Impact of Nonlocal Interaction on Chimera States in Nonlocally Coupled Stuart–Landau Oscillators

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We investigate the existence of collective dynamical states in nonlocally coupled Stuart–Landau oscillators with symmetry breaking included in the coupling term. We find that the radius of nonlocal interaction and nonisochronicity parameter play important roles in identifying the swing of synchronized states through amplitude chimera states. Collective dynamical states are distinguished with the help of strength of incoherence. Different transition routes to multi-chimera death states are analyzed with respect to the nonlocal coupling radius. In addition, we investigate the existence of collective dynamical states including traveling wave state, amplitude chimera state and multi-chimera death state by introducing higher-order nonlinear terms in the system. We also verify the robustness of the given notable properties for the coupled system.

Keywords: Stuart–Landau oscillators, nonlocal coupling, chimera states

1. Introduction

The study of coupled nonlinear oscillators constitutes a strong platform for exploring various oscillatory patterns in physics, chemistry, neuroscience and other related areas. Owing to various coupling scenarios among interacting oscillators with intrinsic properties, a variety of novel collective phenomena can be realized, including

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synchronization, suppression of oscillations, chimera states and so on. In particular, a notion of chimera state has drawn considerable attention toward the study of coupled networks with nonlocal topology [1–6]. It was initially observed in nonlocally coupled dynamical networks [1, 2]. Since then, nonlocal coupling topology has been considered as a necessary condition to induce chimera states. In later studies, chimera states have also been identified in networks with global [7] and local couplings [8–10]. Experimental observations of chimera states have been made in mechanical oscillators with metronomes [11], coupled chemical oscillators [12], optical combs systems [13], coupled electronic oscillators [14], time-varying networks [15], oscillators with more than one population [16–20] and so on.

In addition, Zakharova et al. have observed the emergent phenomenon of chimera states in nonlocally connected identical oscillators with symmetry-breaking coupling [21]. It is characterized by the coexistence of spatially coherent steady states (where neighboring oscillators occupy the same branch of inhomogeneous steady states) and incoherent steady states (where neighboring nodes are randomly distributed among two different branches of the inhomogeneous steady states). In recent works, the present authors have shown that the multi-cluster chimera death state can also be observed in dynamical networks with global coupling [22], structural changes in chimera death states [23] can occur, and different transition routes to chimera death states [24] in nonlocally coupled Stuart–Landau oscillators can arise.

On the other hand, in [25], Daido and Nakanishi investigated a network of globally coupled oscillators with diffusive coupling and reported the phenomenon of the swing of synchronized states. They found that the synchronized state is destabilized with increasing strength of the coupling interaction. It is found to be restabilized by further increases in the coupling strength. Also, they have reported that the globally coupled systems with diffusive coupling can induce a synchronized state that is to be mediated by the so-called cluster states. Recently the phenomenon of the swing of synchronized states that is mediated through amplitude chimera states has been reported in nonlocally coupled Stuart–Landau oscillators [24]. Motivated by these works, in the present paper, we investigate the robustness of swing of synchronized states by introducing higher-order nonlinearity in the system.

In this paper, we study the dynamical behavior of nonlocally coupled Stuart–Landau oscillators with symmetry-breaking form. We observe the basic feature that the nonisochronicity in the system plays an important role in realizing the synchronized states mediated through the amplitude chimera states and that we cannot observe such a phenomenon when there is an absence of nonisochronicity in the system. The other crucial contributor in inducing sway of synchronized states is the coupling radius of nonlocal interaction. Here the presence of nonlocal coupling in the system makes the amplitude chimera states mediate the synchronized states. Moreover, the system transits to the multi-chimera death state in the strong coupling limit. In addition, we analyze whether the phenomenon of a swing of the synchronized state is robust for introducing the higher-order nonlinear term in the system. We find that the higher-order nonlinearity in the system suppresses the phenomenon of a swing of the synchronized state that transits through the amplitude chimera state. Also, we find that the existence of multi-chimera death states is robust in this case.

2. Model

We consider an array of nonlocally coupled identical Stuart–Landau oscillators with symmetry-breaking form in the coupling, whose dynamics can be represented by the following set of equations:

$$\dot{z}_j = z_j - (1 - ic)\left|z_j\right|^2 z_j - b\left|z_j\right|^4 z_j + \frac{\epsilon}{2P}\sum_{k=N-P}^{N+P}\left(\mathrm{Re}[z_k] - \mathrm{Re}\left[z_j\right]\right), \tag{1}$$

where $z_j = x_j + iy_j$, $j = 1, 2, 3, \ldots N$. Here c is the nonisochronicity parameter and N is the total number of oscillators. Further, b is the real parameter and it is chosen as $b = 1$. The nonlocal coupling in the system is controlled by the coupling strength ϵ and the coupling range $r = P / N$, where P corresponds to the number of nearest neighbors in both directions. Here, we have introduced the coupling only in the real parts of the complex amplitude, and so this coupling introduces a symmetry breaking in the system.

In our simulations, we choose the number of oscillators N to be equal to 100, and in order to solve equation (1), we use the fourth-order Runge–Kutta method with time step 0.01 and with symmetric initial conditions between -1 and $+1$, which is necessary for the occurrence of the oscillation death state. In the following sections of the present paper, we investigate the dynamics of the coupled system under two different contexts: (1) when the real parameter $b = 0$; and (2) when the parameter $b \neq 0$.

3. Characterization to Collective Dynamical States and Swing by Mechanism

3.1 Case 1: When $b = 0$

First, we start our study by considering the value of the nonlinear parameter $b = 0$ in equation (1). In order to check whether the system exhibits the swing of synchronization mechanism in any region of the

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parametric space of this nonlocally coupled system, we vary the coupling range r, or equivalently P, and the nonisochronicity parameter c, and observe the dynamical behavior of the system. In order to know the nature of dynamical states in more detail, we look at the strength of incoherence of the system introduced recently by Gopal, Venkatesan and two of the present authors [26], which will help us to detect interesting collective dynamical states such as the chimera state. It is defined as

$$S = 1 - \frac{\sum_{m=1}^{M} s_m}{M}, \quad s_m = \Theta(\delta - \sigma(m)), \tag{2}$$

where

$$\sigma(m) = \left\langle \left(\frac{1}{n} \sum_{j=n(m-1)+1}^{mn} \left| z_j - \overline{z_j} \right|^2 \right)^{1/2} \right\rangle_t$$

with

$$\overline{z_j} = \frac{1}{N} \sum_{i=1}^{N} z_{j,i}(t).$$

δ is the threshold value, which is small, and Θ is the Heaviside step function. The angular bracket $\langle \ldots \rangle_t$ denotes the average over time. Thus the strength of incoherence measures the amount of spatial incoherence present in the system, which is zero for the spatially coherent/synchronized state. It has the maximum value, that is $S = 1$, for the completely incoherent/desynchronized state and has intermediate values between 0 and 1 for chimera states and cluster states. Further, to distinguish the amplitude chimera state and phase chimera state, we find the strength of incoherence in the amplitude domain S_r as different from S. For finding S_r, we use the same procedure as earlier, with z_j in equation (2) replaced by

$$w_j = r_j - r_{j+1} \left(r_j^2 = \sqrt{x_j^2 + y_j^2} \right).$$

Now S_r can be used to clearly distinguish the phase and amplitude chimera state, since the amplitudes of all the oscillators in the system are the same for the phase chimera state and strength of incoherence in the amplitude domain is $S_r = 0$, while S varies between 0 and 1. On the other hand, both S and S_r have values between 0 and 1 for amplitude chimera states. Now using the previous measures of strength of incoherence S and S_r, we identify the different dynamical regions that the system passes through while the coupling radius and nonisochronicity parameter are varied. For this purpose, in Figure 1 we demonstrate the behavior of the strength of incoherence S as well as S_r for the variables x_j with respect to the coupling strength ϵ for coupling radius $P = 10$ with $c = 3$. We find that initially all the oscillators are desynchronized in phase while the amplitudes of the oscillators are the same, where the value of S is found to be maximum and S_r is found to be zero. However, in the region $0.021 \leq \epsilon \leq 0.071$ the system of oscillators attains a synchronized state where $S = 0$ (and also $S_r = 0$). On increasing the coupling strength, in the region $0.072 < \epsilon < 0.11$, both S and S_r oscillate between 0 and 1, implying that the states correspond to an amplitude chimera state. By increasing ϵ beyond 0.11, both S and S_r are found to be zero, which confirms the synchronization among the oscillators. Thus, in this case, we can observe a recurrence of synchronization, where the synchronization in the system disappears with the increase of ϵ, but with a further increase of ϵ, synchronized states again reemerge. This analysis shows that the swing in synchronization in the system is mediated by the amplitude chimera state.

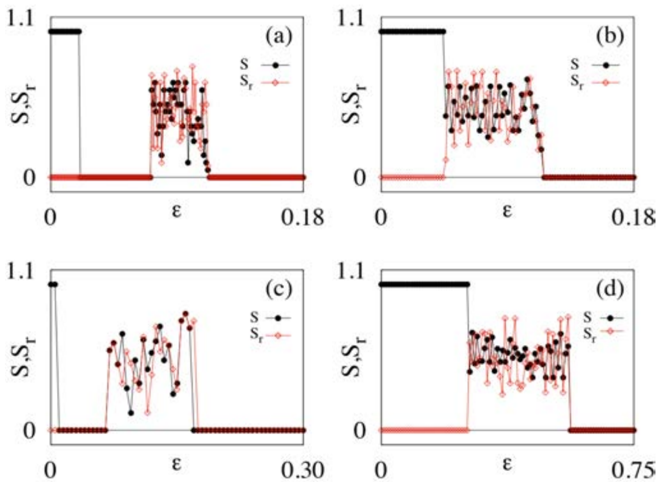

Figure 1. Strength of incoherence S and S_r of the system equation (1) for different values of ϵ for (a) $c = 3$ and $P = 10$; (b) $c = 3$ and $P = 25$; (c) $P = 10$ with $c = 4.5$; and (d) $P = 10$, $c = 7$, respectively.

Now increasing the coupling radius to $P = 25$, we cannot observe this type of sway in synchronization, which is also shown in Figure 1(b). Initially the states are phase desynchronized, where $S = 1$ and $S_r = 0$ in the region $\epsilon < 0.045$, and both S and S_r take values between 0 to 1 in the region $0.046 < \epsilon < 0.11$, indicating the presence of chimera states. For $\epsilon > 0.11$, the states are synchronized, where both S and S_r take the value zero. Thus, we can observe here the absence of recurrence of the synchronized state for large values of nonlocal interaction. Thus, from Figures 1(a) and 1(b), we find that the swing in synchronization occurs for small values of coupling radius (or strength of nonlocal interaction). Now we check whether this type of undulation of synchronization occurs for all values of c.

For this purpose, we fix $P = 10$ and find S for different values of c. The calculated results show that the swing mechanism in synchronization occurs neither for large values of c nor for smaller values of c but can be observed only for the window $2.7 \leq c \leq 5.2$. To illustrate this, we have plotted the strength of incoherence S of the system for two different values of c, namely $c = 4.0$ and $c = 7.0$ in Figures 1(c) and (d), respectively. At $c = 4.5$ (from Figure 1(c)), we can observe that for smaller values of ϵ ($\epsilon < 0.015$), S is found to be unity and $S_r = 0$, which represents the phase desynchronization among the oscillators. By increasing the coupling, both S and S_r reach the value zero, where the oscillators are in synchronization for the values of ϵ between 0.016 and 0.055. For the values $0.056 < \epsilon < 0.165$, we find the occurrence of amplitude chimera states in this region (as both S and S_r take the values between 0 and 1 for these values of ϵ). Both S and S_r are found to reach the minimum value (S, $S_r = 0$) by a further increase of ϵ. Thus for the values $\epsilon > 0.166$, the oscillators return to the synchronized state. This analysis shows that the swing in synchronization in the system is mediated by the amplitude chimera states. Now let us look at whether this type of reappearance occurs for higher values of c also.

The obtained results for $c = 7.0$ are shown in Figure 1(d), which indicates the absence of the given type of synchronized state. In this case, we can observe the phase desynchronization, where S has the maximum value while S_r takes the value zero for small values of ϵ ($\epsilon < 0.25$). Both S and S_r take a value between 0 and 1 for $0.25 < \epsilon < 0.550$, corresponding to an amplitude chimera state. Finally for $\epsilon > 0.550$, S and S_r decrease to zero, corresponding to a synchronized state. Hence the large value of nonisochronicity leads to the absence of swing in the synchronized state. Thus, for $P = 10$, we have the phenomenon of swing by in synchronization for the values of c between $2.7 < c < 5.2$.

3.1.1 Existence of Multi-chimera Death States

Further, we also note that the presence of a symmetry-breaking term in the coupling leads the system to transit to a new dynamical state called a chimera death state for large values of coupling strength.

Chimera death patterns combine the characteristics of both phenomena: chimera state and oscillation death. These patterns consist of coexisting domains of coherent and incoherent populations of the inhomogeneous steady-state branches. Within the incoherent domains, the population of the two branches (upper and lower) follows a random sequence, as shown in the space-time plot in Figure 2(a) for the parameter values $P = 10$, $c = 3$ and $\epsilon = 0.8$. Within the coherent domains, the number of clusters of neighboring nodes that populate the same branch of the inhomogeneous steady state may vary. Snapshots of the variables x_j and their frequency profiles illustrated in Figures 2(b) and (c) confirm the existence of multi-coherent and incoherent domains. Hence this state is designated as the multi-chimera death state. To give a concrete idea about the different dynamical states with transition routes, we extend our study with phase diagram in the next subsection.

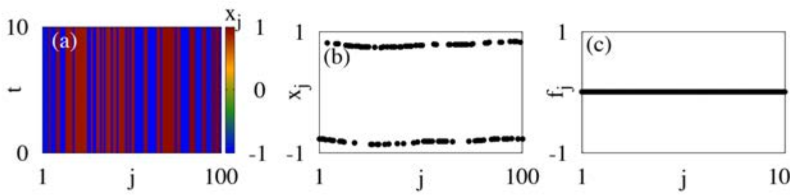

Figure 2. (a) Space-time plot for chimera death states; (b) corresponding snapshot of the variables x_j; and (c) frequency profile for the parameter values $P = 10$, $c = 3$ and $\epsilon = 0.08$.

3.2 Case 2: When $b \neq 0$

Next, we consider the value of the nonlinearity parameter b in equation (1) as nonzero. For simplicity, in our study, we assume the value of b as $b = 1$. To explore the spatio-temporal dynamics of equation (1) in some detail, we start by choosing the system parameter values as $c = 3$ and $P = 10$ for the reason that we explored the results in the previous section for these values. Figures 3 (a–c) are plotted by varying the strength of the coupling interaction to check whether the system exhibits a swing of synchronized states or not. Initially, the system of oscillators is oscillating desynchronously. By varying the coupling interaction, we can observe the traveling wave state for $\varepsilon = 0.03$ in Figure 3(a), and its snapshot is illustrated in Figure 3(d). On further increasing the coupling interaction, the system exhibits an amplitude chimera state (as in Figure 3(b)), which is confirmed through the snapshot of the variables x_j in Figure 3(e) for $\varepsilon = 0.1$. In an amplitude chimera state, oscillators in the coherent domain oscillate with large amplitudes, while the incoherent oscillators have smaller amplitudes. With the further increase of the coupling interaction, the system is seen to attain a multi-chimera death state for a sufficiently large value of coupling strength ($\varepsilon = 0.20$), which is indicated by the space-time plots in Figures 3(c) and (f). The number of oscillators that attain the steady state from the amplitude chimera state increases gradually with increases in the value of the coupling strength. In contrast to the previous case (where $b = 0$), here we cannot observe the phenomenon of the swing of synchronized state. The system attains multi-chimera death states through a traveling wave and amplitude chimera state, irrespective of the actual value of the coupling radius, which is clearly illustrated with the appropriate phase diagram in Figure 4.

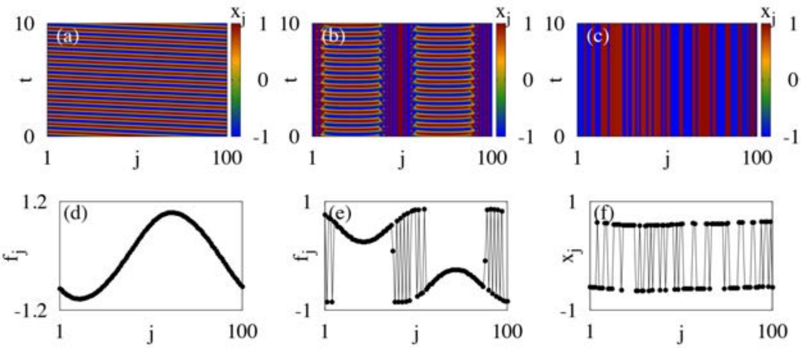

Figure 3. (Color online) Spatio-temporal plots for the variables x_j after leaving transients up to 10^5 time units: (a) traveling wave state for $\varepsilon = 0.03$; (b) amplitude chimera state for $\epsilon = 0.1$; and (c) multi-chimera death state for $\epsilon = 2.0$. Snapshots for the variables x_j: (d) for the traveling wave state at $t = 10$; (e) for the amplitude chimera state at $t = 10$; and (f) multi-chimera death states at $t = 10$. Other parameter values are $P = 10$, $c = 3$ and $b = 1$.

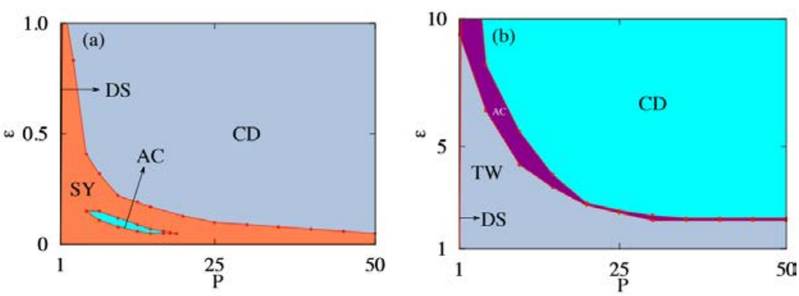

Figure 4. (Color online) Phase diagrams of equation (1) for: (a) $b = 0.0$; and (b) $b = 1.0$. SY, DS, AC, CD, TW regions represent synchronized state, desynchronized state, amplitude chimera state, multi-chimera death state (CD) and traveling wave state (TW), respectively.

4. Collective States in the (P, ε) Parameter Space

In order to give a global picture of the system more clearly, we present a phase diagram of the system for $c = 3.0$ and $b = 0$ in Figure 4(a). It shows that the system exhibits an amplitude chimera state for certain values of coupling radius P ($7 \leq P \leq 13$) with respect to the coupling interaction ε. In this region, the system shows the phenomenon of the swing of synchronized state mediated through the amplitude chimera state and the transition route is represented by

desynchronization → synchronization →
amplitude chimera → synchronization → chimera death.

For other values of the nonlocal coupling radius, the system follows the transition route as

desynchronization → synchronization → multi-chimera death.

Since the desynchronized regions exist in a very small region, it is marked by an arrow in the two-parameter diagram. We can find the existence of multi-stability between the stable amplitude chimera state and the synchronized state in the region AC. That is, we can observe the stable amplitude chimera state for the specific choice of initial conditions. In that region, we can also find that the synchronized solution coexists for initial conditions near the synchronized solution. We can find the multi-stability between the chimera death state and synchronized state in the region CD. Here we can observe the chimera death state for the specific choice of initial conditions. In this region, we can also find that the synchronized solution coexists for the initial conditions near the synchronized solution.

Similarly, Figure 4(b) shows the phase diagram of the system in the (P, ε) parametric space for $c = 3$ and $b = 1.0$. Here the system follows the transition route as

desynchronization →
traveling wave → amplitude chimera → multi-chimera death

with respect to the value of the coupling radius. In the TW region, we find multi-stability between the traveling wave and synchronized state. That is, we can observe the traveling wave state for the specific choice of initial conditions. In this region, we can also find that the synchronized solution coexists for initial conditions near the synchronized solution. We can also find the multi-stability between the stable amplitude chimera state and synchronized state in the region AC and multi-stability between the chimera death state and synchronized state in the region CD.

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5. Conclusion

In summary, we have investigated two cases of the occurrence of synchronized oscillations via amplitude chimera states in nonlocally coupled systems with symmetry-breaking interaction. We illustrated the roles of nonlocal interaction and the strength of nonisochronicity in inducing such types of synchronized states. Our results indicate the fact that the occurrence of characteristic features in synchronization is observed for smaller values of nonlocal interaction in the nonlocally coupled system with symmetry breaking.

Another interesting feature of these nonlocally coupled systems for higher values of coupling interaction is the existence of multi-chimera death states. We also carefully studied the occurrence of different transition routes to the recently observed dynamical state called chimera death while varying the strength of the nonlocal coupling radius. In addition, we have explored the existence of collective dynamical states such as traveling wave state, amplitude chimera state and multi-chimera death state by introducing the higher-order nonlinear term in the system. We find that the phenomenon of swing of the synchronized state is suppressed in this case and the system attains the multi-chimera death state directly from the amplitude chimera state.

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