From Collective Mind to Communication

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"Collective mind" is introduced as a set of simple intelligent units (say, neurons, or interacting agents) that can communicate by exchanging information without explicit global control. Incomplete information is compensated for by a sequence of random guesses symmetrically distributed around expectations with prescribed variances. Both the expectations and variances are the invariants characterizing the whole class of agents. These invariants are stored as parameters of the collective mind, while they contribute to dynamical formalism of the agents' evolution, and in particular, to the reflective chains of their nested abstract images of the selves and nonselves. The proposed model consists of the system of stochastic differential equations in the Langevin form to represent motor dynamics, and the corresponding Fokker–Planck equation to represent mental dynamics. The main departure of this model from newtonian and statistical physics is due to feedback from the mental to the motor dynamics, which makes the Fokker–Planck equation nonlinear. Interpretations of this model from mathematical, physical, biological, and psychological viewpoints are discussed. The model is illustrated by the dynamics of a dialog.

1. Introduction

The concept of "collective mind" has appeared recently as a subject of intensive scientific discussion from economical, social, ecological, and computational viewpoints. It can be introduced as a set of simple units of intelligence (say, neurons, or interacting agents) that can communicate by exchanging information without explicit global control. The objectives of the agents may be partly compatible and partly contradictory, that is, they can cooperate or compete. The information exchanged may at times be inconsistent, often imperfect, nondeterministic, and delayed. Nevertheless, observations of working insect colonies, social systems, and scientific communities suggest that such collectives of agents appear to be very successful in achieving global objectives. They are also good at learning, memorizing, generalizing, and making predictions due to their flexibility, adaptability to environmental changes, and creativity.

The objective of this paper is to introduce a dynamical formalism describing the evolution of communicating agents. All previous attempts

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to develop models for so-called "active systems" have been based upon the principles of newtonian and statistical mechanics [1]. These models appear to be so general that they predict not only physical but also some biological, economical, and social patterns of behavior, exploiting such fundamental properties of nonlinear dynamics as attractors. Notwithstanding indisputable successes of that approach (e.g., neural networks and distributed active systems) there is still a fundamental limitation that characterizes these models: On a dynamical level of description, they propose no difference between a solar system, a swarm of insects, or a stock market. Such a phenomenological reductionism is incompatible with the first principle of progressive biological evolution [2, 3]. According to this principle, the evolution of living systems is directed toward higher levels of complexity, if the complexity is measured by an irreducible number of different parts that interact in a well-regulated fashion. At the same time, solutions to the models based upon dissipative newtonian dynamics eventually approach attractors where evolution stops (until a "master" reprograms the model). Therefore, such models fail to provide an autonomous progressive evolution of living systems.

Let us now turn to the stochastic extension of newtonian models. Actually, it is a well-established fact that the evolution of life has a diffusion-based stochastic nature. This means that the simplest living species must obey the second law of thermodynamics, just as physical particles do. However, then the evolution of living systems (during periods of their isolation) will be regressive since their entropy will increase [4]. As pointed out by Gordon in [5], a stochastic motion describing physical systems does not have a sense of direction, and therefore, it cannot describe a progressive evolution. As an escape from this paradox, Gordon proposed a concept of differentiating waves (represented by traveling waves of chemical concentration or mechanical deformation) which are asymmetric by their nature, and this asymmetry creates a sense of direction toward progressive evolution. Although the concept of differentiating waves itself seems convincing, it raises several questions to be answered: Who or what arranges the asymmetry of the differentiating waves in the "right" direction? And, can their formalism be incorporated into statistical mechanics, providing progressive evolution without violating the second law of thermodynamics? Thus, although the stochastic extension of newtonian models can be arranged in many different ways (e.g., *via* relaxation of the Lipcshitz conditions [6], or by means of opening escape routes from the attractors) the progressive evolution of living systems cannot be provided.

These limitations have been addressed in several publications in which the authors were seeking a "border line" between living and nonliving systems. It is worth noting that one of the most obvious distinctive properties of the living systems; namely, their intentionality, can be formally disqualified by simple counter-examples. Indeed, any mechanical (nonliving) system has an "objective" to minimize action (the Hamilton principle) and any isolated diffusion-based stochastic (nonliving) system has an "objective" to maximize entropy production (the Jayne's principle, [3]). Departure from newtonian models *via* the introduction of dynamics with expectations and feedback from the future has been proposed by Huberman and his associates in [7]. Further departure, which includes learning nested models of multi-agent systems, have been introduced by Vidal in [8]. However, despite the fact that the nonnewtonian nature of living systems in these works was captured correctly, there is no global analytical model that would unify the evolution of the agent's state variables and their probabilistic characteristics such as expectations, self-images, and so forth.

The objective of this paper is to develop a new mathematical formalism, within the framework of classical dynamics, that allows one to capture specific properties of natural or artificial living systems, such as formation of the collective mind. Based upon abstract images of the selves and nonselves, this collective mind is utilized for communicating, predicting future expected characteristics of evolution, and also for making decisions and implementing corresponding corrections if the expected scenario is different from that originally planned. The approach is based upon the author's previous publications [9–11] which postulate that even a primitive living species possesses additional nonnewtonian properties that are not included in the laws of newtonian or statistical mechanics. These properties follow from a privileged ability of living systems to possess a self-image (a concept introduced in psychology) and to interact with it. The mathematical formalism is based upon coupling the classic dynamical system (with random components caused by uncertainties in initial conditions, as well as by the Langevin forces) to represent motor dynamics, with the corresponding Fokker–Planck equation describing the evolution of these uncertainties in terms of their probability density to represent mental dynamics. The coupling is implemented by the information-based supervising forces representing self-awareness. These forces fundamentally change the pattern of the probability evolution, and therefore, lead to a major departure of the behavior of living systems from the patterns of both newtonian and statistical mechanics. Further extension, analysis, interpretation, and application of this approach to collective-mind-based communicating agents will be addressed in this paper.

2. Reflective chains: What do you think I think you think...

We start with the simplest model of two interacting agents, assuming that each agent is represented by an inertionless classical point evolving in physical space. We also assume that the next future position of each

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agent depends only upon its own present position and the present position of its opponent. Then their evolutionary model can be represented by the following system of differential equations:

$$\dot{x}_1 = f_1(x_1, x_2) \tag{1}$$
$$\dot{x}_2 = f_2(x_1, x_2). \tag{2}$$

Here x_1 and x_2 are the state variables for agents 1 and 2, respectively. We will start with the assumption that these agents belong to the same class, and therefore, they know the structure of the whole system. However, each of the agents may not know the initial condition of the other one, and therefore, cannot calculate the current value of the opponent's state variable. As a result, the agents try to reconstruct these values using the images of their opponents. Let us turn first to agent 1, in whose view the system looks like:

$$\dot{x}_{11} = f_1(x_{11}, x_{21}) \tag{3}$$
$$\dot{x}_{21} = f_2(x_{21}, x_{121}) \tag{4}$$

where x_{11} is the self-image of agent 1, x_{21} is agent 1's image of agent 2, and x_{121} is agent 1's image of agent 2's image of agent 1.

This system is not closed since it includes an additional 3-index variable x_{121}. In order to find the corresponding equation for this variable, one has to rewrite equations (3) and (4) in the 3-index form. But it is easy to verify that such a form will include 4-index variables, and so on so that this chain of equations will never be closed. By interchanging the indices 1 and 2 in equations (3) and (4), one arrives at the system describing the view of agent 2. The situation can be generalized from two- to n-dimensional systems. It is easy to calculate that the total number of equations for the mth level of reflection, that is, for the m-index variables, is

$$N_m = n^m. \tag{5}$$

Thus, the number of equations grows exponentially with the number of levels of reflection, and it grows linearly with the dimensionality n of the original system. It should be noted that for each mth level of reflection, the corresponding system of equations always includes $(m + 1)$-index variables, and therefore, it is always open. Hence, for any quantitative results, this system must be supplemented by a closure, that is, by additional equations with respect to extra variables. In order to illustrate how this can be done we first reduce equations (1) and (2) into the linear form

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 \tag{6}$$
$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2. \tag{7}$$

Taking the position of agent 1, we can rewrite equation (6) into the form:

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_{21} \tag{8}$$

in which the unknown state variable x_2 of agent 2 is replaced with its value x_{21} to be predicted by agent 1. Recalling that agents 1 and 2 belong to the same class, it is reasonable to assume that agent 1 knows the expected initial value χ_2^0 as well as the initial variance σ_2^0 of agent 2's state variable x_2. Based upon that, agent 1 can predict current values of agent 2's state variable as:

$$x_{21} = \chi_2 + \sigma_2 L(t) \tag{9}$$

where $L(t)$ is a Langevin force represented by a random function with zero mean and a δ-correlation function. According to this representation, the variable x_{12} has the expected value χ_2 and the variance σ_2. In other words, the prediction consists of random guesses dispersed symmetrically around the expected value, while the expected value x_2, as well as the variance σ_2 characterizing this dispersion, are to be found. Substituting equation (9) into equation (8), one arrives at the following Langevin-type stochastic differential equation [12] of agent 1:

$$\dot{x}_1 = a_{11}x_1 + a_{12}(\chi_2 + \sigma_2 L). \tag{10}$$

A similar equation can be written for agent 2:

$$\dot{x}_2 = a_{21}(\chi_1 + \sigma_1 L) + a_{22}x_2. \tag{11}$$

However, from the viewpoint of agent 1, equation (11) may have two different forms:

$$\dot{x}_2 = a_{21}x_1 + a_{22}(\chi_2 + \sigma_2 L) \tag{11a}$$

or

$$\dot{x}_2 = a_{21}(\chi_1 + \sigma_1 L) + a_{22}(\chi_2 + \sigma_2 L). \tag{11b}$$

Equation (11*a*) expresses that agent 1 assumes that agent 2 knows the state variable of its opponent, that is, x_1. On the contrary, equation (11*b*) expresses that agent 1 assumes that agent 2 does not know the opponent's variable and predicts it in the same way that agent 1 does. From the viewpoint of reflection levels, equations (10) and (11) are on the first level since each agent uses only the image of its opponent, while equations (10),(11*a*) and (10),(11*b*) are on the second level since each agent, in addition, uses the image of the image of the opponent of itself and its opponent. In order to make our point in the simplest way, we will stay with the first level of reflection, that is, with equations (10) and (11).

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Formally these equations are not coupled (unlike their original versions, equations (6) and (7)). However, as will be shown, they are coupled indirectly, *via* the variables $\chi_1, \chi_2, \sigma_1, \sigma_2$. Indeed, since equations (6) and (7) can be considered as the Langevin-type stochastic differential equations, evolution of these variables is governed by the corresponding Fokker–Planck equation [12]:

$$\frac{\partial p}{\partial t} + \frac{\partial}{\partial X_1}[(a_{11}X_1 + a_{12}\chi_2)p] + \frac{\partial}{\partial X_2}[a_{21}\chi_1 + a_{22}X_2)p] = \left(a_{12}^2\sigma_2^2\frac{\partial^2}{\partial X_1^2} + a_{21}^2\sigma_1^2\frac{\partial^2}{\partial X_2^2}\right)p. \quad (12)$$

Here $p(t, X_1, X_2)$ is the joint probability density of distribution of the state variables x_1 and x_2 over the space coordinates X_1 and X_2. This equation must be complemented by the normalization condition

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p\,dX_1 dX_2 = 1 \quad (13)$$

as well as by the definitions of χ and σ

$$\chi_i = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X_i p(t, X_1, X_2)dX_1 dX_2, \qquad i = 1, 2 \quad (14)$$

$$\sigma_i = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (X_i - \chi_i)^2 p(t, X_1, X_2)dX_1 dX_2, \qquad i = 1, 2. \quad (15)$$

The system of equations (12) through (15) is closed, and it can be solved subject to the initial and boundary conditions

$$p(0, X_1, X_2) = p^0(X_1, X_2), \quad (16)$$

$$p(t, \pm\infty, \pm\infty) = 0, \qquad \frac{\partial p}{\partial X_i}(t, \pm\infty, \pm\infty) = 0, \quad i = 1, 2. \quad (17)$$

Substituting equations (14) and (15) (as the known functions of time found from the solution of this system) into equations (10) and (11), one obtains the solutions for the state variables:

$$x_i = \exp(a_{ii}t)\left[\int_0^t a_{ij}(\chi_j + \sigma_j L)dt + x_i^0\right], \qquad i, j = 1, 2; \quad i \neq j. \quad (18)$$

Here x_i^0 is the initial value of the corresponding state variable.

It should be noted that the solutions in equation (18) are random because of the randomness of the Langevin forces L. That is why, for qualitative analysis, it is more convenient to stay with the statistical invariants of these solutions, that is, with the means χ_i and the variances σ_I expressed by equations (14) and (15), respectively.

It should also be noted that the same solution strategy can be applied to the models describing the second level of reflection, that is, equations (10),(11*a*) or (10),(11*b*) with the only difference that these equations express only the view of agent 1. After interchanging the indexes 1 and 2 in these equations, one arrives at the similar system expressing the view of agent 2. Then each agent is supposed to create the image of the model of his opponent, and so on.

3. Dialog as an evolutionary game with incomplete information

In this section we apply the model of two interacting agents presented by equations (10) through (17) to evolutionary games. A *game* is understood as a special type of an interaction between two agents, which follow certain rules and expect a certain outcome. A game is *evolutionary* if the interacting agents change their internal states in order to be successful in the future. Turning to equations (10) and (11), one can see that this model of interacting agents satisfies both of these definitions. Indeed, the state variables x_I can be associated with internal representations of the agents. The right-hand parts of equations (10) and (11) can be considered as the statements made by each agent. Each of those statements depends upon both the state variables and the reactions to the corresponding statements. The left-hand parts of equations (10) and (11) express the changes of the state variables. From these changes the agents calculate the next statements, and that changes the internal states of the agents. It should be noted that the representations, as well as the statements, are arbitrary with respect to what one wants to present. In other words, the model captures the grammar without a semantic. It should be recalled that games can be adversarial or cooperative. In this section we deal only with cooperative games; in particular, the outcome of the game is to approach the common ground (or mutual belief, or shared conception) regardless of the initial conditions. Obviously, if the system of equations (10) and (11) is stable, that is, if

$$a_{11} + a_{22} < 0, \qquad a_{11}a_{22} > a_{12}a_{21}, \tag{19}$$

then any initial conditions x_i^0 $(i = 1, 2)$ will lead to the common ground, or the zero solution. This happens under the condition that each agent has complete information not only about the values of its own state variable, but about the values of the state variable of another agent as well. However, in the case of language communications, information is never complete. Language can always be interpreted in many different ways, unless the sender and receiver have some "expected" mutual belief following from previous knowledge about each other, or about the context of the forthcoming dialog. Hence, it would be reasonable to assume that although each agent does not know the values of the state variable of its partner, it can nevertheless come up with some random guesses that are characterized by known statistical invariants such as the mean χ and the variance σ. Turning to equation (9), one can recognize

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that this is exactly the same representation just mentioned. As shown in section 2, it is sufficient for the agent to know only the initial values of these invariants, since then their current values are uniquely determined by the corresponding Fokker–Planck equation (12). Thus, we have to return to equations (12) through (17) in order to find these invariants. We will demonstrate that for a simple system such as equations (10) and (11), one does not need to find the solution to equation (12) subject to the conditions of equations (13) through (17); instead, the direct equations with respect to the statistical invariants χ and σ can be derived from equation (12). Indeed, let us multiply equation (12) by X_i ($i = 1, 2$) and integrate over the whole space. Then, taking into account the conditions of equations (13) and (17), one obtains the following system of ordinary differential equations (ODEs) with respect to the expected values of the state variables

$$\dot{\chi}_1 = a_{11}\chi_1 + a_{12}\chi_2 \tag{20}$$

$$\dot{\chi}_2 = a_{21}\chi_1 + a_{22}\chi_2. \tag{21}$$

Obviously equations (20),(21) and (6),(7) are identical, that is, the evolution of the state variables (with complete information) and their expectations are described by the same model. (Such a coincidence results from the linearity of the original model.) But it should be noted that stability of expectations does not guarantee stability of equations (10) and (11), that is, the stability of the state variables with incomplete information. Indeed, let us turn to equation (18), which is the solution of equations (10) and (11). Obviously this solution is unstable if $a_{11} > 0$ or $a_{22} > 0$, regardless of the expectations and variances as functions of time, and even if the first inequality in equation (19) is satisfied. Therefore, in order to provide the stability of evolution of the state variables with incomplete information, the expectations must be "more stable," that is, the inequalities are to be stronger than equation (19)

$$a_{11} < 0, \qquad a_{22} < 0, \qquad a_{11}a_{22} > a_{12}a_{21}. \tag{22}$$

In our further analysis we assume that these conditions hold. However, one should note that these conditions are only necessary, but not yet sufficient for the stability of equations (10) and (11). In order to derive the sufficient conditions, one has to analyze the evolution of the variances. For that purpose, let us multiply equation (12) by X_i^2 ($i = 1, 2$), and integrate it over the whole space. After transformations similar to those performed above for expectations, one arrives at a system of ODEs that is nonlinear with respect to variances and is coupled with equations (20) and (21). For better observability, we will simplify this system by assuming that

$$\chi_1^0, \qquad \chi_2^0 = 0. \tag{23}$$

Then the governing equations for the variances can be written as

$$\dot{\sigma}_1^2 = 2a_{11}\sigma_1^2 + 2a_{12}^2\sigma_1^2\sigma_2^2 \tag{24}$$

$$\dot{\sigma}_2^2 = 2a_{22}\sigma_2^2 + 2a_{21}^2\sigma_2^2\sigma_1^2. \tag{25}$$

Although this system is still nonlinear, its stability analysis is simple. Indeed, since it has one attractor

$$\sigma_1 = 0, \qquad \sigma_2 = 0, \tag{26}$$

and one repeller

$$\sigma_1^2 = -\frac{a_{22}}{a_{21}}, \qquad \sigma_1^2 = -\frac{a_{11}}{a_{12}^2} \tag{27}$$

the stability will be provided if the initial variances are inside the basin of attraction, that is, if

$$(\sigma_1^0)^2 < -\frac{a_{22}}{a_{21}^2}, \qquad (\sigma_2^0)^2 < -\frac{a_{11}}{a_{12}}. \tag{28}$$

Thus, equations (22) and (28) are necessary and sufficient for the stability of the system of equations (10) and (11) in the simplified case of equation (23). As follows from equation (28), the incompleteness of the information measured by the initial variances σ_1^0 and σ_2^0 is directly responsible for the divergence of the dialog. At the same time, the stability of the original model in equations (6) and (7) represented by the conditions of equation (22), increases the allowed degree of incompleteness that would still preserve the convergence of the dialog.

To conclude this section, we will make several remarks. First we will analyze the effect of noise upon the stability of the dialog. For that purpose, we add additional Langevin forces to equations (6) and (7):

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + b_1^2\Gamma_1(t) \tag{29}$$

$$\dot{x} = a_{21}x_1 + a_{22}x_2 + b_2^2\Gamma_2(t). \tag{30}$$

Here $\Gamma_1(t)$ and $\Gamma_2(t)$ are random functions with zero means and with correlation functions equal to a δ-function, and the constants b_1^2, b_2^2 represent the strengths of the noise interference for agents 1 and 2 respectively. After transformations similar to those performed earlier, one arrives at the following stability conditions:

$$(\sigma_1^0)^2 < -\frac{a_{22}}{a_{21}^2} + \frac{b_1^2}{a_{21}^2}, \qquad (\sigma_2^0) < -\frac{a_{11}}{a_{12}^2} + \frac{b_2^0}{a_{12}^2}. \tag{31}$$

These conditions demonstrate that noise interference decreases stability of the dialog by decreasing the upper bound of the initial variances (which are proportional to the degree of incompleteness of information) that are sufficient for the convergence of the dialog to the common ground.

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Second, we will assume that each agent is able to stabilize the dialog by applying control (or self-supervising) forces composed of the probability density and its space-derivatives. We will choose the following control forces

$$F_1 = -c_1^2 \frac{\partial}{\partial x_1} \ln p, \qquad F_2 = -c_1^2 \frac{\partial}{\partial x_2} \ln p \tag{32}$$

to be added to the right-hand sides of equations (6) and (7) respectively:

$$\dot{x}_1 = a_{11} x_1 + a_{12} x_2 + F_1 \tag{33}$$

$$\dot{x}_2 = a_{21} x_1 + a_{22} x_2 + F_2. \tag{34}$$

Here the constants c_1^2 and c represent the strengths of the control forces. Their contribution into the stability conditions is

$$(\sigma_1^0)^2 < -\frac{a_{22}}{a_{21}^2} + \frac{b_1^2}{a_{21}^2} - \frac{c_1^2}{a_{21}^2}, \qquad (\sigma_2^0) < -\frac{a_{11}}{a_{12}^2} + \frac{b_2^2}{a_{12}^2} - \frac{c_2^2}{a_{12}^2}. \tag{35}$$

As follows from this, the control forces can completely suppress the effect of noise if

$$c_1 = b_1, \qquad c_2 = b_2. \tag{36}$$

Thus, the proposed model consists of three basic components, (Figure 1). The first component is the dynamical model of the agents interaction (in the form of a dialog) represented by ODEs (see equations (6) and (7)). This model can be associated with the motor dynamics. Since language communication is always incomplete, the motor dynamics have to be complemented by additional information. This information comes from the collective mind that represents a special knowledge base (or a context) composed of the abstract images of the agents in terms of their joint probability density. These images capture general characteristics of the agents, their "habits," expected routes of their evolution, possible deviations of the state variables from their expected values, and so on (see equation (12)). From the collective mind, each agent can extract the evolution of its own (marginal) probability density p_i that represents its mental dynamics. In our model, the marginal densities are approximated by the expectations χ_i and the variances σ_i, and therefore, the mental dynamics of the agents are expressed by equations (20),(24), and (21),(25), respectively. The last component of the model is the feedback from the collective mind (equation (12)) to the motor dynamics (equations (6) and (7)) implemented by the control forces of equation (32) (see equations (33) and (34)). These self-supervising forces can be associated with the agent's self-awareness since they are composed only of the internal parameters characterizing the state of the collective mind, while their goal is to affect the motor dynamics in order to achieve a desired outcome of the "game."

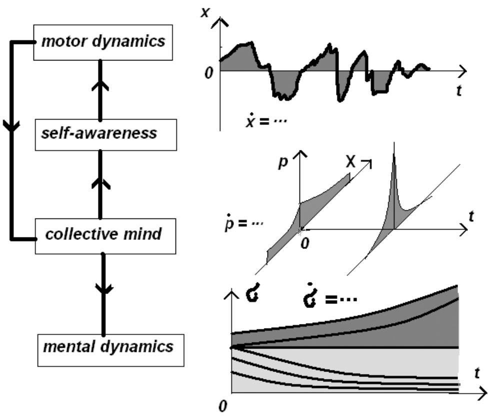

Figure 1. Dynamics of dialog.

The model introduced in this section is, probably, the simplest one that still allows us to present the proof-of-concept using well-observable closed-form analytical solutions. In section 4 we will sketch an extension of this model to the general case when only a numerical approach can be applied.

4. General model

In this section we will generalize our model using the following neural-net-based structure:

$$\dot{x}_i = f_i(x_i) + \phi_i, \qquad \phi_i = \tanh \sum_{i \neq j} T_{ij} x_j + F_i. \tag{37}$$

Here ϕ_i is the sigmoid function, T_{ij} are the synaptic interconnections, F_i are the control (or self-supervising) forces that, in general, depend upon the probability density p, its gradients and functionals, while $i, j = 1, 2, \ldots n$. When each agent gets complete information about all the state variables x_I, then equation (37) is fully deterministic, the control forces vanish, and it represents a regular neural net. However, when the information is incomplete, that is, when each agent knows only the values of its own state variables, which is always the case in language-based communications, the neural-net equation (37) has to be complemented by the "context" as well as by control forces provided by the collective mind. Applying the same strategy introduced in section 3, we will seek for the approximation of ϕ_i in a form similar to equation (9):

$$\phi_i = \chi_i + \sigma_i L_i(t), \quad \chi_i = \int_V \phi_i p dV, \quad \sigma_i = \int_V (\phi_i - \chi_i)^2 p dV, \tag{38}$$

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where $V = \{X_1, X_2, \ldots X_n\}$. Then equation (37) reduces to

$$\dot{x}_i = f_i(x_i) + \chi_i(t) + \sigma_i(t)L_i(t) + F_i\left(p, \frac{\partial p}{\partial x_i}, \ldots\right). \tag{39}$$

Obviously, equation (39) is stochastic due to the random Langevin forces $L_i(t)$ and the control forces F_i. The corresponding Fokker–Planck equation describing the evolution of the joint probability $p(t)$ is

$$\frac{\partial p}{\partial t} + \sum_i \left\{\left[\frac{\partial}{\partial X_i}(f_i + \chi_i + F_i)p\right] - \sigma_i^2 \frac{\partial^2}{\partial X_i^2} p\right\} = 0. \tag{40}$$

This equation should be solved subject to n-dimensional versions of equations (13) through (17), and its solution can be associated with the evolution of the collective mind.

Substituting the solution $p(t, X_1, X_2, \ldots X_n)$ into equation (38), one finds the evolution of the expected values χ_i as well as the variances σ_i of the state variables x_i which represent the mental dynamics of the agents. Substituting this solution, together with the expectations and variances found into equation (39), one arrives at the closed (but stochastic) model of the motor dynamics.

We now discuss in more detail the control (or self-supervised) forces F_i. In the previous model of equation (32) their role was to stabilize the dialog by suppressing noise. In general, their choice depends upon the objective of the cooperating agents. If this objective is formulated in terms of minimization of a functional

$$J = \int \Phi(p, \nabla p, \ldots) dV \to \min, \tag{41}$$

then, applying the formalism of control theory, one can find the corresponding control forces. However, one should note that in classical control theory, the control forces depend upon the state variables [13], while here they are composed of the parameters of the collective mind such as expected values and variances of the state variables, and that is why it is better to call them "self-supervised forces."

When communicating agents simulate human society, some emerging objectives can take over and govern the agent's behavior. As an example, consider the principle of reflectivity introduced by Lefebvre in [14]: "the subject tends to generate a pattern of behavior such that similarity is established and preserved between the subject and his model of the self" while this principle "is a manifestation of a special cognitive mechanism of self-representation rather than a result of the intellectual efforts of the subject consciously thinking about the self." In terms of our formalism, the following self-supervised forces can implement this principle (see [15]):

$$F_i = \gamma_i(\chi_i - x_i)^{1/3}, \qquad \gamma_i = \text{const}. \tag{42}$$

On a large time scale, one can introduce the principle of maximum complexity stating that a human-like community of agents evolves toward the maximum increase of the complexity of its social structure. In terms of our formalism, such an evolution is achieved by an increase in the number of levels of reflection (see equation (5)). However, a natural constraint for such an increase is the exponential growth of the capacity and resources required.

A more practical approach to selecting the self-supervised forces can be adopted from the concept of learning in neural nets. Suppose that these forces are sought in the following parametrized form:

$$F_i = \tanh \sum_{i,j} w_{i,j} p_j, \qquad i, j = 1, 2, \dots k, \tag{43}$$

in which w_{ij} are constant weights, and p_j is the value of the probability density at a fixed point j of the n-dimensional space $X_1 \dots X_n$, while k is the number of points at which the probability density is discretized. Let us assume that our objective is to teach the artificial communicating agents to make correct decisions in response to unexpected changes in external forces or in the objectives. For that purpose, first we have to find an expert whose responses (either rational or intuitive) will be optimal. Then, comparing these responses with the corresponding responses of the model and applying the back-propagation technique, one can find the optimal weights in the self-supervising forces of equation (43).

5. Interpretation of the model

In this section we present and discuss interpretations of the proposed collective-mind-based model of communicating agents from the viewpoints of mathematics, physics, biology, psychology, neuroscience, social dynamics and economy, language communications, control theory, and hardware implementation.

5.1 Mathematical viewpoint

From the mathematical viewpoint the model is represented by the system of the Langevin-type stochastic differential equations (see equations (10), (11), or (39)) and the corresponding Fokker–Planck equation (see equation (12) or (40)). The connection between these equations is the following: equation (39) simulates randomness while equation (40) manipulates the values of its probability. Therefore, if equation (39) is run independently many times, and statistical analysis of the corresponding solutions is performed, then the calculated probability density will evolve according to equation (40). However, the major departure from the classical case here is in the coupling between the Langevin and the Fokker–Planck equations. This coupling is implemented by the

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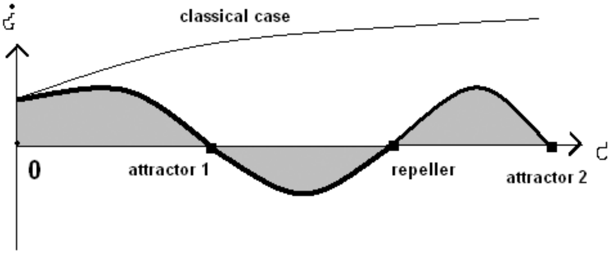
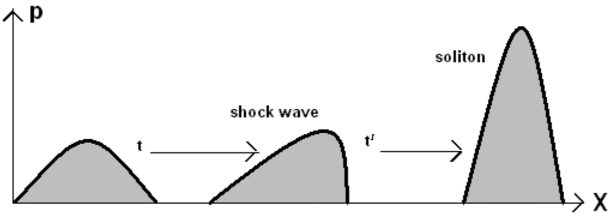


(a) Multiattractor structure of the variance.



(b) Formation of shock waves and solitons.

Figure 2. Nonlinear phenomena in probability space.

self-supervising forces F_i as well as by the expectations χ_i and the variances σ_i of the state variables, (see equation (39)). As a result of this coupling, the Fokker–Planck equation becomes nonlinear with respect to the probability density, and that, in turn, leads to new fundamental phenomena in the probability space [15]. These phenomena include formation of multiattractor limit sets as well as formation of shock waves and solitons, Figure 2.

5.2 Physical viewpoint

From the physical viewpoint, the model represents a fundamental departure from both newtonian and statistical mechanics. Indeed, firstly, in newtonian mechanics the evolution of the probability density (described by the Fokker–Planck equation) is always linear, and it never affects the underlying motion of the corresponding physical system. Secondly, in newtonian mechanics the Fokker–Planck equation only registers the evolution of the probability density without affecting the corresponding equations of motion, and there are no principles that would determine additional feedback forces. Both of these conditions are violated in the proposed model: due to the self-supervising forces F_i, equations (39) and (40) are coupled, and that, in turn, makes equation (40) nonlinear. The same coupling between the evolution of the probability density and the corresponding motion in physical space may cause a decrease of the entropy, or a progressive evolution that is strictly forbidden by the statis-

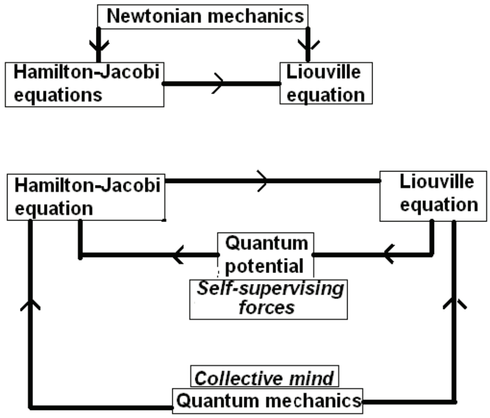


Figure 3. Physical structure of collective mind.

tical mechanics of isolated systems. Since the entropy, loosely speaking, is proportional to the variance of the corresponding probability density, its qualitative evolution can be illustrated by Figure 2(a) where the linear and nonlinear cases are compared. Thus, the proposed model is incompatible with both newtonian and statistical mechanics. At the same time, it is fully consistent with the theory of differential equations and stochastic processes. The only conclusion following from that is that this model can display some "nonnewtonian" features. However, surprisingly, the proposed model has formal mathematical similarity with quantum mechanics. Indeed, let us turn to the Schredinger equation and decompose the wave function into real and imaginary parts [11, 15]. This will lead us to the Madelung equations that consist of the equation of motion (in the Hamilton–Jacobi form) and the equation of evolution of the probability density (in the Liouville form). The Madelung equations differ from the corresponding equations of newtonian dynamics only by an additional term (called *quantum potential*) which is responsible for the fundamental departure of quantum mechanics from newtonian mechanics, Figure 3. The gradient of this term represents a feedback force that couples the motion and the corresponding evolution of the probability density in the same way in which the feedback forces F_i couple equations (39) and (40). It should be noted, however, that the forces F_i are not necessarily potential (in most cases they are dissipative). Nevertheless, the formal structure of the proposed model, that is, equations (39) and (40), is more similar to those of quantum mechanics than to newtonian mechanics. This comparison does not imply that there is any physical connection between quantum mechanics and the dynamics of communicating agents: it just illustrates the usefulness of

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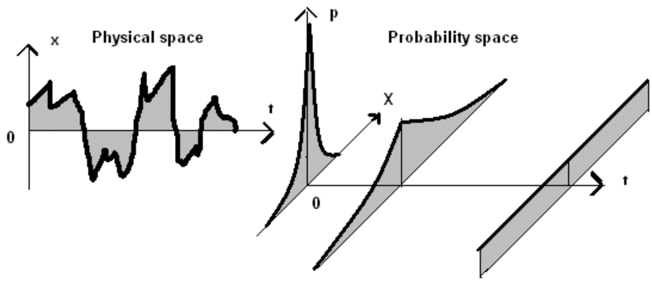
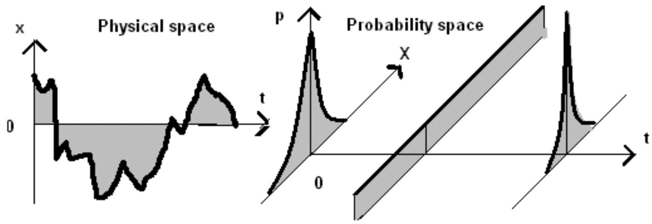
the dynamical systems based upon feedback from the evolution of the probability density to the underlying motion.

5.3 Biological viewpoint

From the viewpoint of evolutionary biology, the proposed model illuminates the "border line" between living and nonliving systems. The starting point of our biologically-inspired interpretation is the second law of thermodynamics, which states that the entropy of an isolated system can only increase. This law has a clear probabilistic interpretation: increase of entropy corresponds to the passage of the system from less probable to more probable states, while the highest probability of the most disordered state (which is the state with the highest entropy) follows from a simple combinatorial analysis [2]. However, this statement is correct only if there is no Maxwell's sorting demon, that is, nobody inside the system is rearranging the probability distributions. But this is precisely what the self-supervising feedback is doing: it takes the probability density p from equation (40), creates functionals or functions of this density, converts them into a force, and applies this force to the equation of motion (see the last three terms in equation (39)). As already mentioned, because of that property of the model, the evolution of the probability density becomes nonlinear, and the entropy may decrease "against the second law of thermodynamics." Obviously the last statement should not be taken literally; indeed, the proposed model captures only those aspects of the living systems associated with their behavior, and in particular, with their motor–mental dynamics since they are beyond the dynamical formalism. Therefore, such physiological processes that are needed for metabolism are not included in the model. That is why this model is in formal disagreement with the second law of thermodynamics while the living systems are not. In order to further illustrate the connection between the life–nonlife discrimination and the second law of thermodynamics, consider a small physical particle in a state of random migration due to thermal energy, and compare its diffusion, that is, physical random walk, with a biological random walk performed by a bacterium. The fundamental difference between these two types of motions (that may be indistinguishable in physical space) can be detected in probability space: the probability density evolution of the physical particle is always linear and it has only one attractor: a stationary stochastic process where the motion is trapped. On the contrary, a typical probability density evolution of a biological particle is nonlinear: it can have many different attractors, but eventually each attractor can be departed from without any "help" from outside, Figure 4. That is how Berg describes the random walk of an *E. coli* bacterium [16]: "If a cell can diffuse this well by working at the limit imposed by rotational Brownian movement, why does it bother to tum-



(*a*) Regressive evolution of nonliving systems.



(*b*) Regressive and progressive evolution of living systems.

Figure 4. Evolution of living and nonliving systems.

ble? The answer is that the tumble provides the cell with a mechanism for biasing its random walk. When it swims in a spatial gradient of a chemical attractant or repellent and it happens to run in a favorable direction, the probability of tumbling is reduced. As a result, favorable runs are extended, and the cell diffuses with drift." Berg argues that the cell analyzes its sensory cue and generates the bias *internally*, by changing the way in which it rotates its flagella. This description demonstrates that a bacterium actually interacts with the medium, that is, it is not isolated, and that reconciles its behavior with the second law of thermodynamics. However, since these interactions are beyond the dynamical world, they are incorporated into the proposed model *via* the self-supervised forces that result from the interactions of a biological particle with "itself," and that formally "violates" the second law of thermodynamics. Thus, the proposed model offers a unified description of the progressive evolution of living systems, and it reconciles this evolution with the second law of thermodynamics. Based upon this model, one can formulate and implement *via* the reflective chains the principle of maximum increase of complexity that governs the large-time-scale evolution of living systems.

5.4 Psychological viewpoint

From the viewpoint of psychology, the proposed model represents interactions of the agent with the self-image and the images of other agents *via* the mechanisms of self-awareness. In order to associate these basic concepts of psychology with our mathematical formalism, we have to recall that living systems can be studied in many different spaces such as physical (or geographical) space as well as abstract (or conceptual) spaces. The latter category includes, for instance, social class space, sociometric space, social distance space, and semantic space. Turning to our model, one can identify two spaces: the physical space x, t in which the agent state variables $x_i(t)$ evolve (see equation (39)) and an abstract space $p(X_1 \ldots X_n, t)$ in which the probability density of the agent's state variables evolve (see equation (40)). The connection with these spaces have already been described: if equation (39) is run many times starting with randomly chosen initial conditions, as well as with random values of the Langevin forces $L_i(t)$, one will arrive at an ensemble of different random solutions, while equation (40) shows the probability for each of these solutions to appear. Thus, equation (40) describes the general picture of evolution of the communicating agents that does not depend upon particular initial conditions. Therefore, the solution to this equation can be interpreted as the evolution of the self- and nonself-images of the agents that jointly constitutes the collective mind in the probability space. Based upon that, one can propose the following interpretation of the model of communicating agents: considering the agents as intelligent subjects, one can identify equation (39) as a model simulating their motor dynamics, that is, actual motions in physical space, and equation (40) as the collective mind composed of mental dynamics of the agents. Such an interpretation is evoked by the concept of reflection in psychology [17]. Reflection is traditionally understood as the human ability to take the position of an observer in relation to one's own thoughts. In other words, reflection is self-awareness *via* interaction with the image of the self. Hence, in terms of the proposed phenomenological formalism, a nonliving system may possess the self-image, but it is not equipped with self-awareness, and therefore, this self-image is not in use. On the contrary, in living systems self-awareness is represented by the self-supervising forces that send information from the self-image to the motor dynamics. Due to this property, which is well-pronounced in the proposed model, an intelligent agent can run its mental dynamics ahead of real time (since the mental dynamics are fully deterministic and do not depend explicitly upon motor dynamics) and thereby, it can predict future expected values of its state variables. Then, by interacting with the self-image *via* the supervising forces, it can change the expectations if they are not consistent with the objective. Such a self-supervised dynamic provides a major advantage for the corresponding intelligent agents, especially for biological species, which due to their ability to predict the future are better equipped for dealing with uncertainties, and that improves their survivability.

5.5 Neuroscience viewpoint

From the viewpoint of neuroscience the proposed model represents a special type of neural net. Indeed, reinterpreting an agent's state variable x_i as a neuron's mean soma potential, and assuming that each neuron receives full information from the other neurons, one arrives at a conventional neural net (equation (37)). It should be recalled that in this case the self-supervising forces are not needed and can be ignored. The departure from the conventional case starts with the incompleteness of information, that is, when a neuron does not receive the values of the mean soma potentials from the rest of the neurons. This incompleteness is compensated for by a "general knowledge" stored in the collective mind (equation (40)) and delivered to the neural net *via* the self-supervising forces F_i. As a result of that, the neural net (equation (39)) becomes random, while the evolution of its statistical invariants is described by the collective mind (equation (49)). In order to illuminate the difference between these two cases we will start with a single continuously updated linear neuron with a dissipation feedback

$$\dot{x} = -x. \tag{44}$$

The state variable x eventually approaches an attractor $x = 0$ regardless of initial conditions x_0

$$x = x_0 \exp(-t). \tag{45}$$

In the general case, a multidimensional nonlinear neural net may converge to one of the several attractors that can be static, periodic, or chaotic as well. However, one fundamental property remains the same: as soon as these attractors are approached, the evolution stops.

Let us turn now to a stochastic extension of a neuron (equation (44)). This can be done in several ways. One way is to relax the Lipschitz conditions [6]. Another way is to introduce a special type of equilibrium points which are attractors in one direction and repellers in the others. In both cases the neuron state variable will perform a brownian motion that can be included in equation (44) *via* the Langevin force $L(t)$:

$$\dot{x} = -x + L(t) \tag{46}$$

which has the solution:

$$x = x_0 \exp(-t) + \int_0^t \exp[-(t - t')]L(t')dt'. \tag{47}$$

Equation (47) describes a stochastic process that characterizes the evolution of the neuron state variable. The evolution of the probability density is described by the corresponding Fokker–Planck equation

$$\frac{\partial p}{\partial t} = \frac{\partial}{\partial X}(Xp) + D\frac{\partial^2}{\partial X^2}p. \tag{48}$$

Its solution for the sharp initial value $p(x, 0) = \delta(x \to 0)$ is represented by a normal distribution

$$p = \frac{1}{\sqrt{2\pi D'}} \exp\left[-\frac{X^2}{2D'}\right], \qquad D' = D[1 - \exp(-2t)]. \tag{49}$$

As $t \to \infty$, the distribution tends to the thermodynamical limit with $D' \to D$. Obviously, $D > D'$, and therefore, the entropy $E = \mathrm{Ln} D\sqrt{2\pi}$ approaches its maximum value at $t \to \infty$. This result can be extended to the general case of multidimensional diffusion-based neural nets. This means that the evolution of such neural nets is always regressive, that is, their entropy can only increase.

Let us introduce a control force to reverse the increase of entropy. Within the framework of the newtonian formalism, the most general control force must depend upon the state variables and time, that is, $F = F(x, t)$. We substitute this force into equation (46), and introduce the corresponding changes into equation (48), in which the drift and the diffusion coefficients are functions of X and t. Then, according to the Boltzman H-theorem [12], the entropy of the system will still monotonously increase regardless of the particular form of the control force. However, the situation is changed if the control is represented by a self-supervised force composed of the probability density and its derivatives. For proof of concept, let us choose this force as follows:

$$F = D\frac{\partial}{\partial x} \ln p \tag{50}$$

which is applied after $t > T$. Then equations (46) and (48) are rewritten as

$$\dot{x} = -x + L(t) + D\frac{\partial}{\partial x} \ln p \tag{51}$$

and

$$\frac{\partial p}{\partial t} = \frac{\partial}{\partial X}\left[\left(X - D\frac{\partial}{\partial X} \ln p\right) p\right] + D\frac{\partial^2}{\partial X^2} p, \tag{52}$$

respectively. After trivial transformations, equation (52) is reduced to the form in which the diffusion term is suppressed

$$\frac{\partial p}{\partial t} = \frac{\partial}{\partial X}(Xp). \tag{53}$$

The solution to this equation that starts from $t > T$ is

$$p = \sqrt{\frac{\beta}{2\pi Y}} \exp\left[\beta t - \frac{\beta X^2 \exp(2\beta t)}{2DY}\right], \qquad Y = 1 - \exp(-2\beta t). \tag{54}$$

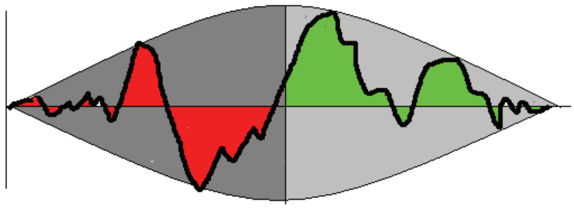
As follows from this solution, the self-supervised force reverses the evolution of the probability density towards decreasing the entropy, and that makes a self-supervised neuron a "messenger of life."

Thus, the fundamental property of the self-supervised neuron is its ability to create the self-image (equation (52)) and interact with this image (equations (53) and (54)). This property can be associated with a recently discovered new type of neuron in the monkey—a mirror neuron—which fires both when performing an action and when the monkey is observing the same action performed by another subject. Indeed, the way in which the self-supervised neuron works is the following: It is assumed that all the communicating agents belong to the same class in a sense that they share the same general properties and habits. This means that although each agent may not know the exact positions of the other agents, it does, nevertheless, know at least such characteristics as their initial positions (to accuracy of initial joint probability density, or, at least, initial expected positions and initial variances). This preliminary experience allows an agent to reconstruct the evolution of expected positions of the other agents using the collective mind as a knowledge base. Hence, a self-supervised neuron representing an agent A can be activated by an expected action of an agent B which may not be in direct contact with agent A at all, and that expresses the mirror properties of the self-supervised neuron, Figure 5.

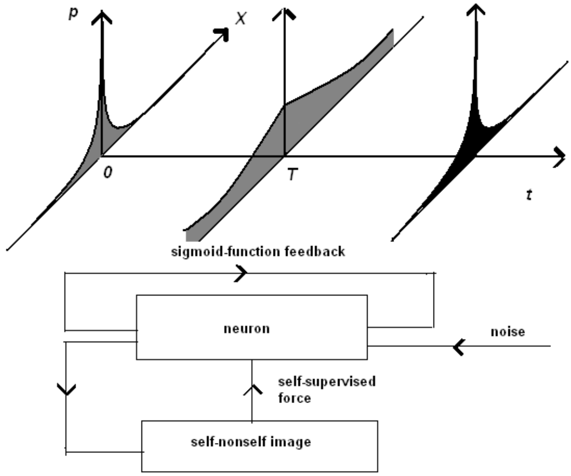
The collective properties of self-supervised neurons; that is, self-supervised neural nets, have a significant advantage over regular neural nets: they possess a fundamentally new type of attractor—the stochastic attractor—which is a very powerful generalization tool. Indeed, it includes a much broader class of motions than static or periodic attractors. In other words, it provides the highest level of abstraction. In addition to that, a stochastic attractor represents the most complex patterns of behavior if the self-supervised net describes a set of interacting agents. Indeed, consider a swarm of insects approaching some attracting pattern. If this pattern is represented by a static or periodic attractor, the motion of the swarm is locked up in a rigid pattern of behavior that may decrease its survivability. On the contrary, if that pattern is represented by a stochastic attractor, the swarm still has a lot of freedom, and only the statistic of the swarm motion is locked up in a certain pattern of behavior. For example, an information-based neural net [15] can approach a stochastic attractor that preserves a prescribed amount of information expressed *via* the entropy E, Figure 6.

5.6 Social and economic viewpoint

One of the basic problems of social theory is to understand "how, with the richness of language and the diversity of artifacts, people can create a dazzlingly rich variety of new yet relatively stable social structures," [18]. Within the framework of the dynamical formalism, the proposed model provides some explanations to this puzzle. Indeed, social events are driven by two factors: individual objectives and social constraints.



(*a*) Motion with diffusion of self-supervised neuron.

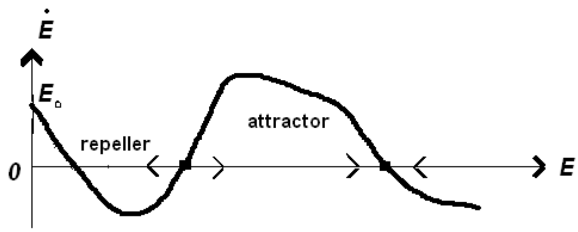


(*b*) Probability density evolution of self-supervised neuron.

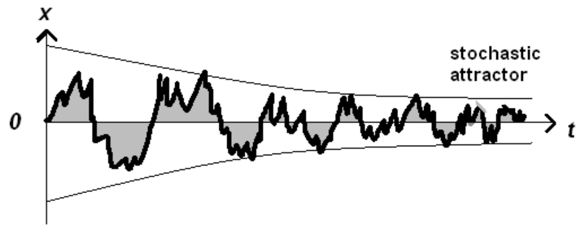
Figure 5. Self-supervised (mirror) neuron.

The first factor is captured by the motor dynamics (equation (39)), while the social constraint is created by the collective mind (equation (40)). A balance between these factors (expressed by stochastic attractors) leads to stable social structures, while an imbalance (expressed by stochastic repellers) causes sharp transitions from one social structure to another (revolutions) or to wandering between different repellers (chaos, anarchy). For an artificial "society" of communicating agents, one can assign individual objectives for each agent as well as the collective constraints imposed upon them and study the corresponding social events by analyzing the governing equations. However, the same strategy is too naïve to be applied to a human society. Indeed, most human members of a society do not have rational objectives: they are driven by emotions, inflated ambitions, envy, distorted self- and nonself-images, and such. At least some of these concepts can be formalized and incorporated into the model. For instance, one can consider emotions to be proportional to the differences between the state variables and their expectations

$$E_m = c(\chi - x). \tag{55}$$



(*a*) Phase space.



(*b*) Physical space.

Figure 6. Self-supervised neural nets.

This equation easily discriminates positive and negative emotions. Many associated concepts (anger, depression, happiness, indifference, aggressiveness, and ambitions) can be derived from this definition (possibly, in combination with distorted self- and nonself-images). But the most accurate characteristic of human nature was captured by cellular automata where each agent copies the behaviors of its closest neighbors (which in turn, copy their neighbors, etc.). As a result, the whole "society" spontaneously moves toward an unknown emerging "objective." Although this global objective is uniquely defined by a local operator that determines how an agent processes the data coming from neighbors, there is no known explicit connection between this local operator and the corresponding global objective: only actual numerical runs can detect such a connection. The creator of this model, S. Wolfram, claims that he discovered the "theory of everything." Notwithstanding the ingenuity of his model, one can see its major limitation: the model is not equipped with a collective mind (or by any other type of a knowledge base), and therefore, its usefulness is significantly diminished in the case of incomplete information. At the same time, our model can be easily transformed into a cellular automata with a collective mind. In order to do that one has to turn to equation (37), replace the sigmoid function by a local operator and the time derivative by the time difference. Then the corresponding Fokker–Planck equation (40) reduces to its discrete version, that is Markov chains [10]. On the conceptual level, the model remains the same as discussed in the previous sections.

From the viewpoint of economics, the proposed model can represent games with incomplete information. Probably the best illustration of that is the so-called minority game, which is a simplified model of conflicting situations observed in financial markets [19]. It describes a system in which an odd number N of agents is allowed to make two possible choices: 1 or 0, and that divides the agents into two groups while the smaller group wins. Clearly, when agents know nothing about the possible strategies of their adversaries, the outcome is totally random. However, if the agents share some global information about each other (such as the history of the game in the form of the sequence of the last winning choices) the dynamics of the game become extremely complex (e.g., it includes such phenomena as phase transitions). Within the framework of our model, the shared information can be stored in the collective mind, and this will provide the agents with the dynamics of interaction between the shared knowledge and the individual strategies.

5.7 Language communications viewpoint

Language represents the best example of a communication tool with incomplete information since any message, in general, can be interpreted in many different ways depending upon the context, that is, upon the global information shared by the sender and the receiver. Therefore, the proposed model is supposed to be relevant for some language-oriented interpretations. Indeed, turning to equation (39), one can associate the weighted sum of the state variables with the individual interpretations of the collective message made by the agents. The sigmoid functions of these sums form the individual responses of the agents to this message. These responses are completed by the self-supervising forces that compensate for the lack of information in the message by exploiting the global shared information stored in the collective mind (see equation (40)). The agent's responses converted into the new values of their state variables are transformed into the next message using the same rules, and so on. These rules determined by the structure of equations (39) and (40) can be associated with the grammar of the underlying language. In particular, they are responsible for the convergence to—or the divergence from—the expected objective. It should be noted that the language structure of the proposed model is invariant with respect to semantics. Hence, in terms of the linguistics terminology that considers three universal structural levels: sound, meaning, and grammatical arrangement [20], we are dealing here with the last one. In our opinion, the independence of the proposed model upon the semantics is an advantage rather than a limitation: it allows one to study invariant properties of the language evolution in the same way in which the Shannon information (that represents rather an information capacity) allows one to study the evolution of information regardless of a particular meaning of the transmitted messages.

Let us now try to predict the evolution of language communication based upon the proposed model. As mentioned earlier, the evolution of living systems is always directed toward the increase of their complexity. In a human society, such a progressive evolution is effectively implemented by increasing the number of reflections in a chain "What do you think I think you think, ..." The society may be stratified into several levels or "clubs" so that inside each club the people will share more and more global information. This means that language communications between members of the same club will be characterized by an increased capacity of the collective mind (see equation (40)), and decreased information transmitted by the messages (see equation (39)). In the theoretical limit, these messages will degenerate into a string of symbols, which can be easily decoded by the enormously large collective mind (a poker club). Language communications across stratified levels will evolve in a different way: as long as the different clubs are drifting apart, the collective mind capacity will decrease while the messages will become longer and longer (legal language). However, the process of diffusion between these two streams (not included in our model) is very likely. Indeed, so far we were dealing only with the "rational" aspects of language ignoring its "artistic" aspects such as poetry. But this is exactly what unites all people *via* the universal language of art, and also what will create the diffusion.

5.8 Control theory viewpoint

The proposed model can be considered as a closed-loop controlled dynamical system as known in engineering control. The only difference is that, unlike engineering control, where the control forces are triggered by the values of the state variables and their time-derivatives, here the control forces are determined by the parameters of the collective mind that implicitly represent the state variables of the underlying system. This type of control can be linked to a so-called "reflective control" introduced in mathematical psychology by Lefebvre in [21] since the system is governed by the reflections, that is, by the parameters characterizing the images rather than real objects. The mathematical consequences of this property have been discussed in section 5.1.

5.9 Applications and implementations

The proposed model has two types of applications that can be associated with science and technology. The first type includes theoretical studies of the behavior of living systems, which can be performed by direct computer simulations of equations (39) and (40). The second type includes the development of artificial living systems that are supposed to simulate and replace some functions of a human (e.g., robots, unmanned spacecrafts, etc.). The most effective way of implementing these systems is by means of analog devices such as VLSI chips used for simulating analog neural nets. As discussed in section 5.5, equation (39) can be treated as regular continuously updated neural nets with additional random forces. Implementation of these forces has been proposed in [15], based upon nonLipschitz dynamics. Equation (40), after approximation of space-derivatives by finite differences, can be easily transformed to a neural-net-like dynamical system that can be implemented by VLSI chips.

6. Conclusion

In summary, we have introduced a new mathematical formalism that offers a rich framework for developing models capturing nonnewtonian properties of living systems. The proposed general approach has been focused on the behavior of communicating agents who compensate an incompleteness of exchanged information by means of the collective mind as a context-type of the global shared knowledge base. Detailed analyses of the proposed formalism, as well as discussion of its scientific and technological applications, have been performed.

Acknowledgment

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References

[1] A. Mikhailov, *Foundations of Synergetics* (Springer, New York, 1990).

[2] I. Prigogine, *From Being to Becoming* (Freeman and Co., San Francisco, 1980).

[3] H. Haken, *Information and Self-organization* (Springer, New York, 1988).

[4] I. Prigogine, *Thermodynamics of Irreversible Processes* (John Wiley, New York, 1961).

[5] R. Gordon, *The Hierarchical Genome and Differential Waves* (World Scientific, 1999).

[6] M. Zak, "Terminal Model of Newtonian Dynamics," *International Journal of Theoretical Physics*, **32** (1992).

[7] B. Huberman, *The Ecology of Computation* (North-Holland, Amsterdam, 1988).

[8] J. Vidal and E. Durfee, "Learning Nested Models in an Information Economy," *Journal of Experimental and Theoretical Artificial Intelligence*, **10**(3) 1998.

[9] M. Zak, "Physical Invariants of Biosignature," *Physics Letters A*, **255**, 1999.

[10] M. Zak, "Dynamics of Intelligent Systems," *International Journal of Theoretical Physics*, **39**(8) 2000.

[11] M. Zak, "Quantum Evolution as a Nonlinear Markov Process," *Foundations of Physics L*, **15** (3) 2002.

[12] H. Risken, *The Fokker–Planck Equation* (Springer, New York, 1989).

[13] S. P. Banks, *Control Systems Engineering* (Prentice-Hall, Englewood Cliffs, NJ, 1986).

[14] V. Lefebvre, "The Law of Self-reflection, Reflexive Processes, and Control," 2002.

[15] M. Zak, "Self-supervised Dynamical Systems," *Chaos, Solitons, and Fractals*, **19** (2004) 645–666.

[16] H. Berg, *Random Walks in Biology* (Princeton Univercity Press, New Jersey, 1983).

[17] M. Arbib and M. Hesse, *The Construction of Reality* (Cambridge University Press, 1986).

[18] D. Challet and Y. Zhang, "On the Minority Game: Analytical and Numerical Studies," *Physica A*, **256** (1998) 514–532.

[19] M. Yaguello, *Language through the Looking Glass* (Oxford University Press, 1998).

[20] V. Lefebvre, *Algebra of Conscience* (Kluwer Academic Publishers, 2001).