

Chemical Excitable Medium in Barcelona Street Network as a Method for Panicked Crowds Behavior Analysis

Andrew Adamatzky

Unconventional Computing Laboratory, UWE, Bristol, England

Corresponding Author: andrew.adamatzky@uwe.ac.uk

Jordi Vallverdú

Universitat Autònoma de Barcelona, Catalonia

Georgios Ch. Sirakoulis

Democritus University of Thrace, Greece

Spacetime dynamics of crowds are proved to be like dynamics of perturbation waves propagating in an excitable nonlinear medium. This analogy is developed further by studying the propagation of excitation—as represented by the two-variable Oregonator model of the Belousov–Zhabotinsky medium—and by then providing a thorough explanatory application study of the dynamics of excitation on Barcelona street networks considering crowd movement. In more detail, two characteristic areas of Barcelona—Gràcia and Raval—recently in the spotlight due to emerging situations, are analyzed in the context of the proposed study. The computer experiments performed show how an excitability of the medium affects the propagation dynamics. It is demonstrated that with decreasing excitability, the spanning of the street networks by an excitation wavefront shows evidence of pruning toward wider and ballistically plausible pathways.

Keywords: Belousov–Zhabotinsky reaction; excitation; street networks; crowds

1. Introduction

Mass or crowd behavior modeling has already been analyzed with several tools [1–5], and it is common to use an analogical comparison between collective human movements and fluid behavior [6–8]. In certain circumstances, crowds behave as excitable media, with evident propagation of wavefronts in the sense of physical movement [9–12] and propagation of information and rumors [13, 14]. Under such conceptual premises, we have considered the use of the Oregonator [15] for the analysis of crowd behavior, as the simplest realistic model [16] of the chemical dynamics of the oscillatory Belousov–Zhabotinsky reaction [17] (henceforth named BZ). Once applied to two realistic

scenarios from a metropolitical center like Barcelona, this model offers us a way to understand crowd behavior in case of an alarm due to emergency conditions, a situation in which people tend to look for the fastest escape route (despite street directions, traffic lights or normal civic norms) [18–21]. The selected spots are related to terrorist attacks in Barcelona: one in the Raval area, where a terrorist used a van to hit people walking through Las Ramblas in August 2017 [22], and a second one at Gràcia-Sagrada Família cathedral, publicly targeted by the same terrorists, but not finally chosen for their attack [23].

Our aim is to apply the Oregonator model [15] to the study of the design of specific Barcelona streets in order to provide a meaningful model for the tentative prevention of undesirable crowd behaviors and for better understanding of the dynamics of panicked crowds during fear contexts. Stampedes and crowd panics are examples of the secondary possible problems of a terrorist attack [24–26]. It is common that human agents become more aggressive, and that they are not afraid of running into others, even chaotically. At the same time, there are studies that indicate psychological disaster myths, and that people could be helpful to each other, providing possible assistance to the ones requesting help [27–29]. The combination of perplexed and even contradictory emotional situations like anxiety, fear, altruism and density is a terrible consequence of immediate agents' reactions.

This BZ model can also provide a way to manage and select the routes of probably injured citizens, as well as the possible escape routes of terrorists. Although BZ has been studied for decades as a classical model of an excitable nonlinear medium [30–32], an imitation of crowd dynamics with BZ has not been addressed before. We consider that the BZ model is able to perform modeling tasks correctly, according to its biologically inspired nature and formal power [33]. The human collective—even constrained by the topology of a city—behaves like a continuous dynamical system [34–37], and that was exactly the inspiration for the design of BZ.

2. Methods

Two fragments of a Barcelona street map including Gràcia (Figure 1(a)) and Raval (Figure 1(b)), each of them of approximately 1.5 and 2.2 km², respectively, were mapped onto a grid of 2500×2500 nodes. Nodes of the grid corresponding to the depicted streets are considered to be filled with a BZ medium, that is, resulting in excitable nodes, while the other nodes are nonexcitable. Distribu-

tions of street nodes at radius r from the perturbation sites are shown in Figure 1(c). Both distributions can be approximated by a second-degree polynomial as follows:

$$\gamma(r) = 6.7073 + 0.95607r - 0.00039843r^2. \quad (1)$$

We use two-variable Oregonator equations [15] adapted to a light-sensitive BZ reaction with applied illumination [16]:

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{1}{\epsilon} \left(u - u^2 - (fv + \phi) \frac{u - q}{u + q} \right) + D_u \nabla^2 u \\ \frac{\partial v}{\partial t} &= u - v. \end{aligned} \quad (2)$$

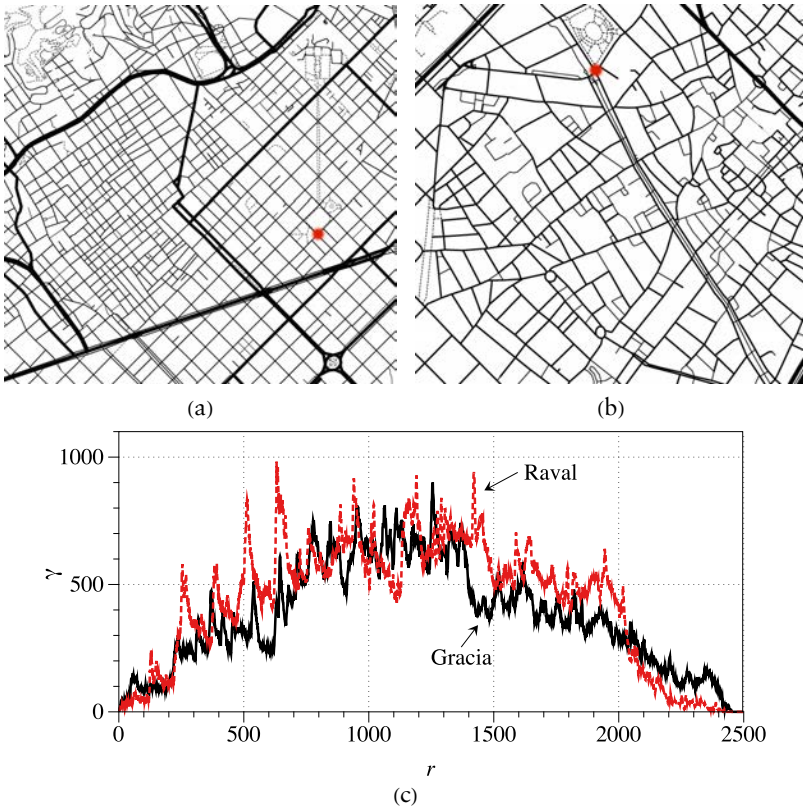


Figure 1. Fragments of Barcelona street networks used in experiments. Sites of initial perturbation are shown by red spots. (a) Gràcia. Initial perturbation site is Sagrada Família. (b) Raval. Initial perturbation site is at the beginning of Las Ramblas. (c) Number of street sites/excitable nodes γ at distance r from perturbation sites.

The variables u and v represent local concentrations of an activator, or an excitatory component of the BZ system, and an inhibitor, or a refractory component. Parameter ϵ sets up a ratio of the timescale of variables u and v , q is a scaling parameter depending on the rates of activation/propagation and inhibition, and f is a stoichiometric coefficient. We integrated the system using the Euler method with a five-node Laplace operator, time step $\Delta t = 0.001$ and grid point spacing $\Delta x = 0.25$, $\epsilon = 0.02$, $f = 1.4$, $q = 0.002$. We varied the value of ϕ from the interval $\Phi = [0.05, 0.08]$, where constant ϕ is the rate of inhibitor production. In a light-sensitive BZ, ϕ represents the rate of inhibitor production proportional to the intensity of illumination. The parameter ϕ characterizes the excitability of the simulated medium; that is, the larger ϕ , the less excitable the medium. With an increase of ϕ from 0.5 to 0.9, the excitation patterns are transformed from classical target waves (Figures 2(a, b)) to long-living wave-fragments (Figure 2(c)), analogous to dissipative solitary waves, to short-living solitons (Figures 2(d, e), respectively).

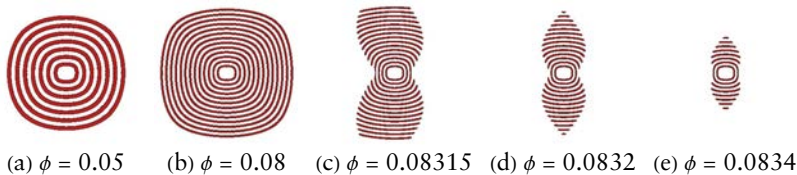


Figure 2. Development of excitation for various values of ϕ . These are time-lapsed snapshots of a single wave-fragment recorded every 150th step of numerical integration. The medium was perturbed by an incentered domain of 20×10 nodes.

To generate excitation waves, we perturb the medium by square solid domains of excitation, 20×20 sites in state $u = 1.0$. The site of the perturbation is shown by red disks in Figure 1. Time-lapse snapshots provided in the paper were recorded at every 150th time step, and we display sites with $u > 0.04$; videos supplementing the figures were produced by saving a frame of the simulation every 50th step of numerical integration and assembling them in the video with play rate 30 fps. It should be mentioned that all figures in this paper show time-lapsed snapshots of waves, initiated just once from a single source of stimulation; these are not trains of waves following each other. For chosen values of parameter ϕ , we recorded integral dynamics and calculated coverage of the street networks by traveling patterns of excitation. Integral dynamics $\alpha(t)$ of excitation are calculated as a number of grid nodes with $u > 0.1$ at each time step t of integration. A value of coverage ζ is calculated as a ratio of nodes, representing

streets, excited ($u > 0.1$) at least once during the medium's evolution to a total number of nodes representing streets. Videos of selected experiments are available at zenodo.org/record/1400783.

3. Results

After a specified site is perturbed, a wave of excitation spreads into the adjacent streets and the streets adjacent to these streets, and so on (Figure 3(a)). Some wavefronts are annihilated when they collide at the junctions; however, they give rise to second-order wavefronts traveling further along the streets. In a highly excitable medium, $\phi = 0.05$ to 0.055 , excitation spans all the streets, for example, Figure 3(a) and Figure 4(a), and then is extinguished. All streets of the networks are spanned by the excitation wavefronts, which is evidenced by nearly 100% coverage (Figure 6(a) and Figures 5(a, e)). Integral activity $\alpha(t)$ for both scenarios is characterized by two major spikes; see Figure 6(b, c). In Gràcia, the spikes reach their peaks at the $14 \cdot 10^3$ th and $19 \cdot 10^3$ th iterations (Figure 6(b)), respectively, and in Raval at the $9 \cdot 10^3$ th and $11 \cdot 10^3$ th iterations, respectively. There is no evidence that the spikes are determined by the number of street nodes available for excitation wavefronts at the mentioned iteration: compare Figures 6(b, c) and Figure 1(c).

With a decrease in excitability—for example, $\phi = 0.065$ and 0.0655 in Gràcia, Figures 3(b, c), and $\phi = 0.073$ in Raval, Figure 4(c)—excitation wavefronts were stopped when attempting to penetrate some narrow streets or streets intersecting with the excited street at an obtuse angle. Still, over 80% of the streets were traversed by their excitation wavefronts (Figure 6(a), Figures 5(b, c) and Figures 5(f–i)). At these values of ϕ , we observe repetitive patterns of excitation: wavefronts travel along cycling pathways in the street networks. Such pathways of the repetitive excitation are seen as nearly solid red areas in Figures 3(b, c) and Figure 4(c). The repetitive excitation is manifested in irregular oscillation of integral activity α observed, for example, for $\phi = 0.065$ in Figure 6(b).

Due to narrow streets at the site of initial perturbation in the Raval street network, the system remains excitable up to $\phi = 0.0669$. For higher values of ϕ , the four narrow streets originating from the perturbation fail to conduct the excitation. Increase of ϕ in experiments with the Raval area leads to a rather gradual pruning of streets (Figure 4(c) and Figures 5(g–j)) until only three remain, and then one. Major streets originating in the perturbation site remain conductive (Figure 4(d) and Figures 5(k, l)).

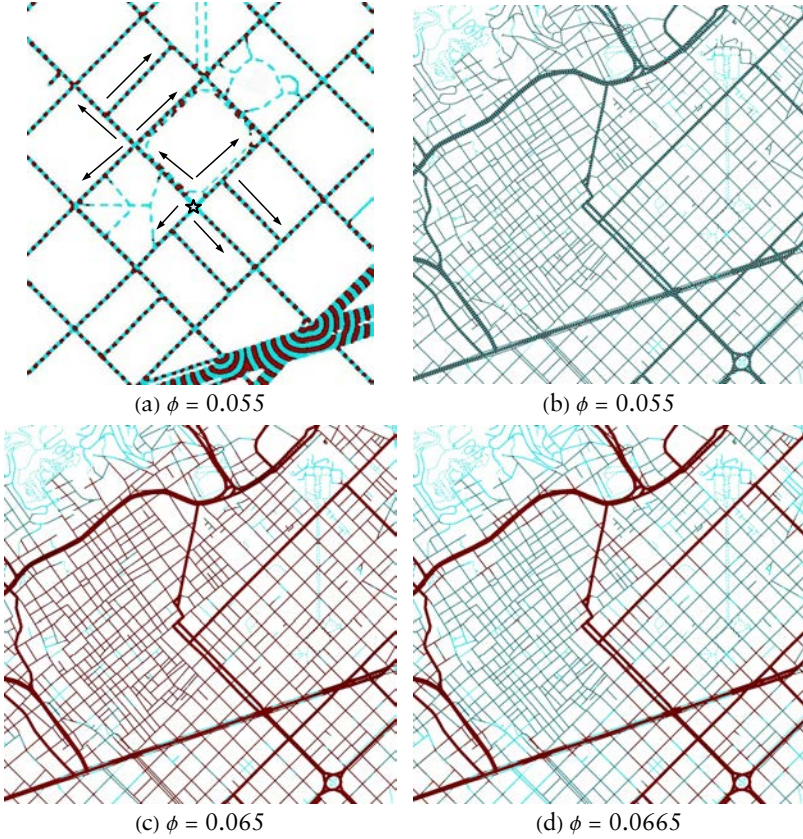


Figure 3. Propagation of excitation in the Gràcia area of Barcelona. Initially a site near Sagrada Família was perturbed (Figure 1(a)). The following parameter ϕ values are considered: (a) and (b) $\phi = 0.055$, (c) $\phi = 0.065$, (d) $\phi = 0.0665$. The zoomed area around the initial perturbation site is shown in (a), where the direction of propagation of the wavefronts along the streets neighboring the site is shown by arrows. These are time-lapsed snapshots of a single wave-fragment recorded every 150th step of numerical integration.

Based on the dynamics of coverage $\zeta_G(\phi)$, we can select several domains of excitability-dependent coverage. These are the following for the Gràcia street network:

$$\zeta_G(\phi) = \begin{cases} 0 - 2\phi + 1, & \phi \leq 0.065 \\ -107\,405\phi^2 + 12\,141\phi - 464, & \phi \in [0.066 - 0.0666] \\ 0.1, & \phi \in [0.067 - 0.0669] \\ 0, & \phi \geq 0.07 \end{cases}$$

and the corresponding ones for the Raval street network:

$$\zeta_R(\phi) = \begin{cases} 0.97, & \phi \leq 0.065 \\ 4077\phi^2 - 583\phi + 22, & \phi \in [0.066 - 0.072] \\ 131\,265\phi^2 - 19\,735\phi + 741, & \phi \in [0.073 - 0.076] \\ 18\phi^2 - 3\phi, & \phi \in [0.077 - 0.08] \\ 0, & \phi > 0.08 \end{cases}$$

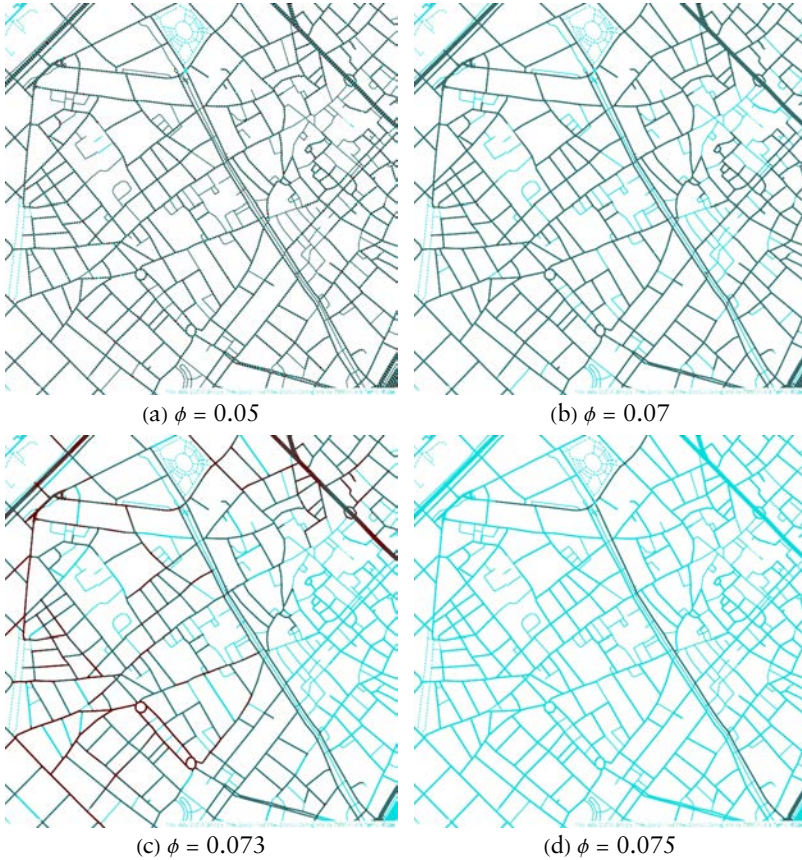


Figure 4. Propagation of excitation in the Raval area of Barcelona. Initially a site at the beginning of Las Ramblas was perturbed (Figure 1(b)). The following parameter ϕ values are considered: (a) $\phi = 0.05$, (b) $\phi = 0.07$, (c) $\phi = 0.073$, (d) $\phi = 0.075$. These are time-lapsed snapshots of a single wave-fragment recorded every 150th step of numerical integration.

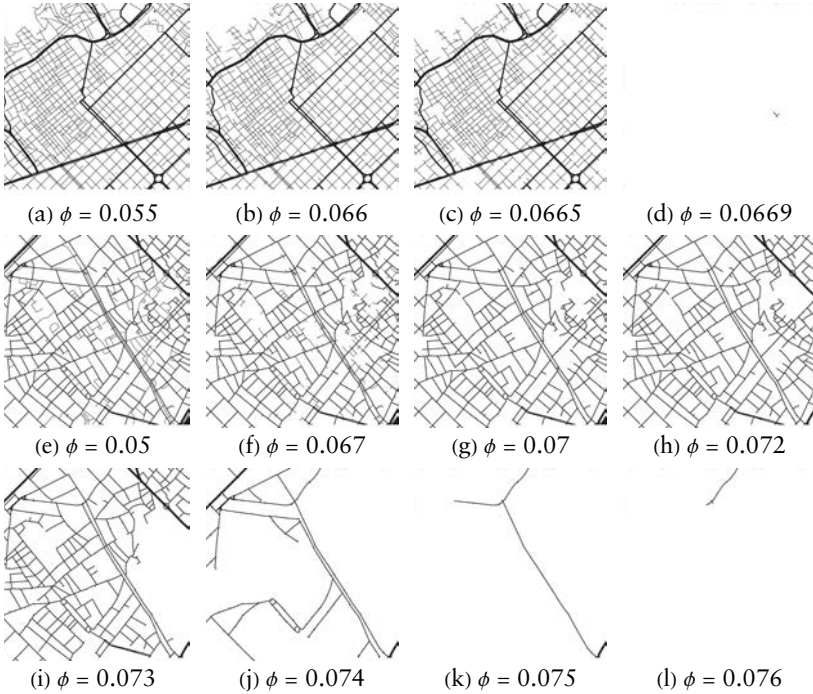
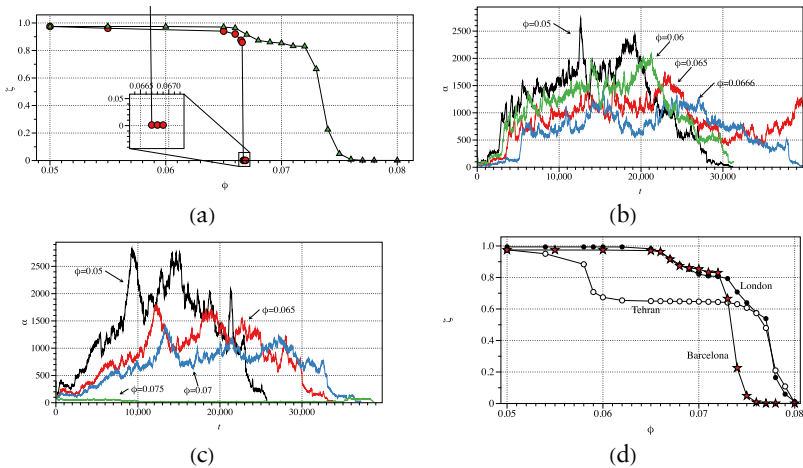


Figure 5. Coverage of street networks by excitation wavefronts for various values of ϕ . (a–d) Coverage of the Gràcia area for (a) $\phi = 0.055$, (b) $\phi = 0.066$, (c) $\phi = 0.0665$ and (d) $\phi = 0.0669$; in (d) we see that excitation is nearly stopped in the vicinity of the initial perturbation site. In correspondence, in (e–l) there is coverage of the Raval area for (e) $\phi = 0.05$, (f) $\phi = 0.067$, (g) $\phi = 0.07$, (h) $\phi = 0.072$, (i) $\phi = 0.073$, (j) $\phi = 0.074$, (k) $\phi = 0.075$ and (l) $\phi = 0.076$, respectively.



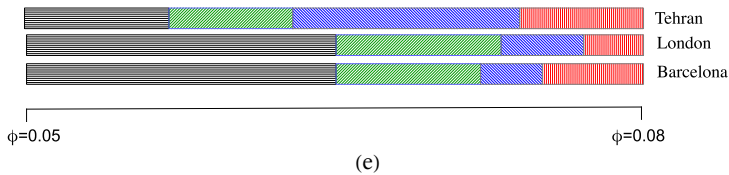


Figure 6. Integral characteristics of the excitation dynamics on Barcelona street networks. (a) Coverage ζ versus ϕ of Gràcia, shown by red disks, and Raval, shown by green triangles. (b, c) Activity α versus time t , in iterations of numerical integration, in the Gracia (b) and Raval (c) areas. (d) Coverage ζ versus ϕ for Barcelona (Raval), London [38] and Tehran [39]. (e) Distribution of phases in covering.

4. Discussion

Using the Oregonator model of an excitable chemical system, we analyzed the geometrical structure of two fragments of Barcelona street networks. We demonstrated that a ratio of the streets traversed by excitation wavefronts shows nonlinear dependence on excitability ϕ of the modeled medium.

In [38], we have shown that the propagation of an excitation wavefront from one street to an intersecting street depends on ϕ and the angle β formed by the intersecting streets (Figure 7(a)). Let β be a maximum angle for which the excitation wavefront traveling along street a , as shown by the arrow in Figure 7(a), propagates into street b . The dependence $\beta(\phi)$, obtained by computer modeling in [38], can be approximated by a linear function $\beta(\phi) = -27857\phi + 2194$. We see that critical value $\phi = 0.075$, as shown in Figure 6, corresponds to $\beta = 90^\circ$. In further studies it might be productive to undertake further analysis of coverage related to a distribution of intersection angles. From the results of [40], we know that such a distribution for Barcelona shows two peaks around the characteristic values of 90° and 180° . Further studies might focus on developing analytical solutions aimed at using such a distribution of intersection angles and excitability parameters to predict coverage of a street network. With regard to relation $\beta(\phi)$, an interesting result has been obtained in experiments with ants in [41]. It was shown that decrease of an intersection angle from 90° to 45° leads to a decrease in the flow of ants escaping from a chamber. This relation between the angle of intersection and the flow is also shown to be nonlinear. Due to these factors, a clear indication of the successful interpretation of crowd behavior that is able to span from altruism to individualism is starting to be established. The specific behaviors can be further stressed by fine-tuning of the aforementioned parameters under the appropriate

investigation to meet various cases referring to crowd behavior and cities' topological characteristics. Thus, the main advantage of the proposed method is its aptitude to deal with nonlinearity and uncertainties as expressed during the evacuation process [42, 43].

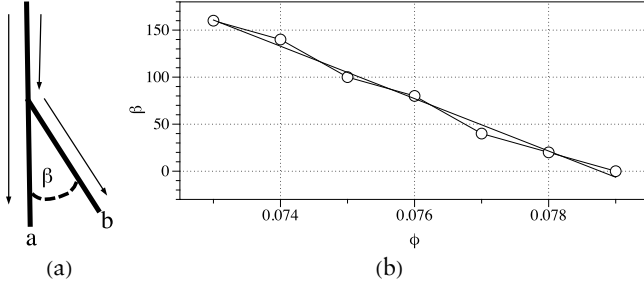


Figure 7. (a) Angle β of intersecting street a and street b . (b) Dependence of a maximum angle β for which an excitation wavefront traveling along street a , as shown by the arrow in Figure 7(a), propagates into street b .

Let us compare the coverage of Barcelona with that of Tehran and London. It should be noticed that in the present study, we are mostly focusing on fragments of the street networks selected and not on the networks of whole cities, as has been done before. The case of Gràcia is rather abnormal due to very narrow streets leading from the initial perturbation site. Therefore, it is more rational that we will compare covering of Raval with covering of London [38] and Tehran [39] (Figure 6(d)). Function $\zeta(\phi)$ behaves similarly on Barcelona and London street networks until $\phi = 0.72$; after that the coverage of Barcelona drops substantially, with just 4% of the network spanned by excitation wavefronts. In contrast, $\zeta(\phi)$ for Tehran shows a dip at $\phi = 0.58$, almost no changes until $\phi = 0.074$ (Figure 6(d)). Comparative distribution of phases of covering for Barcelona, Tehran and London is shown in (Figure 6(e)). We believe that boundaries of the phases might provide a reasonably good characterization of the geometrical features and topology of the street networks. Comparing $\zeta(\phi)$ with results of topological analysis of street networks [40, 44–47] could be the basis for further studies.

Furthermore, the Oregonator model used has been verified by the authors in experimental laboratory studies of the BZ system, and a sufficiently satisfactory match between the model and the experiments has been already demonstrated in [48–51]. Nevertheless, we believe it would be fruitful to analyze specific patterns of oxidation of the BZ mixtures poured into street network templates made from a chemically inert material. This perspective will be also addressed in the future.

Finally, as an early experimental trial and to further explore our theoretical and modeling findings, we have proceeded with the performance of indicating real experiments with a prototype of the Barcelona road network under study, so as to pave the way for the interested reader to better follow the proposed analysis of our approach. The specific prototype corresponding to a three-dimensional Barcelona Raval, smoothed version, was designed as an STL (stereolithography) file with the help of CURA software and with appropriate image processing of the Barcelona Raval graphic by assigning white areas as elevations, 5 mm and base 2 mm as presented in the corresponding snapshots of Figure 8. It was constructed with Generic Standard PETG (Polyethylene Terephthalate Glycol) because it is chemically resistant. The template was filled with BZ solution prepared according to the classical recipe [52].

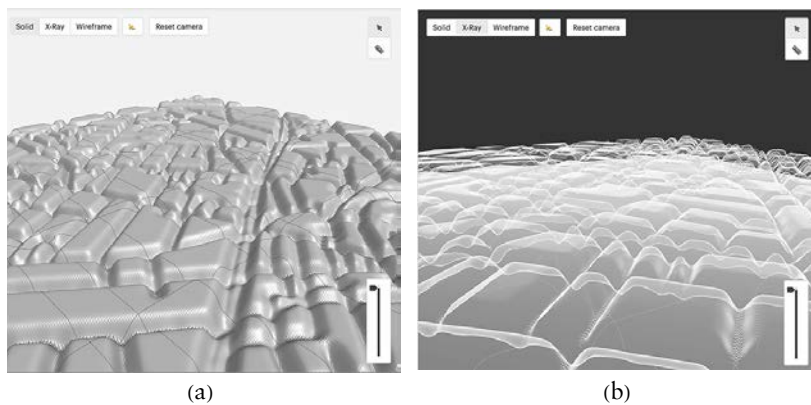


Figure 8. Three-dimensional simulated snapshots for the development of three-dimensional Barcelona Raval.

The first early results of the ongoing experimental exploitation of the proposed study are shown in Figure 9. The oxidation (excitation) wavefronts are bluish, while a few characteristic wavefronts are indicated by the black arrows (Figure 9(a)), and the rest can be also found on the other streets of the template. In the remainder of Figure 9, early wavefronts (Figures 9(b, d)) and later wavefronts (Figures 9(c, e)) are presented for the same experiments in different time steps, respectively. The experimental results confirm the ability of the proposed modeling method to reproduce nonlinear phenomena of increased complexity, as happens with crowd movement. Nevertheless, it is also obvious that there are existing problems in regard to the

experimental protocols that need more adaptive confirmation; namely, in some cases of the conducted experiments, it was found that the template was leaking, the wavefronts were not always sufficiently visible, and also, there were cases where the reaction self-excited in different places of the template. The aforementioned issues can be resolved with the advancement of the template, referring to the material used and the process used to fabricate it, and they will be considered as part of our ongoing and future research, as mentioned in Section 5.

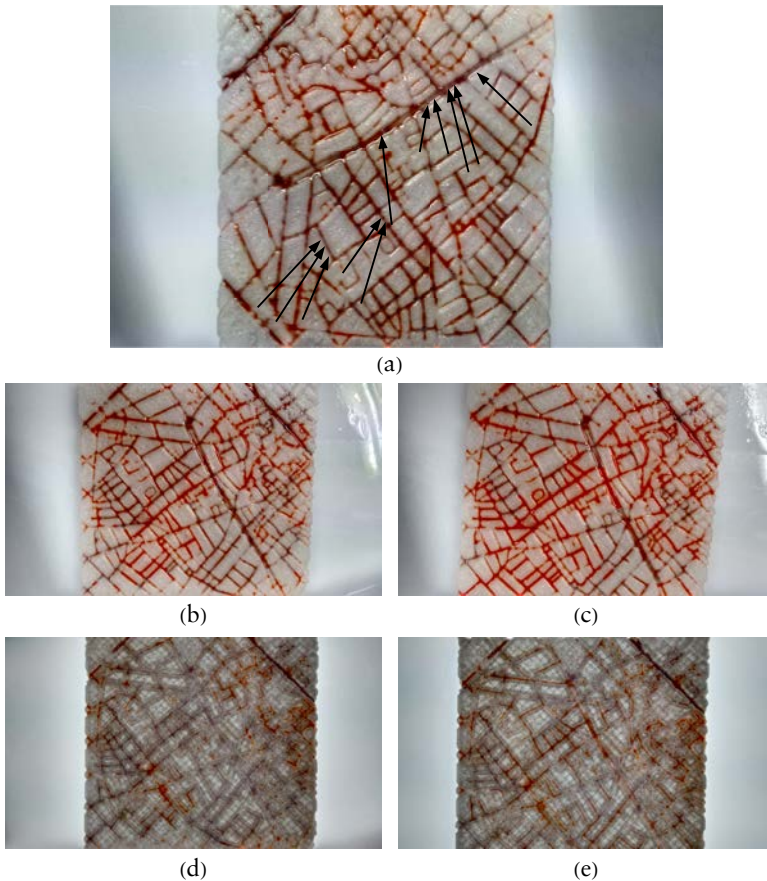


Figure 9. Various experimental results of three-dimensional Barcelona Raval. In (a) the excitation wavefronts are presented in the template and the ones most easily distinguished are indicated by black arrows. Early wavefronts (b, d) and later wavefronts (c, e) are presented for the same experiments in different time steps, respectively.

5. Interpretation and Future Work

This research has delivered many intriguing results: first, it was presented in an efficient manner that the Oregonator model using Belousov–Zhabotinsky (BZ) provides reliable ways for the understanding of complex crowd behavior under stressful situations in contemporary cities. Second, the Oregonator model has enough sensitivity to offer obvious differences between different city areas as well as between different cities. Third, excitation waves in the Gràcia area have made a new behavior visible in relation to our previous Oregonator model applications, because of the important dimensional variations of such a street’s area: the special topography of Barcelona, with several topographic historical designs, opens the possibility of the study of crowd behavior in topographically mixed cities, taking into consideration the various phases of the cities’ development. Hence, this new approach is a reliable tool for the better understanding of crowd behavior under stressful situations in complex topographical scenarios. In future research, we aim to apply this model to new cities with more intriguing characteristics, as well as designing a model that could be applied to micro-architectural structures like buildings or reduced areas (metro stations, bus stations, etc.), which are visited very frequently by users at times that show complex movement pathways.

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