Automatization of Universal Cellular Automaton Discoveries: A New Approach to Stream Duplication

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A crucial step toward an automatization of the demonstration of the universality of cellular automata is studied here. Inspired by Conway's work on the universality of the Game of Life, we propose a new approach of the duplication of streams of information in two-dimensional cellular automata. This approach is based on specific collisions of streams and is illustrated with the example of an automaton that is already able to simulate logical gates. This duplication system could be used with new automata for the simulation of a Turing machine.

Keywords: duplication; cellular automata; Game of Life; computation; automaton R; glider guns

1. Introduction

Emerging computation and universality is a hot topic at the forefront of the theory of complex systems and cellular automata [1–3]. Turing-universality, the capacity to encompass the whole computational power of Turing machines, is considered here as Turing-universal cellular automata and their frequency have been tackled by many [4–6] since Wolfram made this subject one of the 20 problems of the theory of cellular automata [7].

Wolfram asked, How common are computational universality and undecidability in cellular automata? [7] and a promising way to answer this question is to construct an automatic system for the discovery of Turing-universal cellular automata. Computation can be performed by NAND gates as described in [8, 9], but this gate operation does not conserve the input streams. A way to duplicate streams seems to be necessary and was used in the demonstration of universality of the Game of Life [9], hereafter described as Life. Conway notably considered duplication of streams as a difficult problem. Duplication also is a central process in biology [10] and at the forefront of the theory of complex systems and cellular automata [11].

In this paper, we propose a general method, based on a geometric study of collisions of streams, for identifying specific collisions allowing cellular automata to duplicate streams of information. This

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general method is described with the example of the automaton $\mathcal{R}$ [12].

The framework of our study is presented in Section 2. Section 3 describes in detail the duplication of streams in Life. Section 4 describes the automaton $\mathcal{R}$. The proposed method of stream duplication is presented in Section 5 using the example of automaton $\mathcal{R}$. Finally, in the last section we summarize our results and discuss directions for future research.

2. Framework

We explore here only binary isotropic cellular automata evolving in a two-dimensional squared matrix with a transition rule only taking into account the eight direct neighbors of a cell for the current generation, so as to determine its states for the next generation.

Streams of information are streams of mobile self-localized patterns of non-resting states [39] called gliders that can be generated by glider guns, which, when evolving alone, periodically recover their original shape after emitting a number of gliders.

3. Duplication Streams in Life

Conway, Berkelamp and Guy proved the universality of Life [9] by isolating a set of components allowing the automaton to simulate a Turing Machine. This section reviews how some of these components are used in order to duplicate glider streams.

3.1 Used Forms

The duplication of streams in Life is based on three principles described here: kickback reaction, vanishing collision and eater.

The kickback reaction is a collision between two gliders that causes the destruction of one glider and the reflection of the other in the opposite direction (Figure 1).

The vanishing collision of two streams at right angles has the following result:

- The destruction of both gliders if a glider is present in each stream.
- The survival of a glider, if there is a glider in one stream and not in the other.

This collision also allows Life to simulate the gates AND, NOT and OR. The eater stops a stream by absorbing gliders.

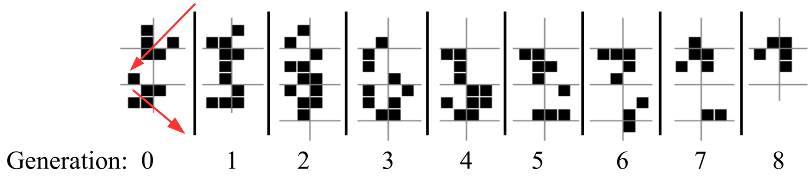

Figure 1. The kickback reaction generation by generation.

3.2 Thinning a Stream

To use the stream duplication described in [13], gliders need to be spaced further apart than those of the stream created by a glider gun. Figure 2 shows a means, using kickback reaction, of spacing the gliders of a stream. The relative position of the various forms allows the deletion of a certain number of gliders in the original stream, thus we can create a thinned stream. If one glider over n survives, the new stream is a n-thinning stream as shown in Figure 2.

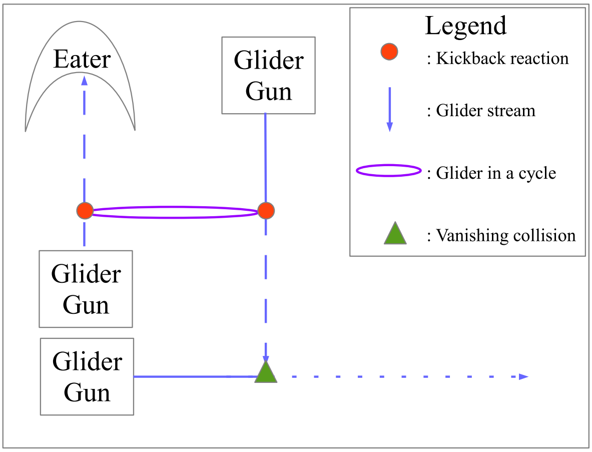

Figure 2. Thinning a stream with the kickback reaction.

3.3 Duplication in Life

In Life, 8-thinning streams can be duplicated. This duplication is presented in Figure 3 with an 8-thinning input stream $0000000A$ where A is the first bit of information of the stream (a glider or no glider). An OR gate is used to add a glider after the first bit of information. This new stream crashes with a stream S created by a gun. The first three gliders of S will disappear in the collision if $A = 1$, whereas the first glider will survive and the next three gliders will disappear if $A = 0$ as shown in Figure 3. The input stream carries its run, keeping the bit of information A. A stream emitted by a 8-thinning glider gun crashes into S to create a duplicate stream, thanks to a vanishing collision.

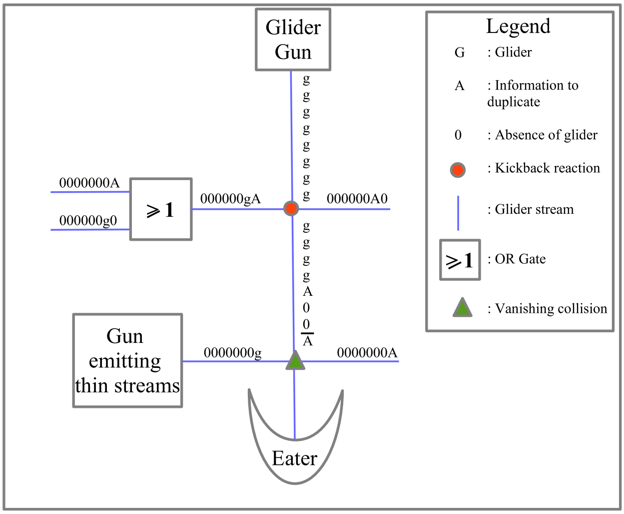

Figure 3. Duplication of glider streams in Life.

4. Presentation of $\mathcal{R}$

In [14, 15], an evolutionary algorithm (stochastic optimization algorithm [16]) is used to find two-dimensional binary isotropic cellular automata supporting gliders. Some automata were found, including one in which glider guns emerge spontaneously through the evolution of random configurations of cells shown in Figure 4.

Figure 4. Evolution of random configurations of cells under the automaton $\mathcal{R}$ after 100 generations (in lighter color: a glider gun).

Later, in [8], another evolutionary algorithm modified this automaton in such a way that some components, as used in [13], appear, allowing it to simulate logic gates. The new automaton is called $\mathcal{R}$.

5. Duplication of a Stream

We propose, using the example of automaton $\mathcal{R}$, a method for stream duplication simpler than Conway's method [13]. This technique requires the use of specific collisions, called duplicative collisions.

5.1 Duplicative Collision

A duplicative collision of a stream of information A and a stream of gliders B at right angles has the following result:

- The destruction of the glider of B and the glider of A if a glider is present in each stream.
- The survival of the glider of B, if there is not a glider in the stream A.

Two orthogonal streams are thus produced:

- The first one is the stream A, which continues its run.
- The second one is the complement of the stream A (i.e., each glider of the stream A corresponds to an absence of glider in the output stream and vice versa).

5.2 Duplication with the Duplicative Collision

A duplicative collision can be used in order to duplicate a stream; an input stream crashes into a stream emitted by a gun (Figure 5).

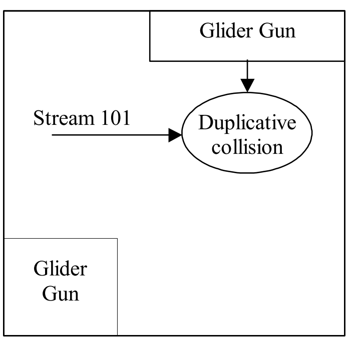

Figure 5. First step of the duplication of the stream 101.

This crash produces two streams:

- The first one is the input stream, which continues its run.
- The second one, which is the complement of the input stream (i.e., each glider of the input stream corresponds to an absence of glider in the other stream and vice versa; for example, the complement of 1011 is 0100), crashes into another stream emitted by a gun (Figure 6). This collision produces a replication of the input stream, which is parallel to the first output stream.

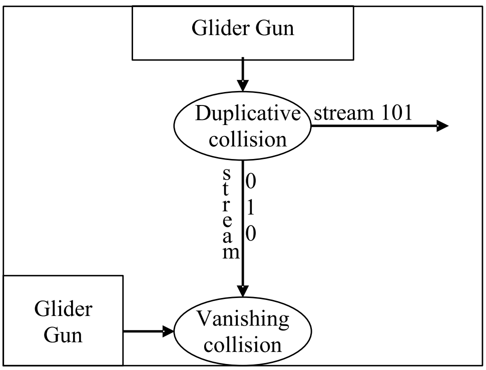

Figure 6. Second step of the duplication of the stream 101.

5.3 Studying the Collisions

We enumerate the possible collisions with two orthogonal streams in order to find a duplicative collision in a cellular automaton. The detail of this research is shown in the example of the streams of automaton $\mathcal{R}$. The number of cells between two consecutive gliders, the period of gliders and the number of cells gliders are shifted after a period will determine how many different collisions can exist.

5.3.1 Number of Possible Collisions

There are nine cells between each glider in a stream. There are four distinct phases in the evolution of a stream, as it is the period of a glider. The four phases of a vertical stream are shown and numbered in Figure 7.

In the following, we will mention streams in phase 1, 2, 3 or 4 (phase 5 would be identical to phase 1). In order to determine the number of possible collisions between two orthogonal streams, we choose to consider the horizontal stream shown in Figure 7 static and to determine how many distinct positions the vertical stream can take. The position of a stream is specified by the coordinates of the cell identified by a white point in the heading glider. For example, in Figure 7, the stream in phase 2 is located at coordinates (34, 17).

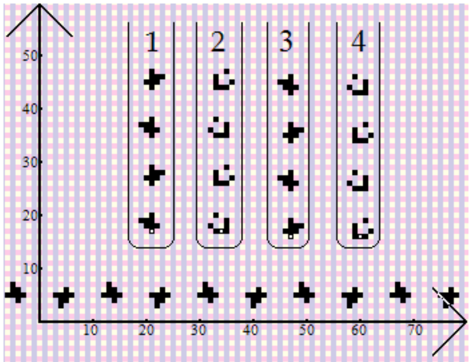

Figure 7. Possible collisions of two streams of gliders in $\mathcal{R}$.

As the period of gliders is 4 and their speed is 2 cells per period, a stream is reproduced with a translation of two cells after four generations. A collision with a stream in coordinates (x, y) has therefore the same effect as a collision with a stream in coordinates $(x + 2, y + 2)$ (as the horizontal stream will also be reproduced with a translation of two cells). Then, we only need to consider crashes into streams in ordinates y and $y + 1$.

The horizontal stream remains identical after an abscissa translation of 18 cells, therefore a collision with a glider stream and a collision with the same stream translated by 18 cells on the abscissa axis would have the same result. So we only need to consider crashes into streams situated in abscissas x to $x + 17$.

After two generations, a stream in phase 1 located in coordinates (x, y) becomes a stream in phase 3 located in coordinates $(x, y - 1)$. The horizontal stream evolution during two generations is equivalent to a left translation of eight cells. The collision with a stream in phase 1 in coordinates (x, y) is equivalent to a collision with a stream in phase 3 in coordinates $(x + 8, y - 1)$. Using this same idea, we can then say that a collision with a stream in phase 2 in coordinates (x, y) is equivalent to a collision with a stream in phase 4 in coordinates $(x + 8, y - 1)$. Therefore we only need to consider streams in phases 1 and 2.

5.3.2 Results of All Possible Collisions

Collisions with streams in phases 1 and 2, in abscissas from 21 to 38 and in ordinates from 16 and 17, are simulated. Six duplicative collisions are discovered, while vanishing and thinning collisions are listed. Figure 8 shows the mentioned collisions with their effects.

Stream at phase 1																		
	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
17	X	X	V	V	V	V	V	V	D	4r	X	X	X	V	V	V	V	X
16	X	X	5r	V	V	V	V	X	X	X	V	V	D	V	V	X	X	X
Stream at phase 2																		
	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
17	X	D	X	X	V	X	D	X	X	5r	X	V	X	X	D	V	X	X
16	X	X	V	X	V	V	X	V	X	4r	D	V	V	V	X	X	X	X

Figure 8. Result of collisions between the horizontal stream of Figure 12 and vertical streams in indicated phases and coordinates: "*n* r" stands for a collision that is "*n*-thinning," "D" stands for the duplicative collision, "V" stands for the vanishing collision, and "X" stands for a collision with another effect.

6. Synthesis and Perspectives

We proposed a way to duplicate glider streams in the cellular automata, using collisions of two streams of gliders with orthogonal directions.

It is proven here that a glider gun, a duplicative collision and a vanishing collision are sufficient to duplicate streams of gliders in a cellular automaton. An automatic search for the duplicative and vanishing collisions is presented.

Our solution, different from the one used by Conway for the demonstration of universality of Life, can be applied to several cellular automata, including the automaton $\mathcal{R}$, used here as an example.

The duplication of glider streams is a step toward an automatic system for the discovery of Turing-universal cellular automata. Such a system may give an answer to the question, How common are computational universality and undecidability in cellular automata?, which remains unsolved and addresses both dynamical systems theory and computation theory aspects of cellular automata [7].

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