# Personal Space and Collective Behavior in Crowd Disasters

**Igor Lugo**
*Centro Regional de Investigaciones Multidisciplinarias (CRIM)*
*Universidad Nacional Autonóma de México (UNAM)*
*Av. Universidad s/n, Circuito 2*
*Cuernavaca, Morelos, 62210, México*

**Martha G. Alatriste-Contreras**
*Facultad de Economía*
*Universidad Nacional Autonóma de México (UNAM)*
*Circuito Escolar, C.U. Coyoacán*
*Ciudad de México, 04510, México*

The importance of personal space (PS) in densely crowded situations is commonly underestimated. However, self-awareness of PS can help prevent harm and reduce risky situations. This paper explores the use of PS as the key factor for designing simple computational models to prevent crushing events. An agent-based model is developed to relate transitional rules with Schelling's spatial proximity model of segregation. An exploratory data analysis and subsequent model validation find that the dynamics of crowd events associated only with the individual perception of PS show significant statistical regularities between density situations. These results suggest that crushing events in densely crowded situations are highly probable and that lower density levels are key for delaying the onset of such disasters.

*Keywords*: personal space; collective behavior; crowd disasters; complex systems; agent-based model; Schelling's spatial model; cellular automata; data analysis

## 1. Introduction

The dynamics of extremely dense crowds or crowd turbulence are characterized by sudden variations that push people around, generating uncontrollable and unpredictable large-scale events [1, 2]. Such events can be triggered by a small group of individuals in close proximity to one another with no harmful intent. Unfortunately, recent well-known events in Etah, Uttar Pradesh, India, on July 2, 2024, during a religious event, and in South Korea on October 29, 2022, during a Halloween celebration, showed a disturbing lack of understanding in preventing fatal injuries. In particular, the crowd crushing event in South Korea occurred without an ex-ante mass panic [2].

https://doi.org/10.25088/ComplexSystems.35.3.237

Contrasting with these lamentable events, there are other examples in which overcrowding did not result in tragedy. For example, the celebration of Argentina winning the soccer World Cup on December 20, 2022, resulted in an auto-organized crowd across the country without fatal crushing events. The incidents were separated by two months and showed completely different outputs.

We propose a theoretical approximation for modeling the dynamics of crowd crushing in social gatherings based on Schelling's model of segregation [3, 4]. Our particular interest is in understanding the role of the area surrounding an individual in a dense crowd—that is, the personal space (PS)—for identifying and preventing crushing events [5, 6, 7]. To achieve this, we use an agent-based model (ABM) to simulate complex and emergent behaviors in densely crowded situations.

PS can be related to studies of human behavior that consider social cohesion as the key to understanding local interactions between individuals [6, 8]. PS can also be associated with levels of psychological comfort or discomfort [5, 9]. Our aim is to use the idea of PS in the ABM to generate simple local rules on a flat surface without obstacles and identify the conditions and dynamics associated with different densely crowded venues, such as music concerts, religious meetings and sport events [10]. Then, if we consider PS as the principal issue to identify and prevent crushing events in dense crowds, we can describe the local reaction of a person to move or stay in place without external motivations to go in a particular direction [11].

Therefore, we are interested in the following questions: What are the density situations in which a crushing event becomes unstoppable? How probable are crushing events in such crowded gatherings? What are the perceived levels of comfort or discomfort for being surrounded by similar people in such situations? We answer these questions using an interdisciplinary approach. We use the idea from sociocultural studies that each individual has a natural instinct for self-preservation in different crowd situations [7, 12, 13]. Based on the idea of self-preservation, we use an ABM based on Schelling's model of segregation to describe how individuals utilize their perceived PS, react with each other and generate large-scale collective behaviors in dense crowds [3].

Even though this type of ABM is not commonly used in modeling crowd dynamics, it has the advantage of generating simple models to study crushing events as emergent phenomena [14]. In particular, instead of using a reducibility approach related to force-based or pedestrian models compressed into a single formulation [15], we apply a computational irreducibility approach in which the complex process of crowd dynamics must be explicitly simulated or performed.

Next, we use an exploratory data analysis and model validation to evaluate the model outputs [16, 17]. In particular, we design an exploratory data analysis to clarify the importance of PS and the different density levels in situations that form crushing events. Furthermore, the model validation is intended to predict the existence of crushing events under those initial conditions that are outside of the realm of observed conditions [18, 19].

Then, we use a probabilistic framework based on a Monte Carlo approximation to look for statistical regularities associated with the dynamics of such initial conditions.

We hypothesize that fatal crushing events can happen in any cultural and regional context. Fatal crushing events can be identified and prevented based on PS. Individuals with a high tolerance threshold of being surrounded by people in near-body proximities are more vulnerable to being fatally crushed in overcrowded events. That is to say, if a person tolerates the violation of their PS, the occurrence of a fatal crushing event due to an unresponsive action is more probable. On the other hand, individuals who have a lower tolerance threshold of being surrounded are less vulnerable to crushing. That is, if a person has a lower tolerance of discomfort associated with the violation of their PS, the chance of being fatally crushed is smaller. Despite the simplicity of PS, it can nevertheless exhibit complex behaviors in dense crowds related to the presence of fatal or nonfatal cases of crowd crushing. Therefore, the practical relevance of this paper is to provide a simple way of identifying dangerous crushing events, to communicate to attendants of dense social gatherings the higher risk of being crushed and to prevent uncontrollable crowd turbulences and their fatal crushing events.

The paper proceeds as follows: In Section 2, we give references to the main studies of collective behaviors and crowds in different disciplines, as well as their computational approximation. In Section 3, we discuss the programming libraries for coding, generating and analyzing our ABM. In Section 4, we explain our ABM based on Schelling's model of segregation to describe the configuration and transitional rules in different situations related to PS. Furthermore, we describe computing the nearest neighbors measure, implementing different density scenarios, the procedure for the data analysis and model validation, and the initial conditions. In Section 5, we display the findings based on the model validation and sensitivity analysis. Finally, in Section 6, we give our final remarks and conclusions.

## 2. Literature Review

In the diverse and extensive literature related to collective behaviors, we can identify different methods for crowd simulation and modeling.

https://doi.org/10.25088/ComplexSystems.35.3.237

Following the work of Yang et al. [20] and Dang et al. [21], we agree that there are different levels of crowd modeling, for example, microscopic and macroscopic levels, and they depend on the scale of analysis. To be effective, models do not have to be realistic in the sense of describing every real-life detail, but instead have to be tools for "providing insightful explanations of known facts, and revealing the unreal ized consequences of widely held beliefs." [22]. Therefore, we use a simple approximation of crowd modeling based on PS [14, 23, 24].

Important characteristics of collective behaviors include the types of coordination and the trigger events that stress a group. In particular, studies in biology, such as flocking in birds, point out the dynamics behind the effect of local actions and the formation of large-scale patterns within groups [25]. A highlighted feature in these studies is the efficient information transfer that prevents crushing events [26]. This is because there is enough space between each individual and their near neighbors at the level of the group that effectively communicates the sudden changes of direction to prevent any type of crushing incident. Even if those changes were generated by warning signs of a possible predator around the group, the group reaction is to avoid the dangerous situation by impulsive but coordinated dispersion. Therefore, the physical space between individuals in animal groups stands as one key for coordinating movements to prevent crashing or crushing events.

On the contrary, collective behaviors related to groups of people show singular attributes of crowd dynamics [2, 27]. In particular, we are interested in chaotic and uncoordinated movements with unknown trigger events. In this respect, there are studies that have considered empirical and theoretical approaches to describe the dynamics in the motions of extremely dense crowds. The work of Helbing et al. [2] analyzed video recordings of the crowd disaster in the "Muslim pilgrimage in Mina/Makkah, Saudi Arabia." They identified turbulent flows that are similar to earthquakes because they initiated as sudden changes of pressure, and these changes generated unstoppable and chaotic movements in the crowd. In addition, the work of Helbing et al. [10] studies the evacuation dynamics of classrooms by experiments and simulations. They identified jamming of students at the exit door that affected the evacuation dynamics. In this study, the trigger event was a signal alarm shouted by the camera operator. In addition to these findings, the study of Nakayama et al. [28] analyzed the two-dimensional formation of pedestrian and granular flows. They suggested the presence of instabilities in pedestrian flows. Therefore, in this type of approximation, it is obvious by now that the density level is another key for understanding a crowd disaster.

With respect to the method, the work of Yang et al. [20] closely reviewed the models used in crowd simulation. They suggested that

the presence of a mesoscopic level of modeling can improve the simulation of more realistic crowds. In the same line, the work of Dang et al. [21] extensively reviewed models of pedestrian simulation in high-density situations. They pointed out the importance of considering different levels and integration models for simulating crowds. Compared with these approaches, the work of Schelling [3] about segregation, which we use in this study, offers an alternate simple approach for modeling crowd behaviors.

Schelling's model of segregation shows the fundamental principles of individual decisions based on their self-interest and group preference [29]. This model commonly uses the cellular automaton (CA) framework for generating an ABM [29]. Consequently, Schelling's model describes crowd behaviors based on a simple psychological process of social groups. In addition, it is worth mentioning a few studies that use cellular automata (CAs) for modeling crowd events. For example, the work of Fukui and Ishibashi [30] suggested the use of CA models for understanding pedestrian flow and "the high density crowd behavior in overcrowding places." Moreover, the work of Muramatsu and Nagatani [31] and Burstedde et al. [32] applied a CA model to describe pedestrian dynamics using traffic flow dynamics. They identified that the jamming transitions emerge from an increase in density. These methods indicate the importance of incorporating simple and significant factors for developing models about collective behaviors.

There are very few published studies using PS that consider its local effect on different groups of people. For example, Welschetal [5] used a field-theoretical framework to show the response of violating PS, finding that such a violation is immediate. Then, if your PS constantly changes, there is no tolerance for violations. The work of Thompson et al. [9] suggested the existence of a comfort or tolerance zone of around 180–240 cm. Finally, the work of Lugo and Alatriste-Contreras [19] used the PS concept to validate applying the classic Moore neighborhood in the CA. They suggested that PS is a fundamental concept for modeling the interpersonal distance in spatial interactions. These studies show the potential to complement the findings of collective behaviors by explicitly including PS in theoretical and empirical analyses.

These references indicate the usefulness of considering an interdisciplinary approach that integrates new ideas into different scales and methods of analysis. In particular, PS should be part of the literature related to crowd simulation and modeling because it is a fundamental concept for understanding crushing events. We present the materials related to the programming libraries used in our ABM in the next section.

https://doi.org/10.25088/ComplexSystems.35.3.237

## 3. Materials

Because our ABM simulates known facts of crushing events at different levels of density, the materials use different programming libraries. Program code is one of the best tools for exploring the presence of simple local rules that cannot be seen in real-life behaviors to explain complex outputs in a crowd [14, 15]. We use the Python programming language (software version: Python 3.8.16; random seeds: seed=42) and its third-party libraries to generate our ABM, analyze its outputs and show its results. In particular, we used numpy.org/NumPy, matplotlib.org/matplotlib and scipy.org/SciPy libraries. We placed all our data, code and results in the Open Science Framework (OSF). For a detailed specification of the programming code, see the OSF project [33].

## 4. Methods

As mentioned earlier, we use Schelling's model of segregation to generate our ABM [3]. Compared with complex models (some of them mentioned in Section 2) that try to describe reality with all the details, our proposed model identifies core assumptions and a key variable that can represent the behavior of the system. In particular, we assume that each individual shows self-awareness of their PS in different crowd density scenarios. Computing the nearest neighbors measure will provide the basic information to understand and describe crushing events in flat areas, indoors or outdoors, without physical obstacles such as chairs, guardrails, trees or stairs and so on. Therefore, we use Schelling's model of segregation to trace the effects of PS related to two types of individuals, those that are tolerant or intolerant of being surrounded by similar individuals, in different levels of crowd density.

Our ABM describes individuals who change location or stay in the same place according to the presence of individuals with different or similar levels of tolerance. Tolerance or intolerance is defined in terms of PS as the individual perception of comfort or discomfort associated with the number of surrounding individuals with a similar perception. The model describes spatial segregation in which an individual will change location in response to the sense of discomfort associated with the presence of surrounding individuals, or an individual will stay in place in response to the alignment of common preferences with surrounding individuals. The following subsections describe the model configuration and the individual behavior when surrounded by others, as well as the proposed data analysis and model validation.

### 4.1 Configuration

To build our theoretical ABM, we use the attributes of each cell and the shape of the individual's neighborhood as references for defining the configuration of a matrix. In this paper, we use basic CAs for modeling two-dimensional crowd events. CAs can represent a wide variety of dynamic systems related to crowd modeling and its physical and temporal scales [34]. In particular, CAs consist of a grid of cells that can take different values and are updated in discrete time steps based on transitional rules depending on the neighborhood of a cell [35] (Figure 1(a)). The configuration of the grid of cells or matrix is used to define different spatial and temporal situations; for example, local environments related to indoor locations or large-scale movements associated with transport systems [18, 19]. In this paper, the matrix has a closed boundary that represents real-life situations in which the crowd is limited by physical boundaries, such as walls, buildings and natural structures. These types of boundaries are closely related to locations and areas—indoors and outdoors—of crowd events, such as mass gatherings and festivals.

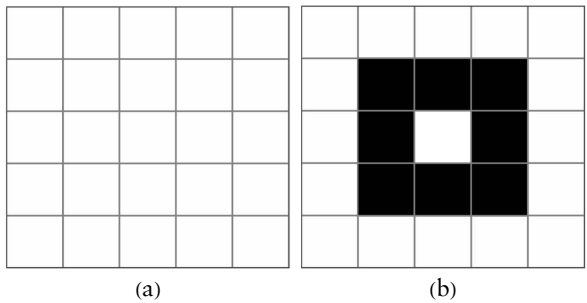



**Figure 1.** Configurations. (a) A simple representation of a matrix with closed boundaries. (b) The shape of the neighborhood associated with PS. The black cells represent the neighbors around a single cell.

Next, each individual is placed in a cell and moves or stays in place based on the presence of surrounding individuals that occupied a cell and their level of tolerance (Figure 1(b)). Based on Schelling's model of segregation, we see the emergence of collective behaviors associated with crushing events as a result of the relatively modest decisions of individuals who act in their self-interest and group preference [29]. In this respect, we consider the concept of "psychological crowd" for describing the presence of different groups of people within a physical crowd linked by a clear social coherence [11, 36]. Then, in this case, we define individuals as crowd members that identify with each other based on their similar opinion or preference [37]. This preference can

be associated with two values of occupied cells by individuals: "tolerance" or "intolerance" of being surrounded by others. In addition, each individual shows the attribute of movement around their unoccupied cells. This attribute considers the possibility of evading the formation of a self-overpopulated neighborhood or staying with similar individuals. Therefore, each cell shows the attribute of occupied (tolerance or intolerance) and unoccupied cell.

Let us now consider the neighborhood of an individual. In particular, we use the Moore neighborhood to describe PS (Figure 1(b)). Because PS can be seen as a "circular area surrounding the person" [5], the Moore neighborhood can be a good approximation for modeling the interpersonal distance around a person in near-body or extremely close interactions [35, 38]. Using the Moore neighborhood and Schelling's model of segregation [3], we can associate the concept of "social psychology" with crowd situations [39, 11]. In particular, the behavior and thoughts of an individual to move or stay in place in a crowd can be influenced by heuristics—rule of thumb for making decisions with limited information [40]. Based on these ideas, we identify and set the number of neighbors who affect the preference of an individual to be surrounded by their similars or move to another unoccupied cell. These situations can describe the individual psychology behind the physical reaction of a person within a crowd.

### 4.2 Rules

We now describe the transitional rules related to an individual's behavior based on their own self-interest and group behaviors. Following the notation in Batty [29], we define each cell in an $N \times N$ matrix $m_{N\times N}$ as $C_{ij}(t)$, in which each cell $C$ shows the location $(i, j)$ at time $t$. The notation $(i, j)$ is referred to as a tuple of values related to the coordinates of $m$, and $t$ is the number of iterations for updating $m$. Each $C_{ij}(t)$ has the attribute of occupied or unoccupied. Occupied cells are related to individuals who have the attribute of tolerance $C_{ij}^{\text{tol}} = 1$ or intolerance $C_{ij}^{\text{intol}} = 0$. Unoccupied cells are defined as $C_{ij}^{\text{unocc}} = 3$. The tolerance and intolerance attribute will not change as $t$ moves forward. The change of each individual will be in their location $(i, j)$ based on their neighbors. We define a neighborhood of eight cells in which the number of neighbors related to tolerant or intolerant individuals follows:

$$\overline{C}_{ij}^{\text{tol}}(t) = \sum_{\substack{k_{ij}\in\Omega \\ k_{ij}=\text{tol}}} C_{k_{ij}}(t) \quad \text{and} \quad \overline{C}_{ij}^{\text{intol}}(t) = \sum_{\substack{k_{ij}\in\Omega \\ k_{ij}=\text{intol}}} C_{k_{ij}}(t), \tag{1}$$

where $k_{ij}$ is a cell location occupied by an individual in the neighborhood $\Omega$, and $k_{ij}$ shows the attribute of tolerance or intolerance.

Then, the movement of an individual is related to the number of unoccupied cells and the number of similar individuals around it. Therefore, we update each cell as follows:

$$C_{ij}(t+1) = \begin{cases} C_{k_{ij}}(t) & \text{if } C_{k_{ij}}(t) = C_{ij}^{\text{unocc}} \text{ and } \overline{C}_{ij}^{\text{tol}}(t) < \text{tol_cells} \\ & \text{elif } C_{k_{ij}}(t) = C_{ij}^{\text{unocc}} \text{ and } \overline{C}_{ij}^{\text{tol}}(t) = \text{tol_cells} \\ C_{k_{ij}}(t) & \text{if } C_{k_{ij}}(t) = C_{ij}^{\text{unocc}} \text{ and } \overline{C}_{ij}^{\text{intol}}(t) < \text{intol_cells} \\ & \text{elif } C_{k_{ij}}(t) = C_{ij}^{\text{unocc}} \text{ and } \overline{C}_{ij}^{\text{intol}}(t) = \text{intol_cells} \\ C_{ij}(t) & \text{if } C_{k_{ij}}(t) = C_{ij}^{\text{tol}} \text{ or } C_{k_{ij}}(t) = C_{ij}^{\text{intol}} \end{cases}, \tag{2}$$

where $C_{k_{ij}}(t)$ is a neighbor cell; $\overline{C}_{ij}^{\text{tol}}(t)$ and $\overline{C}_{ij}^{\text{intol}}(t)$ are the sum of the values of neighbors related to an individual with tolerance and intolerance attributes, respectively; and tol_cells and untol_cells are parameters for specifying different levels of tolerance and intolerance—that is, high and medium levels. Following the ideas of Thompson et al. [9] and Welsch et al. [5] that there is an immediate discomfort associated with a near-body or extreme close-proximity presence, we describe equation (2) as follows. The first and second situations in equation (2) set the rules of movement and immobility of an individual based on the existence of an unoccupied cell and similar individuals around it. In particular, the first and second rules describe an individual with a tolerance or intolerance attribute that will change their location in the neighborhood or stay in place based on the number of available cells and the number of similar individuals around them. The third rule describes a situation in which an individual does not move due to the lack of empty space around them.

Based on the parameters tol_cells and untol_cells, we can define three types of criteria related to a similar number of surrounding individuals, each with different levels of tolerance or intolerance. If we consider the Schelling model [3] with a mild threshold of 30%—individuals that are highly tolerant of being surrounded by different ones—the dynamics of the system generate a global segregation. In our case, tol_cells and untol_cells are related to the thresholds of 2, 4 and 7 with highly tolerant (between 25% and 30%), midpoint tolerant (50%) and lower tolerant (between 87.5% and 60%) individuals.

The first criterion (Algorithm 1) is related to the medium level of tolerance and intolerance among individuals. This criterion considers the same discomfort associated with the violation of PS for the two

types of individuals. The particular parameters using equation (2) are tol_cells = 4 and untol_cells = 4.

**Algorithm 1.** Rules for moving or staying in a place based on a similar number of tolerant and intolerant individuals.

**Require:** $m$ = matrix
**Ensure:** $C_{ij}$ = Occupied cell
  **if** $C_{ij}$ has unoccupied neighbors $> 1$ **then**
    list neighbors to move
    **if** $C_{ij} == 1$ and similar neighbors $< 4$ **then**
    move new location ▷ Tolerant
    **else if** $C_{ij} == 1$ and similar neighbors $== 4$ **then**
    stay in location
    **else if** $C_{ij} == 0$ and similar neighbors $< 4$ **then**
    move new location ▷ Intolerant
    **else if** $C_{ij} == 0$ and similar neighbors $== 4$ **then**
    stay in location
    **end if**
  **end if**

The second criterion (Algorithm 2) uses different levels of tolerance and intolerance. In particular, we specify tol_cells = 7 for tolerant individuals and untol_cells = 4 for intolerant ones. This situation describes the case of high levels of tolerant individuals who will move their location or stay in place until they are almost caught by neighbors. Meanwhile, there is a medium level of intolerant individuals.

**Algorithm 2.** Rules for moving or staying in a place based on highly tolerant and medium-intolerant cells.

**Require:** $m$ =matrix
**Ensure:** $C_{ij}$ = Occupied cell
  **if** $C_{ij}$ has unoccupied neighbors $> 1$ **then**
    list neighbors to move
    **if** $C_{ij} == 1$ and similar neighbors $< 7$ **then**
    move new location ▷ Tolerant
    **else if** $C_{ij} == 1$ and similar neighbors $== 7$ **then**
    stay in location
    **else if** $C_{ij} == 0$ and similar neighbors $< 4$ **then**
    move new location ▷ Intolerant
    **else if** $C_{ij} == 0$ and similar neighbors $== 4$ **then**
    stay in location
    **end if**
  **end if**

The third criterion (Algorithm 3) is associated with extreme levels of tolerance and intolerance. In particular, we specify tol_cells = 7 for tolerant individuals and untol_cells = 2 for intolerant ones. This

situation describes the case of high levels of tolerant and intolerant individuals.

**Algorithm 3.** Rules for moving or staying in a place based on highly tolerant and highly intolerant cells.
**Require:** $m$ = matrix
**Ensure:** $C_{ij}$ = Occupied cell
**if** $C_{ij}$ has unoccupied neighbors $> 1$ **then**
list neighbors to move
**if** $C_{ij} == 1$ and similar neighbors $< 7$ **then**
move new location ▷ Tolerant
**else if** $C_{ij} == 1$ and similar neighbors $== 7$ **then**
stay in location
**else if** $C_{ij} == 0$ and similar neighbors $< 2$ **then**
move new location ▷ Intolerant
**else if** $C_{ij} == 0$ and similar neighbors $== 2$ **then**
stay in location
**end if**
**end if**

### 4.3 Nearest Neighbors Measure and Density Scenarios

We compute a nearest neighbors measure using a neighborhood of 12 cells to describe the local density of people around an individual [41] (Figure 0). In terms of PS, the nearest neighbors measure shows how well or badly the individual manages their self-perception of comfort or discomfort in a group behavior. For example, if one or two people are around, the individual could well manage their self-perception of comfort or discomfort without feeling threatened. This measure is computed by the function cnt_neighbors (see [33]).

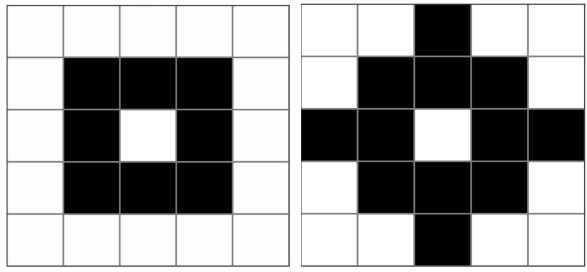



**Figure 2.** PS and the nearest neighbors measure. (a) The neighbors related to the individual. (b) The neighbors associated with computing the nearest neighbors measure.

The nearest neighbors measure can identify dangerous and potentially fatal situations when an individual is jammed by their eight or

12 nearby occupied cells. This measure can provide critical information about possible crushing situations; no matter if the neighbors are similar or dissimilar, if you are jammed by people you are in a dangerous location.

To explore different levels of crowd density, we incorporate different proportions of occupied or unoccupied cells into the initial conditions of a fixed $N \times N$ matrix. As mentioned in Sections 4.1 and 4.2, we consider three types of cell attributes for specifying those proportions: $C_{ij}^{\text{tol}} = 1$, $C_{ij}^{\text{intol}} = 0$ and $C_{ij}^{\text{unocc}} = 3$. Depending on the initial matrix dimensions, we can set the value of each cell randomly based on particular probabilities associated with those cell attributes. Instead of using a uniform distribution with the same probabilities of selecting those attributes, we generate a nonuniform random sample (see [33]). For example, if we generate a $10 \times 10$ matrix based on the values of 0, 1 or 2, we can define a scenario in which there are more occupied cells than unoccupied as (0.4, 0.4, 0.2). The first decimal is the probability of occupied cells with the tolerance attribute, the second decimal is the probability of occupied cells with the intolerance attribute, and the third decimal is the probability of unoccupied cells, ($\text{Occupied}_{\text{tol}}$, $\text{Occupied}_{\text{intol}}$, Unoccupied). Such a tuple of decimals must sum to 1 for purposes of normalization. Then, the tuple (0.4, 0.4, 0.2) defines a density of 80% in a total of 100 cells. Therefore, we assume three density levels that cover the most generative cases: 40% (0.2, 0.2, 0.6), 60% (0.3, 0.3, 0.3) and 80% (0.4, 0.4, 0.2).

### 4.4 Data Analysis and Model Validation

Based on Section 4.3, we apply an exploratory data analysis to validate the model [16, 42]. In particular, the exploratory data analysis searches for trends and relationships between our nearest neighbors measure and the density scenarios to explain public perceptions and scientific knowledge about the crushing events. For example, how accurate is the public perception of an increasing concern about the dangers of overcrowded events? and how accurate is our local measure for describing harmful effects of overcrowded events? To determine whether our theoretical model describes the presence of crushing events, we evaluate the dynamics of the nearest neighbors measure based on different sets of initial conditions (not only the density scenario, but also the tolerance or intolerance parameters in the transitional rules).

We propose a conceptual model validation in which the theory and assumptions generate consistent and justifiable results [17]. To validate our model, we use a probabilistic framework based on a Monte Carlo approximation to look for statistical regularities associated with the initial conditions. The presence of such regularities can validate the initial conditions when the model output agrees with the

observed data of crushing events. Therefore, our model validation process supports our conclusions.

In particular, a sensitivity analysis determines the relative influence of the initial conditions and the parameters on the model output. We are interested in exploring which combination of those parameters is related to the current public perceptions and scientific knowledge of crushing events. To do this, we use the following procedure (Figure 3).

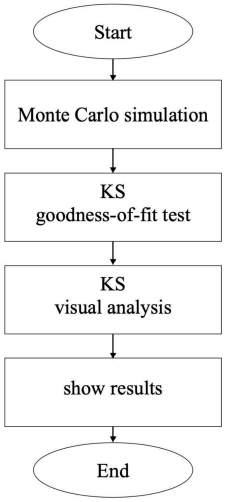



**Figure 3.** Flow chart of the model validation.

We first apply a Monte Carlo simulation to test how probable the presence of a crushing event is in certain density situations. The number of trials is set at 10 000. Then, we present a best-fit analysis of the density levels and different parameters associated with group behaviors to identify the statistical distribution that best describes the data generated. In particular, this second process uses the Kolmogorov–Smirnov (KS) goodness-of-fit test to identify the statistical distributions that best describe the data [43]. This test compares each realization of data with a set of theoretical distributions. In this respect, we follow the procedure in Lugo and Martinez-Mecker [18] and Lugo and Alatriste-Contreras [19] and use the normal, log-normal, exponential, Pareto, Gilbrat, power law and exponentiated Weibull as the main theoretical distributions. Then, we review the KS test statistics and their visual representations, which are based on the empirical and theoretical cumulative density functions of a sample. Finally, we report our results.

https://doi.org/10.25088/ComplexSystems.35.3.237

### 4.5 Initial Conditions

Based on the given processes, we explore different initial conditions related to density levels and the tolerance or intolerance parameters in the transitional rules. For simplicity, the total number of cells is set by a matrix with a fixed dimension of (50, 50), that is 2500 cells. Recall that this matrix shows a closed boundary that represents physical obstacles to free movement. It is important to note that this analysis can accurately represent increasing scales. Therefore, there is a consistency between our theoretical assumptions, the generated synthetic data, the public perceptions and scientific knowledge about the crushing events.

As mentioned earlier, the following set of tuples is related to different scenarios based on the proportions of tolerant, intolerant and unoccupied cells, respectively: (0.33, 0.33, 0.34), (0.2, 0.2, 0.6) and (0.4, 0.4, 0.2). The first scenario has a similar proportion of occupied and unoccupied cells, the second has decreasing and similar proportions of occupied cells but an increased proportion of unoccupied cells, and the third has increasing and similar occupied cells but a decreasing proportion of unoccupied cells.

In addition to these density levels and based on the three criteria mentioned in Section 4.2, we are interested in the nature of relationships between individuals in a group. We explore three sets of parameters related to tolerant and intolerant individuals. In the first set, the discomfort or comfort associated with the violation of the PS is the same for tolerant or intolerant individuals. In the second set, tolerant individuals show high levels of comfort associated with the violation of the PS. The third set shows a presence of high levels of comfort and discomfort associated with tolerant and intolerant individuals, respectively.

Finally, the number of iterations in each realization is fixed to 60, as a reference to time in minutes. In particular, we assume that crushing events could happen within a range of 60 minutes, in a window of 15 minutes [41, 44, 45].

## 5. Results

In this section, we present our findings and display graphical and statistical results based on the model validation. We use sensitivity analysis to explore the dynamics of the nearest neighbors measure based on different sets of initial conditions and parameters. These sets identify the current public perceptions and scientific knowledge about the importance of being tolerant or intolerant to similar neighbors and the consequences of interpersonal distance in overcrowding events. The figures display the results that draw attention to the most representative of cases.

Figure 4 shows the general results of the sensibility analysis that point out the importance of density situations over group behaviors. It seems that variations in density levels considerably affect the dynamics that generate crushing events. For example, if we select Figure 4(a), which is characterized by the similar behavior of individuals, we can see that increasing the proportion of unoccupied cells affects the nearest neighbors measure showing overcrowding values. That is, the first entry in this figure shows the lowest values of the nearest neighbors measure. This indicates that if we control the number of people in a crowd and give more space in which people can move with out jamming, the probability of generating crushing is lower (see Figure 5).

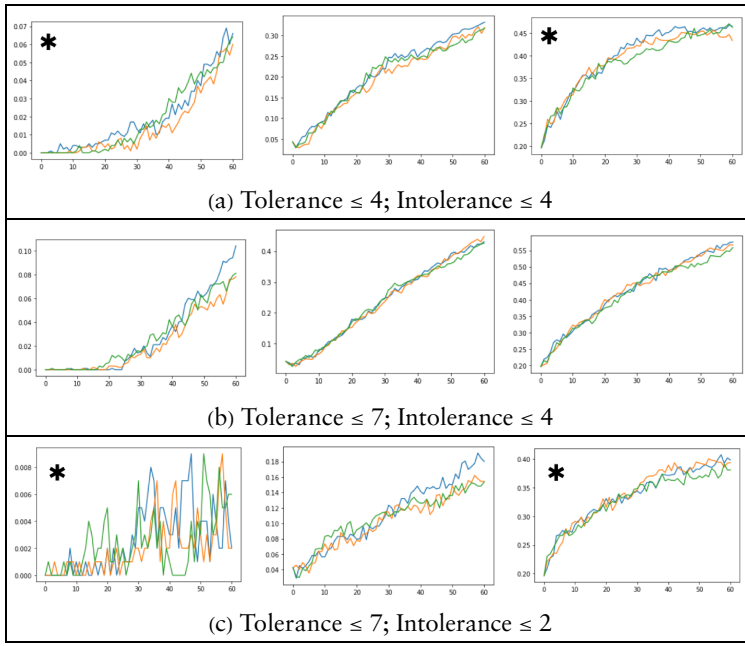



**Figure 4.** Sensitivity analysis. The random seeds were set to obtain the same initial values of the matrix. To exemplify the results of a total of 10 000 realizations, each subfigure displays three realizations per case. Subfigures in rows show variations in the group behaviors (tolerance and intolerance parameters), and columns show variations in the proportion of occupied and unoccupied cell parameters (density levels). The $x$ axis in each subplot shows the number of iterations, and the $y$ axis shows the values of the nearest neighbors measure based on cells of 12 surrounding neighbors. The asterisk boxes represent the cases that we use for an in-depth analysis in Figures 5 and 6.

Now, looking at Figure 4(c), which is characterized by dissimilar behaviors of individuals—that is, extremist positions of tolerant or

intolerant individuals—we can see a sharp drop in the proportion of cells with high values of the nearest neighbors measure. This result is particularly important because it suggests that even if there is an extreme preference for being surrounded by others, the drastic position of being intolerant produces lower values of the nearest neighbors measure. That is, the crushing events are less probable (see Figure 5).

Columns 2 and 3 in Figure 4 display the presence of unstoppable crushing events. All of these cases show a marked increase in values of the nearest neighbors measure. Even if there is a similar density between occupied and unoccupied cells, as well as the presence of extremist preferences of being tolerant or intolerant, the values of the nearest neighbors measure rapidly and dangerously increase. These results suggest that crushing events are more probable than we had thought at the beginning (see Figure 5).

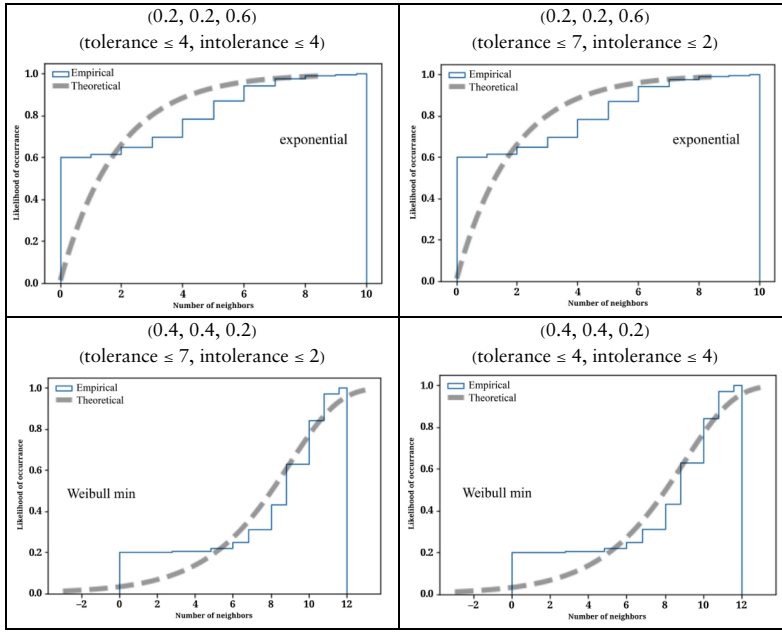



**Figure 5(a).** Minute 0. Best-fit analysis of four selected cases of the sensitivity analysis at a particular given time step. Subfigures are the visual results of the KS test (see Table 2 for the probability density functions, PDF). These results are based on a Monte Carlo simulation of a total of 10 000 realizations per case. The subfigures show a particular time in minutes for selected cases of the sensitivity analysis. The $x$ axis in each subplot shows the value of the nearest neighbors measure, and the $y$ axis shows the likelihood of particular values of the nearest neighbors measure.

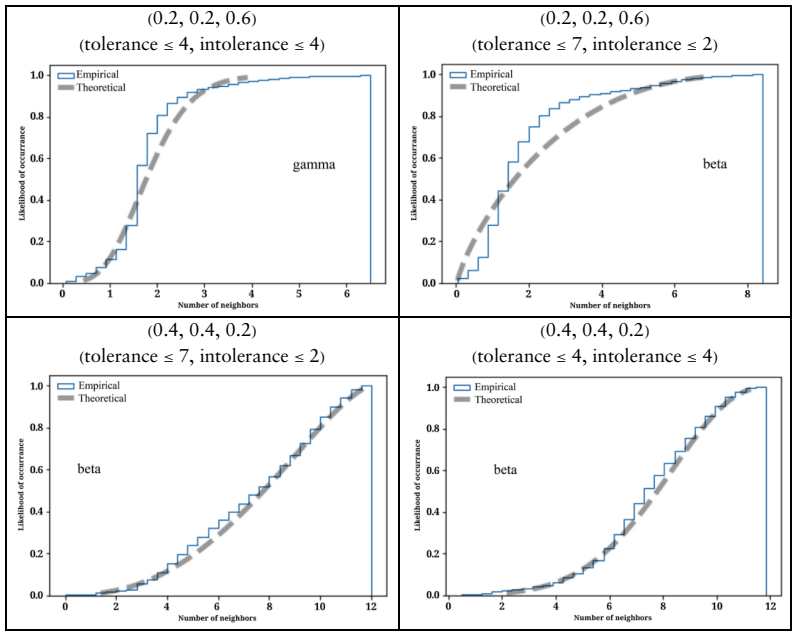



**Figure 5(b).** Minute 15.

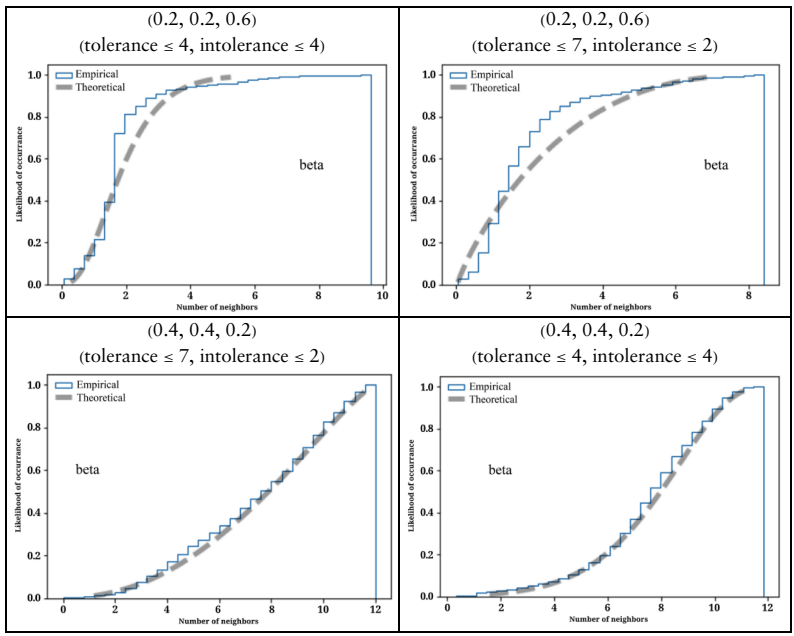



**Figure 5(c).** Minute 30.

https://doi.org/10.25088/ComplexSystems.35.3.237

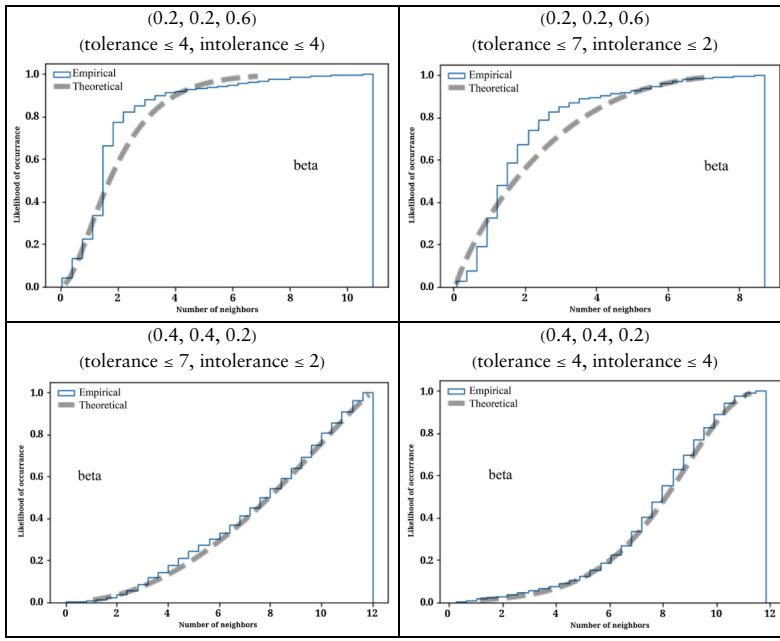



**Figure 5(d).** Minute 45.

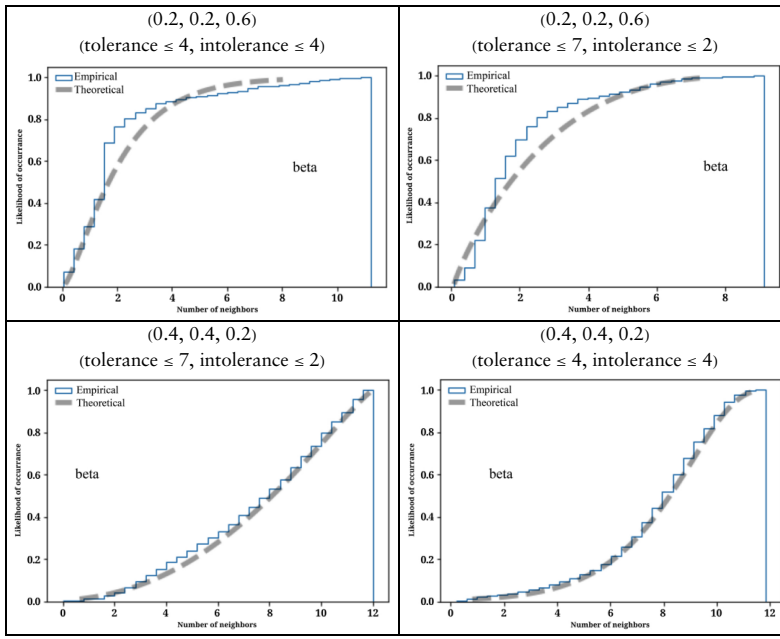



**Figure 5(e).** Minute 60.

| (0.2, 0.2, 0.6) (tolerance ≤ 4, intolerance ≤ 4) | (0.2, 0.2, 0.6) (tolerance ≤ 7, intolerance ≤ 2) | (0.4, 0.4, 0.2) (tolerance ≤ 7, intolerance ≤ 2) | (0.4, 0.4, 0.2) (tolerance ≤ 4 intolerance ≤ 4) |
|---|---|---|---|
| Minute 0 | | | |
| (0.0210, 0.2128) (0.0, 1.8592) (1.2886, 1.8592, 3.4566, 2.0, 6.0) | (0.0210, 0.2128) (0.0, 1.8592) (1.2886, 1.8592, 3.4566, 2.0, 6.0) | (0.0210, 0.2128) (118 250 608.8906, −307 748 410.8082, 307 748 419.8257) (8.0636, 7.5152, −21.0296, nan, nan) | (0.0210, 0.2128) (118 250 608.8906, −307 748 410.8082, 307 748 419.8257) (8.0636, 7.5152, −21.0296, nan, nan) |
| Minute 15 | | | |
| (0.0209, 0.2193) (10.8212, −0.6161, 0.2276) (1.7721, 1.8475, 0.5609, 0.6079, 0.5544) | (0.0128, 0.7947) (0.8075, 2.4988, 0.0369, 8.3864) (1.6323, 2.0853, 3.0147, 0.9311, 0.1429) | (0.0105, 0.9415) (2.5750, 1.5102, −0.1535, 12.1656) (7.7941, 7.5147, 6.7816, −0.4001, −0.6405) | (0.0125, 0.8187) (6.7570, 2.8738, −2.8736, 14.8900) (7.7877, 7.5731, 4.3662, −0.4940, −0.1378) |
| Minute 30 | | | |
| (0.0163, 0.5104)(3.4379, 173 936 740 089.8359, −0.0697, 101 457 801 056.7912) (1.7448, 1.9355, 1.1697, 1.0786, 1.7452) | (0.0152, 0.6034) (0.8611, 2.5301, 0.0497, 8.3639) (1.7475, 2.1736, 3.0180, 0.8789, 0.0387) | (0.0167, 0.4804) (2.2461, 1.2968, −0.1105, 12.1137) (7.9017, 7.5692, 7.4954, −0.4277, −0.6844) | (0.0124, 0.8264) (13.0214, 2.9999, −10.9775, 22.9707) (7.9944, 7.6918, 4.7176, −0.7341, 0.4505) |
| Minute 45 | | | |
| (0.0149, 0.6226)(1.9387, 8 943 638 380.0540, 0.0211, 9 405 569 749.6733) (1.7222, 2.0600, 2.1442, 1.4363, 3.0947) | (0.0124, 0.8298) (0.8583, 2.7209, 0.0592, 8.7588) (1.7163, 2.1598, 3.0541, 0.9348, 0.1998) | (0.0123, 0.8328) (2.0921, 1.1966, −0.0934, 12.0948) (7.9657, 7.6006, 7.8948, −0.4432, −0.7062) | (0.0124, 0.8309) (32.6266, 3.1779, −35.5496, 47.5424) (8.1350, 7.7730, 4.9671, −0.9281, 1.1044) |
| Minute 60 | | | |
| (0.0187, 0.3414)(1.5031, 2 665 795 063 131.341, 0.0242, 3 759 673 881 016.624) (1.6969, 2.1441, 2.9897, 1.6313, 3.9917) | (0.0161, 0.5243) (0.8786, 2.2031, 0.0679, 9.2604) (1.7496, 2.1611, 3.1623, 0.9658, 0.3128) | (0.0147, 0.6449) (2.4231, 1.2023, −1.0739, 13.0755) (8.1027, 7.6653, 8.1927, −0.5469, −0.5246) | (0.0146, 0.6524) (6159.2526, 3.3250, −7681.2806, 7693.2654) (8.2414, 7.8339, 5.1782, −1.0956, 1.7994) |

**Table 1.** KS test, estimated parameters and first moments of the selected cases. KS test: (stat, p-value), estimated parameters based on the KS test: (parameter1, loc, scale), first moments: (median, mean, variance, skewness, kurtosis). These results are based on a Monte Carlo simulation of a total of 10 000 realizations per case.

| Name | PDF |
|---|---|
| exponential | $f(x) = \exp(-x)$, for $x \geq 0$ |
| gamma | $f(x, a) = \frac{x^{a-1}e^{-x}}{\Gamma(a)}$, for $x \geq 0$ and $a > 0$ |
| beta | $f(x, a, b) = \frac{\Gamma(a+b)x^{a-1}(1-x)^{b-1}}{\Gamma(a)\Gamma(b)}$, for $0 \leq x \leq 1$, $a > 0$, $b > 0$ |
| Weibull minimum | $f(x, c) = cx^{c-1}\exp(-x^c)$, for $x > 0$, $c > 0$ |

**Table 2.** Statistical distributions of the Kolmogorov–Smirnov (KS) test and their probability density functions (PDF). Name of the statistical distributions and their PDF used for the KS test.

With respect to our fourth particular case, we can see the behavior of the nearest neighbors measure in different time periods. Figures 5 and 6 and Table 1 display the KS test per time period related to the nearest neighbors measure. Figure 5 reveals that there is a clear difference between the best-fit statistical distributions of the initial conditions and the rest of the periods. It suggests a differentiated

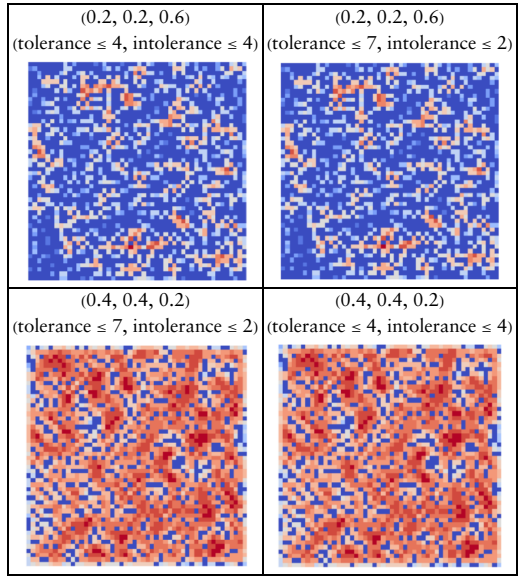



**Figure 6(a).** Minute 0. Nearest neighbors measure of selected cases of the sensitivity analysis at a particular time. We use the matplotlib "coolwarm" palette to display the result. From dark blue to bright red, we can see the range of zero to 12 neighbors. These results are based on a Monte Carlo simulation of a total of 10 000 realizations per case. Subfigures show a particular time in minutes for selected cases of the sensitivity analysis.

behavior of individuals that produces a marked transition between statistical distributions. Furthermore, Figure 5 shows that there has been a marked presence of the beta distribution in most of the time periods. The preponderance of these results suggests the presence of highly skewed statistical distributions behind this type of collective behavior.

Finally, we emphasize that the case with the lowest probability of generating crushes is when there are dissimilar behaviors of individuals and a low density of unoccupied cells. This result is clearly displayed in row one, column two of Figures 5 and 6. This particular case shows a few dispersed clusters of individuals in all periods of time, suggesting that this particular situation delays the onset of crushing events.

Therefore, it seems that the density of unoccupied cells matters the most. An increased proportion of unoccupied cells or a lower density is preferable to an increased proportion of individuals with extreme preferences among them.

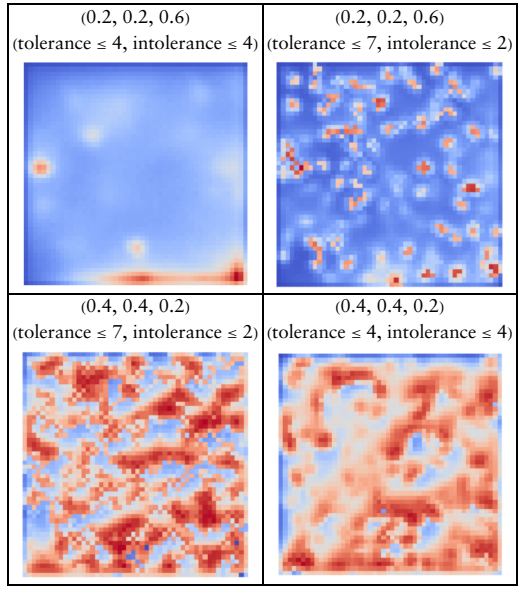



**Figure 6(b).** Minute 15.

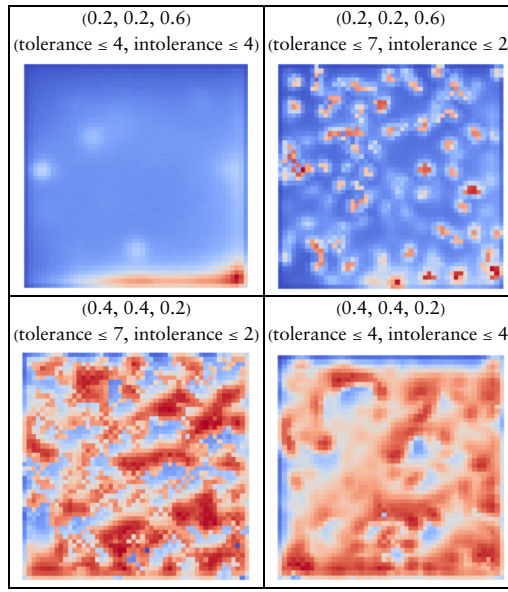



**Figure 6(c).** Minute 30.

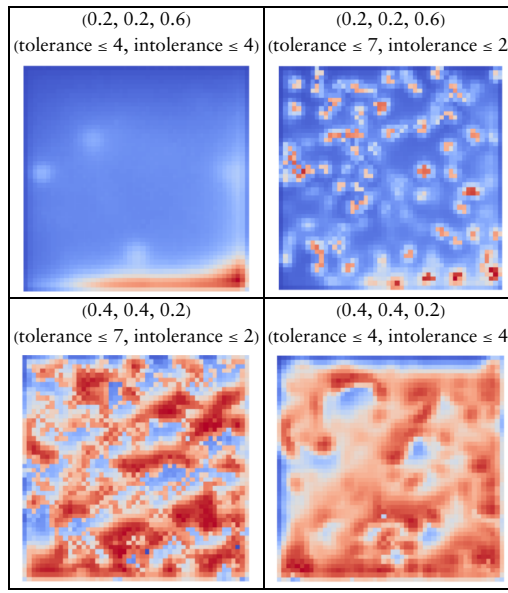



**Figure 6(d).** Minute 45.

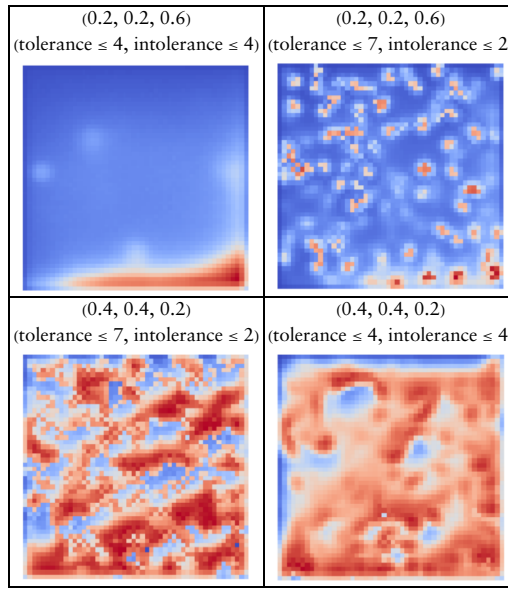



**Figure 6(e).** Minute 60.

## 6. Discussion

As stated in Section 1, we want to identify situations in which a crushing event is unstoppable, to show the probability of crushing events and to differentiate the group behavior between tolerant and intolerant individuals surrounded by similar neighbors. Based on our findings, we reach the following interpretations.

We found that the initial density, rather than the group behaviors of individuals that are surrounded by similar neighbors, can turn densely crowded situations into highly probable unstoppable crushing events. The number of unoccupied cells can diminish the presence of crushing events. Meanwhile, the group behavior of tolerant and intolerant individuals conditions the development speed of crushing events. Contrary to expectations, the sole presence of extreme preferences of being tolerant or intolerant does not ensure effective control of crushing incidents in densely crowded situations. Therefore, our hypothesis is partially true. This implies that differences in individual preferences for being surrounded by people in near-body proximity may explain the generation speed of crushes. But, unfortunately they cannot prevent or stop them.

Our theoretical results allow us to make predictions of probable scenarios. For example, our main findings have implications for understanding the dynamics behind densely crowded events. In particular,

independently of the type and scale of social gatherings, the presence of uncontrollable factors can cause severe personal injuries. Social gatherings, indoor or outdoors, can represent the perfect situation for a fatal crushing event. In this respect, controlling the density could be the only way to prevent risky situations. Based on the sensitivity analysis, we showed that variations in the density parameter can generate integrated and controlled crowd scenarios or segregated and crushing patterns. Therefore, we suggest the following actionable guidelines for crowd control professionals and nonprofessionals:

- To inform assistants about the importance of PS and crowd density for preventing and handling risky situations.
- To inform assistants to look out every 15 minutes for any sign of stressful or chaotic behavior around them.
- To organize an event in which the crowd density can be closed to 40% of individuals and 60% of free space.
- To organize an event in flat areas with a smaller number of physical obstacles.

It is worth mentioning that our results are based on a computational irreducibility approach instead of the well-established force-based or pedestrian model. The pedestrian models study crowd movements using classical mechanics—that is, systems of equations to determine pedestrian motion. However, these models are based on a reducibility approach, in which the output can be predicted or calculated. One of its main assumptions but, at the same time, one of its disadvantages, is the presence of a representative agent that assumes that all individuals behave identically. Therefore, we consider that computational irreducibility is a better approach for determining final outputs of complex processes—characterized by dependency to the trajectory, emergence, nonlinearity, decentralized control, adaptability and evolution—related to crowd disasters, because it simulates or performs the entire process. In particular, the fact that well-established pedestrian models do not consider exits, obstacles or directional goals in a flat environment is a limitation. In contrast, our computational irreducibility approach has the advantage that it can capture fundamental integrated and segregated patterns without explicitly setting those obstacles in the code.

With specific reference to the computational efficiency or scalability of our theoretical model, we observe stability in the processing time of each execution per scenario. In particular, if the size of the array increases, the processing time increases. Despite these variations of the initial matrices, from small to large scales, the dynamics of the system remain similar. Therefore, our model shows consistency between the public perception and our simple modeling of crushing

disasters. Consequently, our model is useful for the purpose of describing the effects of PS in densely crowded scenarios.

Despite these promising results, questions remain open. In particular, what are the possible cultural drivers associated with crushing scenarios? how do the daily interpersonal interactions affect our natural instinct for self-preservation in different crowd situations? what are the effective practices for communicating the risks of social gatherings? how can the current technology of mobile applications prevent possible crushing events in social gatherings? Future work is needed to generate an application for predicting fatal situations. We are considering exploring the generation of a warning signal in which every mobile device can be used to prevent fatal crushing. Similar to the warning signal of earthquakes, it is possible to generate a mobile application that indicates a warning signal of probable crushing events in real time.

In addition, it is important to generate simple and generative models that consider the presence of external factors, such as weather, time of day and event time. These factors can explore the effects of the generation speed of crushing events.

### 6.1 Conclusion

Based on the Schelling model of segregation, crowd crushing events in social gatherings might show a high probability of occurrence. In this context, personal space (PS) is a key variable for modeling densely crowded situations. Meanwhile, density can be considered the core element for preventing and handling risk situations in extremely dense crowds. The best empirical situation related to our theoretical results could be having enough space to ensure that the individual PS of attendees is always free in a social gathering. This situation could reduce the risk of crushing events. If this condition is not feasible, there are high probabilities of harmful incidents and fatal crushing events. Therefore, every person should be aware of the high risk of attending events in extremely dense crowds.

## References

[1] D. Helbing, A. Johansson, J. Mathiesen, M. H. Jensen and A. Hansen, "Analytical Approach to Continuous and Intermittent Bottleneck Flows," *Physical Review Letters*, **97**(16), 2006 168001. doi:10.1103/PhysRevLett.97.168001.

[2] D. Helbing, A. Johansson and H. Z. Al-Abideen, "Dynamics of Crowd Disasters: An Empirical Study," *Physical Review E*, **75**(4), 2007 046109. doi:10.1103/PhysRevE.75.046109.

[3] T. C. Schelling, *Micromotives and Macrobehavior*, New York: Norton, 1978.

[4] D. Silver, U. Byrne and P. Adler, "Venues and Segregation: A Revised Schelling Model," *PLoS One*, **16**(1), 2021 e0242611. doi:10.1371/journal.pone.0242611.

[5] R. Welsch, C. von Castell and H. Hecht, "The Anisotropy of Personal Space," *PLoS One*, **14**(6), 2019 e0217587. doi:10.1371/journal.pone.0217587.

[6] R. Gifford, "Environmental Psychology," *Encyclopedia of Human Behavior*, 2nd ed. (V. S. Ramachandran, ed.), San Diego: Academic Press, 2012 pp. 54–60. doi:10.1016/B978-0-12-375000-6.00150-6.

[7] D. E. Battle, "Communication Disorders in a Multicultural and Global Society," *Communication Disorders in Multicultural and International Populations*, 4th ed. (D. E. Battle, ed.), St. Louis: Mosby, 2012 pp. 1–19. doi:10.1016/B978-0-323-06699-0.00010-8.

[8] M. Bonnes and G. Carrus, "Environmental Psychology, Overview," *Encyclopedia of Applied Psychology* (C. Spielberger, ed.), Amsterdam: Elsevier, 2004 pp. 801–814. doi:10.1016/B978-0-12-809324-5.05554-1.

[9] D. E. Thompson, J. R. Aiello and Y. M. Epstein, "Interpersonal Distance Preferences," *Journal of Nonverbal Behavior*, **4**(2), 1979 pp. 113–118. doi:10.1007/BF01006355.

[10] D. Helbing, M. Isobe, T. Nagatani and K. Takimoto, "Lattice Gas Simulation of Experimentally Studied Evacuation Dynamics," *Physical Review E*, **67**(6), 2003 067101. doi:10.1103/PhysRevE.67.067101.

[11] S. D. Reicher, "The St. Pauls' Riot: An Explanation of the Limits of Crowd Action in Terms of a Social Identity Model," *European Journal of Social Psychology*, **14**(1), 1984 pp. 1–21. doi:10.1002/ejsp.2420140102.

[12] D. E. Battle, ed., *Communication Disorders in Multicultural Populations*, 2nd ed., Boston: Butterworth-Heinemann, 1998.

[13] K. Y. L. Ku, S. Phillipson and S. N. Phillipson, "Educational Learning Theory," *International Encyclopedia of the Social & Behavioral Sciences*, 2nd ed., (J. D. Wright, ed.), Elsevier, 2015 pp. 238–245. doi:10.1016/B978-0-08-097086-8.92150-0.

[14] S. Wolfram, "A Class of Models with the Potential to Represent Fundamental Physics," *Complex Systems*, **29**(2), 2020 pp. 107–536. doi:10.25088/ComplexSystems.29.2.107.

[15] S. Wolfram, *A New Kind of Science*, Champaign, IL: Wolfram Media, Inc., 2002.

[16] R. G. Sargent, "Simulation Model Validation," in *Simulation and Model-Based Methodologies: An Integrative View*, (T. I. Ören, B. P. Zeigler and M. S. Elzas, eds.), Heidelberg: Springer, 1984 pp. 537–555.

[17] L. A. Kerr and D. R. Goethel, "Simulation Modeling as a Tool for Synthesis of Stock Identification Information," *Stock Identification Methods*, 2nd ed., (S. X. Cadrin, L. A. Kerr and S. Mariani, eds.), San Diego: Academic Press, 2014 pp. 501–533. doi:10.1016/B978-0-12-397003-9.00021-7.

[18] I. Lugo and G. Martínez-Mekler, "Theoretical Study of the Effect of Ports in the Formation of City Systems," *Journal of Shipping and Trade*, **7**(1), 2022 16. doi:10.1186/s41072-022-00117-6.

[19] I. Lugo and M. G. Alatriste-Contreras, "Intervention Strategies with 2D Cellular Automata for Testing SARS-CoV-2 and Reopening the Economy," *Scientific Reports*, **12**(1), 2022 13481. doi:10.1038/s41598-022-17665-3.

[20] S. Yang, T. Li, X. Gong, B. Peng and J. Hu, "A Review on Crowd Simulation and Modeling," *Graphical Models*, **111**, 2020 101081. doi:10.1016/j.gmod.2020.101081.

[21] H.-T. Dang, B. Gaudou and N. Verstaevel, "A Literature Review of Dense Crowd Simulation," *Simulation Modelling Practice and Theory*, **134**, 2024 102955. doi:10.1016/j.simpat.2024.102955.

[22] R. E. Klosterman, "Simple and Complex Models," *Environment and Planning B: Planning and Design*, **39**(1), 2012 pp. 1–6. doi:10.1068/b38155.

[23] K. Lewin, "The Research Center for Group Dynamics at Massachusetts Institute of Technology," *Sociometry*, **8**(2), 1945 pp. 126–136. doi:10.2307/2785233.

[24] K. L. Dion, "Group Cohesion: From 'Field of Forces' to Multidimensional Construct," *Group Dynamics: Theory, Research, and Practice*, **4**(1), 2000 pp. 7–26. doi:10.1037/1089-2699.4.1.7.

[25] C. C. Ioannou and K. L. Laskowski, "A Multi-scale Review of the Dynamics of Collective Behaviour: From Rapid Responses to Ontogeny and Evolution," *Philosophical Transactions of the Royal Society B: Biological Sciences*, **378**(1874), 2023 20220059. doi:10.1098/rstb.2022.0059.

[26] D. J. T. Sumpter, "The Principles of Collective Animal Behaviour," *Philosophical Transactions of the Royal Society B: Biological Sciences*, **361**(1465), 2006 pp. 5–22. doi:10.1098/rstb.2005.1733.

[27] E. Üsten, H. Lügering and A. Sieben, "Pushing and Non-pushing Forward Motion in Crowds: A Systematic Psychological Observation Method for Rating Individual Behavior in Pedestrian Dynamics," *Collective Dynamics*, **7**, 2022 pp. 1–16. doi:10.17815/CD.2022.138.

[28] A. Nakayama, K. Hasebe and Y. Sugiyama, "Instability of Pedestrian Flow and Phase Structure in a Two-Dimensional Optimal Velocity Model," *Physical Review E*, **71**(3), 2005 036121. doi:10.1103/PhysRevE.71.036121.

[29] M. Batty, *Cities and Complexity: Understanding Cities with Cellular Automata, Agent-Based Models, and Fractals*, Cambridge, MA: MIT Press, 2005.

[30] M. Fukui and Y. Ishibashi, "Self-Organized Phase Transitions in Cellular Automaton Models for Pedestrians," *Journal of the Physical Society of Japan*, **68**(8), 1999 pp. 2861–2863. doi:10.1143/JPSJ.68.2861.

[31] M. Muramatsu and T. Nagatani, "Jamming Transition in Two-Dimensional Pedestrian Traffic," *Physica A: Statistical Mechanics and Its Applications*, **275**(1), 2000 pp. 281–291. doi:10.1016/S0378-4371(99)00447-1.

[32] C. Burstedde, K. Klauck, A. Schadschneider and J. Zittartz, "Simulation of Pedestrian Dynamics Using a Two-Dimensional Cellular Automaton," *Physica A: Statistical Mechanics and Its Applications*, **295**(3–4), 2001 pp. 507–525. doi:10.1016/S0378-4371(01)00141-8.

[33] I. Lugo and M. G. Alatriste-Contreras. "Crowd Behavior and Social Gatherings." Open Science Framework (OSF). (Aug 24, 2026) doi:10.17605/OSF.IO/DHGUJ.

[34] D. G. Janelle, "Time–Space Modeling," *Encyclopedia of Social Measurement* (K. Kempf-Leonard, ed.), New York: Elsevier, 2005 pp. 851–856. doi:10.1016/B0-12-369398-5/00347-9.

[35] N. H. Packard and S. Wolfram, "Two-Dimensional Cellular Automata," *Journal of Statistical Physics*, **38**(5), 1985 pp. 901–946. doi:10.1007/BF01010423.

[36] S. Reicher and J. Drury, "Collective Identity, Political Participation, and the Making of the Social Self," *Identity and Participation in Culturally Diverse Societies: A Multidisciplinary Perspective* (A. E. Azzi, X. Chryssochoou, B. Klandermans and B. Simon, eds.), Oxford, England: Wiley-Blackwell, 2010 pp. 158–176. doi:10.1002/9781444328158.ch8.

[37] A. Templeton, J. Drury and A. Philippides, "From Mindless Masses to Small Groups: Conceptualizing Collective Behavior in Crowd Modeling," *Review of General Psychology*, **19**(3), 2015 pp. 215–229. doi:10.1037/gpr0000032.

[38] L. A. Hayduk, "Personal Space: An Evaluative and Orienting Overview," *Psychological Bulletin*, **85**(1), 1978 pp. 117–134. doi:10.1037/0033-2909.85.1.117.

[39] L. S. Vygotsky, *Mind in Society: The Development of Higher Psychological Processes* (M. Cole, ed.), Cambridge, MA: Harvard University Press, 1978.

[40] H. A. Simon, "A Behavioral Model of Rational Choice," *The Quarterly Journal of Economics*, **69**(1), 1955 pp. 99–118. doi:10.2307/1884852.

[41] M. Moussaid, D. Helbing and G. Theraulaz, "How Simple Rules Determine Pedestrian Behavior and Crowd Disasters," *Proceedings of the National Academy of Sciences*, **108**(17), 2011 pp. 6884–6888. doi:10.1073/pnas.1016507108.

[42] J. T. Leek and R. D. Peng, "What Is the Question?," *Science*, **347**(6228), 2015 pp. 1314–1315. doi:10.1126/science.aaa6146.

[43] F. J. Massey, Jr., "The Kolmogorov–Smirnov Test for Goodness of Fit," *Journal of the American Statistical Association*, **46**(253), 2020 pp. 68–78. doi:10.1080/01621459.1951.10500769.

[44] *The Times of Israel* (2021). "Eleven Minutes at Meron: Video Shows How Deadly Crush Unfolded (May 6, 2021) www.timesofisrael.com/eleven-minutes-at-meron-video-shows-how-deadly-crush-unfolded.

[45] O. Slow, "South Korea: How the Halloween Tragedy Unfolded." BBC. (Sep 2, 2026) www.bbc.com/news/world-63448040.