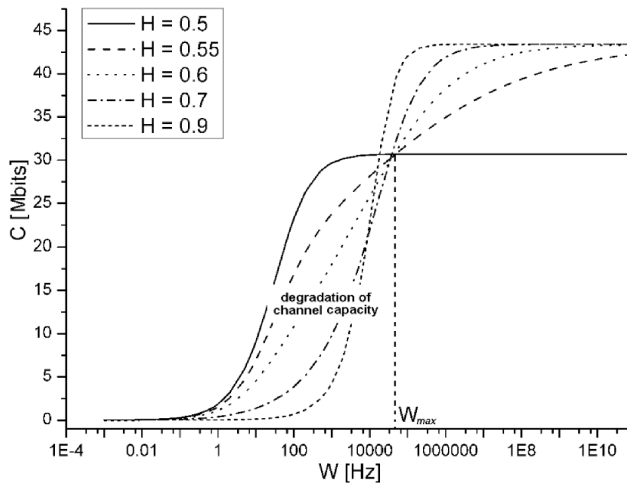
**Figure 3.** Channel capacity influenced by noise with long-range dependencies via Shannon's theorem with equation (15). White noise is the worst case.

But when the assumption that both white noise and  $1/f^\alpha$  noise have an equal power (or  $1/f$  noise has a smaller power), it is possible to imagine the situation that can be satisfied only for some  $f = W_{\max}$ , that is, the power of  $1/f$  noise will be greater than white noise. This situation is natural if  $1/f^\alpha$  noise is considered an excess noise that occurs when the dynamic behavior of each g-r process starts exhibiting long-range dependencies. In other words, such a noise occurs dynamically as the excess noise due to the long-range dependent relations (governed by a power law) between the relaxation times of each g-r process. In the case of a channel, this means that dynamic switching of paths with different noise levels will be governed by long-range dependencies (e.g., due to traffic self-similarity or problems with queuing). In such cases white noise cannot totally cover the excess  $1/f^\alpha$  noise. Thus, for frequencies smaller than  $W_{\max}$ , the power of  $1/f^\alpha$  noise will dominate over the white noise and the capacity of a channel disturbed by such a dynamically occurring noise will be even smaller. It can be said that a degradation of capacity appears in the channel. In such a case the capacity  $C$  equals [12]:

$$C = W \log \left( 1 + \frac{S}{N_1} \right) = W \log \left( 1 + \frac{S}{e^{2H-1} W N_0 (2-2H)} \right), \quad (20)$$

where  $e^{2H-1} W N_0 (2 - 2H)$  is the entropy power for  $1/f^\alpha$  noise and  $N_0$  is the entropy power for white noise.

Figure 4 shows the phenomenon of channel capacity degradation for values  $W < W_{\max}$ . This is due to the existence of excess  $1/f^\alpha$  noise that covers white noise, and the power of its interference is a few times larger than in the case of white noise.



**Figure 4.** Channel capacity degradation resulting from the existence of dynamically appearing excess  $1/f^\alpha$  noise.

#### 4. Conclusions

As has been observed, the classical information theory that comes from Shannon's theorems cannot fully describe all phenomena that influence channel capacity. The existence of excess  $1/f$  noise is an inherent feature of each complex system that is governed by processes with long-range dependencies. Their existence in communication systems, due to possible dynamics of path switching, leads to channel capacity degradation, so a broader frequency band  $W$  is needed, especially when the phenomenon of traffic self-similarity occurs. It seems that the current transmission channels are also complex systems and their complex properties need to be included because they have a significant influence on channel capacity. The degradation that appears is a primary problem for quality of service.

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