El Botellón: Modeling the Movement of Crowds in a City

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A simulation of crowd movement in a city is studied under various assumptions about interactions between people. We find, in general, that there are two modes of steady-state behavior. The crowd may be distributed across the city, or it may end up gathered in one place. A mathematical model describes the long-term behavior and shows that this change in behavior is sensitive to a critical parameter setting in our model. Some alternative interpretations of the results are formulated.

1. Introduction

In Spanish cities, on summer nights, crowds of young people wander the streets in search of a party. Often accompanied by "binge" drinking, this phenomenon, known as *El Botellón* (the large bottle) is perceived by local governments as a significant social problem [1]. One of the key problems is that it is impossible to predict where a party will take place. The people walk from square to square, meeting their friends and stopping to drink. When a critical mass of people happen to arrive in the same square at the same time, a party spontaneously erupts.

In an attempt to understand the dynamics of the crowds of people as they move through a city, we have developed a simulation tool [2] for running experiments. This has led to a mathematical model of the social system that helps us to understand the conditions under which parties self-organize. It turns out there is a critical threshold in the number of people that can be accomodated by any area of a city. Below this threshold, people can move freely about and are distributed around the different squares. Above the threshold, large crowds start gathering in a few places, which then act as bottle-necks. Our model helps predict the conditions under which this critical phenomenon occurs.

The use of agent-based systems has been increasing within the social sciences in recent years [3]. In some systems, agents are complex pieces of code with sophisticated rules of behavior and interaction [4, 5]. However, we take the view that one should try to come up with the simplest

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possible set of rules that generate the desired behavior (at least qualitatively). Examples of similar work on the movement of people (though more fine-grained than ours) can be found in [6, 7].

Notation

Throughout this paper, vectors will be in **bold type**. The notation [*expr*] evaluates to one if the enclosed expression *expr* is a true statement, otherwise it evaluates to zero.

2. Simulating crowd movement

We model the city as an undirected graph. The vertices are the squares where people gather. Vertices are connected by edges if the corresponding squares are directly connected by a street. People are simulated by "agents" using the following simple set of rules.

1. Agents are located in the squares.
2. The probability that an agent remains in a square depends on how many other agents are in the same square.
3. Some squares have bars. This increases the probability that an agent remains.
4. If an agent decides to move, it moves to a neighboring square.
5. An agent is more likely to move to a square with a bar, than one without a bar.

Let us first consider a simple model in which there are no bars. To capture an agent's behavior, we need to specify the probability that it will remain in a square. We suppose there is a parameter c (called the *chat probability*) which represents the probability that two agents will talk to each other. If an agent finds no one to talk to in a square, it leaves and moves to a neighboring square. If a square contains $x > 0$ agents the probability that an agent leaves is given by $(1 - c)^{x-1}$. In the case that an agent leaves, one of the neighboring squares is chosen uniformly at random.

Now consider the effect of adding bars to the model. These alter things in two ways: (1) They make a square more attractive to remain in. (2) They make a square more attractive to move to. We measure the attractiveness of bars by a parameter α. The chat probability for an agent in a square with a bar is αc. The probability that a moving agent goes to a square with a bar is α times that of a square without a bar.

A computer simulation of this system has been implemented. Some experimental results were presented in [2]. These results are analyzed in more detail below, using a mathematical model.

3. Predicting steady-state behavior

We number the vertices of the graph $\{1, 2, \ldots, n\}$. Let p_i be the number of agents in square i. Then the state of the system at any discrete time step is given by the vector $\mathbf{p} = (p_1, p_2, \ldots, p_n)$. The total number of people in the city is $N = \sum_{i=1}^{n} p_i$. We let the attractiveness of square i be denoted by α_i.

Assume, to begin with, that there are no bars, that is, $\alpha_i = 1$ for all i. Then the expected number of agents that will leave a square at a given time step t is given by the function

$$f_i(t) = p_i(t)(1 - c)^{p_i(t)-1} \tag{1}$$

and we write $\mathbf{f}(t) = (f_1(t), f_2(t), \ldots, f_n(t))$.

Let A be the matrix with i, j entries

$$A_{i,j} = [i \text{ is connected to } j]/d_j \tag{2}$$

where d_j is the degree of vertex j. $A_{i,j}$ thus gives the probability that an agent will move from square j to square i. We can then write the expected distribution of agents at a given time step as:

$$\mathbf{p}(t + 1) = \mathbf{p}(t) - \mathbf{f}(t) + A\mathbf{f}(t). \tag{3}$$

It is clear from equation (1) that if the number of agents p_i in a square is large, then the number that are expected to leave is very small ($f_i \to 0$ as $p_i \to \infty$). This means that once a sufficient number of people have gathered in a square, it will take a long time for the crowd to disperse. However, there is also the possibility of a different kind of steady-state behavior. Equation (3) tells us that we will have a steady state when $A\mathbf{f} = \mathbf{f}$. That is, when $\mathbf{f}$ is an eigenvector of the matrix A corresponding to eigenvalue one. We show that $\mathbf{d} = (d_1, d_2, \ldots, d_n)$ is such an eigenvector:

$$(A\mathbf{d})_i = \sum_j A_{i,j} d_j = \sum_j [i \text{ is connected to } j] = d_i. \tag{4}$$

So a steady-state behavior of the system arises when the number of people leaving a square (which must equal the number of people arriving) is proportional to the number of streets connected to that square. It is interesting to observe that this steady-state distribution does not depend on the detailed topology of the graph: it only depends on the degrees of the vertices, irrespective of how they are connected.

We now have two possible long-term behaviors for the system: (1) All the agents can gather into large clumps in a few squares (corresponding to the spontaneous emergence of a party). (2) They can flow freely around the city, in a way described by the eigenvector just calculated. It is of great importance for the social problem of *El Botellón* to understand the conditions under which each of these states arises. We

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can answer this question in part by considering the *stability* of the fixed point $\mathbf{p}$, which is the solution to the steady-state equations

$$p_i(1-c)^{p_i-1} = d_i. \tag{5}$$

Formally, we define an operator $G : \mathbb{R}^n \to \mathbb{R}^n$ by

$$G(\mathbf{p}) = \mathbf{p} - \mathbf{f}(\mathbf{p}) + A\mathbf{f}(\mathbf{p}) \tag{6}$$

so that the expected distribution at time $t+1$ is $\mathbf{p}(t+1) = G(\mathbf{p}(t))$. The steady-state distribution given by equation (5) is then a fixed point of G, that is, it satisfies $G(\mathbf{p}) = \mathbf{p}$. We now give a condition under which this fixed point becomes *unstable*, under the assumption that the graph is regular (i.e., all vertices have the same degree).

Theorem 1. Suppose the graph of the city is regular, and that there are no bars. Then the fixed-point distribution given by equation (5) is unstable if $c > n/N$.

Proof. Suppose each vertex has degree d. Calculating the entries of the Jacobian of G gives us:

$$(\partial G)_{i,j} = \frac{\partial G_i}{\partial p_j} = \begin{cases} 1 - f_i' & \text{if } i = j \\ [i \text{ is connected to } j]\frac{f_i'}{d} & \text{if } i \neq j \end{cases} \tag{7}$$

where $f_i' = f_i(\log(1-c) + 1/p_i)$. Now, at the fixed-point distribution, the number of agents in a square depends only on the degree of that square. Since these are equal for all squares, the population must be distributed evenly throughout the city, that is, at the fixed point $p_i = N/n$ for all i. Moreover, we know that $f_i(\mathbf{p}) = d$ at the fixed point for all i. So at the fixed point, the entries of the Jacobian are:

$$(\partial G)_{i,j} = \begin{cases} 1 - \beta & \text{if } i = j \\ [i \text{ is connected to } j]\beta/d & \text{if } i \neq j \end{cases} \tag{8}$$

where $\beta = d(\log(1-c) + n/N)$.

Let λ be any eigenvalue of the Jacobian matrix ∂G evaluated at the fixed point. By the Gershgorin Circle Theorem (see page 685 of [8]), there exists a k such that

$$\left|(\partial G)_{k,k} - \lambda\right| \leq \sum_{j \neq k} \left|(\partial G)_{k,j}\right|. \tag{9}$$

Therefore

$$|1 - \beta - \lambda| \leq \sum_{j \neq k} \left|[k \text{ is connected to } j]\frac{\beta}{d}\right| = |\beta|. \tag{10}$$

Therefore, if we can show that $\beta < 0$, then we have that all the eigenvalues are greater than or equal to one, which makes the fixed point unstable. Now

$$\begin{aligned} c > n/N &\Longrightarrow c > 1 - e^{-n/N} \\ &\Longrightarrow \log(1-c) + n/N < 0 \\ &\Longrightarrow \beta < 0. \blacksquare \end{aligned} \tag{11}$$

We consider how accurate this bound is experimentally in section 4.

To adapt this model to include bars, we have to update the formula for the number of people expected to leave a square (equation (1)):

$$f_i(t) = p_i(t)(1 - \alpha_i c)^{p_i(t)-1} \tag{12}$$

and the probabilities of moving from one square to another (equation (2)):

$$A_{i,j} = \frac{[i \text{ is connected to } j]\alpha_j}{v_j} \tag{13}$$

where $v_j = \sum_k [k \text{ is connected to } j]\alpha_k$ is a normalizing factor. With these changes, the equation of dynamics (equation (3)) still applies, and there is still a fixed point satisfying $A\mathbf{f} = \mathbf{f}$. We now show that the vector $\mathbf{v} = (v_1, v_2, \ldots, v_n)$ is such an eigenvector of A (with bars) with eigenvalue one:

$$(A\mathbf{v})_i = \sum_j A_{i,j} v_j = \sum_j [i \text{ is connected to } j]\alpha_j = v_i. \tag{14}$$

This fixed point tells us that, in the steady state, there will be more traffic through squares connecting directly with squares of high attractiveness (i.e., with bars), and that more people will be found in squares of high attractiveness. The stability of this fixed point is more difficult to analyze, but there again seems to be a critical transition value for c.

4. Empirical observations

We have conducted a number of experiments with different city topologies, varying the number of agents, and their probability of chatting. Our simulation is written in Java, using the RePast libraries.[1] We have collected data from many different runs under different parameter settings, in an attempt to understand the characteristics of such systems.

As predicted, we observe two distinct modes of long-term behavior. If the chat probability c is sufficiently small, then the agents will tend to diffuse around the city. The number of agents that, on average, are in a square depends on the number of streets leading to that square, and whether or not it contains a bar. A square will have a higher long-term

[1] http://repast.sourceforge.net/

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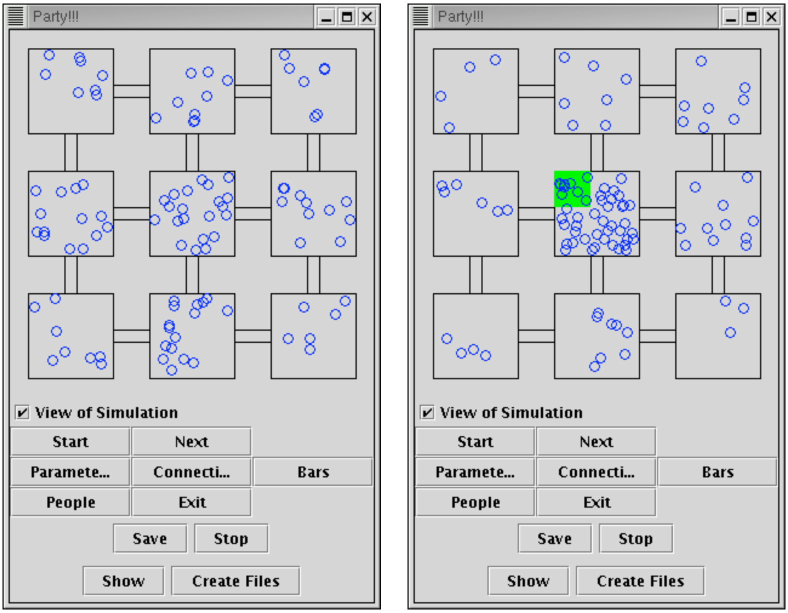

Figure 1. Screenshots of simulator showing long-term behavior with a low chat probability. Left: with no bars. Right: with one bar.

population if it is highly connected to other squares, and if it contains a bar. This situation corresponds to a quiet night in the city, where people wander freely. This is illustrated by the results in Figure 1. In this example, there are nine squares in a three-by-three grid, with no bars. The population size is 100 and $c = 0.005$. The population of each square is shown for a typical run. The effect of adding a bar to the central square is also shown. The population is still distributed about the city but with more people gathered in the square with the bar.

However, if the chat probability c is above a critical value, then all the agents gather very quickly together into a single square. This corresponds to the spontaneous self-organization of a party. This phenomenon occurs even if there are no bars around which such clusters might be expected to form.

To test our critical bound $c > n/N$ for the emergence of such clusters in regular graphs we ran experiments on a city comprising only four squares connected in a ring. We varied the population size and the chat probability. For each run, we began with the entire population in a specified square. Throughout the run (which took 100,000 ticks, each tick corresponding to one action by a single agent), we observed the proportion of the population that remained in that square. This number was averaged over the entire run. The results are shown in Figure 2. A clear phase transition is demonstrated. The bound $c = n/N$

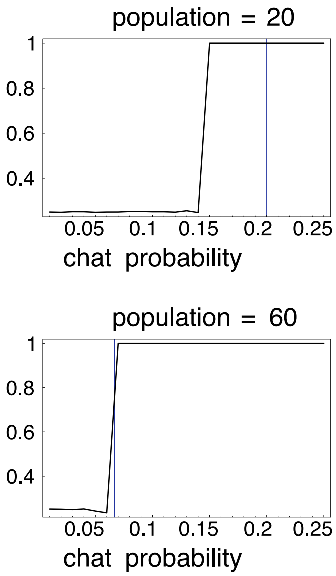

Figure 2. Proportion of population remaining in their initial square (averaged over entire run). The vertical line indicates the estimate of the critical chat probability in each case.

gives a good estimate for the critical value of the chat probability, except when the population is small, in which case it overestimates it.

5. Conclusions

We have presented a multi-agent model of crowd movement through a city, in which the behaviors of the agents depend on their mutual interactions (determined by the probability c which measures their willingness to interact). It is found that this parameter c has a critical threshold which separates two distinct modes of behavior. Below the threshold, the agents move freely around the city. Above the threshold, they clump together in a few isolated squares. This emergent behavior captures (qualitatively, at least) the spontaneous self-organization of parties, known as *El Botellón*.

There are, however, alternative interpretations. One can view the vertices of the graph as representing different possible states in which the agents may find themselves. Edges between graphs represent possible changes of state. The parameter c then represents the *conservatism* of an agent, that is, the tendency of an agent to remain in the same state as others. Under this view, the system may be seen as a model of consumer decisions between competing products, the vertices representing product choices. An agent moves from one vertex to another if it changes its

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choice of product. The ability of this system to collapse into a single product (rather than support a diversity of competitors) is similar to Arthur's economic model of "increasing returns" [9].

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