Exponential Period of Neuronal Recurrence Automata with Excitatory Memory

René Ndoundam[*]
Maurice Tchuente[†]
Department of Computer Science,
Faculty of Science, University of Yaounde I,
P.O. Box 812 Yaounde, Cameroon

The sequences generated by a neuronal recurrence equation with memory of the form $x(n) = \mathbf{1}[\sum_{i=1}^{h} a_i x(n-i) - \theta]$, where h is the size of the memory, are studied. It is shown that in the case where all the parameters $(a_i)_{1 \le i \le h}$ are positive reals, there exists a neuronal recurrence equation of memory length h that generates a sequence of period $\Omega(e^{\sqrt[3]{h(\ln(h))^2}})$. This result shows that in the case where all the weighting coefficients are positive reals, the neuronal recurrence equation exhibits a complex behavior.

1. Introduction

In [1] it is suggested that the dynamic behavior of a single neuron with memory length k that does not interact with other neurons can be modeled by the neuronal recurrence equation:

$$x(n) = \mathbf{1}\left(\sum_{i=1}^{k} a_i x(n-i) - \theta\right) \tag{1}$$

where we have the following.

- $x(n)$ is a variable representing the state of the neuron at time $t = n$, $x(n) \in \{0, 1\}$.
- k is the memory length, that is to say, the state of the neuron at time $t = n$ depends on the state assumed by the neuron at the k previous steps $t = n-1, \ldots, n-k$.
- The values a_i $(i = 1, \ldots, k)$ are real numbers called the *weighting coefficients*; a_i represents the influence of the state of the neuron at time $n-i$ on the state of the neuron at time n. That influence is said to be *excitatory* if $a_i > 0$, *inhibitory* if $a_i < 0$, and *null* if a_i is equal to zero.

[*]Electronic mail address: ndoundam@uycdc.uninet.cm. Also affiliated with LRI, Université Paris-Sud XI, Bât 490, 91405 Orsay Cedex, Paris, France and The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy.
[†]Electronic mail address: tchuente@camnet.cm.

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- θ is a real number called the *threshold*.
- $\mathbf{1}[u] = 0$ if $u < 0$, and $\mathbf{1}[u] = 1$ if $u \geq 0$.

The system obtained by interconnecting several neurons is called a neural network (NN). Such networks were introduced in [2], and are quite powerful. Indeed, it can be shown that they can be used to simulate any Turing machine. More recently, NNs have been studied extensively as tools for solving various problems such as classification, speech recognition, and image processing. The application field of the threshold functions is large. The spin moment of the spin glass is one example from solid state physics that has been widely simulated by NNs. In electronics, for instance, a threshold function represents a transistor; in social science a threshold function is often used to represent vote laws.

Let p and T be two positive integers such that $p > 0$ and $T \geq 0$. Equation (1) is said to be of period p and transient T if and only if:

- $Y(p + T) = Y(T)$
- $\nexists\, T'$ and $p'(T', p') \neq (T, p)$ $T \geq T'$ and $p \geq p'$ such that $Y(p' + T') = Y(T')$

where $Y(t) = (x(t), x(t-1), \ldots, x(t-k+2), x(t-k+1))$. The period and transient of sequences generated by a neuron are good measures of the complexity of the behavior of the neuron.

We are interested in the longest period $LP(k)$ that can be generated by a neuronal recurrence equation with memory k. In [3], it was conjectured that if $(a_i)_{1\leq i\leq k} \in \mathbb{R}$, then $LP(k) \leq 2k$. This conjecture has been disproved. The best known lower bound in $LP(k)$ is $O(e^{\sqrt{k\ln(k)}})$, which was proved in [4].

When all the weighting coefficients are positive, the influence of the previous states of a neuron (at time $n-k, n-k+1, \ldots, n-2, n-1$) on its state at time n is excitatory, and from a physiological point of view, it is important to know the behavior of that class of neuron. In [3], it was also conjectured that if $\forall\, i, i = 1, \ldots, k,\ a_i \in \mathbb{R}^+$ (i.e., $a_i \geq 0$), then $LP(k) \leq k$. This conjecture has been disproved in [5] where a neuronal recurrence equation of memory length k and of period $O(k^3)$ has been exhibited.

In this paper, we exhibit a neuronal recurrence equation of memory length h where all the weighting coefficients are strictly positive that generates a sequence of period $\Omega(e^{\sqrt[3]{h(\ln(h))^2}})$. This result more strongly contradicts the conjecture than the aforementioned counter example [5].

2. Neuronal recurrence equation with positive weighting coefficients

Let k be a positive integer. For a vector $a \in \mathbb{R}^k$, a real number $\theta \in \mathbb{R}$, and a vector $z \in \{0, 1\}^k$ we define the sequence $\{x(n) : n \in \mathbb{N}\}$ by the following recurrence

$$x(t) = \begin{cases} z(t) & t \in \{0, \ldots, k-1\} \\ \mathbf{1}\left(\sum_{i=1}^{k} a_i x(t-i) - \theta\right) & t \geq k. \end{cases} \quad (2)$$

We denote by $S(a, \theta, z)$ the sequence generated by equation (2) and $T(a, \theta, z)$ its period.

Let m be a positive integer greater than one, we denote the cardinality of the set $\mathcal{P} = \{p : p$ prime and $2m < p < 3m\}$ by $\rho(m)$. Let us denote with $p_1, \ldots, p_{\rho(m)}$ the prime numbers lying in $\{2m+1,\ 2m+2, \ldots, 3m-2, 3m-1\}$ and the sequence $\{\alpha_i : 1 \leq i \leq \rho(m)\}$ is defined as $\alpha_i = 3m - p_i$, $1 \leq i \leq \rho(m)$.

It is easy to check that $\{2m+1, 2m+2, \ldots, 3m-2, 3m-1\}$ contains at most $\lceil (m-1)/2 \rceil$ odd integers. It follows that

$$\rho(m) \leq \left\lceil \frac{m-1}{2} \right\rceil. \quad (3)$$

We set $k = (6m-1)\rho(m)$ and $\forall\, i \in \mathbb{N}$, $1 \leq i \leq \rho(m)$, we define:

$$\mu(m, \alpha_i) = \left\lfloor \frac{k}{3m - \alpha_i} \right\rfloor$$
$$\beta(m, \alpha_i) = k - ((3m - \alpha_i)\mu(m, \alpha_i)).$$

From the previous definitions, we have $k = ((3m - \alpha_i)\mu(m, \alpha_i)) + \beta(m, \alpha_i)$.

It is clear that $\forall\, i \in \mathbb{N}$, $1 \leq i \leq \rho(m)$:

$$2m + 1 \leq 3m - \alpha_i \leq 3m - 1.$$

This implies

$$\frac{(6m-1)\rho(m)}{3m-1} \leq \frac{k}{3m - \alpha_i} \leq \frac{(6m-1)\rho(m)}{2m+1}.$$

Therefore

$$2\rho(m) < \mu(m, \alpha_i) < 3\rho(m). \quad (4)$$

$\forall\, i \in \mathbb{N}$, $1 \leq i \leq \rho(m)$, we want to construct a neuronal recurrence equation $\{x^{\alpha_i}(n) : n \geq 0\}$ with excitatory memory of length k that evolves as follows:

$$\underbrace{00\ldots0}_{\beta(m,\alpha_i)}\underbrace{100\ldots0}_{3m-\alpha_i}\underbrace{100\ldots0}_{3m-\alpha_i}\cdots\underbrace{100\ldots0}_{3m-\alpha_i}\cdots\underbrace{100\ldots0}_{3m-\alpha_i}\cdots \quad (5)$$

and which describes a cycle of length $3m - \alpha_i = p_i$.

$\forall\, i \in \mathbb{N}$, $1 \leq i \leq \rho(m)$, let $w^{\alpha_i} \in \{0,1\}^k$ be the vector defined by

$$w^{\alpha_i}(0)\ldots w^{\alpha_i}(k-1) = \underbrace{0\ldots0}_{\beta(m,\alpha_i)}\underbrace{\underbrace{10\ldots0}_{p_i}\cdots\underbrace{10\ldots0}_{p_i}}_{\mu(m,\alpha_i)p_i}. \quad (6)$$

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In other words, w^{α_i} is defined by:

$$w^{\alpha_i}(j) = \begin{cases} 1 & \text{if } \exists\, l,\ 0 \le l \le \mu(m, \alpha_i) - 1 \text{ such that } j = \beta(m, \alpha_i) + l p_i \\ 0 & \text{otherwise.} \end{cases}$$

Let γ be a real number satisfying $\gamma > 0$. We define the neuronal recurrence equation $\{x^{\alpha_i}(n) : n \ge 0\}$ by the following recurrence:

$$x^{\alpha_i}(t) = \begin{cases} w^{\alpha_i}(t) & t \in \{0, \ldots, k-1\} \\ \mathbf{1}\left(\sum_{j=1}^{k} \bar{a}_j x^{\alpha_i}(t-j) - \bar{\theta}\right) & t \ge k \end{cases} \tag{7}$$

where

$$\bar{a}_j = \begin{cases} \gamma & \text{if } j \in F \\ 0 & \text{if } j \in G \end{cases}$$

$$\begin{aligned} \operatorname{Pos}(\alpha_i) &= \{j p_i : j = 1, \ldots, 2\rho(m)\} \\ &= \{p_i, 2p_i, \ldots, (-1 + 2\rho(m))p_i, 2\rho(m)p_i\}, \quad 1 \le i \le \rho(m) \\ D &= \{i : i = 1, \ldots, k\} = \{1, 2, \ldots, k-1, k\} \\ F &= \bigcup_{i=1}^{\rho(m)} \operatorname{Pos}(\alpha_i) \\ G &= D \setminus F \\ \bar{\theta} &= \rho(m)\gamma. \end{aligned}$$

By definition $\operatorname{Pos}(\alpha_i)$ represents the set of indices j, $1 \le j \le k$ such that $x^{\alpha_i}(k-j) = 1$.

From the definition of $\operatorname{Pos}(\alpha_i)$ and from equation (6), one can easily verify that

$$j \in \operatorname{Pos}(\alpha_i) \Longrightarrow x^{\alpha_i}(k-j) = 1 \tag{8}$$

$$j \in D \setminus \operatorname{Pos}(\alpha_i) \Longrightarrow x^{\alpha_i}(k-j) = 0. \tag{9}$$

$\forall\, d \in \mathbb{N}$, $0 < d < p_i$, we also denote $\operatorname{PPos}(\alpha_i, d)$ as the set of indices j such that $x^{\alpha_i}(k + d - j) = 1$, in other words:

$$\operatorname{PPos}(\alpha_i, d) = \{j : x^{\alpha_i}(k + d - j) = 1 \text{ and } 1 \le j \le k\}.$$

$\forall\, i, d \in \mathbb{N}$, $1 \le i \le \rho(m)$, and $0 < d < p_i$, we denote:

$$\begin{aligned} Q(\alpha_i, d) &= \{d + j p_i : j = 0, 1, \ldots, \mu(m, \alpha_i)\}, \quad 0 < d \le \beta(m, \alpha_i) \\ Q(\alpha_i, d) &= \{d + j p_i : j = 0, 1, \ldots, -1 + \mu(m, \alpha_i)\}, \quad \beta(m, \alpha_i) < d < p_i \\ E(\alpha_i, d) &= Q(\alpha_i, d) \cap F \\ z(\alpha_i, d) &= \operatorname{card} E(\alpha_i, d). \end{aligned}$$

The neuronal recurrence equation $\{x^{\alpha_i}(n) : n \geq 0\}$ with excitatory memory of length k is defined by equations (6) and (7).

We will show that the neuronal recurrence equation $\{x^{\alpha_i}(n) : n \geq 0\}$ evolves as specified in equation (5). An important property is presented in the following proposition.

Proposition 1. $\forall\, i \in \mathbb{N}$, $1 \leq i \leq \rho(m)$ and $\forall\, d \in \mathbb{N}$, $1 \leq d < p_i$:

$$z(\alpha_i, d) \leq \rho(m) - 1.$$

Proof. The proof will be done by contradiction. Let us suppose that:

$$z(\alpha_i, d) \geq \rho(m). \tag{10}$$

By considering the fact that $0 < d < p_i$, we deduce that:

$$Q(\alpha_i, d) \cap \mathrm{Pos}(\alpha_i) = \varnothing. \tag{11}$$

Equations (10) and (11) imply that $\exists\, l \in \mathbb{N}$, $1 \leq l \leq \rho(m)$ and $l \neq i$ such that $\mathrm{card}(Q(\alpha_i, d) \cap \mathrm{Pos}(\alpha_l)) \geq 2$.

Consequently, there exists $j1, j2 \in \{0, 1, \ldots, \mu(m, \alpha_i)\}$, $j1 \neq j2$ such that $d + (j1 \times p_i) \in \mathrm{Pos}(\alpha_l)$ and $d + (j2 \times p_i) \in \mathrm{Pos}(\alpha_l)$. We have the following.

- $d + (j1 \times p_i) \in \mathrm{Pos}(\alpha_l)$ implies that $\exists\, c1 \in \mathbb{N}$, $1 \leq c1 \leq 2\rho(m)$ such that:

$$d + (j1 \times p_i) = c1 \times p_l. \tag{12}$$

- $d + (j2 \times p_i) \in \mathrm{Pos}(\alpha_l)$ implies that $\exists\, c2 \in \mathbb{N}$, $1 \leq c2 \leq 2\rho(m)$ such that:

$$d + (j2 \times p_i) = c2 \times p_l. \tag{13}$$

- $j1 \neq j2$ implies that $c1 \neq c2$. From equations (12) and (13), we deduce that:

$$(j1 - j2)p_i = (c1 - c2)p_l. \tag{14}$$

- $1 \leq c1, c2 \leq 2\rho(m)$, and $c1 \neq c2$ imply that:

$$0 < |c1 - c2| \leq 2\rho(m) \leq 2\left\lceil \frac{m-1}{2} \right\rceil. \tag{15}$$

- $j1, j2 \in \{0, 1, \ldots, \mu(m, \alpha_i)\}$ and $j1 \neq j2$ imply that:

$$0 < |j1 - j2| \leq \mu(m, \alpha_i) \leq 3\rho(m) \leq 3\left\lceil \frac{m-1}{2} \right\rceil. \tag{16}$$

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From equation (14), we can deduce that:

$$j1 - j2 \text{ is a multiple of } p_l \tag{17}$$

$$c1 - c2 \text{ is a multiple of } p_i. \tag{18}$$

From the fact that $2m + 1 \leq p_i, p_l \leq 3m - 1$, from equation (15), equation (16), equation (17), and equation (18) we have a contradiction. ■

The following lemma characterizes the evolution of the sequence $\{x^{\alpha_i}(n) : n \geq 0\}$ at time $t = k$.

Lemma 1. $x^{\alpha_i}(k) = 1$.

Proof. We have:

$$\begin{aligned}
\sum_{j=1}^{k} \bar{a}_j x^{\alpha_i}(k-j) &= \sum_{j \in \mathrm{Pos}(\alpha_i)} \bar{a}_j x^{\alpha_i}(k-j) + \sum_{j \in D \setminus \mathrm{Pos}(\alpha_i)} \bar{a}_j x^{\alpha_i}(k-j) \\
&= \sum_{j \in \mathrm{Pos}(\alpha_i)} \bar{a}_j x^{\alpha_i}(k-j) \quad \text{by application of equation (9)} \\
&= \sum_{j \in \mathrm{Pos}(\alpha_i)} \bar{a}_j \quad \text{by application of equation (8)} \\
&= \mathrm{card}(\mathrm{Pos}(\alpha_i)) \times \gamma \\
&= 2\rho(m)\gamma. \qquad (19)
\end{aligned}$$

It follows that:

$$\begin{aligned}
x^{\alpha_i}(k) &= \mathbf{1}\left(\sum_{j=1}^{k} \bar{a}_j x^{\alpha_i}(k-j) - \bar{\theta}\right) \\
&= \mathbf{1}\left(2\bar{\theta} - \bar{\theta}\right) \quad \text{by application of equation (19)} \\
&= 1. \ ■
\end{aligned}$$

From Lemma 1 and equation (6), it is easy to verify that

$$\mathrm{PPos}(\alpha_i, 1) = Q(\alpha_i, 1). \tag{20}$$

From the definition of $E(\alpha_i, 1)$, from equation (6), from equation (20), and from Lemma 1, we check easily that:

$$l \in E(\alpha_i, 1) \Longrightarrow x^{\alpha_i}(k+1-l) = 1 \quad \text{and} \quad \bar{a}_l = \gamma \tag{21}$$

$$l \in D \setminus E(\alpha_i, 1) \Longrightarrow x^{\alpha_i}(k+1-l) = 0 \quad \text{or} \quad \bar{a}_l = 0. \tag{22}$$

The values of the sequence $\{x^{\alpha_i}(n) : n \geq 0\}$ at time $t = k+1, \ldots, k-1+p_i$ are given by the following lemma.

Lemma 2. $\forall\, t \in \mathbb{N}$ such that $1 \leq t \leq 3m-1-\alpha_i$, we have $x^{\alpha_i}(k+t) = 0$.

Proof. The result follows from the definitions of $E(\alpha_i, d)$, D, $z(\alpha_i, d)$, and by application of Proposition 1. ■

It is easy to verify that $\forall\, i \in \mathbb{N}$, $1 \le i \le \rho(m)$, we have:

$$\mathrm{PPos}(\alpha_i, j) = Q(\alpha_i, j)\ \forall\, j, \quad 1 \le j \le 3m - 1 - \alpha_i.$$

Lemma 3. There exists $\bar{a}$, $w^{\alpha_i} \in \mathbb{R}^k$ with $\bar{a}_j \ge 0$ for every $j = 1, \ldots, k$, and $\bar{\theta} \in \mathbb{R}$ such that

$$T(\bar{a}, \bar{\theta}, w^{\alpha_i}) = p_i.$$

Proof. By application of Lemmas 1 and 2 we deduce the result. ■

We showed that the recurrence neuronal equation $\{x^{\alpha_i}(n) : n \ge 0\}$ with excitatory memory of length k describes a cycle of length p_i and evolves as described in equation (5).

From line 11 to line 15 of page 15 in [6], it is written:

> This shows that, if there is a neuronal recurrence equation with memory length k that generates sequences of periods $p_1, \ldots, p_r$, then there is a neuronal recurrence equation with memory length kr that generates a sequence of period $\mathrm{lcm}(p_1, \ldots, p_r)r$, where lcm denotes the least common multiple.

This allows us to write the following fundamental lemma of composition of a neuronal recurrence equation.

Lemma 4. [6] If there is a neuronal recurrence equation with memory length k that generates sequences of periods $p_1, p_2, \ldots, p_r$, then there is a neuronal recurrence equation with memory length kr that generates a sequence of period $r \cdot \mathrm{lcm}(p_1, \ldots, p_r)$.

The following lemma is also shown.

Lemma 5. [7] If there is neuronal recurrence equation with memory length k that generates a sequence $\{x^j(n) : n \ge 0\}$, $1 \le j \le g$ of transient length T_j and of period p_j, then there is a neuronal recurrence equation with memory length kg that generates a sequence of transient length $g \cdot \max(T_1, T_2, \ldots, T_g)$ and period of length $g \cdot \mathrm{lcm}(p_1, p_2, \ldots, p_g)$.

Now, we want to build a neuronal recurrence equation with excitatory memory of length $k\rho(m)$ that describes a cycle of length $\rho(m) \cdot \mathrm{lcm}(p_1, p_2, \ldots, p_{\rho(m)})$. Let us denote $\lambda(m) = \prod_{i=1}^{\rho(m)} p_i$ and $h(m) = k\rho(m) = (6m - 1) \cdot (\rho(m))^2$.

Corollary 1. For every positive integer m, $m \ge 2$, there exists $c \in \mathbb{R}^{h(m)}$, $\bar{\theta} \in \mathbb{R}$, and $v \in \{0, 1\}^{h(m)}$ such that $c_i \ge 0$ for every $i = 1, \ldots, h(m)$ and with $T(c, \bar{\theta}, v) = \rho(m)\lambda(m)$.

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Proof. From Lemma 3, we know that for $3m - \alpha_i \in \mathcal{P}$ we have that $T(\bar{a}, \bar{\theta}, w^{\alpha_i}) = 3m - \alpha_i$ with $\bar{a}_j \geq 0$ for every $j = 1, \ldots, k$. We construct the vector c as in the fundamental lemma of composition of automata [6]. By construction, the vector c satisfies $c_i \geq 0$, for every $i = 1, \ldots, h(m)$. From w^{α_i} with $3m - \alpha_i \in \mathcal{P}$, we construct v as in the fundamental lemma of composition of automata [6]. By application of Lemma 4 or 5, we deduce that $T(c, \bar{\theta}, v) = \rho(m)\lambda(m)$. ■

The technique used in Corollary 1 defines several coefficients c_i as zero. We now show that it is possible to modify the coefficients (c_i) and ensure that all of them are strictly positive.

Corollary 2. For every positive integer m, $m \geq 2$ there exists $d \in \mathbb{R}^{h(m)}$, $\theta' \in \mathbb{R}$, and $v' \in \{0, 1\}^{h(m)}$ such that $d_i > 0$ for every $i = 1, \ldots, h(m)$ and with $T(d, \theta', v') = \rho(m)\lambda(m)$.

Proof. It suffices to apply Proposition 1 of [5] to Corollary 1. ■

Notation 1.

- $\pi(x)$ is the number of prime numbers less than or equal to x.
- $\vartheta(x) = \sum_{p \leq x,\ p \text{ prime}} \ln(p)$.

In [8], it is estimated that:

$$\vartheta(x) < x\left(1 + \frac{1}{2\ln(x)}\right) \text{ for } 1 < x \tag{23}$$

$$x\left(1 - \frac{1}{\ln(x)}\right) < \vartheta(x) \text{ for } 41 \leq x. \tag{24}$$

From these estimations, we deduce that for $x \geq e^5$, we have:

$$\frac{2}{10}x < \vartheta(3x) - \vartheta(2x) < \frac{17}{10}x. \tag{25}$$

This allows us to state the result in Corollary 3.

Corollary 3. $\forall\, m \geq e^5, \quad e^{0.2m} < \prod_{2m<p<3m,\ p \text{ prime}} p < e^{1.7m}$. ■

From estimations in [8], we have:

$$\pi(x) < \frac{1.25506x}{\ln(x)} \quad \text{for } 1 < x. \tag{26}$$

It follows that

$$\begin{aligned} h(m) &= (6m-1)\,(\rho(m))^2 \\ &< 6m \times (\pi(3m))^2 \ \text{ because } \rho(m) \le \pi(3m) \\ &< 6m\left(\frac{1.25506 \times 3 \times m}{\ln(3m)}\right)^2 \\ &< \frac{85.0595 m^3}{\left(\ln(3m)\right)^2} \\ &< \frac{85.0595 m^3}{\left(\ln(m)\right)^2}. \end{aligned}$$

We deduce

$$\begin{aligned} m^3 &> \frac{h(m)\left(\ln(m)\right)^2}{85.0595} \\ m &> \sqrt[3]{\frac{h(m)\left(\ln(m)\right)^2}{85.0595}} \\ m &> \sqrt[3]{\frac{h(m)\left(\ln\left(\sqrt[3]{\frac{h(m)(\ln(m))^2}{85.0595}}\right)\right)^2}{85.0595}} \\ m &> \sqrt[3]{\frac{h(m)\left(\ln\left(\frac{h(m)(\ln(m))^2}{85.0595}\right)\right)^2}{3^2 \times 85.0595}}. \end{aligned}$$

This allows us to deduce that:

$$m = \Omega\left(\sqrt[3]{h(m)(\ln(h(m)))^2}\right). \tag{27}$$

On the basis of Corollary 3 and equation (27), we deduce Corollary 4.

Corollary 4. $\displaystyle\prod_{2m<p<3m,\ p \text{ prime}} p = \Omega\left(e^{\sqrt[3]{h(m)\ (\ln(h(m)))^2}}\right).$

Theorem 1. For every positive integer m there exists $d \in \mathbb{R}^{h(m)}$, $\theta' \in \mathbb{R}$, and $v' \in \{0,1\}^{h(m)}$ such that $d_i > 0$ for every $i = 1, \ldots, h(m)$ and with $T(d, \theta', v') = \Omega\left(e^{\sqrt[3]{h(m)\ (\ln(h(m)))^2}}\right)$.

Proof. From Corollaries 4 and 2, we deduce the result. ■

3. Conclusion

The existence of a neuronal recurrence equation of memory length $h(m)$ which describes a cycle of length $\Omega\left(e^{\sqrt[3]{h(m)\ (\ln(h(m)))^2}}\right)$ shows that

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the behavior of neuronal recurrence equations is complex when all the weighting coefficients are positive. The technique used is inscribed in the framework of structural construction. Structural construction methods are the general and more powerful tools used in the study of sequences generated by neuronal recurrence equations [7, 9].

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