

A Cellular Automaton–Based Technique for Estimating Mineral Resources

Soumyadeep Paty

*Department of Mining Engineering
Kazi Nazrul University, Asansol
West Bengal, India
email: soumyadeep.paty@gmail.com*

Supreeti Kamilya

*Department of Computer Science and Engineering
BIT Mesra, Ranchi
Jharkhand, India
email: kamilyasupreeti779@gmail.com*

A significant contribution to the economic growth of a nation comes from the mineral industries. Therefore, the concentration of metallic or nonmetallic minerals in different regions of Earth's crust is important to determine. The present paper studies the grade and thickness estimation of iron and coal deposits, respectively, by applying two-dimensional cellular automata (CAs). Krigging is a popular method for the estimation of mineral resources. However, krigging results in complex mathematical calculations if the number of sample points increases. Here, each cell of the cellular automaton (CA) is represented as a block. Using CAs, the grade values and thickness are estimated in a simpler and faster way. Two-dimensional CAs are used in this paper where the local rule is the ordinary krigging estimator function using the spherical variogram model. The total weight of iron as well as coal is calculated using the CA-based technique. A comparative analysis between the estimated weight of minerals and the actual extracted mineral is also given.

Keywords: block model; krigging; cellular automata (CAs); grade estimation; thickness estimation; iron deposit; coal seam

1. Introduction

The importance of mineral industries for a country's growth is undebatable. Minerals are naturally present at the outermost shells of the Earth. For instance, hematite, bauxite, chalcopyrite and galena are the metallic minerals of iron, aluminium, copper and lead, respectively. Some metallurgical processes may be required for extracting the metals from the minerals, whereas nonmetallic minerals like coal or limestone can be mined and directly used without further processing in most cases. *Ores* are the naturally occurring rocks that contain minerals to a certain concentration that can be economically extracted. The

grade of an ore represents the percentage of metallic or nonmetallic elements it contains. Estimating mineral resources is a crucial step to check if mining is feasible. In general, estimating mineral resources indicates estimating ore grades. For *stratified deposits*, sediments are deposited in layers (also known as *strata*), one above another. For a coal deposit, the stratum that contains coal is also known as a *coal seam*. There is almost no grade variability in a stratum of a stratified deposit. That is, the grade is the same throughout a stratum. Therefore, grade estimation of a stratified deposit is not required. Instead, estimating the thickness of the stratum gives an idea of how much mineral it contains. In the present paper, grade estimation is done on iron ore deposits, while thickness is estimated in a coal reserve because it is a stratified deposit.

A *block model* depicts a comprehensive representation of a mineral deposit in a three-dimensional space. In general, a number of uniform-sized cubical blocks are arranged in a block model where each block has a certain grade value. Direct determination of the grade value of each and every block is not possible before the mining operation. However, *geological exploration* is a way of identifying grade values of a small number of blocks. Grade values are spatially correlated, as each grade value is connected to some coordinates (or spatial location). Therefore, a small number of known blocks carry information of unknown blocks and hence they help in estimating the grades of unknown blocks [1]. In the early 1950s, Dan Krige popularized the study of spatially correlated data [2]. Later, the mathematical formulation of the theory on regionalized variables (that deals with spatial data) was done by Matheron [3, 4] in the 1960s. The mathematical formulation helps in representing the data by regionalized structures. Such data can also be analyzed by statistical methods. In the 1990s, spatial characteristics of the regionalized variables were proposed by Kim [5]. The author of the paper illustrates that the regionalized variables are highly variable in space; however, they are not completely independent.

Geostatistics is a discipline that largely deals with spatially correlated data. Various geostatistical methods exist to regionalize the spatial variables, such as variogram, correlation coefficient and covariance. Among them, the most commonly used method that estimates mineral grades is the variogram. In 1963, a method called *kriging* (after Dan Krige) was proposed, in order to estimate the regionalized variables. Kriging is based on known data points and variogram modeling [3] and involves a number of computationally expensive mathematical equation-solving techniques. A simplified mathematical tool, *cellular automata* (CAs), is put in place that estimates the grade values of minerals while reducing the computational complexity. To

the best of our knowledge, very few papers have been reported that apply the cellular automaton (CA) model in mineral industries [6].

A CA is a D -dimensional lattice of cells where each cell changes its state depending on a local function (or rule) and its neighbors. Relying on the concept of a Turing machine, von Neumann became interested in the theory of self-reproductive automata in the 1940s and worked on the concept of a self-reproductive machine [7–9]. Later, Stanislaw Ulam suggested the use of a regular lattice to mathematically model the self-reproduction, which is presently known as CAs [10].

We introduce the CA-based evaluation of mineral resources in terms of grade values in [11]. The grade value estimation is done for metallic elements. However, in the case of stratified deposits like coal or limestone, grade estimation is not necessary for feasible mining. Along with the grade estimation of metallic minerals, the present paper evaluates thickness estimations of the stratified minerals (coal deposit) using two-dimensional CAs. For estimating metallic mineral grade, we take each horizontal slice of a three-dimensional block model and apply a two-dimensional CA on an individual slice. The evaluation is performed on the data of an iron deposit in the eastern part of India. The study is done on a 520-meter north-south and 360-meter east-west region of mineral deposit, at a depth of 140 to 260 meters. We subdivide the block model into 20 m^3 blocks. The mineral grade for each block is calculated by applying the CA on a single horizontal slice. In the case of a coal deposit, we construct a two-dimensional block model for each coal stratum on the basis of horizontal projection of the stratum. The two-dimensional model is subdivided into a 20 m^2 square block. Each square consists of the depth value of its center. The study of thickness estimation is done on the coal deposit data of a coal mine in the Jharia coal fields (Jharkhand state of India). The study is performed on the data spreading over a 460-meter north-south, 940-meter east-west region, at a depth of 80 to 115 meters. We use the variogram function as the local rule of the two-dimensional CA for estimating both grade and thickness.

The paper is organized as follows. Section 2 discusses the basics of CAs, krigging and variogram models. In Section 3, CA-based grade estimation of iron deposits is shown. CAs are also applied on the data of a coal deposit to estimate the thickness of the coal seam in Section 4. Section 5 shows the computational time for an ordinary krigging and CA-based approach. Also, a comparative study between the estimated value of iron and actual iron produced from the mine is provided. Finally, the paper concludes with Section 6.

2. Preliminaries

2.1 Cellular Automata

Let us formally define a CA.

Definition 1. A CA, as defined here, is a quadruple $(\mathcal{L}, S, \mathcal{M}, f)$ where,

- $\mathcal{L} \subseteq \mathbb{Z}^D$ is a D -dimensional cellular space, $\vec{v} \in \mathcal{L}$
- S is the set of states a cell can assume
- $\mathcal{M} = (\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m)$ is the neighborhood vector of each $\vec{v}$ of the lattice, and $(\vec{v} + \vec{v}_i) \in \mathcal{L}$,
- $f : S^m \rightarrow S$ is the transition function, called the local rule of the CA.

Definition 2. A configuration is a mapping that assigns states to the cells $c : \mathcal{L} \rightarrow S$.

This indicates that $S^{\mathcal{L}}$ is the set of all possible configurations. A CA can also be defined by a function $G : S^{\mathcal{L}} \rightarrow S^{\mathcal{L}}$, which is known as the global transition function.

In the present paper, we consider a finite size of CAs with $D = 2$. In general, the neighborhood of a two-dimensional CA is determined in two ways: (1) von Neumann neighborhood and (2) Moore neighborhood. The CA proposed by von Neumann is a two-dimensional CA with square grids (Figure 1(a)).

The Moore neighborhood [12] of two-dimensional CAs follows a nine-neighborhood dependency, where four nonorthogonal cells are additionally considered as neighbors. Figure 1(b) shows the Moore neighborhood dependency of a CA. The Moore neighborhood structure has been exploited for sketching the famous Game of Life, a CA introduced by John Conway and popularized by Martin Gardner [13]. Here also, we use a Moore neighborhood of a two-dimensional CA.

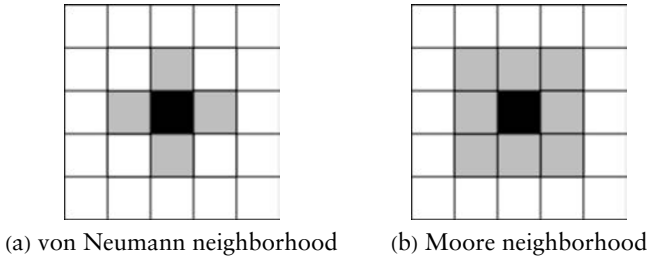


Figure 1. Neighborhood dependencies of two-dimensional CAs where the black cell is the cell under consideration, and the black cell changes its state depending on its neighbors marked in gray.

2.2 The Variogram Model and Kriging

The variogram model depicts the relationship between spatially related data with distance. The semivariogram function was initially defined in [3], which is calculated as

$$\gamma(h) = \frac{1}{2 * N(h)} \sum_{i=1}^{N(h)} [z(x_i) - z(x_i + h)]^2. \quad (1)$$

Here, $z(x_i)$ and $z(x_i + h)$ are the grade values at locations x_i and x_{i+h} , respectively. The locations are a distance h apart from each other, and $N(h)$ is the number of pairs $z(x_i)$ and $z(x_i + h)$.

A variogram model can be constructed using lag distance h and the corresponding semivariogram function $\gamma(h)$. A plot of the variogram model is shown in Figure 2.

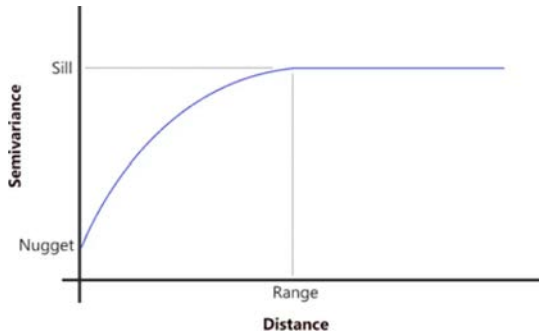


Figure 2. A variogram model.

A variogram model has several parameters, such as range, nugget effect, sill and spatial variance.

- *Range* (a). Range is the distance within which spatial data is correlated and beyond which no correlation exists.
- *Nugget effect* (C_0). Nugget effect shows the measurement error or microscale variation of data. The value of the nugget effect can be calculated from $\gamma(h)$ when $h = 0$.
- *Sill* (C). Sill is the variogram value at $h = \text{range}$.
- *Spatial variance* (C_1). Spatial variance is the difference between the sill and nugget effects.

Kriging is an estimation method for spatially related data. The data depends on its neighboring data. The estimation is done by the following equation:

$$z(x_0) = \sum_{i=1}^n \lambda_i z(x_i) \quad (2)$$

where $z(x_0)$ is the estimated mineral grade at x_0 and λ_i is the weight of each data point. It is calculated from the covariance matrix [6]. n is the number of samples and x_i is the position of the i^{th} sample data point.

For ordinary krigging purposes, the following $n + 1$ number of equations is used to calculate the weight λ_i :

$$\gamma_{1,1} \cdot \lambda_1 + \gamma_{1,2} \cdot \lambda_2 + \cdots + \gamma_{1,n} \cdot \lambda_n + \mu = \gamma_{1,0}.$$

Similarly, for the n^{th} equation, we can write

$$\gamma_{n,1} \cdot \lambda_1 + \gamma_{n,2} \cdot \lambda_2 + \cdots + \gamma_{n,n} \cdot \lambda_n + \mu = \gamma_{n,0}.$$

The $(n + 1)^{\text{th}}$ equation can be defined as

$$\lambda_1 + \lambda_2 + \cdots + \lambda_n = 1.$$

Here, μ is the Lagrangian parameter.

3. Estimating Iron Deposit Grade Values Using Cellular Automata

In this section, we briefly elaborate on estimating the grade values of iron deposits using a mathematical model, that is, CAs, which has been studied in [11]. The conventional way of estimating our collected data is by means of ordinary krigging.

In the case of the iron ore deposit, samples are collected from 71 exploratory boreholes. Exploratory boreholes are drilled to collect physical samples. A visual representation of 71 boreholes is shown in Figure 3.

From the sample data, a spherical variogram model is constructed in the study. The solid red line of Figure 4 shows the spherical variogram model for our sample data. In the figure, the spherical variogram model (solid red line) is fitted with the experimental data (represented by the solid black line).

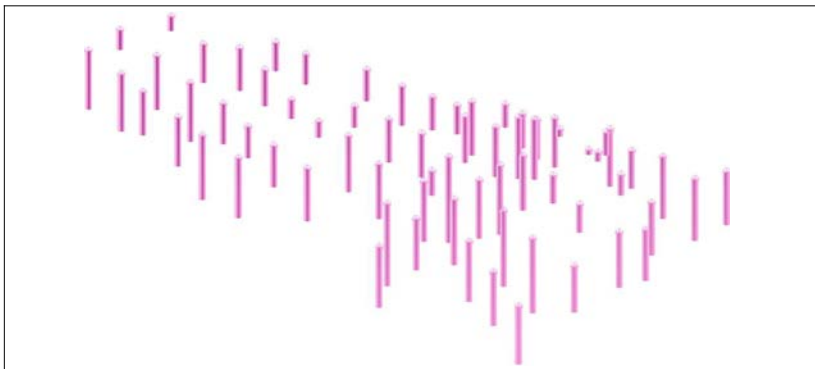


Figure 3. A visual representation of 71 boreholes.

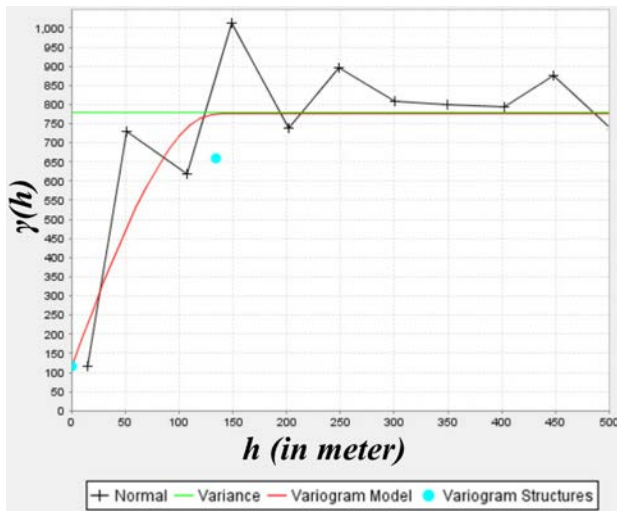


Figure 4. Experimental variogram model of the sampled data.

Here, the range a , sill value and nugget effect C_0 of the variogram for the grade value of the mineral are 142.6 meters, 778.4 and 115.9, respectively. So, the spatial variance, C_1 (difference between sill and nugget effect), is 662.5. From this model, the variogram for distance h can be calculated as:

$$\gamma(h) = \begin{cases} C_1 + C_0 \cdot \frac{h}{a} \left[1.5 - 0.5 \left(\frac{h}{a} \right)^2 \right] & \text{if } h \leq a \\ C & \text{otherwise.} \end{cases} \quad (3)$$

Grade values of the unknown blocks are estimated by using equations (2) and (3). Figure 5 shows the krigging estimated values of a horizontal slice (with a depth of 280 meters) of the deposit.

The krigging method needs to solve a large number of equations. Hence, the method is computationally complex, so we estimate the values using two-dimensional CAs. The size of the CA is equal to the size of a horizontal slice of the block model. Cells of a CA represent blocks of minerals. The cells interact with their neighboring cells and evolve to their next states depending on the local transition function. The ordinary krigging function is used as the local transition function of the CA. Considering the variogram function, as given in equation (3), the krigging function is applied to obtain the new state of a cell (i, j) . The grade values of sampled data are used as the initial configuration of the CA. The state of the cell is considered as 0 if no sample data exists in a cell. The state of the cells may change from 0 to

some grade value in the next time step of transition. The grade values, after the first evolution, are shown in Figure 6. It can be observed that the grade values converge to 59.77 and 60.44. That is, after the grade value reaches these points, the evolution will halt.

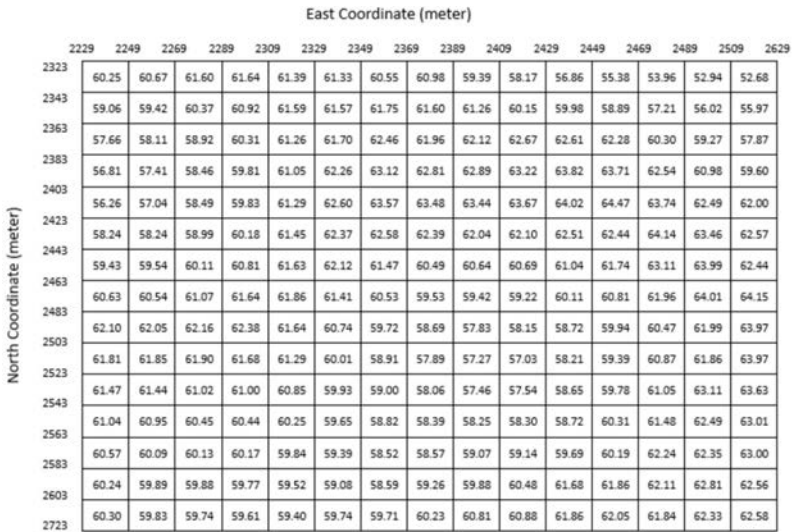


Figure 5. A snapshot of kriging estimated values of a horizontal slice.

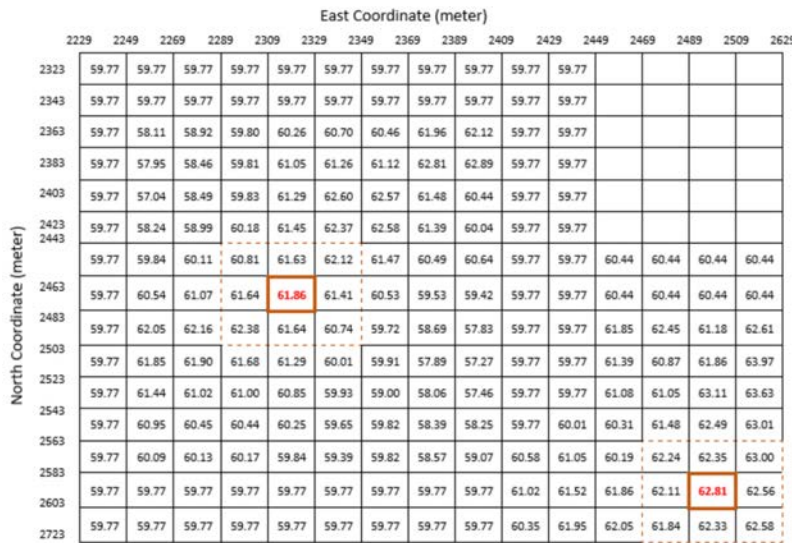


Figure 6. Grade values after the first stage of CA evolution.

The values of neighboring cells of cell (i, j) are kept fixed after the first stage of evolution, and the transition rule is applied for evolution. The fixed neighboring cells are shown by dotted lines in Figure 6. Again, Figure 7 shows the fixed cells and the cell values after the second evolution. The same method is applied in the next stages, where the next neighboring cells of the fixed cells are kept constant. The transition function is applied until the fixed values of all the cells are obtained.

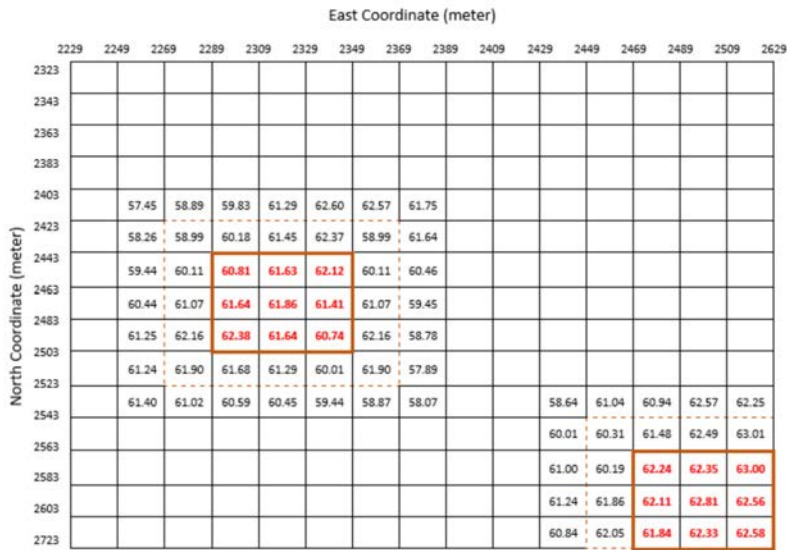


Figure 7. Grade values after the second stage of CA evolution.

A total of six stages of evolution is needed to get the fixed values of all the cells. The final grade values are given in Figure 8.

It can be observed that the values obtained by using the CA are the same as those of the ordinary krigging method because the same variogram model is used in both cases. However, the computational complexity is reduced in the CA case, compared to the krigging method. Also, it is easier to use CAs for such an estimation. After estimating grade values in the horizontal two-dimensional slices, the multiple slices are combined into a three-dimensional structure. The three-dimensional structure is shown in Figure 9. The grade value of each block is represented by different colors.

		East Coordinate (meter)																	
		2229	2249	2269	2289	2309	2329	2349	2369	2389	2409	2429	2449	2469	2489	2509	2629		
North Coordinate (meter)	2323	60.25	60.67	61.60	61.64	61.39	61.33	60.55	60.98	59.39	58.17	56.86	55.38	53.96	52.94	52.68			
	2343	59.06	59.42	60.37	60.92	61.59	61.57	61.75	61.60	61.26	60.15	59.98	58.89	57.21	56.02	55.97			
	2363	57.66	58.11	58.92	60.31	61.26	61.70	62.46	61.96	62.12	62.67	62.61	62.28	60.30	59.27	57.87			
	2383	56.81	57.41	58.46	59.81	61.05	62.26	63.12	62.81	62.89	63.22	63.82	63.71	62.54	60.98	59.60			
	2403	56.26	57.04	58.49	59.83	61.29	62.60	63.57	63.48	63.44	63.67	64.02	64.47	63.74	62.49	62.00			
	2423	58.24	58.24	58.99	60.18	61.45	62.37	62.58	62.39	62.04	62.10	62.51	62.44	64.14	63.46	62.57			
	2443	59.43	59.54	60.11	60.81	61.63	62.12	61.47	60.49	60.64	60.69	61.04	61.74	63.11	63.99	62.44			
	2463	60.63	60.54	61.07	61.64	61.86	61.41	60.53	59.53	59.42	59.22	60.11	60.81	61.96	64.01	64.15			
	2483	62.10	62.05	62.16	62.38	61.64	60.74	59.72	58.69	57.83	58.15	58.72	59.94	60.47	61.99	63.97			
	2503	61.81	61.85	61.90	61.68	61.29	60.01	58.91	57.89	57.27	57.03	58.21	59.39	60.87	61.86	63.97			
	2523	61.47	61.44	61.02	61.00	60.85	59.93	59.00	58.06	57.46	57.54	58.65	59.78	61.05	63.11	63.63			
	2543	61.04	60.95	60.45	60.44	60.25	59.65	58.82	58.39	58.25	58.30	58.72	60.31	61.48	62.49	63.01			
	2563	60.57	60.09	60.13	60.17	59.84	59.39	58.52	58.57	59.07	59.14	59.69	60.19	62.24	62.35	63.00			
	2583	60.24	59.89	59.88	59.77	59.52	59.08	58.59	59.26	59.88	60.48	61.68	61.86	62.11	62.81	62.56			
	2603	60.30	59.83	59.74	59.61	59.40	59.74	59.71	60.23	60.81	60.88	61.86	62.05	61.84	62.33	62.58			
	2723																		

Figure 8. The final grade values using CAs.

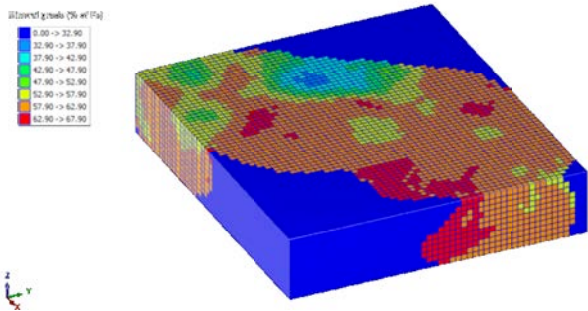


Figure 9. The final three-dimensional structure. Color represents the grade values of the deposit.

4. Estimating the Thickness of a Coal Deposit Using Cellular Automata

In this paper, we estimate the thickness of a coal deposit by means of the krigging method because a coal seam depth is not randomly varied and it is spatially correlated. We collect data from 55 exploratory boreholes that are basically the thickness of each seam at the particular point of a borehole. The known thicknesses are given as the initial configuration of the two-dimensional CA (as shown in Figure 10). The unknown thickness is considered as 0. Here also, the transition

function of the CA is the ordinary kriging function. Similar to the grade estimation technique, we fix the neighboring cells of cell (i, j) where the value of cell (i, j) is known, after the first step of evolution. Figure 11 shows a snapshot of the CA after the first step of evolution. In the next step, neighboring cells of the previously fixed cells are made fixed. The same procedure is continued until all the cells of the CA become fixed. Figure 12 shows a snapshot of the CA after final evolution.

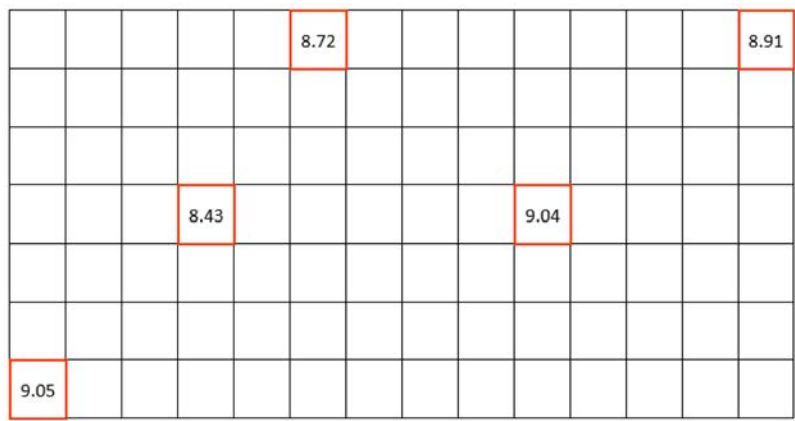


Figure 10. Initial configuration of the CA that represents the thickness of a coal seam.

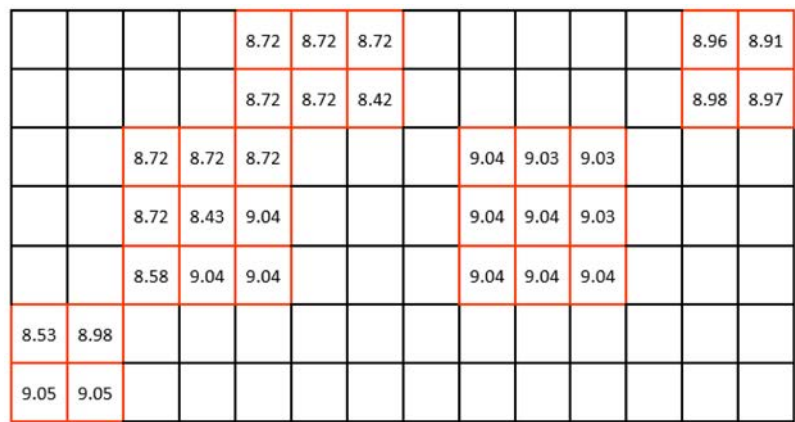


Figure 11. A snapshot of the CA after the first step of evolution.

8.72	8.72	8.72	8.72	8.72	8.72	8.72	8.71	8.80	9.03	9.02	9.00	8.96	8.91
8.73	8.72	8.72	8.72	8.72	8.72	8.42	9.04	9.04	9.03	9.02	9.01	8.98	8.97
8.73	8.72	8.72	8.72	8.72	8.68	9.04	9.04	9.04	9.03	9.03	9.02	9.01	9.00
8.73	8.73	8.72	8.43	9.04	9.04	9.04	9.04	9.04	9.04	9.03	9.03	9.02	9.02
8.73	8.73	8.58	9.04	9.04	9.04	9.04	9.04	9.04	9.04	9.04	9.03	9.03	9.03
8.53	8.98	9.04	9.04	9.04	9.04	9.04	9.04	9.04	9.04	9.04	9.04	9.04	9.04
9.05	9.05	9.04	9.04	9.04	9.04	9.04	9.04	9.04	9.04	9.04	9.04	9.04	9.04

Figure 12. A snapshot of the CA after the final step of evolution.

In order to compare the newly established CA-based technique with the ordinary kriging method, the thickness of a seam is estimated by applying ordinary kriging to determine the thickness of the unknown points. It can be observed that the estimated thickness of the seam at different points is the same for both the ordinary kriging method and the CA-based method. However, the CA-based estimation can be performed in a much simpler and faster way compared to the ordinary kriging method.

5. Results and Discussion

For the calculations in the present paper, we used an AMD dual core processor with 8 GB of RAM with the C language. It can be observed that the CA model executed faster compared to the kriging model. The computer took 127 seconds to estimate the iron grade values using the ordinary kriging-based technique, whereas it took 42 seconds for the CA-based technique. On the other hand, estimating the coal seam took 51 seconds for ordinary kriging and 12 seconds for the CA-based estimation.

On the basis of the estimation done in Section 3, we now calculate the weight of the total iron present in the deposit under study as:

$$w_i = \sum \left(v_b \times \frac{g_b}{100} \times \rho_{\text{iron}} \right). \quad (4)$$

Here, w_i is the weight of iron, v_b is the volume of each block, g_b is the grade of the block and ρ_{iron} is the density of iron. By applying equation (4), we obtain that the total iron present in the deposit is 195 million tonnes. However, 161 million tonnes of iron have been extracted

by actual mining. The loss of 17.4% is due to the mining operation and mineral processing. However, it is within an acceptable range.

In the case of the coal deposit, the total weight of coal is estimated as

$$w_c = \sum (A_b \times t_b \times \rho_{\text{coal}}). \tag{5}$$

Here, w_c is the weight of coal, A_b is the area of each block, t_b is the thickness of the block and ρ_{coal} is the density. The total coal obtained by applying equation (5) is 9.1 million tonnes, whereas the actual weight of coal extracted from the mining operation was 6.2 million tonnes. So, the recovery of coal is 68.1%, which can be considered as a good recovery in the case of coal mining. Table 1 shows a comparative study of estimated weights and actual extracted weights of iron and coal deposits from the mines under study.

	Estimated Weight (in MT)	Actual Weight (in MT)	Recovery (%)	Mining and Processing Loss (in MT)	Loss (%)
Iron	195	161	82.6	34	17.4
Coal	9.1	6.2	68.1	2.9	31.9

Table 1. A comparative analysis of estimated weight and actual extracted weight of iron and coal deposits.

6. Conclusion

The current paper presents a cellular automaton (CA)-based technique for estimating the grade of an iron deposit as well as estimating the thickness of a coal seam. A two-dimensional Moore neighborhood CA has been used for the estimation. Each block of mineral represents a cell of the CA. A popular way to do such an estimation is the ordinary krigging method; however, it needs a larger number of mathematical calculations. Using the CA makes it simpler and faster. Using the CA-based estimation, the total weight of iron as well as coal was calculated. A comparative analysis between the estimated weight of the minerals and the actual extracted amount was also shown. As a future area of study, functions of other geostatistical techniques that are used for interpolation can be applied and compared with the CA-based estimation method.

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