

# A Model of City Traffic Based on Elementary Cellular Automata

David A. Rosenblueth\*

Carlos Gershenson†

*Departamento de Ciencias de la Computación*

*Instituto de Investigaciones en Matemáticas Aplicadas y en Sistemas*

*Universidad Nacional Autónoma de México, Ciudad Universitaria*

*Apdo. Postal 20-726, 01000 México D.F. México*

*\*drosenbl@servidor.unam.mx*

*†cgg@unam.mx*

---

Several highway traffic models based on cellular automata have been proposed. The simplest one is elementary cellular automaton rule 184. We extend this model to city traffic with cellular automata coupled at intersections using only rules 184, 252, and 136. We study the model properties by simulating a single intersection. We describe the different dynamical phases of the model with velocity-density and flux-density diagrams. The model is useful for studying the problem of traffic light coordination for very large systems.

---

## 1. Introduction

A mathematical model is an abstraction of a system. Ideally, such an abstraction should be as simple as possible, provided that the essential properties of the system are preserved. Hence, the complexity of a model faces a compromise between simplicity and usefulness. A complex model provides an accurate description of the system's behavior, but may also bring added difficulty to its representation, computation, and analysis. In the study of vehicular traffic, for example, numerous models have appeared in the literature [1–10]. Such models are useful for different purposes depending on their complexity. Here we propose a vehicular model that is as simple as possible while reproducing city traffic behavior. The purpose of our model is not predictive but explanatory.

Our model is based on elementary cellular automata (ECAs), which we briefly present in Section 2. This is followed by a summary of previous traffic models based on cellular automata (CAs) in Section 3. Our model is presented in Section 4 and our results are shown in Section 5. Potential refinements to the model are given in Section 6 and conclusions close the paper in Section 7.

## 2. Elementary Cellular Automata

---

CAs were perhaps first studied by Stanisław Ulam and John von Neumann [11] as a tool for modeling biological systems. Later, John Conway devised his well-known “Game of Life” [12] using such a formalism. More recently, the work of Stephen Wolfram [13, 14] showed applications to many areas of science, further increasing the interest in CAs. As a modeling tool, CAs are especially valuable because of the simplicity of their specification, on the one hand, and the complexity in their behavior, on the other hand.

An ECA is a collection of cells arranged on a one-dimensional array. A cell in such an automaton has only two possible states (0 or 1, say). Time is discrete and all cells’ states are updated synchronously. Moreover, the state of a cell in the next time step, or “tick,” depends only on the present states of that cell and those of its nearest neighbors. As a result, the behavior of an ECA can be described by a table specifying the state a given cell will have in the next “generation” based on the state of the cell to its left, the state of the cell itself, and the state of the cell to its right. Such a table has as input these three current states and as output the state of a cell in the next generation. Wolfram names each ECA with the binary numeral, called its “rule,” resulting from reading the output of the table when the inputs are lexicographically ordered. Because cells’ states are updated synchronously, an ECA can be readily simulated with only two arrays of bits.

## 3. Models of Vehicular Traffic and Cellular Automata

---

In this section, we first give an account of vehicular traffic models in general and then proceed to summarize models based on CAs.

In empirical observations of highway traffic, it is possible to notice two different regimes [15]. For low densities (number of vehicles per length unit), the flux (number of vehicles per time unit) shows an approximately linear behavior. For higher densities, however, the flux exhibits strong fluctuations resulting in a complex behavior that is still not clearly understood [7]. First, such fluctuations prevent the use of a functional model. Second, hysteresis has been noticed, where the flux is greater when the density increases than when the density decreases. Third, metastable states (i.e., states in a precarious equilibrium) have been observed. Authors normally distinguish between at least two different congested (jammed) regimes: the synchronized and the stop-and-go phases [7].

As a result of this complexity, myriad highway traffic models have appeared in the literature. Macroscopic models view traffic as a one-dimensional compressible fluid [7]. The microscopic approaches, by contrast, model each individual vehicle. Within the microscopic treat-

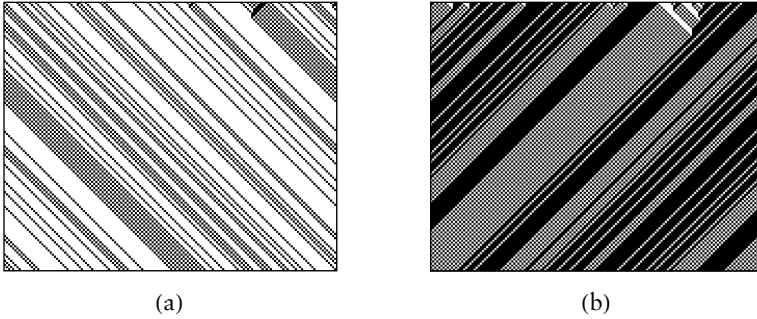
ments, kinetic theories model traffic as a gas in which each particle represents a vehicle [16]. The class of follow-the-leader models represents each vehicle with a motion equation in a system of interacting classical particles [7]. Coupled-map lattice models treat time as a discrete variable and the dynamical equations for each vehicle become a discrete dynamical map [17]. Finally, CA models play a prominent role. One of the main reasons is that these models are computationally cheap.

Possibly the first traffic model that could be viewed as a CA was that of Cremer and Ludwig [18]. By using binary one-dimensional arrays, this model represents the presence/absence of a vehicle with each of the two states of a cell, so that each vehicle occupies exactly one cell. These authors used bitwise Boolean operations together with shifts to update the state of the automaton. Successive applications of different operations can simulate acceleration, deceleration, lane changing, passing, and turning. Observe that updating the state of each cell requires the state of neighbors that are not the nearest ones to such a cell. This CA, therefore, is not elementary.

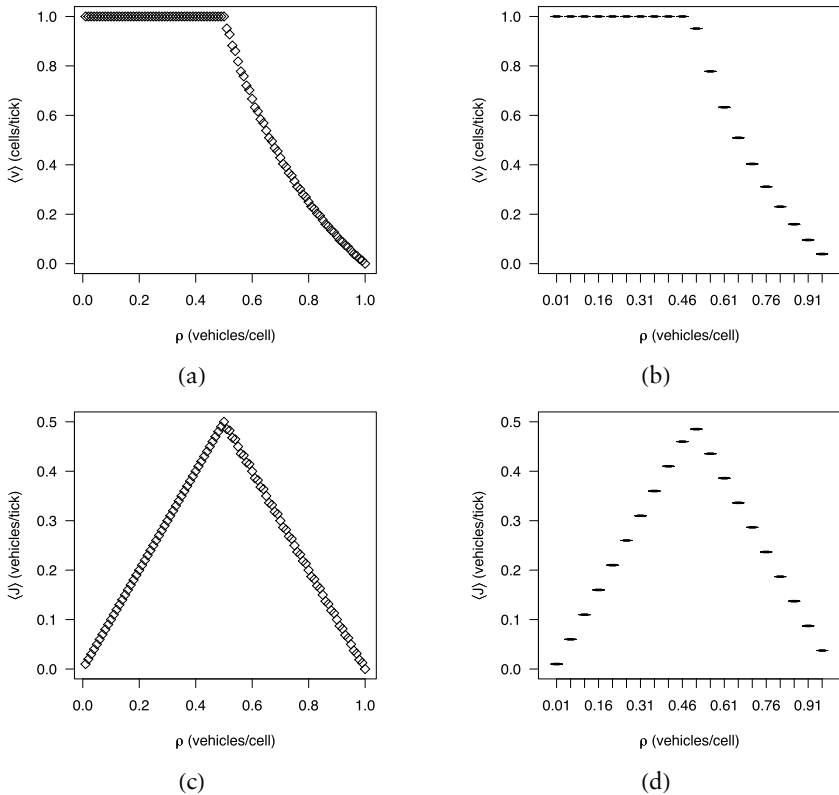
ECAs, nonetheless, can model traffic as well. We can assume that whether a vehicle moves forward or not depends only on the presence or absence of another vehicle just in front. Supposing that a vehicle moves one cell to the right if and only if such a cell to the right is empty, then rule 184 corresponds to traffic moving to the right [9]. Apparent movement is created as follows: if the state of a cell is 1 and that of its right neighbor is 0, then this rule assigns 0 to the cell's state in the next generation, eliminating the vehicle from the current cell (which accounts for 110 and 010). Similarly, if the current cell's state is 0 and its left neighbor is 1, then the cell's next state is 1, thus apparently making the vehicle in the left neighbor move one cell to the right (covering cases 100 and 101). Two other situations must be dealt with. First, a vehicle cannot move if there is another vehicle directly in front (i.e., 111 and 011). Second, if there is no vehicle in the cell or its left neighbor, then in the next generation the cell will have no vehicle either (i.e., 001 and 000), see Table 1 and Figure 3 in Section 4.1.

By randomly placing vehicles on the streets, this CA exhibits two kinds of behavior, that is, phases, depending on the initial density  $\rho$  of vehicles. For  $\rho < 50\%$ , vehicles stabilize in a flow phase, without interacting with each other and are constantly moving. By contrast, for  $\rho > 50\%$ , the traffic flow is jammed, as congestion waves move to the left, that is, in the opposite direction of traffic. The temporal evolution of rule 184 in both phases is shown in Figure 1. Average velocities and fluxes for different densities are shown in Figure 2.

Rule 184 has also been used to model surface deposition [19], ballistic annihilation [20], context-free parsing (for  $\rho = 50\%$ ) [14], and surface heating [21].



**Figure 1.** Evolution of rule 184. Black cells (1) represent vehicles, white cells (0) represent spaces. Traffic flows to the right, time flows to the bottom. (a) In the free-flow phase ( $\rho = 0.25$  shown) all vehicles flow at a velocity of one cell per tick. (b) In the jammed phase ( $\rho = 0.75$  shown) jams move to the left, as vehicles can only advance when there is a free space ahead of them.



**Figure 2.** Simulation results for rule 184: (a,b) average velocity  $\langle v \rangle$  and (c,d) average flux  $\langle J \rangle$  for different densities  $\rho$ : (a,c) single runs and (b,d) box plots of 50 runs per density.

The traffic model of Nagel and Schreckenberg [22] (NaSch) can be seen as an elaboration of rule 184 with the following extensions: (1) a variable (discrete) velocity is associated with each vehicle, (2) acceleration (tending to attain the maximum velocity), (3) deceleration (due to the presence of other vehicles), and (4) a random tendency to slow down (an attempt to model a human tendency to overreact when decelerating). The NaSch model reproduces the appearance of spontaneous, also called “phantom,” traffic jams. Depending both on the global density and the probability to slow down, such spontaneous traffic jams either disappear or survive indefinitely.

Note that a vehicle can have a velocity greater than one cell per tick. This fact implies that the NaSch traffic model, just as that of Cremer and Ludwig, is not an ECA. The reason is that the next state of a cell depends not only on those of its immediate neighbors, but also on those of other cells.

Many variants of the NaSch model have appeared, each with different degrees of realism. For example, Nagel and Paczuski [23] inhibit the random slowdown for vehicles traveling at the maximum velocity. This variation eliminates spontaneous traffic jams for the free-flow regime by representing automatic cruise control.

Fukui and Ishibashi [24], by contrast, only have a random behavior component for vehicles traveling at the maximum velocity. Such a random slowdown of maximum-velocity vehicles attempts to model the fact that drivers traveling at high velocity (without cruise control) cannot continue at that speed indefinitely. In addition, accelerations in this model are instantaneous.

A two-dimensional CA modeling traffic was devised by Biham, Middleton, and Levine [25] (BML). This model is interesting because of being remarkably simple and yet exhibiting self-organization as well as two distinct phases. Cells form a bidimensional array. Each cell can be viewed as representing either an empty space or a vehicle that is traveling either to the right or upwards. Except for the initial random placement of the vehicles (i.e., the initial condition) this automaton is deterministic. Boundary conditions are periodic, so that the number of vehicles of each kind is preserved. On even ticks, only upward-facing vehicles move, whereas on odd ticks, only vehicles facing right do so, unless there is a nonempty space just in front.

Self-organization emerges when consecutive rows or columns have vehicles moving one cell ahead or behind the next row or column, thus forming a diagonal of vehicles. This pattern minimizes collisions and therefore maximizes speed. Above a certain density, a global cluster appears that rapidly includes all vehicles, showing a “sharp” phase transition.

The BML model is interesting because it exhibits complex behavior, but it is not a realistic model of city traffic. More realistic city traffic models have been developed as a common generalization of the NaSch and BML models [26–29]. Such generalizations are essentially extensions of the BML model so that streets have an arbitrary length

(instead of only one cell), and vehicles traveling in between crossings behave according to the NaSch model. Traffic lights are incorporated by making vehicles decelerate or halt not only because of a vehicle being in front, but also because of approaching a red traffic light.

An approach closer to ours is that of Chopard, Luthi, and Quelo [30]. Each street has two lanes, each of which is bound for the opposite direction. Traffic within streets follows rule 184. At intersections, however, roundabouts mimic traffic lights as follows. On the one hand, vehicles within a roundabout have priority. On the other hand, each vehicle stays in the roundabout for a certain number of ticks, after which it leaves. This model exhibits metastability and gridlocks. A vehicle entering a roundabout must check not only the state of a cell, but also its “flag,” indicating whether or not the vehicle in such a cell is to remain in, or exit from, the roundabout. Therefore, this CA is not elementary nor does it deal with traffic lights, our main purpose of study.

#### 4. An Elementary Cellular Automata Model of an Intersection

In this section, we first extend the highway traffic model consisting only of rule 184 [7, 9, 31]. Next, we show how to measure the value of several parameters in such an extended model. Finally, we relate the scale of our model with that of real traffic.

##### 4.1 Extension

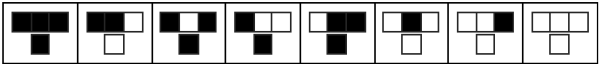
The behavior of streets has already been modeled by rule 184. If we draw the cells horizontally, the vehicles will flow to the east. We can obtain the other directions by rotating the street arrays by 90, 180, and 270 degrees. Our extension lies in modeling traffic lights at the intersections. We can include traffic lights by considering several coupled inhomogeneous ECAs, where rules change around the intersection, depending on the state of the traffic light.

Note that the combination of ECAs has to be conservative [32–34], that is, the number of ones needs to be constant. Otherwise, the model would be equivalent to having vehicles appearing or disappearing in the middle of the simulation.

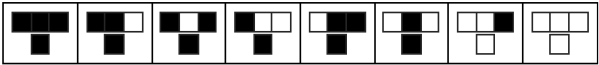
If a street has a green light, then all its cells use rule 184. If there is a red light, then all cells also use rule 184, with two exceptions: the cells immediately before and after the intersection. Let  $L$  be the cell immediately before the intersection.  $L$  has to stop traffic from going into the intersection. If there is a vehicle in  $L$  (cases 010, 011, 110, 111, where the value of  $L$  is in the middle), it will stay there, so the next state of  $L$  will continue to be 1. If there is no vehicle in  $L$  and there is a vehicle in the previous cell (100, 101), then that vehicle will advance, so the next state of  $L$  will also be 1. Only if there are no vehi-

cles in  $L$  or its previous cell will the next state remain 0. This is rule 252. The cell immediately after the intersection has to allow vehicles to leave, but must not allow vehicles in the intersection (flowing in the perpendicular direction) to enter. Thus, the next state will be 1 only if its state is already 1 and the cell immediately ahead is blocked (011, 111). This is rule 136. Table 1 lists the transition tables for the three rules used by the model. Figure 3 shows the corresponding rule icons.

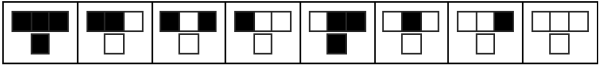
The intersection cell is a special case, as it has four potential neighbors. The rule never changes (184). What changes is the neighborhood, that is, it takes as nearest neighbors only the two cells on the street with a green light (also using rule 184). A diagram of the cells around an intersection is shown in Figure 4.



(a) 184



(b) 252

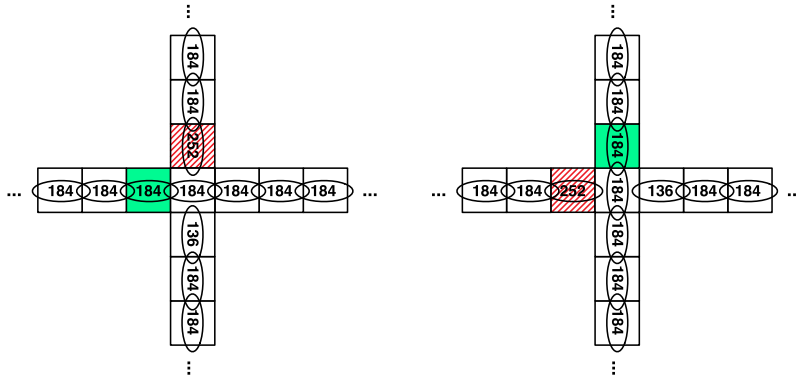


(c) 136

**Figure 3.** Rule icons for (a) 184, (b) 252, and (c) 136. Taken from <http://atlas.wolfram.com>.

$t - 1$	$t_{184}$	$t_{252}$	$t_{136}$
000	0	0	0
001	0	0	0
010	0	1	0
011	1	1	1
100	1	1	0
101	1	1	0
110	0	1	0
111	1	1	1

**Table 1.** ECA rules used in the model.



**Figure 4.** Diagram for different rules (shown within cells) and neighborhoods (indicated by ovals) used around intersections, depending on the state of the traffic light. For green lights (indicated by a green solid cell), rule 184 is used, and the intersection cell has as neighbors cells on the street with the green light. For red lights (indicated by a diagonally red striped cell), rule 252 is used for the cell before the intersection and rule 136 for the cell after the intersection. The rest of the cells use rule 184.

If at time  $t$  a traffic light is meant to switch, the model needs to ensure that the intersection cell is empty. Otherwise, the vehicle in the intersection would “turn” into the crossing street. To avoid this situation, the actual switching of rules and neighborhood is made only when the intersection is clear.

Certainly, cells using rules 252 and 136 could be simplified to depend only on two cells, as their state is not affected by the intersection cell. However, we prefer to have redundancy in the rules but homogeneity in the neighborhood sizes.

To model flow in different directions, we could use mirror rules (e.g., 226 is equivalent to rule 184 by reflection, with vehicles flowing to the left), but it is simpler to invert neighborhoods. Thus, the model is reduced to the combination of only three ECA rules.

It is also worth noting that rules 252 and 136 belong to the same equivalence class. To be more specific, they belong to the same semi-symmetric rule cluster [35, p. 23], that is, rule 252 results from rule 136 by performing negative and reflective transformations, or vice versa [35, pp. 21–22]. The behavior of rules 252 and 136 is complementary, since rule 252 prevents vehicles from moving into an intersection, while rule 136 prevents vehicles from appearing after an intersection. From this rule cluster, rules 192 and 238 could be used together with rule 226 to model traffic flowing to the left. The other four rules in the cluster (rules 3, 17, 63, and 119) do not conserve density, so they are not useful for traffic modeling. This is also the case for the other two rules in the fully asymmetric cluster of rules 184 and 226 (rules 71 and 29) [35, p. 24].

From all 256 ECA rules, there are 88 equivalence classes [35, p. 21]. One might wonder how many of these or combinations of these could be relevant to traffic modeling. In our current model we use only three, but this does not prevent the use of more or different rules or combinations of rules. Still, the resulting system has to be conservative [32–34], that is, the density must remain constant for all initial conditions. This restricts considerably the rule space for potential models of traffic.

Just like in the rule 184 model of highway traffic, there is a certain symmetry between vehicles and spaces. This is a quantitatively unrealistic property, but it has provided us with interesting qualitative insights of city traffic [36].

## 4.2 Measures

The behavior of the model will depend strongly on the vehicle density  $\rho \in [0, 1]$ . Trivial cases are the extremes  $\rho = 0$ , where there are no vehicles and  $\rho = 1$ , where all cells are occupied by vehicles, so there is no space to move and flow is stopped. The density can be easily calculated by dividing the number of cells with 1 (i.e., total number of vehicles,  $\sum s_i$ ) by the total number of cells ( $|S|$ ):

$$\rho = \frac{\sum s_i}{|S|}. \quad (1)$$

The performance of the system can be measured with velocity  $v \in [0, 1]$ , which is simply the number of cells that changed from 0 to 1 divided by the total number of vehicles:

$$v = \frac{\sum (s'_i > 0)}{\sum s_i} \quad (2)$$

where  $s'_i$  is the derivative of state  $s_i$ . If  $s'_i = 1$ , the cell changed from 0 to 1. If  $s'_i = -1$ , the cell changed from 1 to 0.  $s'_i = 0$  when there is no change in the state of  $s_i$ , that is, either there is no vehicle in  $s_i$ , or the vehicle in  $s_i$  has stopped.

The flux of the system represents how much of the space is used by moving vehicles. It can be obtained by multiplying the vehicle density by the velocity:

$$J = \rho v. \quad (3)$$

In the rule 184 highway traffic model, the maximum possible flux is  $J = 0.5$ , at a density  $\rho = 0.5$ . This is because vehicles need at least one cell between them to move. If there are fewer vehicles, the flux will be lower, since there is no movement in free space. If there are more vehicles, then the flux will also be lower, since stopped vehicles do not move (see Figures 2(c) and 2(d)).

From equation (2), the average waiting time and the number of stopped vehicles can be inferred as

$$t_{\text{wait}} = \int (1 - v) dt \quad (4)$$

and

$$\text{vehicles}_{\text{stopped}} = (1 - v) \sum s_i. \quad (5)$$

The average waiting time of equation (4) is calculated by integrating over time the complement of the velocity ( $1 - v$ ). At every tick, if a vehicle is not flowing, then it is waiting, so  $1 - v$  captures how many vehicles are waiting.

The number of stopped vehicles from equation (5) is calculated by counting all vehicles (1) in the environment ( $\sum s_i$ ) and multiplying such a quantity by the complement of velocity ( $1 - v$ ), which indicates how many vehicles are not moving.

The percentage of stopped vehicles would be simply:

$$\% \text{ vehicles}_{\text{stopped}} = 100 (1 - v). \quad (6)$$

These measures can be efficiently calculated from the states of the automaton.

### 4.3 Scale

Even when the time and space are abstract and discrete, we can assume that one cell represents 5 meters, roughly the space occupied by a stopped vehicle. Thus, 1 kilometer of a street is represented by 200 cells. If each tick represents 1/3 second, then a velocity of one cell per tick is equivalent to 15 m/s, that is, 54 km/h, roughly the speed limit in cities (e.g., in continental Europe it is 50 km/h in many countries; in Russia it is 60 km/h). A maximum density of  $\rho = 1$  is equivalent to 200 vehicles per kilometer.

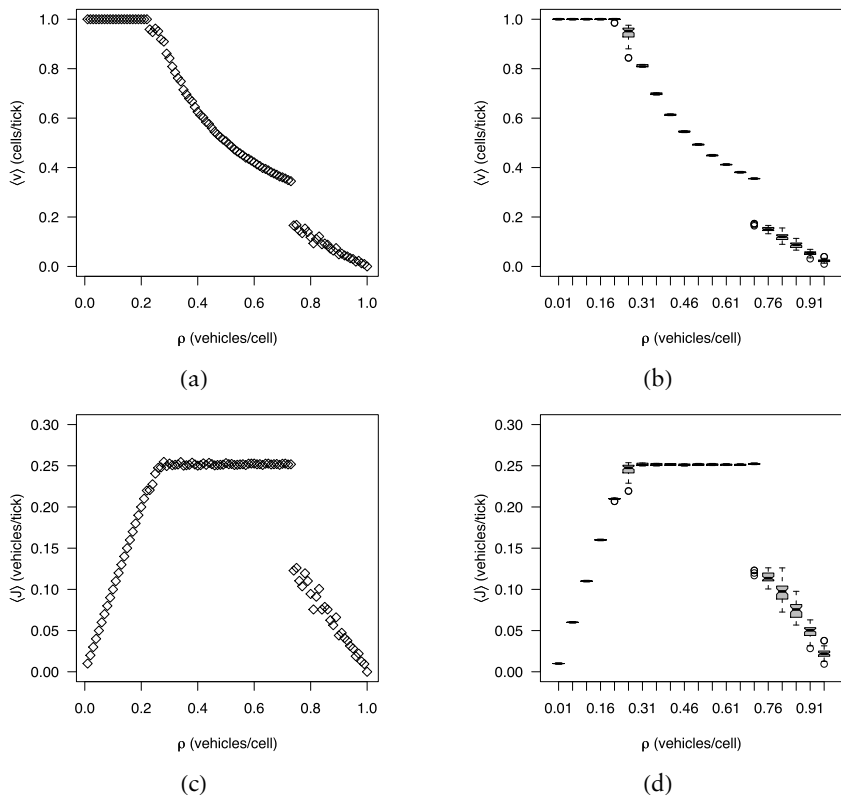
## 5. Simulations

We developed a computer simulation in NetLogo [37] to implement our model. The reader is invited to access the simulation via web browser at <http://turing.iimas.unam.mx/~cgg/NetLogo/trafficCA.html> (for short, <http://tinyurl.com/trafficCA>). The environment consists of two cyclic streets with periodic boundaries: one eastbound and one southbound. Each street is 160 cells long, with one cell shared: the intersection. Thus, a maximum density  $\rho = 1$  implies 319 vehicles.

A fixed period  $T = 160$  ticks was used for the traffic light. This implies that the eastbound street has a green light for 80 ticks and the southbound street has a green light for the following 80 ticks. Thus, if

a single vehicle in the simulation encounters a red light, it will stop until the light turns green. Afterward, the vehicle will always flow, since the time required to go once around the torus is equal to the period  $T$ .

For the experiments, each run consisted of an initial 30 minute (5400 ticks) simulation for random initial conditions. Since the vehicles are placed randomly, one street may have a slightly higher density than another. After these initial 5400 ticks, the system is considered to have stabilized, that is, gone through a transient, so another 30 minutes are simulated. The velocities of the second 30 minutes are averaged to obtain the average velocity  $\langle v \rangle$  and average flux  $\langle J \rangle$ . The results are shown in Figure 5.



**Figure 5.** Simulation results for a single intersection: (a,b) average velocity  $\langle v \rangle$  and (c,d) average flux  $\langle J \rangle$  for different densities  $\rho$ : (a,c) single runs and (b,d) box plots of 50 runs per density.

It can be seen that the phase transition from *free-flow* ( $v = 1$ ) to an *intermittent* phase occurs at  $\rho = 0.25$ . Recall that for a single street with no intersections, that is, using rule 184 as a model of highway traffic, a similar transition (to jammed traffic) occurs at  $\rho = 0.5$  when there is exactly one free cell between vehicles. However, when an inter-

section is added, vehicles coming from both streets pass through the intersection cell. They have to share this resource, reducing the maximum flux to  $J = 0.25$ . Thus, in order to have free-flowing traffic (apart from setting  $T$  carefully), there should be space available in one street while vehicles in the other street are crossing. Otherwise, they have to stop behind a red light, leading to intermittent traffic. Notice that the average velocity and flux are reduced slightly before the phase transition at  $\rho = 0.25$ . This is because there is a certain probability that one street will have a density  $\rho > 0.25$ . Thus, not all the vehicles on that street will be able to cross the intersection in one period and will have to wait, while the other street will have free-flow.

There is a second phase transition at  $\rho = 0.75$  toward an *interfered* phase. This occurs when the traffic jams (traveling in the opposite direction of traffic at the velocity of one cell per tick) are long enough to reach the intersection around the torus and block it momentarily. This interference affects vehicles in the crossing street, reducing noticeably the average flux  $J = 0.25$  before the phase transition to  $J < 0.125$  after it. The difference lies in the fact that when the interference of one street is dissipated, that same street will have the green light again, so in practice the traffic in one street will be completely stopped. This explains why the flux  $J$  is reduced to 0.5.

In the intermittent phase ( $0.25 < \rho < 0.75$ ), traffic jams form behind red lights and travel in the opposite direction of vehicles, but the jams dissipate before reaching the intersection around the torus again. This phase is characterized by a flux equal to the maximum of the model  $J = 0.25$ , that is, there are always vehicles crossing the intersection.

Since there is always some free space if  $\rho < 1$ , the *gridlock* situation ( $v = 0$ ) is only reached when  $\rho = 1$ .

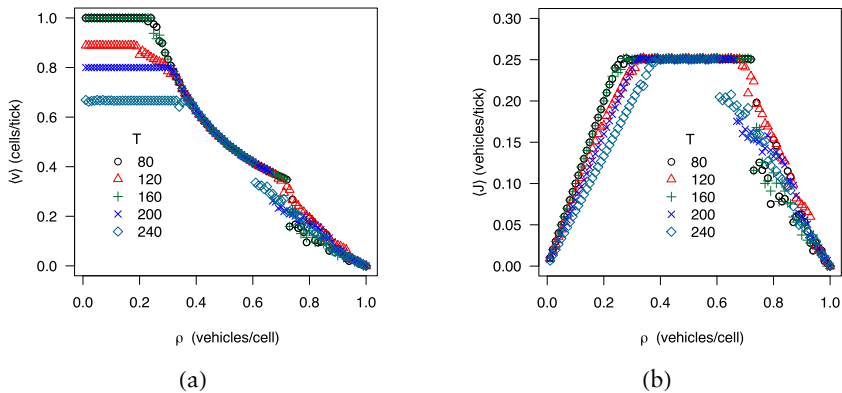
Observe that the variance is small for both the free-flow and intermittent phases, compared with that of the phase transitions and the interfered phase. As a consequence, few simulations are needed to obtain significant values for both of these first two phases.

We performed experiments where  $T$  was varied, as shown in Figure 6. It can be seen in Figure 6(a) that the value of  $T$  affects the existence of a free-flow phase. The appearance of such a phase requires a synchronization of the traffic light period  $T$  with the travel time around the torus. This is achieved in our model only when  $T$  is a multiple of the street length ( $T = 80, 160$ ), a condition for free-flow. Otherwise, the period of the traffic light and the period of the vehicles are not synchronous. In this case, vehicles need to wait for a green light and the average velocity is less than 1. In spite of this lack of synchrony, the average velocity is constant for low densities. The reason is that platoons are small, and the velocity does not depend on the density. We will call this the *constant-velocity, intermittent* phase. Once the density reaches a certain threshold, however, the velocity does decrease as the density increases.

Note however, that now the flux is independent of the density (see Figure 6(b)). We will therefore call this the *constant-flux, intermittent* phase. Remark that there is a symmetry in the flux diagram: the  $T$  values that reach the maximum flux capacity earlier will degrade later, that is, values that reach the constant-flux, intermittent phase earlier will reach the interfered phase later. Interestingly, there is a density interval for which the flow is constant not only with respect to  $\rho$ , but also with respect to  $T$ . For this density interval, the velocity  $v$  is also independent of  $T$ . This is because there is always a vehicle crossing the intersection. Statistically, it does not matter which street has a green light. Some vehicles will be moving and some will be stopped, but on average the flux and velocity will be the same.

There is a peculiarity for  $T = 120$ . Unlike for other values of  $T$ , the velocity decreases slightly before leaving the constant-velocity, intermittent phase. This phenomenon can be explained as follows. When  $T = 120$  and the density is small (i.e.,  $\rho < 0.2$ ), vehicles go one time around the torus without stopping and another time stopping. This is because vehicles take 160 ticks to go around the torus. If a vehicle starts at time  $t = 0$  when the light just switched to green, that is, phase  $\varphi = 0$ , it will next cross the intersection when the phase  $\varphi = 40$ , that is,  $160 \bmod 120$ , still with a green light. The second time, however, there will be a red light because  $\varphi = 320 \bmod 120 = 80$ , so it will have to wait until  $t = 360$ , that is,  $\varphi = 360 \bmod 120 = 0$ . Hence, vehicles will go twice around the torus every three periods  $T = 120$ . This leads to the formation of three platoons in each street in the constant-velocity, intermittent phase. The interaction of these platoons with the traffic light causes platoons to alternate position: the first two platoons will go through the intersection, but the third one will not. After the first two platoons go through the intersection and around the torus, the light changes, and the third platoon, which was last in the queue, now appears at the head of the queue. The reason is that each of the three groups of vehicles are synchronized with a different phase of the period  $T = 120$ . When the density reaches a certain threshold, two of the three platoons merge, leaving a total of two platoons. This causes some vehicles to wait slightly more than in the three-platoon case, being delayed one phase of the three occupied by platoons with  $T = 120$ . This leads to a phase particular of  $T = 120$  between the constant-velocity, intermittent and constant-flux, intermittent, where there is intermittent traffic but neither velocity nor flux are constant.

When platoons are even larger, then a phase transition occurs from the intermittent to the interfered phases. Beyond such a transition, the synchronization of  $T$  with vehicular travel time is counterproductive, because in most cases one street will be blocked. The other, unsynchronized,  $T$  values give a better performance because—even when flow is interfered—all vehicles are able to move. As a consequence, there is no single  $T$  value that gives the best performance across all densities  $\rho$ .



**Figure 6.** Simulation results for a single intersection varying period  $T$ . (a) Average velocity  $\langle v \rangle$  and (b) average flux  $\langle J \rangle$  for different densities  $\rho$ .

## 6. Possible Refinements

Our traffic model is as simple as possible while preserving the essential properties of flow around an intersection with a traffic light. These are possible improvements that could be made to contemplate more realistic traffic situations.

- There are no yellow lights in the current model. The behavior theory behind yellow lights is equivalent to red lights, that is, vehicles should stop. This could be implemented in our model by adding a “red-only” phase, where both streets have a red light to allow clearing the intersection. This can be achieved as follows: (a) the cell previous to an intersection on a street with a light that is about to turn red should change its rule ( $184 \rightarrow 252$ ) and (b) the intersection should change its rule ( $184 \rightarrow 136$ ). During the red-only phase, the cell after the intersection continues using rule 184, to allow clearing the intersection of vehicles in that direction. At the end of the red-only phase (and if the intersection is cleared): (a) the cell after the intersection should change its value ( $184 \rightarrow 136$ ), (b) the intersection and cells in the street that is turning green should change their rules to 184, and (c) the intersection should change neighbors to those in the street with the green light.
- Turns can be modeled with the rules already implemented in this model. The rules should be rearranged to allow a turning vehicle to temporarily switch the rules for the cell directly after the intersection for the street it was on before the turn ( $184 \rightarrow 136$ ) and the cell which the vehicle will turn into ( $136 \rightarrow 184$ ). This change is restored when the turning vehicle leaves the intersection.
- Multiple-lane streets could be modeled with parallel arrays of CAs, with further rules for lane changing; for examples, see [38]. This would also increase the number of cells that form intersections, so more changes should be made to ensure the clearance of vehicles in the direction they were heading.

## 7. Conclusions

There are several advantages of simple traffic models. Such models are, on the one hand, easy to implement and reproduce, and on the other hand, computationally cheap. Also, by abstracting most details from real traffic, one can observe properties more clearly.

The simplicity of our cellular automaton (CA) model allows for simulations with many intersections. In [36] we report simulation results comparing two traffic-light control methods: an optimized fixed cycle method and a self-organizing method. We simulated a Manhattan-style grid with 100 intersections. The dynamical phases of a system with several intersections can be much richer than that for the single intersection case, since there is feedback between intersections. For example, the self-organizing method exhibits six phase transitions [36].

With our simple model we do not aim to make realistic predictions. Our goal is to find better explanations of city traffic properties. It is possible, nonetheless, to improve the realism of our model while at the same time preserving its simplicity. Kanai [39] proposes associating each cell of a CA with more than one vehicle. Each cell then represents a number of vehicles varying between one and two. Representing more than two vehicles per cell is unnecessary, because it is possible in such a situation to reduce the cell size by one-half. By calibrating our model using Kanai's method, we plan to obtain more realistic simulations within the simplicity of elementary cellular automata (ECAs).

## Acknowledgments

We gratefully acknowledge the facilities provided by IIMAS, UNAM. C. G. was partially supported by SNI membership 47907 of CONACyT, Mexico. We also thank Genaro Juárez-Martínez, Taivo Lints, Todd Rowland, and an anonymous referee for useful comments and suggestions.

## References

- [1] I. Prigogine and R. Herman, *Kinetic Theory of Vehicular Traffic*, New York: Elsevier, 1971.
- [2] D. E. Wolf, M. Schreckenberg, and A. Bachem, eds., *Workshop on Traffic and Granular Flow '95*, Singapore: World Scientific, 1996.
- [3] M. Schreckenberg and D. E. Wolf, eds., *Traffic and Granular Flow '97*, Singapore: Springer, 1998.

- [4] D. Helbing, H. J. Herrmann, M. Schreckenberg, and D. E. Wolf, eds., *Traffic and Granular Flow '99: Social, Traffic, and Granular Dynamics*, Berlin: Springer, 2000.
- [5] D. Helbing, *Verkehrsdynamik*, Berlin: Springer, 1997.
- [6] D. Helbing and B. A. Huberman, "Coherent Moving States in Highway Traffic," *Nature*, **396**, 1998 pp. 738–740. doi:10.1038/25499.
- [7] D. Chowdhury, L. Santen, and A. Schadschneider, "Statistical Physics of Vehicular Traffic and Some Related Systems," *Physics Reports*, **329**(4–6), 2000 pp. 199–329. doi:10.1016/S0370-1573(99)00117-9.
- [8] A. Di Febbraro, D. Giglio, and N. Sacco, "Urban Traffic Control Structure Based on Hybrid Petri Nets," *IEEE Transactions on Intelligent Transportation Systems*, **5**(4), 2004 pp. 224–237. doi:10.1109/TITS.2004.838180.
- [9] S. Maerivoet and B. de Moor, "Cellular Automata Models of Road Traffic," *Physics Reports*, **419**(1), 2005 pp. 1–64. doi:10.1016/j.physrep.2005.08.005.
- [10] J. Maroto, E. Delso, J. Felez, and J. M. Cabanellas, "Real-Time Traffic Simulation with a Microscopic Model," *IEEE Transactions on Intelligent Transportation Systems*, **7**(4), 2006 pp. 513–527. doi:10.1109/TITS.2006.883937.
- [11] J. von Neumann, *The Theory of Self-Reproducing Automata* (A.W. Burks, ed.), Urbana, IL: University of Illinois Press, 1966.
- [12] E. R. Berlekamp, J. H. Conway, and R. K. Guy, *Winning Ways for Your Mathematical Plays, Volume 2: Games in Particular*, London: Academic Press, 1982.
- [13] S. Wolfram, *Theory and Applications of Cellular Automata*, Singapore: World Scientific, 1986.
- [14] S. Wolfram, *A New Kind of Science*, Champaign, IL: Wolfram Media, Inc., 2002.
- [15] F. L. Hall, B. L. Allen, and M. A. Gunter, "Empirical Analysis of Freeway Flow-Density Relationships," *Transportation Research Part A: General*, **20**(3), 1986 pp. 197–210. doi:10.1016/0191-2607(86)90094-4.
- [16] K. Huang, *Statistical Mechanics*, 2nd edition, New York: Wiley, 1987.
- [17] K. Kaneko, ed., *Theory and Applications of Coupled Map Lattices*, New York: Wiley, 1993.
- [18] M. Cremer and J. Ludwig, "A Fast Simulation Model for Traffic Flow on the Basis of Boolean Operations," *Mathematics and Computers in Simulation*, **28**(4), 1986 pp. 297–303. doi:10.1016/0378-4754(86)90051-0.
- [19] J. Krug and H. Spohn, "Universality Classes for Deterministic Surface Growth," *Physical Review A*, **38**(8), 1988 pp. 4271–4283. doi:10.1103/PhysRevA.38.4271.
- [20] V. Belitsky and P. A. Ferrari, "Ballistic Annihilation and Deterministic Surface Growth," *Journal of Statistical Physics*, **80**(3–4), 1995 pp. 517–543. doi:10.1007/BF02178546.
- [21] V. J. Peridier, "Estimating Transient Surface Heating Using a Cellular Automaton Energy-Transport Model," *Complex Systems*, **16**(2), 2005 pp. 139–153.

- [22] K. Nagel and M. Schreckenberg, "A Cellular Automaton Model for Freeway Traffic," *Journal de Physique I*, 2(12), 1992 pp. 2221–2229. doi:10.1051/jp1:1992277.
- [23] K. Nagel and M. Paczuski, "Emergent Traffic Jams," *Physical Review E*, 51(4), 1995 pp. 2909–2918. doi:10.1103/PhysRevE.51.2909.
- [24] M. Fukui and Y. Ishibashi, "Traffic Flow in 1D Cellular Automaton Model Including Cars Moving with High Speed," *Journal of the Physical Society of Japan*, 65(6), 1996 pp. 1868–1870. doi:10.1143/JPSJ.65.1868.
- [25] O. Biham, A. A. Middleton, and D. Levine, "Self-Organization and a Dynamical Transition in Traffic-Flow Models," *Physical Review A*, 46(10), 1992 pp. R6124–R6217. doi:10.1103/PhysRevA.46.R6124.
- [26] P. M. Simon and K. Nagel, "A Simplified Cellular Automaton Model for City Traffic," *Physical Review E*, 58(2), 1997 pp. 1286–1295. doi:10.1103/PhysRevE.58.1286.
- [27] D. Chowdhury and A. Schadschneider, "Self-Organization of Traffic Jams in Cities: Effects of Stochastic Dynamics and Signal Periods," *Physical Review E*, 59(2), 1999 pp. R1311–R1314. doi:10.1103/PhysRevE.59.R1311.
- [28] A. Schadschneider, et al., "A New Cellular Automata Model for City Traffic," in *Traffic and Granular Flow '99: Social, Traffic, and Granular Dynamics* (D. Helbing, H. J. Herrmann, M. Schreckenberg, and D. E. Wolf, eds.), Berlin: Springer, 2000.
- [29] E. Brockfeld, R. Barlovic, A. Schadschneider, and M. Schreckenberg, "Optimizing Traffic Lights in a Cellular Automaton Model for City Traffic," *Physical Review E*, 64(5), 2001 p. 056132. doi:10.1103/PhysRevE.64.056132.
- [30] B. Chopard, P. O. Luthi, and P.-A. Quelo, "Cellular Automata Model of Car Traffic in a Two-Dimensional Street Network," *Journal of Physics A: Mathematical and General*, 29(10), 1996 pp. 2325–2336. doi:10.1088/0305-4470/29/10/012.
- [31] S. Yukawa, M. Kikuchi, and S. Tadaki, "Dynamical Phase Transition in One Dimensional Traffic Flow Model with Blockage," *Journal of the Physical Society of Japan*, 63(10), 1994 pp. 3609–3618. doi:10.1143/JPSJ.63.3609.
- [32] N. Boccaro and H. Fukś, "Cellular Automaton Rules Conserving the Number of Active Sites," *Journal of Physics A: Mathematical and General*, 31(28), 1998 p. 6007. doi:10.1088/0305-4470/31/28/014.
- [33] A. Moreira, "Universality and Decidability of Number-Conserving Cellular Automata," *Theoretical Computer Science*, 292(3), 2003 pp. 711–721. doi:10.1016/S0304-3975(02)00065-8.
- [34] A. Moreira, N. Boccaro, and E. Goles, "On Conservative and Monotone One-Dimensional Cellular Automata and Their Particle Representation," *Theoretical Computer Science*, 325(2), 2004 pp. 285–316. doi:10.1016/j.tcs.2004.06.010.
- [35] A. Wuensche and M. J. Lesser, *The Global Dynamics of Cellular Automata: An Atlas of Basin of Attraction Fields of One-Dimensional Cellular Automata*, Santa Fe Institute Studies in the Sciences of Complexity, Reading, MA: Addison-Wesley, 1992.

- [36] C. Gershenson and D. A. Rosenblueth, “Adaptive Self-Organization vs. Static Optimization: A Qualitative Comparison in Traffic Light Coordination,” Technical Report 2010.03, C3, 2010.
- [37] U. Wilensky. “NetLogo.” (May 2, 2011)  
<http://ccl.northwestern.edu/netlogo>.
- [38] W. Knospe, L. Santen, A. Schadschneider, and M. Schreckenberg, “A Realistic Two-Lane Traffic Model for Highway Traffic,” *Journal of Physics A: Mathematical and General*, 35(15), 2002 pp. 3369–3388. doi:10.1088/0305-4470/35/15/302.
- [39] M. Kanai, “Calibration of the Particle Density in Cellular-Automaton Models for Traffic Flow,” *Journal of the Physical Society of Japan*, 79(7), 2010 pp. 075002. doi:10.1143/JPSJ.79.075002.