Implementing Cellular Automata for Dynamically Shading a Building Facade

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We present a practical cellular automaton (CA) implementation from the field of architecture that drives the modular shading system of a building facade. Some CAs produce patterns that seem to live their own life and may therefore please the human eye. Probably the most important quality of a good design is integrity of elements. By nature, a CA is an essence of integration, where all elements are interconnected and locally related to each other. Due to the computational irreducibility of CAs [1], controlling them to perform purposeful actions [2] is often challenging. Nevertheless, visual effects in the patterns created often develop intriguing complexity that is difficult to achieve by means of artistic will, whim, or chance. The four classes of CA behavior are presented with conjunction to the problem of average grayness of a pattern. Two CA classes are analyzed for potential practical use: 2-color, 1-dimension, range-1 (2C-1D-R1) and 2-color, 1-dimension, range-2 (2C-1D-R2). One problem discussed is the linear gradual change of average grayness as a function of the sequence of initial conditions. Another problem discussed is choosing a sequence of initial conditions to cause a desired change in the opacity of the shading array. A proposed realization includes a mechanical scheme that could be made inexpensively by using coupled polarized film. A rotation of one polarized film by 90 degrees causes a change in the element's transparency.

1. Introduction

Interesting qualities of the cellular automaton (CA) have been studied for decades, but practical (physical) applications other than generating pretty pictures are still sparse. A CA is a collection of colored cells on a grid of specified shape that evolves through a number of discrete time steps according to a set of rules based on the states of neighboring cells. The rules are then applied iteratively for as many time steps as desired. In the 1940s, Stanislaw Ulam studied the growth of crystals using a simple lattice network as his model while working at the Los Alamos National Laboratory. John von Neumann was one of the first to consider such a model, and incorporated a cellular model into his "universal constructor" [3]. CAs were studied in the early 1950s as a possible model for biological systems. CAs can be viewed as the

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simplest model of life and as such, often despite their striking underlying simplicity, produce puzzling results (Figure 1). In a nutshell, these are the only requirements of a CA: a regular grid, a set of rules, and an initial state.

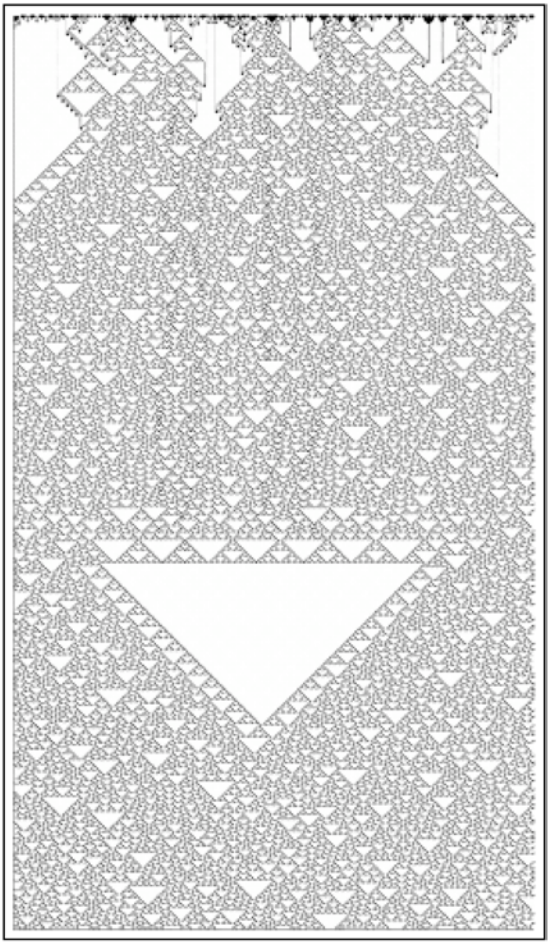

Figure 1. The combination of a regular grid, a set of rules, and an initial state results in the so-called "behavior" of a CA.

2. Concept

The proposed modular shading system changes the average opacity of a building facade and takes visual advantage of the emerging behavior of a CA (Figure 2).

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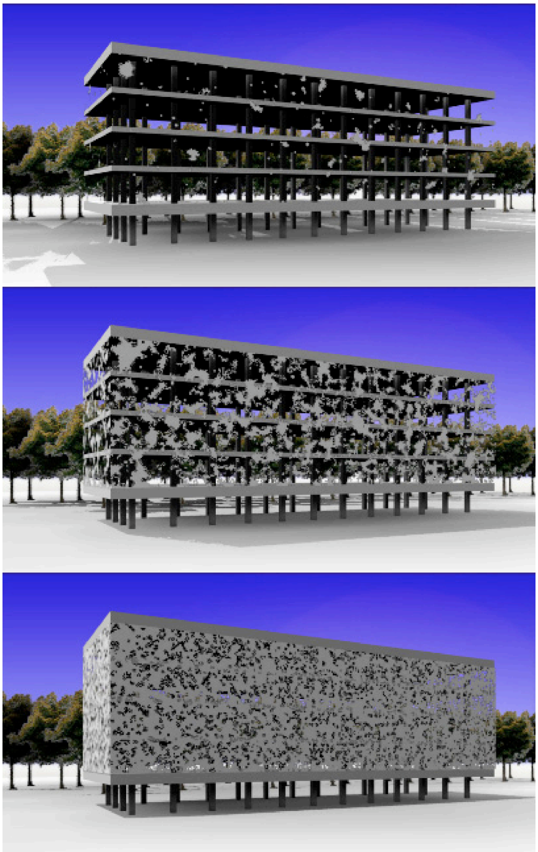

Figure 2. Visualization showing the organic behavior of a building facade, where opacity is controlled in relation to the daylight conditions. The facade evolves to maintain a constant level of light indoors with changing outdoor luminosity levels.

3. General Approach

The general approach is based on an opto-mechanical system of square plates made of polarized glass. The coupled plates are transparent and become opaque when one of them rotates by 90 degrees (Figure 3). The white cell (value 0) is equivalent to the transparent state of a module (facade element); similarly, a black cell (value 1) represents an opaque state. The notion of average grayness is the ratio between the number of black cells (value 1) to all cells in the array. For the practicality of a physical implementation, one-dimension (1D) CAs are considered and the top row of the array is set directly to give the initial conditions. The rest of the cells evolve down the array

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according to the CA rule (Figure 4). Periodic geometry is usually used to avoid boundary problems at the edges of the grid, so the leftmost column is virtually adjacent to the rightmost column of the grid. In this case, to avoid visual confusion caused by the interaction of virtually adjacent cells (possible changes to the left or right end of an array caused by cells on the opposite end), a nonperiodic geometry was applied. That is, in the case of a range-1 (R1) CA, cells in the extreme columns have only one neighbor, whereas an R2 has two neighbors. In the inner columns each cell has either two or four neighbors, respectively.

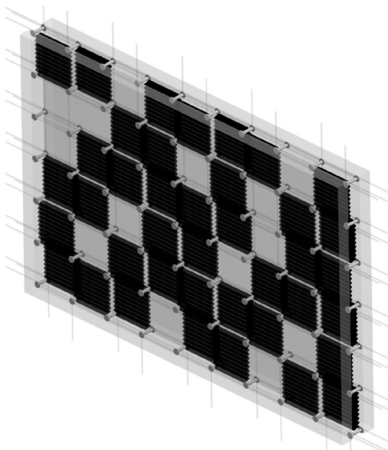

Figure 3. Schematic of a part of the facade with the opacity controlled by a CA.

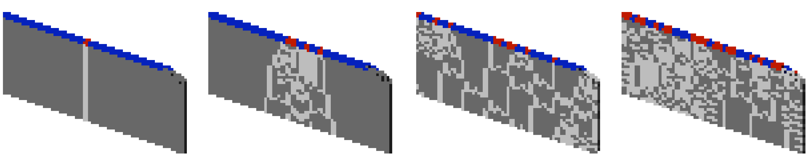

Figure 4. The overall grayness of the array is controlled by the top row. Four different initial conditions are shown with 1, 4, 10, and 20 transparent cells in the top row.

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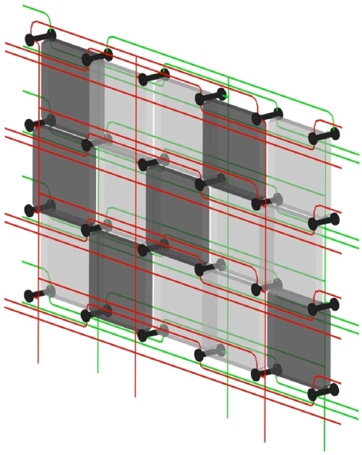

Figure 5. Detailed visualization showing the wiring for an R2 CA.

4. From Simple to Complex

There are four main classes of CA behavior: constant, repetitive (and nested), (pseudo)random, and complex. These classes are already present in the simplest, nontrivial case of a 2C-1D-R1 CA. Here is a description of the naming convention being used: two-color (2C, or binary; possible states of a cell are black for value 1 or white for value 0), one-dimension (1D; the CA is a simple one-unit-high stripe of cells and the cell state changes every cycle, so it is convenient to show the history of the changing states as a series of stripes together forming an array), range-1 (R1; a cell's neighbors are defined as the adjacent cells on either side, i.e., a cell and its two neighbors form a neighborhood of three cells). The term *general* describes a CA where each value of neighboring cells is an input, as opposed to *totalistic*, where the input is an average value of the neighboring cells. The simplest general CAs are referred to as *elementary*. Figures 6 through 10 show examples of all these classes of behavior. The example rule is given along with the corresponding grayness function that shows the relationship between the number of black cells in the initial condition and the number of black cells in the whole array. The initial conditions of neighboring arrays differ by one randomly removed black cell.

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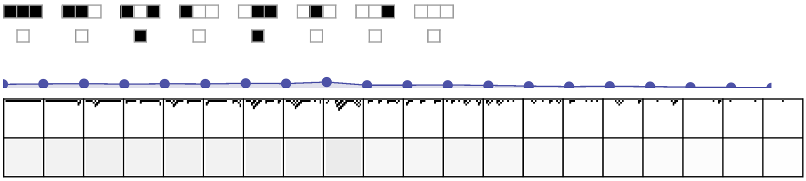

Figure 6. Class 1. Constant, any initial condition produces a uniform pattern (rule 40).

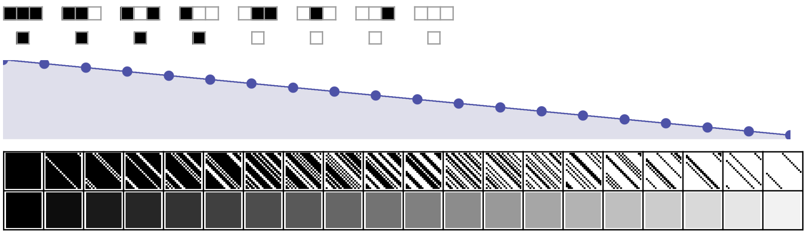

Figure 7. Class 2. Repeating patterns with loops and stripes (rule 240). In some instances of class 2 CAs the gray of the whole array is proportional to the gray of the initial condition's row. These CAs are well suited for shading purposes.

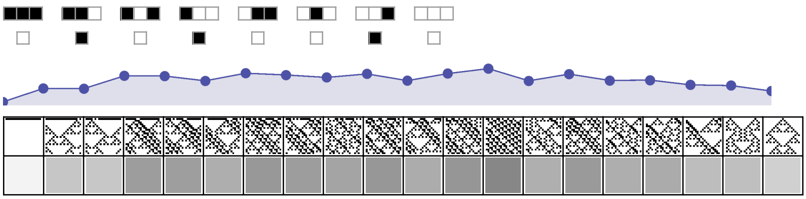

Figure 8. Class 2A. Nested, with regular fractal patterns (rule 82). Nesting is clearly visible at almost any set of initial conditions but there is usually very little variation in the average grayness.

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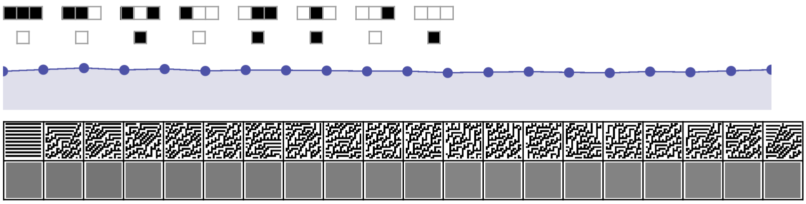

Figure 9. Class 3. Pseudorandom with a seething pattern (rule 45). The average grayness is nearly the same regardless of the initial conditions because of noise.

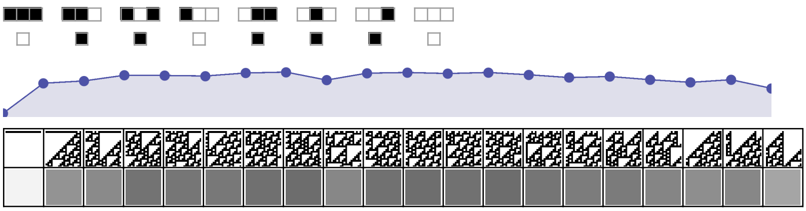

Figure 10. Class 4. Complex patterns with gliders, some rules support universal computation (rule 110). There is an interesting mixture of order and chaos, but as in the case of random behavior, the average ratio between black and white cells usually remains fairly constant.

5. Two-Color One-Dimension Range-1 Elementary Cellular Automata

The search for an appropriate CA started in the simplest nontrivial class of 2C-1D-R1 CAs. There are $2^3 = 8$ possible patterns for a neighborhood. This gives $2^8 = 256$ possible rules, which is not too many. The search for interesting rules can be done by simple simulation and browsing through all representative possibilities. Figure 11 shows the most appropriate CAs that have fairly proportional grayness curves. This selection from all of the 256 CAs was done by comparing the grayness function charts. Interestingly, in the rule sets of the chosen CAs, the number of black cells usually equals the number of white cells with a deviation of at most 1. Here is a list of the CAs chosen and the corresponding number of black cells: [142, 4], [154, 4], [162, 3], [166, 4], [170, 4], [174, 5], [176, 3], [180, 4], [184, 4], [200, 3], [204, 4], [208, 3], [210, 4], [212, 4], [226, 4], [236, 5], [240, 4], [244, 5]. An explanation of this notation is given in Figure 12. In accordance with our intuition, the grayness curve usually reflects the balance between black and white cells in the CA rule set.

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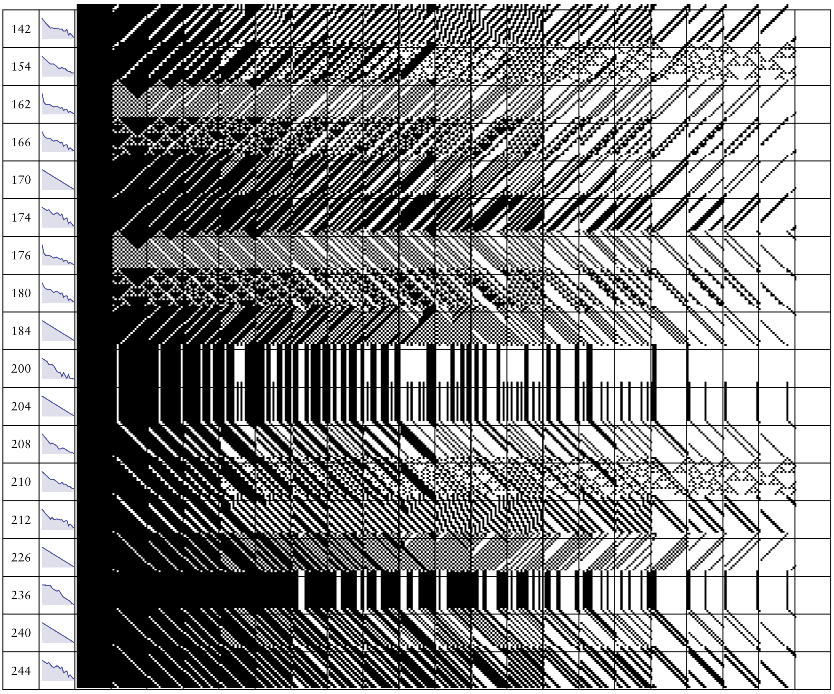

Figure 11. Table of all 1D elementary CAs that have an appropriate grayness function. Column 1 is the rule number. Column 2 shows the array grayness as a function of initial gray (top row). The remaining columns show 20 patterns generated by the given CA showing the history of 20 steps of evolution from the given initial conditions. From left to right each initial condition differs by one randomly removed black cell starting from 19 black cells (1 white) and ending on 0 black cells (20 white).

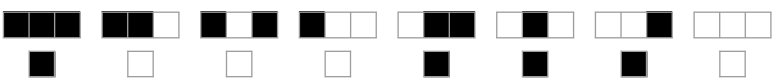

Figure 12. Example: [142, 4] = Rule 142 ($10\,001\,110_2 = 142_{10}$) and four black cells in the set of rules.

If there are four black cells in the given set of initial conditions (i.e., equal to the number of white cells), then the grayness function is fairly linear. If the number of black cells in the set of rules is lower (3) or higher (5), then in general the curve becomes concave or convex respectively, as shown in Figure 13. All of these CAs have good control of grayness, but since they do not belong to the complex class 4, they may not be the most interesting visually.

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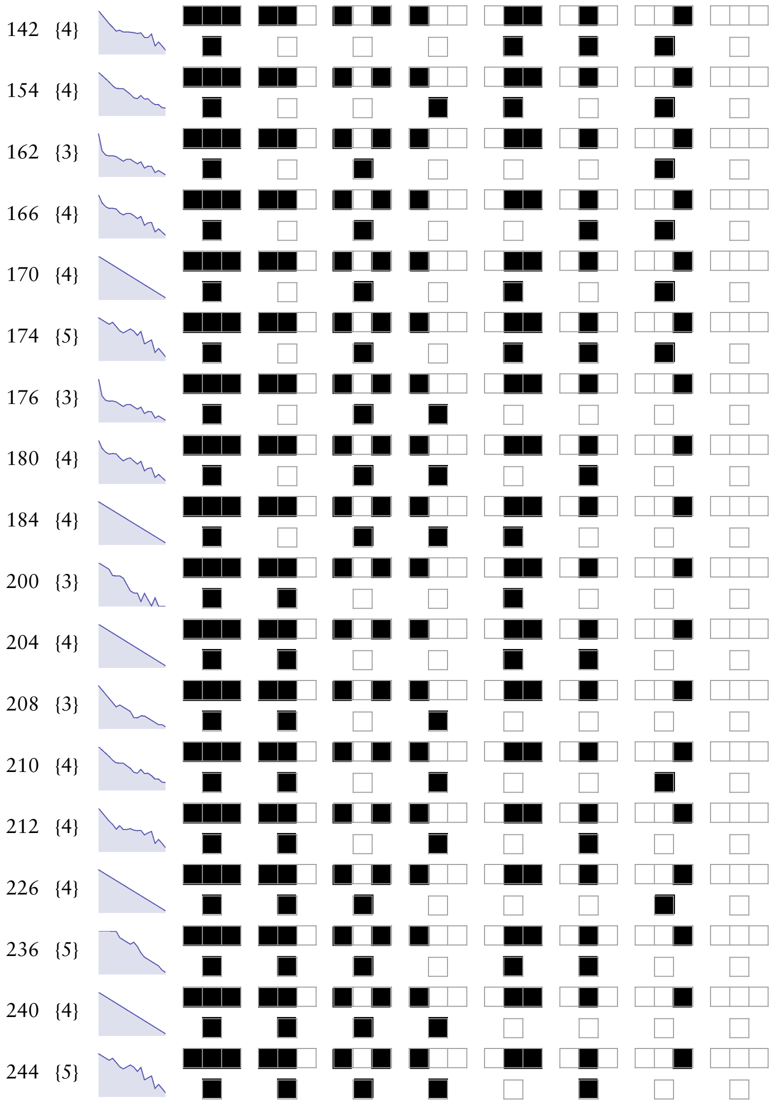

Figure 13. The curvature of the grayness function depends on the proportion of black to white cells in the rule set and usually becomes convex if black cells are in the majority and concave if in the minority.

Among the 256 elementary CAs only two belong to class 4. For the same sequence of initial conditions, the grayness function is nonmonotonic and resembles class 3 (pseudorandom) behavior as shown in Figure 14. Perhaps it is possible to set the sequence of initial conditions for the two complex-behaving CAs so that the overall grayness of the array would gradually change, but the purpose of this project is to

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find CAs that produce visually interesting patterns with straightforward control over their grayness.

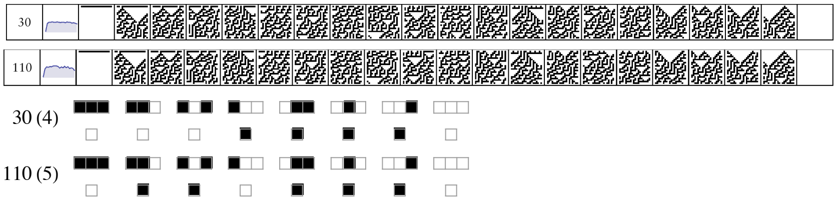

Figure 14. In the set of 2C-1D-R1 CAs, there are two in the complex class 4, but the grayness curve is nonmonotonic.

Since searching through the elementary CAs did not bring satisfactory results, the investigation moved toward more complicated rules, hoping that in a greater number of possible CAs there will be more that meet both of the given criteria. This could be done by increasing the number of possible states of a cell (colors), by increasing the dimension, or as in this project, by increasing the size of the neighborhood.

6. Two-Color One-Dimension Range-2 General Cellular Automata

The only difference from the previous category of CAs and the 2C-1D-R2 CAs is the neighborhood size. Range-2 (R2) means that the cell's state in the next generation depends on the cell's own state, as well as the state of the closest and the second-closest cells on each side. Examples of such CAs are shown in Figures 15 and 16.

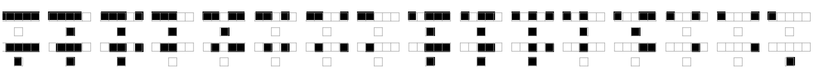

Figure 15. Set of rules for a sample R2 CA: 2029576417_{10} ($01111000111110001110000011100001_2$).

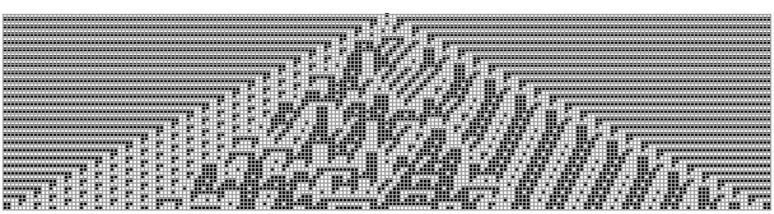

Figure 16. Fifty steps of evolution of the CA {2029576417, 2, 2} starting from a single black cell.

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There are $2^5 = 32$ possible tuples for a neighborhood with five binary cells. There are then $2^{32} = 4\,294\,967\,296$ possible rules. This is a substantially greater number than before, therefore different search methods are used. The new method was based on rule symmetry, which means that for inverted initial conditions, the generated patterns will be exactly inverted: $(0 \to 1, 1 \to 0)$. Figures 17 and 18 show examples of two R2 CAs that produce interesting geometrical patterns, but only the latter one is symmetric.

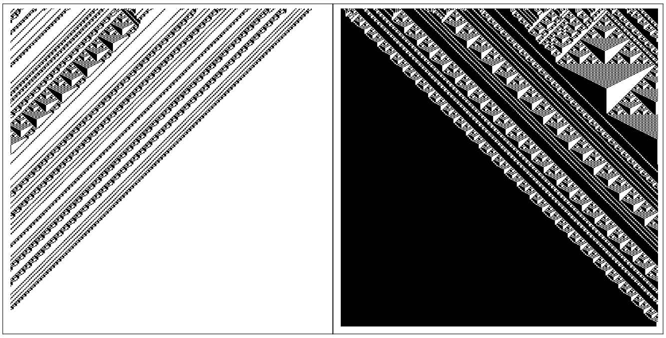

Figure 17. A nonsymmetric rule; an inversion of the initial conditions does not invert the whole pattern.

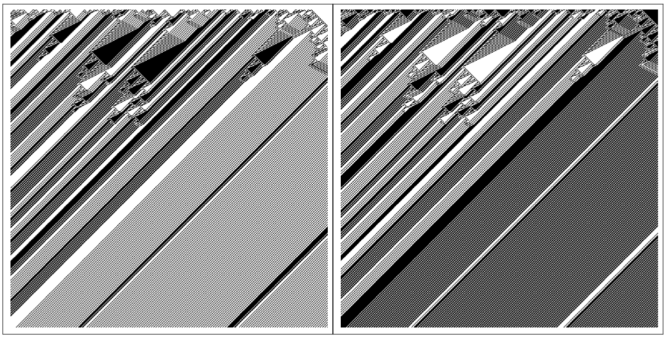

Figure 18. A symmetric rule; an inversion of the initial conditions produces a negative image.

The search was limited to rules having an equal number (16) of black and white cells in the rule set. There are $32!\,/\,16!^2 = 601\,080\,390$ such rules, which is approximately 14% of all the 2C-1D-R2 general CA rules (Figure 19).

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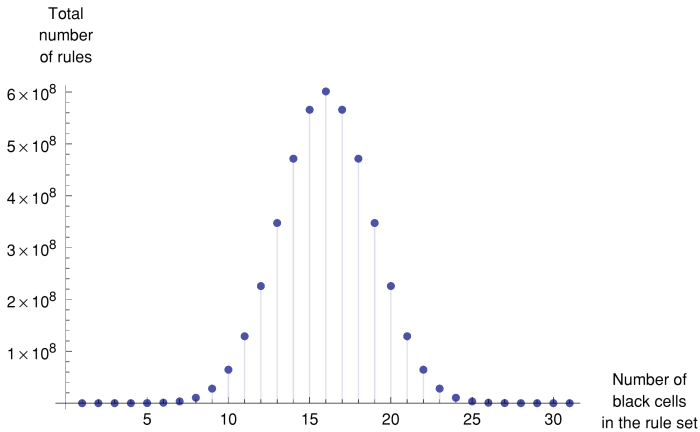

Figure 19. The distribution of 2C-1D-R2 general CA rules as a function of the number of black cells in the rule set.

However, most of these rules produce patterns that are neither interesting (noncomplex) nor applicable (nonproportional grayness function, as shown in Figure 20).

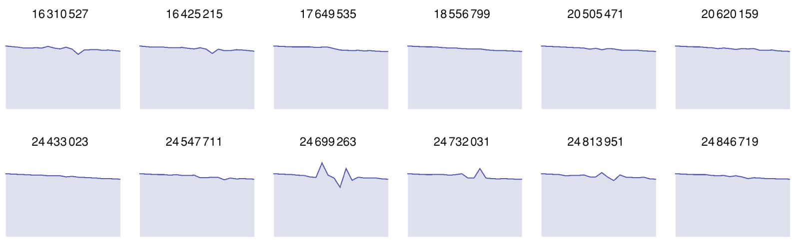

Figure 20. Some 2C-1D-R2 general CAs shown with their corresponding grayness functions produced from the same sequence of initial conditions. The grayness curves are either nonmonotonic or nearly constant.

The next step is to search through all of these rules looking for those that produce a wide range of grays and have a fairly proportional grayness function (Figure 21).

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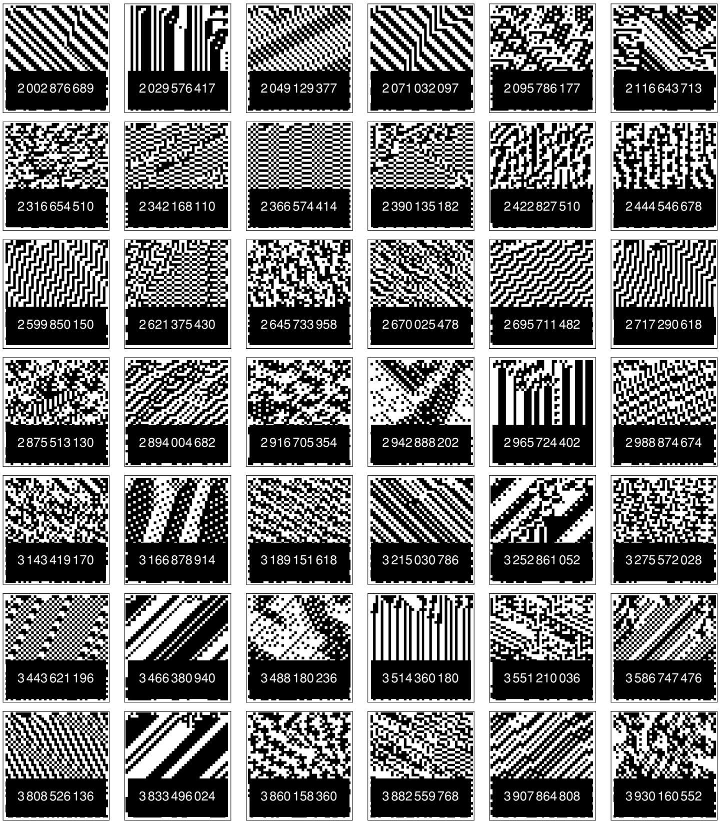

Figure 21. Some promising 2C-1D-R2 general CAs with sample patterns produced from the same initial conditions.

7. Rules {3471517836, 2, 2} and {3818817080, 2, 2}

Two interesting CAs were found: CA 3471517836 (Figure 22), which produces a wide range of grays at many random initial condition sequences with fairly interesting patterns, and CA 3818817080, which produces highly complex patterns, but in which controlling the grayness function is problematic (Figure 23).

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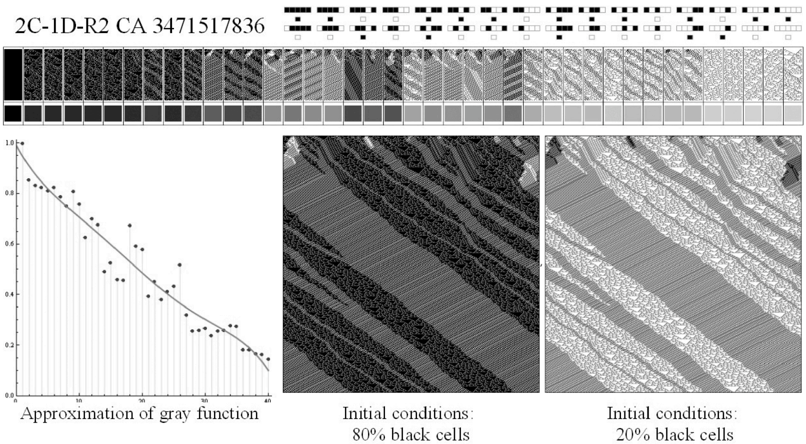

Figure 22. Rule {3471517836, 2, 2}. (a) The rule set. (b) Incremental change from 100% to 5% black cells in the initial conditions showing the patterns and their average grayness below. (c) From the left: the grayness curve and two symmetric sample patterns with initial conditions having 80% and 20% black cells.

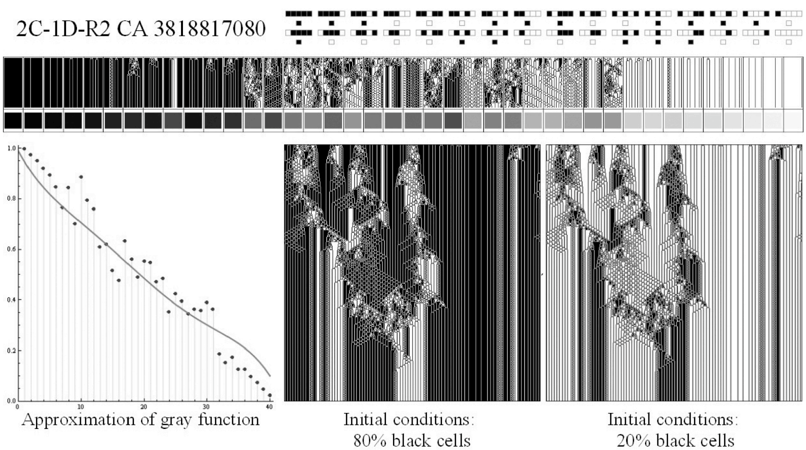

Figure 23. Rule {3818817080, 2, 2} with (a), (b), and (c) the same as Figure 22.

8. The Sequence of Initial Conditions

Setting the initial conditions is as important as finding the appropriate rule and must meet two constraints: the k^{th} initial condition has ex-

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actly one black cell less than $(k-1)^{\text{th}}$ and all the remaining black cells are preserved. Such constraints ensure that changes in the shading array will not appear excessively chaotic or disturbing and the transition from one state to another can be understood and rationally interpreted by an observer. Figure 24 shows an example of an array that is five cells wide, which at the beginning has five black cells. The black cells turn white in five steps in given order: [1, 2, 3, 4, 5]. Generally, if the changing sequence is more scattered, more interesting patterns are usually produced than in such a consecutive way. In this example, the change from black to white can be made in $5! = 120$ different sequences (Figure 25). Figure 26 shows their grayness curves. Among them there are some nonmonotonic patterns (e.g., as marked with a black rectangle) and some fairly proportional patterns (e.g., as marked with a gray rectangle). More detailed analysis is shown in Figure 27.

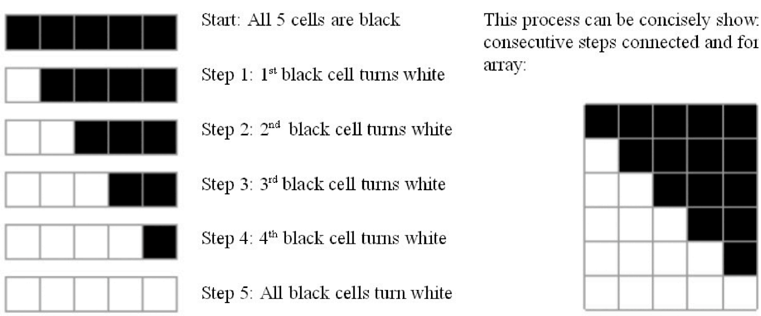

Figure 24. The change of initial conditions in the sequence 1, 2, 3, 4, 5.

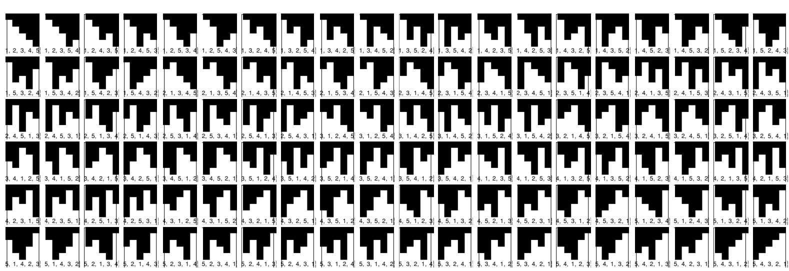

Figure 25. All possible sequences of turning one of the five cells at a time from black to white.

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Figure 26. Grayness curves of the CA 3818817080 at the 5×5 array corresponding to the sequences of initial conditions shown in Figure 24.

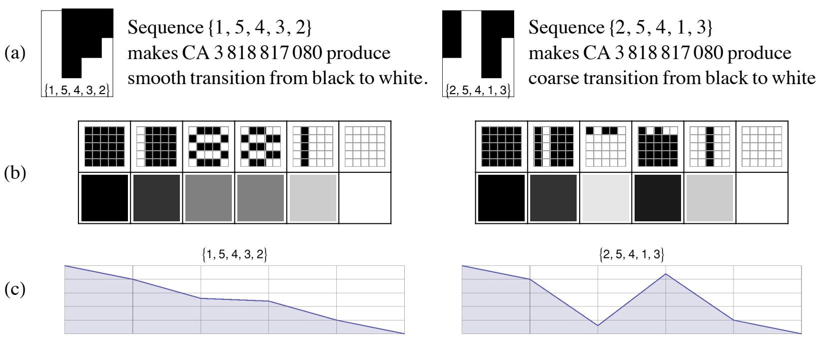

Figure 27. The left column shows a sequence that produces a smooth and nearly proportional grayness curve. The right column shows a coarse grayness curve. (a) The transition sequence. (b) The upper chart shows the pattern produced by the CA and the lower is an averaged gray of that pattern. (c) Grayness curve with values corresponding to the average gray values of a pattern.

For a moderate size array, in the range of a few dozen cells, a simple backtracking search method can be used. In the case of larger array sizes (applicable in reality), the number of possible initial condition sequences grows astronomically, therefore heuristic methods should be applied.

9. Some Practical Issues

Since the concept is new, many technical and practical issues arise.

- If certain parts of the building facade are opaque and some transparent, will it cause a glare problem?

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- Is it possible to adjust the pattern locally?
- How does the system respond to the outdoor lighting conditions?
- Is it possible to use different CAs that serve the same purpose but generate different patterns?
- What should be the size of the cell? The larger the cell, the more easily the pattern can be seen, but the smaller the cell, the higher the precision of the average opacity that can be achieved.

These, among other problems, will be addressed in a forthcoming paper concerning the fabrication of a prototype for such a shading device.

10. Conclusions and Future Work

- It is possible to control the average opacity of a shading array and create very interesting patterns at the same time with CAs.
- Simple search methods for setting initial conditions fail at larger array sizes, so heuristic methods should be applied.
- Possible application of a simple 2D CA. In the case of 2C-2D-R1 (2-color, 2-dimension, range-1) general CAs with a Moore neighborhood, the wiring becomes much simpler, reaching only four immediate neighbor cells. A Neumann neighborhood is more likely to create interesting patterns with the slightly more complicated wiring of eight neighbors. In both cases the wiring is much simpler than in the case of a 1D-R2 CA (Figure 28).
- Possible application for a totalistic CA. In this case the minimum number of states rises to three. But, as is the case for all CAs more complicated than the elementary ones, the number of totalistic CA rules is significantly smaller than for general CAs of the same category and the complexity of patterns produced is sufficient for the discussed shading concept. Figure 28 shows a table with CAs that could be used for this purpose.
- A different grid, for example, an implementation of a hexagonal grid with a circular cell placed between two glass panels.
- A prototype is under consideration. The logic circuit being investigated is shown in Figure 29.

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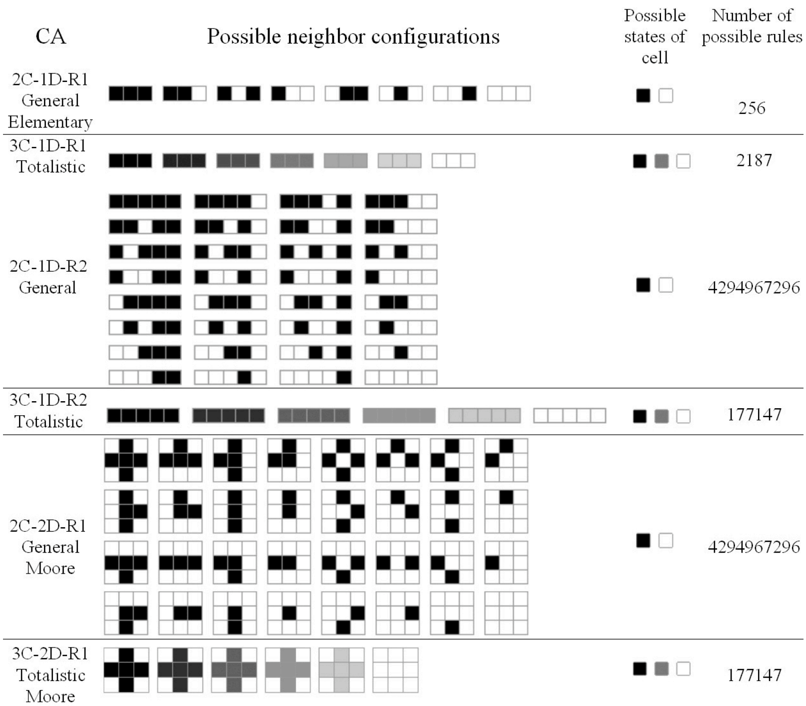

Figure 28. This table shows the simplest CA categories that could be used for shading purposes.

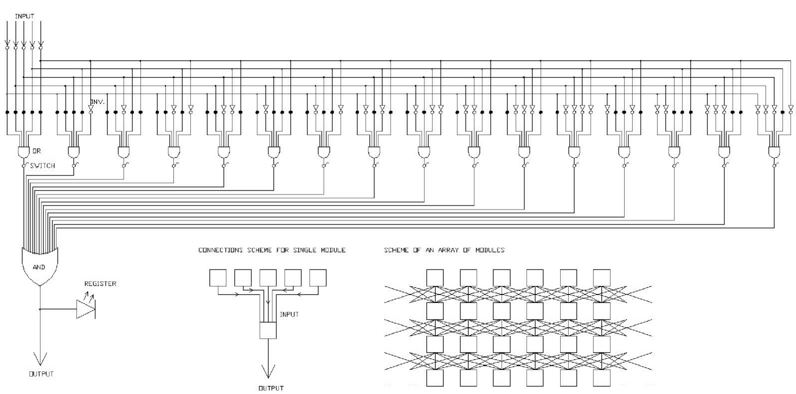

Figure 29. General logic scheme of an electric circuit for an R2 CA module.

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