

Hello, everyone. Welcome to another episode of Q&A about science and technology for kids and others. I think we haven't done one of these for a little while. We have a number of questions saved up, but happy to get lots more.

So I see a question here from Elsa.

Is the three-body problem really unsolvable, or just computationally irreducible? Okay, so lots of things to define here.

What is the three-body problem?

It has to do with... you have three

Planets, stars, other things that have gravity between them.

And they're orbiting each other, moving around, their forces of gravity attracting these things, planets, stars, whatever, to each other. And the question is, how do they move?

So the classic example of this is the Earth-Moon-Sun combination. The Earth is basically just going around the Sun, but the Moon is going around the Earth, and there's gravity affecting the Moon from the Sun, as well as from the Earth.

And so on. And so, this was a problem that was originally kind of, started to be thought about in the 1600s, and, people... it was kind of a top mathematical physics problem in the 1800s to solve the three-body problem.

And what does it mean to solve it? Well, the two-body problem of just a single planet orbiting a single star

that problem can be, sort of, solved in terms of somewhat easy mathematics. And the answer is that the planet will always move around if it... assume it's an idealized star, an idealized planet.

So how does that work? Well, assume that the star and the planet are both of zero size, they have a certain mass, and then there's the inverse square law of gravitation that says that the force exerted by gravity between two objects of masses M_1 and M_2 , let's say, is M_1 times M_2 divided by,

R squared, the distance between them squared, multiplied by a thing called the gravitational constant, which just has a fixed value.

So, the force of attraction is proportional to the 1 over the square of the distance between the objects.

And you would think, well, so in the case of the two-body problem, it's possible to solve the equations that describe the motion of that deal with momentum and angular momentum and that force of gravitational attraction.

The result of solving that problem is the planet moves in an ellipse around the star.

and an ellipse. So in a circle, you have just one point in the middle, and let's say you have a piece of string attached to a peg in the middle, you could make a circle by just going around with that one piece of string.

An ellipse, instead of having one center, you have two central foci.

foci of the ellipse. And if you put a piece of string that is attached

to one focus and the other focus, and then you're sort of having... you make kind of a V shape, and you have your pencil at the tip of the V, and you move your pencil around. That will make an ellipse.

As you move the foci together so that they coincide, that will correspond to a circle. As you move the foci apart, you increase the eccentricity of the ellipse, basically how squashed the ellipse is.

So, in the case of our solar system, for example, most of the planets in our solar system orbit the Sun in pretty close to circular orbits. They have a little bit of eccentricity, not too much.

Pluto, which was sort of rejected as being a planet about a couple of decades ago now, Pluto has a much more... much higher eccentricity orbit. It gets, if you had a circular orbit with zero eccentricity, you'd always be the same distance away from the Sun.

But, if you have a... the more eccentric your orbit is, the more variation there is from the so-called perihelion, the closest approach to the Sun, to the aphelion, the furthest distance from the Sun. Those become more different.

So...

In any case, the two-body problem of how does an idealized planet orbit an idealized star, that problem was solved basically in the late 1600s, and the solution is the planet goes in an ellipse. Okay, so how hard can it be to add in a third object?

let's say Earth, Moon, Sun, where you've got, you know, a planet and its moon, or you have two stars, and a planet orbiting both stars, or all kinds of configurations.

The... from the point of view of gravity, it doesn't really matter what the things are, all that matters is their mass, and then you're calculating this gravitational force that's proportional to 1 over the square of the distance between the things.

Okay, so, how do you solve the three-body problem? What does it even mean to solve the three-body problem? Can one give a description that's like saying, oh, the things don't move in an ellipse, they move in a something or other thing?

Where that's something or other thing, you can easily work out what that will produce any time in the future. So, if you know that the planet's orbiting in an ellipse, you can just say, okay, a million revolutions from now, where's the planet going to be? Well, you just know it's going to be orbiting around in this ellipse over and over again, and it's easy to work that out.

What happens with the three-body problem is if you solve the equations of the three-body problem, you'll find that sometimes rather simple things happen, like, for example, one of the objects will be ejected, and the other two will just

be like a two-body problem, just orbiting around each other. But a thing that can happen quite often is you make a picture of what the trajectories of these things look like, and it's a big tangle of wool. The thing is kind of wiggling around and going all over the place for a very long time. Eventually, there's some outcome. For example, the three objects just go off and neither... none of them orbit each other, they just independently go off to infinity. Or another possibility, as I mentioned, is two of them are orbiting, one of them is ejected. There are a variety of possibilities like that.

So...

what would it mean to solve the three-body problem? It would mean if I tell you the masses of these objects, and I tell you how they start off, where they are when they start, what direction their velocity is going in, what direction they're moving in, then to solve the three-body problem would be to say, I can basically give you a formula for what will happen forever.

In this configuration of objects.

Okay, so people tried to figure out how to do that, and it turned out to be really difficult. And basically, well, so one of the big approaches was to find what are called constants of the motion.

So, in, when...

What is a constant of the motion? It means, for example, momentum is conserved. So if you have different objects, the momentum is roughly the mass of the object times its velocity.

That when you have, objects colliding and combining and so on, the momentum is conserved.

And so, similarly, when you have objects in orbit and so on, the, essentially, the angular momentum is conserved. That's the... that's the momentum associated with, kind of, circular motion.

Similarly, the energy, the total energy of the system is conserved.

And it turns out that in the case of the two-body problem, there are enough things that are conserved that you can kind of nail down the fact that the thing has to follow some simple orbit. There's a... in addition to energy and angular momentum, there's a third more exotic, thing that's conserved in the two-body problem, it's usually called the Runge-Lenz vector. And what it is, is this. If you have a two-body system.

Where...

that one body is, you know, the planet is orbiting the star, and the planet is following an elliptical trajectory as it has to around the star. The big question is, does that ellipse stay fixed in space, every revolution, just retracing its steps, or does the ellipse process around?

So that the axis of the ellipse moves as time goes on.

So, as a pretty famous case, the advance of the perihelion of mercury.

That's the question. The perihelion is the place of closest approach to the Sun, and so the question is, does the perihelion of Mercury happen in a sort of a fixed position relative to the rest of the solar system and so on, or does the perihelion of Mercury, the closest approach to the Sun, does it, on successive orbits, does it slowly move? Does it process on successive orbits?

So the answer is the perihelion mercury does precess, it does move, and... but what turns out to be the case is if you have a two-body problem, and you're dealing with a sort of ordinary gravity that has exactly inverse square law forces.

then there is no precession of the perihelion, there is no... this vector that describes, kind of, the axis of the ellipse, stays fixed in space, the orbit doesn't... just keeps on retracing its steps. So in the sort of perfect two-body problem, you have that additional conservation law.

The reason that Mercury has, doesn't

Have that, that has its... that its orbit processes Is actually the biggest effects

have to do with the fact that the Sun isn't a point mass, the Sun is big, certainly big relative to the orbit of Mercury, and the gravitational field of the Sun is... is not... it doesn't... it doesn't... it's not like a perfect... it's not as it would be if the Sun was a single point at the center, and so that affects the motion of Mercury around the Sun.

There's another effect that's very famous about the precession of the perihelion of mercury, and that's an effect that, well, it had been measured quite carefully in the 1800s, and it was... and features of the shape of the sun had been measured and so on.

And, also the effects of other planets have been measured, and it didn't agree. There was an additional piece, it's only, I don't know what it is, 10%, 5% or something.

additional precession that Mercury shows relative to what you would expect.

And that turned out to be evidence of general relativity, Einstein's theory from 1915, that sort of goes beyond the simple inverse square law of gravity, the Newtonian law of gravity. What actually happens, I could explain.

In... normally, when you are dealing with gravity, you're dealing with, kind of, this force of attraction that doesn't depend on the speed of things. It's just a force of attraction that just depends on mass and distance, and so on.

And that's a little bit like in electromagnetism, in electricity and magnetism and so on. If you have two electric charges that are just stationary, they have a certain force of attraction between

them, or force of propulsion if they're the same charge between them. It's also an inverse square law force.

And that's all well and good, but when the charges are moving, there's an additional effect that has to do with magnetism that goes beyond the effect from pure electrical forces.

And those magnetic forces, well, they're important if the charges are moving. It turns out there are also analogs of magnetic forces for gravity, and

Those are the main effect that leads to a change in the way that gravity works for things, particularly close to the Sun and so on, and that's what leads to the additional precession of the perihelion of Mercury, and it was fairly accurately reproduced by general relativity.

Okay, so back to the two-body problem. So the two-body problem, there are these constants of the motion that are energy and angular momentum, the Rangi-Lenz vector, it, those... just knowing that those things are conserved.

that you started off with some energy, you started off with some momentum, it will stay with that energy, that angular momentum. That's... it turns out that there are enough of these things which are conserved that that pretty much tells you what the form of the solution, what the form of the orbit.

For the two-body problem has to be like.

The three-body problem, a big obsession in the 1800s was to find additional constants of the motion in the three-body problem. In other words, find things where you can sort of look at the configuration, you could say it's always going to do that, forever. Just like in the case of the ellipse, it's always going to retrace its steps, it's never going to... the ellipse, it's never going to process around.

Okay, so no additional constants of the motion were found in the three-body problem.

And so then the question was, well, were there... the three-relie problem can be described by a set of differential equations. So, equations in general say this thing is true about some variable.

Like, you could say $X^2 + 2X + 1$

No, I'm gonna mess it up. $X^2 = 10$ or something. And then you say, what X satisfies $X^2 = 10$? And the answer is, well, X can be either plus the square root of 10 or minus the square root of 10 in that case. But in general, an equation states some relation among some... about some quantity. It says.

you know, $X^3 + 12x - 4 = 7$ or something, and then what X satisfies that equation? Differential

equations go a bit further than that. They say not only there's this relation between the values of variables.

But you're also imagining that these things, these variables are changing, so they might represent the position of something as a function of time. These variables are changing, and a differential equation contains not just the values of the variables.

but also the rates of change of the variables, the derivatives of the variables. And differential equations relate variables, their rates of change, the rates of change of rates of change, and so on. And the differential equations are all very complicated to solve. I mean, in Wolfram language, we put lots of effort over the years into being able to solve lots of kinds of differential equations, and we make... do a great job of it.

But the point is, the three-body problem can be stated in terms of differential equations.

And in the end, what you're doing to figure out where those objects move is you're solving those differential equations.

And then the question is, well, what's the solution to the differential equations? I'll give you an example of a differential equation. So, the equation rate of change of rate of change of X plus X equals 0.

So, that equation, it's a rather simple differential equation, that equation, the solution to that equation is a sine wave that just wiggles back and forth.

So that equation is pretty easy to solve. If I change that equation to be rate of change of rate of change of X plus, oh, I don't know, X squared

times... something else. That equation very quickly becomes very difficult to solve.

it becomes... there isn't a mathematical function that's well known that is the solution to that equation. You kind of ascend rapidly. So people might have heard, you know, of sines and cosines from trigonometry. Slightly more complicated differential equations give things called Bessel functions.

Other equations give so-called hypergeometric functions. You get these more and more exotic functions, which have names, because they show up in a lot of places, and there are lots of variations of differential equations and other things that give those same functions.

So... but then, rather quickly, you ascend to the point where there's no standard known named mathematical function that represents the solution to that equation. If you are doing things purely in terms of numbers, you can work out the solution to the equations, you can

Sorry, who can solve the equation numerically? You just say, I'm going to say, I've got the values of X right now, what are the values of X going to be just a small moment later? And that's a fairly easy thing to find out, at least approximately, and just keep doing that, and can solve the equations. But the question is, what

is there some mathematical function? If there is a mathematical function, it typically means one has worked out, sort of, all the properties, or a bunch of properties of that function. One can immediately say, hey, this is the value of this function in the case of representing positions of things as a function of time a long time in the future.

Okay, so people tried to find, sort of, mathematical functions that would represent the solutions to the three-body problem. They didn't succeed.

Eventually, in the early 1900s, somebody found a so-called series solution to the three-body problem.

The, okay, so this gets us into a different area. When...

If you are trying to represent some mathematical function, you've got some, you know, X as a function of time or something, and it's wheeling around all over the place. If you say, well, what does it do right near time equals zero? What is the sort of, what is it doing right there?

Well, you could say the first approximation is, it has this value at zero. The next approximation is it has this value at zero, and it has a small variation. It's going... it's just got a slope at zero.

Next approximation, it's got, you're looking at the slope of the slope, the second derivative, and so on.

you can kind of ask, sort of, you can make these approximations, and you can move away from just what it is at zero. You can say, well, what does it do a little bit further away from zero? You can make kind of a series approximation. You can say, well, it does... roughly, it's like a constant.

Then it's plus something that's just...

Linear in time, plus something that goes like the square of time, plus something that goes like the cube of time, and so on.

And that might only work for time very close to zero, but you can make this kind of series approximation

to say, what does it look like right around zero? That's something that actually Isaac Newton already figured out when he was first developing calculus in the late 1600s, this idea of series approximations. Okay, so...

You can, in the case of the three-body problem, or celestial mechanics in general, the motion of objects in astronomy in general.

It's a little bit more complicated, because what is usually happening is that things are in orbit, so there's a... there's a periodic thing happening. It's coming back in its orbit, and it's... it's coming around again.

And so, there are more elaborate forms of series approximation that get used in those cases, very often things called Poisson series.

And so on. Those are series that, that kind of take account of the fact that it's always going to repeat, so they're kind of representing things in terms of functions that always repeat.

Actually, most of celestial mechanics is done in terms of those kinds of series of things that always repeat. It's kind of like, you can have an orbit, and you could say this orbit wiggles once, wiggles twice as it goes around the orbit, and so on. That's roughly what you're doing in making those series. And when...

Usually, the... even when we solve the equations for celestial mechanics.

Using some fancy numerical computer technique or something, you approximate the solution using a series of some kind, and you're then using, you know, 100 terms in the series or something to get a very good approximation to where this planet is at this time.

So anyway, in the early... in the early 20th century, somebody did actually find a series approximation to the three-body problem. So in some formal sense, there is a solution to the three-body problem in terms of the series, but it's an infinite series, so you have to keep adding terms forever. And it turns out the series, the successive terms, it doesn't, they don't get smaller very quickly, so you have to... in practice, it's an absurd number of terms that you have to use to make use of the solution to the three-body problem, so it isn't really really a solution.

So now the question is, well, what...

What does it even mean for the three-body problem to be unsolvable?

In practical terms, it has meant there are...

Well, okay, this is again a little bit complicated. There are no so-called algebraic constants of the motion.

So that means that if there is something... what was proved, actually, in the 1800s, is that there are no algebraic constants of the motion.

So that means that when you compute something like the energy associated with the two-body problem, you're taking, kind of, the velocities of things in the two-body problem, you're computing things using the kinetic energy formula, half MV squared, and so on.

It depends... the thing you're computing, in that case energy.

depends, it's just a polynomial, it's just a, you know, it's just the square of the velocity in that case. It's something that you can write down as an algebraic function, a function that involves just powers and addition and so on, division and such like, in terms of the sort of raw quantities that appear there. So, you could imagine a... so, in the case of the two-body problem, the constants of the motion are all algebraic.

And in the three-body problem, it's known that there can't be algebraic constants of the motion. There could be a constant of the motion, it's not excluded by the theorems that are known. It's some exotic computation in terms of some very bizarre function that isn't an algebraic function, that isn't something that just involves powers and addition and so on.

But that's not known right now.

But then the question is, well, the sort of real question of the three-body problem is, you know, given a certain initial set of conditions for the three bodies that are traveling at this... in this direction, at this speed, and this position, and so on, what will happen in the end?

So you could use a computer, you could solve it, you could, you know, figure out step by step, you know, how the bodies move, and they're tangling around, and eventually they escape, or whatever else. You can do that by sort of trying to go through step-by-step like that.

The question is, given the input, is there some quick way that you can sort of guarantee to just say, oh, I know this is going to end up in this form?

Nobody knows how to do that.

It is my strong suspicion that it simply isn't possible. That the only way to find out what's going to happen in the three-body problem is essentially to go through every step of the actual behavior of the three-body problem and see what happens. That's kind of a phenomenon of computational irreducibility. You can't sort of outrun the actual computation that the system itself is doing.

If you ask, what is it going to do in the end, after an infinite time.

that's a question that can be undecidable in the sense that there's no finite computation that will guarantee to give you the answer. So it's my very strong suspicion that the three-body problem is, in fact, computationally irreducible, will show undecidability if you ask the infinite time question, and so on.

How would you show that? Well, essentially, the way you show that is that you have to be able to show that somehow the three-body problem is doing complicated enough things that you can sort of encode any computation in what the three-body problem does.

You know, we have computers that do all kinds of computations based on their electrical signals and their CPU chips and things like this, but we know that we can also make things that will do computation using very different kinds of, in some cases, concrete, in some cases, abstract.

kinds of constructions. You know, you can have a cellular automaton, one of my favorite kinds of things, that's just a row of black and white cells.

where the next color of a cell just depends on the colors of its neighbors. Just using that kind of setup, you can make an arbitrary universal computer. So that's an example of being able to do that.

In the three-body problem, the question is, can you set up a configuration of the three-body problem that will encode any computation so that the actual behavior of the three-body problem will give you the answer to that computation?

So, it's... the three-body problem does complicated enough things that you might think, oh, it's able to do any computation, but proving that that's the case is really hard.

And actually, I've even been looking at it a bit. Recently, I've looked at it at various times over the course of the last few decades. Why is it hard? I could describe a little bit why it's hard.

When you're dealing with a computer, you're dealing with

digital... a digital computer. There are, you know, bits, ones and zeros. It's a discrete thing. The computer represents the number 7, or it represents the number 8. It's not representing something that's just a tiny bit different from 7.

and so on. It's something where there are discrete digital states of the computer.

Okay, in the three-body problem, the way the mathematics is set up.

there... it is all continuous. That is, these differential equations are talking about the positions of the objects and their speeds and so on, and everything is just an arbitrary real number, a number with... you can state to any amount of precision you want.

So the problem is that it's quite hard to kind of make the sort of connection between arbitrary, continuous real numbers and the kind of discrete things that we normally deal with in computers.

And the... my guess is the way to actually show what's going on

Is to have some way of representing real numbers so that when you run the three-body problem on those real numbers, you can kind of see how the real numbers are changing in a way where you can kind of take them apart and see that they're doing a computation.

So the most obvious way to represent a real number is by a sequence of digits. In principle, the number can have, you're stating it, with infinite precision, so it has an infinite number of digits. You know, if you take a number like pi or square root of 2,

Those are specified in sort of the short specifications I just gave them, but if you write out their digits, their digits will just go on forever.

And, so the question is, if you're using ordinary digits, when you run the three-body problem, when you start the system off.

with this particular configuration, with the thing that's at square root of 2 position, and square root of 7 position, and things like this. And you let the three-body problem run. You do... you solve these equations to represent the motions of these objects.

Those numbers get hopelessly tangled up. You kind of can't trace anything about what happened to the digits. They just, in an instant, the digits are just completely scrambled relative to what they were at the beginning.

So, it doesn't really... to use digits, it doesn't play nice with the kind of process of actual, sort of, evolution, actual behavior of the three-body problem. The question is, is there a representation of those numbers

That would play nice with the actual operation of the three-value problem. So there are many other representations you can imagine for numbers. A pretty well-known one is so-called continued fractions.

So you represent a number, so in the case of digits, you say, you know, you know, 0.734 or something means 7 tenths plus 3 hundredths plus 4 thousandths, and so on. You're adding up these terms. In the case of a continued fraction, what you're doing is you're saying the number is 1 over

One plus.

1 over 2 plus, 1 over 3 plus, and you're making this big, sort of, they always look rather nice when you write them out, there's this big stacked collection of fractions.

And continued fractions have all sorts of interesting properties. You can represent any number in terms of continued fractions. Some numbers have very regular continued fraction representations.

All square roots of whole numbers, like square root of 2, square root of 7, and so on, they all have continued fractions where those numbers that appear in the continued fraction repeat periodically.

It's an example of a kind of a simple form of a continued fraction. So what you might hope for is that maybe continued fractions don't get scrambled up by the actual running of the three-body problem. They do, that doesn't work. The question is, is there some way of representing numbers that doesn't get scrambled up

In the actual running of the three-way problem? I don't know yet. There are things one can do, but they're a bit artificial, and it's not clear... it's not clear how robust what's going on is, it's... it's all rather complicated. But it's something we might know in the... in the not-too-distant future.

I think one thing to just mention about the three-body problem, and in general the n-body problem with any number of objects interacting with each other under gravity, is one of the reasons people were really interested in that in the 1800s and beyond is the question of the stability of the solar system.

So, if you have your Earth-Moon-Sun system, does it remain stable for all time? Does the Moon keep going around the Earth and the Earth around the Sun, or can it become unstable where the moon escapes?

For example. And it's generally believed that in the early history of our solar system, there were a bunch more planets

And typically, several things happen. They either get ejected and sort of are wandering out in interstellar space, a rather lonely journey, or with actual planets, the other thing that can happen is that they can crash into each other. That doesn't happen with idealized mathematical kinds of things, because in the idealized mathematical setup, you're dealing with point masses, masses of zero size.

which is equivalent, as far as their gravity is concerned, to larger objects. It's just a feature of the way that inverse square law rules work, that it doesn't matter. If you have a sphere, it will have the same gravity outside it.

as a point mass at the center of the sphere. But when it comes to actual planets, if the planet gets closer than the actual size of the planet, it'll smash into each other, which doesn't happen, obviously, when the masses are points.

So, that was a little bit of a story, about, the three-body problem. I see some...

Questions, from people here following up on that.

Jamie asks, what would seasons look like on a planet orbiting multiple stars? Well, the first question is, why do we have seasons on Earth?

And the... why isn't it the case that, at any given... okay, so... so, on the surface of the Earth. your... you know, what makes different parts of the Earth different temperatures is the sun is kind of of different intensity in different places. Why is that? So let's first of all imagine that we just have a nice spherical Earth, and it's always... its axis... it's always... its axis of rotation is always kind of just straight up and down relative to the way it orbits the Sun.

So imagine, then, that the North Pole is, it never changes its orientation relative to the Sun. At the North Pole.

the sun is kind of your... you're looking at the sun to the side. You don't... so, at the North Pole, any... okay, so actually, let's start at the equator.

at the equator, in this kind of idealized Earth.

Where it's always the... the axis is always pointed sort of up, down. At the equator, the sun is always overhead.

And the sun, and if you have a,

if you imagine that there's sort of light coming from the sun, and this kind of a cone of light coming from the sun, then that cone of light, when it sort of hits the surface of the Earth, it will just make a... it will make a circle on the surface of the Earth, at the place where the cone hits the Earth.

Well, now imagine you move towards the North Pole. At the North Pole, it won't make a... the cone of light from the sun won't make a circle, it will make a very, very elongated ellipse. So, in other words, if you're... if the question is, there's a certain amount of energy coming from the sun in that cone.

At the equator, that energy gets kind of deposited in your little circle that is, you know, depending on which cone you're looking at, let's say it's a one mile across cone at one mile across Circle. At the pole.

the... your... that same amount of energy is deposited from the sun, but it's deposited in this very, very, very elongated ellipse.

So if you ask, well, how much does that amount of energy heat the surface of the Earth?

At the equator, all that energy is deposited in your one-mile radius circle. At the pole, that energy is deposited in an incredibly elongated region, much bigger than... whose area is much bigger than the area of the one that was at the equator.

So that means that... that the... that energy is going to be distributed over a much larger area, which means that it's going to heat the surface of the Earth much less. It's going to... the Sun will seem a lot dimmer, because it's, per area of the surface of the Earth.

a lot less energy gets deposited. So that's why the poles are colder than the equator, is because you're kind of looking at the sun at a... you're kind of looking at a slanted sun, so to speak, and not as much energy is being deposited there.

So, why does the Earth have seasons? Why does it vary? Why does the amount of solar insulation, it's called, the rate of, sort of, radiation from the sun, why does that vary through the year?

The reason it varies through the year is that the Earth does not have its axis pointing sort of straight up and down relative to the way it orbits the Sun. It's actually pointed at 23 degrees from the, from... from the... from sort of straight up and down. That 23 degrees is responsible for our seasons. That means that in the course of the year.

The, the, the, the, because that, that,

the axis of the Earth kind of remains fixed as it goes around the Sun. You're... you're varying, in the course of the year.

a point on the... on the Earth

is... it has a sort of variable angle relative to the Sun, and so you're getting a different amount of solar radiation there, solar energy coming to the Earth, and that's why our seasons vary.

it's... it's, so, I mean, on... on other planets.

the obliquity, it's called, of the angle that the rotation, the planet's rotating on its axis, the angle that that axis of rotation makes with the so-called plane of the ecliptic, the plane in which the planet is orbiting the sun.

that angle is different for different planets. I think, let's see, I think...

Uranus, the planet... it's tipped so much that the planet is kind of rotating in the opposite direction relative to what it is for the Earth. But those obliquities are different for different planets, and so the seasons will be different on different planets. So that's how seasons basically work.

In the case of a planet orbiting multiple stars.

So, all of the exoplanets that are known.

I believe that all the ones that are known in multiple star systems... so, I mean, the Sun is a single star. There are lots of stars that aren't single stars, that are binary stars, where there are two stars. Those stars are orbiting each other. Our nearest star other than the Sun, Alpha Centauri, is a triple star system, where... but a triple star system, which is a great example of a three-body problem at work.

book. I believe in that case that,

the, the A and B stars are kind of orbiting each other quite nearby, and the third star... the third star is quite a long way away. So it's really, like, two things are just orbiting each other, and the third one is orbiting those two.

So, in, in the case of, but...

I believe the exoplanets that have been discovered are all kind of orbiting outside of where... whatever the stars on the inside are. In other words, even a triple star system, the planets are kind of... it's kind of like there are three suns in the sky, so to speak.

And... but they're all kind of close together. There are no planets that are sort of interweaving between the stars.

Most likely, orbits that interweave between the stars will not be stable. Most likely, when you solve the equations and you take into account other effects of kinds of... other kinds of stellar winds and so on that push on planets and things like this, that that orbit won't be stable, and so if it started, you know, a billion years ago in that orbit.

It wouldn't still be in that orbit today, it would have crashed into one of the stars, or escaped from the star system, or whatever else.

So I think the, the story...

Now, okay, if you have the sun, the sun, doesn't, you know, it's a small disk in the sky from the Earth.

You know, it's a much bigger disk if you were on Mercury, close to the Sun, it's a much smaller disk if you're on Mars. But on the Earth, it's, you know, it's a disk about the same size as the Moon, as it turns out, in the sky.

Now, if you were,

And the way that seasons work and so on depends on the fact that it's... that there's a sort of a point source of light that's making this cone and things, you work out all the math from that. If you had 3 stars in your sky, and they were

they were kind of far apart in your sky, then you would have a different kind of setup for how, even if your planet had some obliquity relative to the plane of its orbit around these stars.

The, there would be sort of a slightly different piece of math wouldn't be one cone, it would be several cones, and so on. I can't instantly work out what would happen. My guess is that these seasons, you would have kind of less extreme seasons. I mean, you get... you get, well, you get the... I guess the most extreme seasons you would get if you had obliquity of 90 degrees, then... then your planet would be kind of rotating on its axis one way and rotating around in a different way around the star. But my guess is it's the equivalent of having lower obliquity if you had more stars in the sky, so to speak.

You know, it's... it's... the kinds of things that can happen with a three-body problem are pretty, it's pretty diverse what can happen. So, a famous example is if you have, two stars that are orbiting each other.

And then you have a third object planet, let's say, that's just going up and down, up and down, between those two stars that are orbiting each other. And let's say that planet has no mass of its own. It's just... it's feeling the gravity, feeling the gravitational attraction from the stars that are orbiting, but it's not producing a gravitational attraction itself.

So it turns out that problem, that situation, you can go up and down, it just... if you set it up correctly, the thing will always just go straight up and straight down as it goes up through the plane of the planets, the stars that are orbiting each other.

So it turns out that if you want to know how many... as it goes up and down through this plane, how many times have those stars orbited

between the time... how many orbits have they gone around between the time the thing last went through the plane of those stars? It turns out that by setting up the initial position and velocity of that sort of third object.

you set it up, and you can... there's always a way to set it up so that you'll get any sequence of numbers as the answer. How many times do the stars orbit between the time when this other object went through their plane? So you can... you can set it up, you can start it so that it will have, you know, two orbits, 7 orbits, 1 orbit, 17 orbits, et cetera, et cetera, et cetera.

Any sequence of numbers can be achieved by sufficiently, closely, precisely setting up the initial conditions for that system. That's an example of kind of an exotic thing that can happen in the three-body problem. I mean, another exotic thing, just to mention it.

Is that there are configurations of the three-body problem

that are sort of strange and stable. So one... one famous one is a triangular configuration, where things are... let's see, I guess it's just orbiting... just the triangle is just moving around. That's the... that's the thing that happens with the...

the Trojan asteroids, in, in,

in orbit around the Sun, and also feeling the gravity of Jupiter. But that's... that's one configuration. There are a bunch of other, kind of, specific, kind of, periodic configurations, where there are sort of exotic, orbit-like things, where things sort of interweave between themselves that are known in the three-body problem.

Let's see... Greg is asking, would stellar flares and radiation be dramatically worse in a multi-star system?

So... the sun, is, in some sense, a big ball of fire. It's got fire is...

A state of matter where atoms have their electrons removed.

And, that's what happens when things get too hot. The surface of the sun is a great big plasma, and it has a lot of complicated effects going on. A lot of things with sunspots, where you have these

sort of regions in... I mean, it's a story of the interaction between, kind of, heat, magnetism, dynamics of gases. It's pretty complicated what happens on the surface of the sun.

Occasionally, there will be things like the so-called corona of the sun, the outer region of the sun.

There'll be coronal mass ejections where, sort of, there's a blurb, and a whole bunch of plasma gets sent out into space from the sun. There's also, all the time.

So, so there's a lot of complicated dynamics, and solar flares are part of the complicated dynamics of this very kind of turbulent magnetohydrodynamics, it's called, the kind of fluid dynamics, but with magnetism included, that happens in the plasma on the surface of the sun.

I... I think that in the... in the...

I don't think we know what

kind of the analog of solar flares, these, sort of blurps of stuff from the, from the plasma, on the outer layers of the sun. I don't think we know what that's like for other stars.

I think, and it's quite a hard thing to calculate much at all about it. My guess is that at different stages in the history of a star, that there will be dramatically different amounts of,

of sort of stuff ejected from the surface of the star. By the way, the other kind of thing that happens is the so-called solar wind, which is not so much big blurps of plasma, more it's just individual particles, protons and things like that, streaming out from the sun.

And it is known that for different kinds of stars, like giant stars, for example, I think are much stronger stellar winds than the sun has. So it's known that at the level of these individual particles, there are real differences

Between the amounts of radiation that's... that's, streaming out from stars.

depends on the stage that the star is at and the evolution of the star. I mean, stars typically go through,

a, well, it depends on the mass of the star. The thing that determines the evolution of a star is mostly its mass, also what's called its metallicity, which is kind of the amount of heavy elements that exist in the star. In the very early universe, the only elements that were generated were hydrogen and helium, but as a result of actual nuclear reactions in stars, heavier elements get produced

And stars that were sort of made from gas that came from earlier generations of stars will have higher numbers of those heavy elements, and that can affect how the stars operate. But mostly, it's just the mass of the star that matters. And so, there is the idea of so-called main-sequence stars.

which go through a series of, that just, depending on their mass, go through a sort of standard series of stages. I mean, our sun is about 4.5 billion years old, it's about halfway through its lifespan.

In its total lifespan's around 10 billion years, so 5 billion years from now, it will be turning into a white dwarf.

That's what a star of the mass of the Sun does. The Sun is a... on the scale of stars, the Sun is actually reasonably big. There are much bigger stars, but the Sun has a fairly respectable mass. There are a lot of stars that have much lower mass than the Sun.

Out there. But, and what happens is, as the mass gets too low.

the force of gravity is not sufficient to kind of push things close enough that they start undergoing nuclear reactions, and the star actually lights up and does its star thing, so to speak. If things are too small, then you'll just have a lump of stuff, and it'll be more like a planet.

And if it's, you know, if you took Jupiter and you made it, I don't know how many times bigger, I'd have to work that out, but significantly much bigger, Jupiter would sort of spontaneously become a star, just because of its sort of force of gravity pulling the pieces of it together would crush it to the point where nuclear reactions would start.

And it would become a star, so to speak.

But in any case, the, the thing that, that you can ask, I mean, in, in,

So, radiation to the Earth there are cosmic rays.

that hit the atmosphere of the Earth, those are mostly coming from the Sun. They're... they're basically related to the solar wind, most of them.

And, they have a, a, they're mostly protons, and that sort of radiation, kind of, that, that hits the atmosphere of the Earth, actually hits the magnetosphere of the Earth, the kind of,

The... and the... the... But...

that radiation does not reach the surface of the Earth. It's always those particles from the Sun always collide with other things in the upper atmosphere, or they are deflected by a magnetic field of the Earth.

And we don't get to experience them on the surface of the Earth. I mean, there are cosmic rays that get to the surface of the Earth, mostly, if you're at sea level, mostly muons. If you're at a mile high or something, there's a lot more protons that make it down, but it's still a tiny fraction of the protons that hit the top of the atmosphere.

And they don't really have, despite people looking for this, because there are more particles in the polar regions than there are in other regions, because the magnetic field of the Earth causes particles to spiral in around the poles. That leads to the aurora and things like this.

But so people have looked for, you know, are there... are people getting cancer and other things more when they're near the poles than elsewhere, and I don't think there's any evidence for that. So the radiation that we get on the surface of the Earth

doesn't seem to do us much harm, or at least we've evolved to deal with that. And so if the question is for another kind of star, or a star in a different stage of its evolution, would there be so much

cosmic radiation, basically, cosmic rays, that enough would get to the surface, even if the atmosphere was the same as the atmosphere on our Earth, that would cause trouble, probably the answer to that is yes. I think there might be all kinds of interesting effects. I mean, there might be, you know, the aurora.

Comes from interaction of, of,

cosmic... well, the solar wind, I guess, of the charged particles from the sun, interacting with mostly, I think, oxygen in the upper atmosphere, and leading to, kind of, these particles hitting oxygen atoms that cause the oxygen atoms

to go to higher energy states, and as the oxygen atoms kind of relax down to their lower energy states, they emit photons, and those photons show up as, you know, the the reds and greens and so on of the aurora. I suspect if there was a lot more cosmic radiation, there would be a glow at night, all over the Earth.

you would get kind of an intense version of the aurora all over the Earth, from charged particles, from particles like protons and electrons and so on, high-energy such particles coming from the star, hitting the upper atmosphere of the Earth.

Let's see...

Well, there's all kinds of questions here.

Gosh, gosh, let's see.

Oh, okay.

Well, there's lots of questions here.

Alright, one from Paul.

Can you explain why particles that produce Cherenko radiation do not violate special relativity?

Boy, you guys are asking sophisticated questions.

So... The, well, let me... let me explain. If... if you...

Cherenko radiation is a strange, usually bluish light that you see where... when high-energy particles, interact with, materials.

So, it's pretty common, I think, if you, if you were to look into the nuclear reactor of a nuclear submarine, which I think is a water-moderated reactor, usually, you would see this glow, this blue glow from Cherenkov radiation.

Trinkor radiation comes when you have, particles, like electrons or protons or alpha particles or whatever, that are, that are going... that are high energy and going fast in the material. Okay, so...

his... Okay. Turnco radiation is the analog for light of a sonic boom for sound.

So, normally, if you have A thing, your car, your plane, Going through the air.

it will... there will be sound waves that will be produced, and the sound waves might be, produced just by, you know, the noise of the engine, or whatever else. And those sound waves will be just... if the car is kind of idling.

then the sound waves will be going outwards, kind of just in a circular fashion. So you'll have the source of sound, and you'll kind of have waves just going out, you know, the waves are going up and down, but they're going... it'll be... they're going outwards equally in all directions.

Okay, let's say that your car is traveling more quickly.

Then, let's say it's a... it's some emergency vehicle with a siren.

And it's producing sound through its siren. Well, people know that there's a so-called Doppler effect that means that when the... when the thing is approaching you, the sound will be higher pitched. When it's going away from you, it will be lower pitched.

That's because in... a sound wave comes because there are these successive high pressure, low pressure, high pressure, low pressure, you know, a thousand times a second or something.

You're, you know, when you, for speech, it'll be roughly, it is around, a few hundred times a second to a thousand times a second, there will be a kind of a compression expansion, compression expansion of the molecules in the air. That's what produces the sound. that wave of compression and expansion will go out equally in all directions if the thing that's producing it is just sitting there at rest. If the thing is moving, then kind of if you think a little bit about the geometry of it, what happens is

That the... the kind of... the peaks and troughs of those waves will be kind of scrunched up if you're... if it's approaching you, and they'll be elongated if it's going away from you.

And that's what means, when they get scrunched up, that means it's a higher frequency, and when they get expanded, it's a lower frequency. So that's kind of why this Doppler effect occurs.

Okay, that scrunching, the amount of scrunching, depends on how fast the thing is going relative to the speed of sound.

If the thing was going at the speed of sound.

those waves would never be able to... the object will always be catching up with those waves.

And so what will happen is that instead of those waves just sort of going outwards as sound waves, they'll be all sort of accumulating on the object, if the object's going at the speed of sound.

The, or actually also above the speed of sound. Those waves, instead of escaping from the object and it's going out of sound waves, they'll be sort of accumulating on the object.

And so, in a sense, that's making more and more intense

Sound, and that's essentially what you hear when you hear a sonic boom.

a little bit more detail, the sonic boom comes from... well, okay, so if you have a plane, and it's, it's kind of... as the plane goes through the air, it's kind of having to, as... as it... the...

The presence of the plane is changing the pressure of the air, and it makes... if it is going through the air at the speed of sound or above the speed of sound, it essentially makes a discontinuous change of pressure.

Anyway, the result is that you get this kind of accumulation of sound waves, and the accumulation of sound waves will start at the tip of the nose of the plane, and it'll make a cone

starting at the tip of the nose of the plane, it'll make a big cone of kind of all kind of bunched-up sound waves that aren't... that are just going outwards in this cone, they're not going outwards the way that sound waves normally go outwards. And the way planes work, the way this works is every time there's sort of a change in pressure associated with the plane going through the air. you'll get one of these kind of shock waves that is associated with this piling up of sound waves. And that happens at the tip of the plane, it'll happen at the tail of the plane, it'll happen on the wings of the plane, all kinds of parts of the plane, and sort of the engineering of trying to reduce, sort of, the amount of that buildup has to do with sort of smoothing out pieces of the plane so that you don't have

These places where there's sort of a discontinuous change of pressure. Actually, discontinuous change of velocity, also.

So, okay, so sonic boom, there's this kind of accumulation of sound

That is in this cone, and the plane's flying along. If you are on the ground.

where the cone, kind of intersects the ground. It'll typically intersect in a parabola on the ground.

If you're right at the place where the cone is intersecting, you'll hear a boom as the... as the plane goes by, from sort of those accumulated sound waves.

So that's... that's...

kind of the idea of sonic booms. I guess one doesn't hear them very often. When I was a kid, and the Concorde was out and about, flying around, you would hear sonic booms, in England at least, from the Concorde from time to time. It's a very kind of cracking sound.

It's, notably, I took one trip once on the Concorde. Of course, from inside the plane, you don't hear any of that. It's only the... it's the... it's what happens outside the plane that, that, that has this... has this effect.

So, you get these shock waves, you get this kind of, this sort of cone of discontinuity that happens for sound, for anything going faster than the speed of sound.

Okay, the same thing happens for light.

But... the,

It's sort of the same effect, but you say, but how can it happen for light? Because the speed of light is the ultimate limit on the speed that anything can go at.

That's true, that the speed of light in a... that,

The speed of light in a vacuum is whatever it is, you know, 300 million, meters per second, or whatever, 186,000 miles a second, whatever. The, the

That speed is the speed of light in a vacuum.

In a material, like water or glass.

The effective speed of light is slower.

What's happening when light goes through, let's say, water is

It's not the photons of light aren't just going straight through. The photons of light keep getting absorbed by an atom, by a molecule of water, by one of the atoms in the molecule of water, and then a little, very short time later, the photon will be re-emitted.

So there's a delay. It keeps on getting delayed. As it interacts with the material, it keeps on getting delayed. In a transparent material, the photon is successfully re-emitted. In a non-transparent material, the photon just gets absorbed by the atoms and doesn't get re-emitted. But in a transparent material, the photon is re-emitted, but there's a delay before it's re-emitted. And that delay causes

The effective speed of light in that material to be slower than the speed of light in a vacuum.

And so, for example, for water, it's 1.33 times slower. For glass, it's typically about 1.5 times slower. For diamond, it's 2.7 times slower, and so on. That's the so-called refractive index of the material, is the amount slower that light goes in it relative to the speed that light goes in a vacuum.

Okay, so how does Cherenko radiation work?

What happens is that you can have something like a proton, and as the proton can be going... let's say you accelerated the proton in a big particle accelerator, or even it came from the sun, and it was accelerated by magnetic fields near the sun, or whatever else, and it's going really close to the speed of light, the speed of light in a vacuum.

Now it ends up going into some water.

Well, it's still going close to the speed of light in a vacuum, but that means it's going faster than the speed of light in water, which is 1.33 times slower than the speed of light in a vacuum. So that means that it's going faster than the local speed of light in the medium that it's in. And then whenever it produces light.

Whenever this proton, as it interacts with the water, some light is produced, that light will kind of bunch up in a kind of cone, and that cone is what leads to Cherenko radiation.

So whenever a particle is going faster than the local speed of light in the medium that it's in, you will get Cherenko radiation.

So, the,

you know, it's an effect, as I say, with a nuclear reactor, because you have a bunch of radioactivity there that's producing particles that are going faster than the radioactivity, is particles.

That, you know, were kind of spat out of a nuclear reaction and are going quite fast, quite close to the speed of light.

Going faster than the local speed of light in the water, that's why you see this bluish glow. Other places Cherenkov radiation shows up

astronauts in, who aren't, sort of, shielded by the Earth's atmosphere, end up reporting seeing flashes of light. They close their eyes, they'll occasionally see flashes of light. Those are cosmic ray protons going through their eyeballs. And it's, you know, usually when you see light in your eyes, it's light that comes from outside your eye.

And your eye lens focuses that light so that it just takes something which is a small object that you're seeing will end up as a small point of light on your retina, and so you can kind of see the image of this thing. With Cherenko radiation, the light is actually originating inside your eyeball, and all you'll see is this kind of diffuse flash of light.

That comes from the particle going through your eyeball.

People use Cherenko radiation to detect particles. There are,

This is, well, it's a common technique, because if you have, oh, I don't know, let's say you're trying to detect,

trying to detect some neutrinos which interact very weakly, or you're trying to detect dark matter, which I don't think you're going to succeed in detecting, because I don't think the theories of that are right right now, but that's a different story. But what people do, then.

Is they'll have a place where they have a big volume of liquid, like water.

That is, sort of

very well shielded from... from the ordinary kind of... it's kind of not just got the atmosphere on top of it, it's got miles of water, and it's in the deep ocean, or it's underneath Antarctica, and it's

got, you know, a mile of ice on top of it. Or it's in a deep mine, and it's got, you know, a mile of rock on top of it.

But then you have a big tank of water, or something like water, and what you do is you try and detect those flashes of light that come from particles going... going...

Through your... these high-energy particles going through the water.

And so, for example, people looked for proton decay. This is, like, 40 years ago now, because it wasn't observed at that time. You have this big volume of something, of water or something, in a mine.

And you have sensitive photodetectors, sensitive detectors of light, and you basically see, if you've just got water sitting in a mine, and it's a deep enough mine that all the cosmic rays that might have been coming from the sun have been absorbed by the rock.

well, you still have some neutrinos that get through and they cause you trouble, but if you can deal with that, then what would make a flash of light in that situation where there's nothing coming from the outside? Well, it would have to be something in the water itself kind of,

producing high-energy thing. Well, that can happen if your water isn't perfectly pure, and you have, some... something that happens to be radioactive. Or it can happen because the rock around your... around your thing has radioactive elements, has a tiny amount of radioactive elements, or radioactive gas, like radon.

Is, the, that, You can have,

or you have a small amount of uranium or something in the rock that's around it, then you can get kind of particles from that source. But otherwise, otherwise, if you get rid of all those sources, the only thing that can be going on is that the protons and neutrons and so on In your actual water or something are themselves somehow decaying.

So protons are believed to be stable.

Protons, protons and electrons are both believed to be completely stable. That is, if you start an electron at the beginning of the universe, it'll still be around today.

And, what's known is that the lifetime of the proton, if it does decay, is more than, I think it's 10 to the 32, years, which is, so the age of the universe is,

About, 10 billion years, and so 14 billion years or so, and so that means that that is, let's see, 10 to 10, so that's,

a 10 to the 20th times the age of the universe is the... if the proton was unstable, its lifetime must be at least longer than that. What would happen if a proton decayed? Well, if

If all other things weren't crazy, a proton, which is a charge plus one particle, would decay into a positron, which is the antiparticle of an electron, together with probably neutrinos.

That's what would happen. If that happened, the proton is 1,800 times heavier than an electron or a positron.

And so, when that... when it decays, it's putting all its... all the energy associated with its rest mass into the kinetic energy of that... that positron, so that positron is going to be zipping with very high speed, and it'll produce a whole bunch of Triangle radiation.

never been seen. So it seems that protons are stable, and, the, the, so that's, that's not a thing that has been observed.

Just for the sake of completeness, you might wonder, you know, there's protons, there's electrons, and there's neutrons in nuclei. Are neutrons stable? The answer is neutrons are absolutely not stable. Neutrons... if you just have a free neutron that isn't kind of all attached inside a nucleus.

A free neutron can actually decay into a proton, an electron, and an antineutrino, and it decays in about a thousand seconds.

It's, so a free neutron is unstable, but it turns out, when a neutron is bound inside a nucleus, it doesn't... it's... it's kind of... it's being sort of pulled into the nucleus sufficiently hard that it doesn't... that...

Essentially, it doesn't... it can't...

its energy is... its effective energy, its effective mass is reduced to the point where it can no longer decay into that proton, electron, and antineutrino. Its kind of effective mass is too low to be able to do that, and that happens because it's bound into a nucleus. So neutrons bound into nuclei are still stable.

A free neutron, unlike a free proton, is unstable.

Let's see...

Means is commenting, proton decay would lead to a very bad day.

Well, yes. If suddenly the protons in the universe all decided to decay, that would be a bad thing.

The way that decay processes work

is... There's always exponential decay, which means that

The... at any given moment, there's a certain chance of a particular particle decaying, and if you have a whole bunch of particles, the number of particles

that you will have left will decrease exponentially over time. So that's why the concept of a half-life is, at what time are... how long is it for half the particles to have decayed?

And if you say, let's say the half-life is 10 years, then in 20 years, you'll have half of half of the particles left, so a quarter, and it keeps going. So that's the... that's the definition of a half-life.

There's some...

there's also the notion of a mean life, which is a 1 over λ , Rather than a one-half.

Thing, but in any case, so what would it mean if protons decayed? Well, actually, proton decay is something that

A lot of theories of fundamental physics

imply that protons should decay with some lifetime.

his... Well... Essentially, the reason for that

Let's see how to explain this. So...

Well, let's just say that the,

The particles that we know exist, the quarks, the electrons, the muons, things like this, there's... actually, let's go in a slightly different direction. So... so, in the standard model of particle physics.

There are things like quarks, electrons, muons, and then there are so-called gauge bosons.

The gauge bosons are the things that are associated with, kind of, forces, like electromagnetism, like the so-called weak force, like... so, those are...

Those are things like photons, which, and those are the things that are essentially exchanged between other particles, leading to various kinds of forces between those particles. So there's photons that lead to electromagnetic forces.

There's gluons that lead to the strong nuclear forces that hold nuclei together, and so on, and hold protons together, actually. And then there's the so-called weak force.

which is responsible for a common kind of radioactivity, beta decay, and its... its particles are the W boson and the Z boson, which are, which are much more massive, they're... they're more like 100 times

The mass of a proton, rather than being zero mass, like the photons and the gluons.

Okay, so the, the, the way... Let's see...

What are gauge bosons? How do they work? What is their, sort of, point?

Here's a way to understand it.

So, imagine you have an electron.

It has an electric charge.

The... that electric charge

There are electric field lines that are the... so you have this electric charge, and it produces an electric field.

This kind of lines of force of the electric field go out radially from that electron, so there's kind of like an idealized hedgehog going out from that electron, of all those lines of force.

Okay.

So now the question is, you're looking at the... you move the electron a bit.

Well, at some point, so that means that, sort of, its hedgehog of lines of force will move a bit.

But how does something far away from the electron, how does it know that something... that the electron moved? So, eventually, far away from the electron, the lines of force better be... you know, they have to be realigned for the new position of the electron.

How does... how does that happen?

In other words, you've got the electron at one place, the lines of force go, you know, one orient... one... one... one way. After you've moved the electron, eventually the lines of force better, however far out you are, have better be realigned for the new position of the electron.

Okay, well, what actually happens is, as you move the electron, there's this kind of wave that goes out that is, that is an electromagnetic wave that is the thing that's sort of explaining that you're reorienting things. So it's sort of inevitable, once you have these electric lines of force. That you'll end up with... with this kind of, this, this thing that is... that propagates outwards. And that's a very rough explanation of, of kind of how it's sort of inevitable, once you have kind of,

Once you have this kind of notion of an electric field, that you will have, you will have things like photons.

It gets a little more elaborate than this, so... gosh, can I explain this?

I want to explain a concept, a thing called local gauge invariance.

which is kind of related to what I just explained.

Okay.

So... Okay.

Roughly.

Okay, so there are... there are features of physics where things are invariant under certain transformations. So, for example, oh, I'm realizing that I...

been... oh, no, I started later than usual. The,

Things like rotating, if I take an object and I rotate it.

the... we expect that the object will sort of behave the same when it's rotated, if there's nothing else around. If it's just, you know, in the universe, if sort of... there's no other objects near you. Purely rotating won't have an effect. You'll still behave the same way as you did before.

So, rotation, global rotation, is kind of a thing that all physics is invariant under that rotation.

Okay, so there are forms of rotation that are not physical rotation in physical space. There are forms of rotation in kind of an abstract form of space. Those

That... you can imagine rotation not with respect to physical space, but with respect to some kind of abstract space.

And the issue is whether... if you have that rotate... if you have something that's rotated one way.

you can have sort of the same thing, that there's invariance under a global rotation in that kind of abstract space. But the thing that can also happen is you can have so-called local gauge invariance. That is, it can be the case that different sort of places in space are rotated different amounts in this internal space. You can imagine, it's kind of like having a bunch of, sort of clocks... let's not say clocks, because that brings time in. A bunch of pointers in different places in space, and those pointers are sort of adjusted differently in different places, so the zero of those pointers is different for different places in space.

So, if you have that setup, Then, it is if... there is... If it is the case.

That it sort of doesn't matter

where those pointers are aligned in different places in space. If... if sort of physics is set up. so that, sort of, things even themselves out, because, sort of, there's a compensating effect for the fact that those pointers were set differently, then you have this invariance. So what happens is, these gauge bosons, like photons and so on, provide well...

The electromagnetic field, which is kind of a more sort of larger-scale version of photons, provides a so-called connection between these different places in space where you have these different settings for these pointers.

And... and so this... if... if there is sort of invariance under how these pointers are set, even locally, that is achieved necessarily by having a field that can be associated with a particle, like an electron. And all of these different, kind of, all these different particles, so photons are associated with the so-called U1 local gauge invariance. U1 is the name of the sort of group of rotations. It also can be known as SO2. It's a, it's a, it's the possible rotations around a single axis.

The, the gluons.

are associated with a group called SU3, which is a slightly more complicated thing that is, again, sort of a rotation-like idea that involves complex numbers and some other things, and that invariance under that

Is what, means that local imbalance under that is what means that gluons have to exist.

the W bosons and Z bosons that are associated with weak forces, they're associated with an SU2 symmetry group. So, for example.

So that actually means that in the standard model, it is SU3 cross SU2 cross U1, which means there's SU3 symmetry for the gluons, there's SU2 symmetry for the weak bosons, there's U1 symmetry for the photon.

people think it's very inelegant, that there isn't just one kind of group that represents sort of this internal rotation associated with all these different kinds of gauge bosons. And so it turns out SU3 across SU2 across U1 is a subgroup, in the mathematical sense, of a group called SU5, Which is in turn a subgroup of SO10, which is in turn a subgroup of E6. These are... these are all so-called Lie groups, Lie groups. They're just... they're sort of the way of abstractly representing things like all possible rotations. In three dimensions, that would be the group SO3. represents all possible rotations. Okay, so the standard model is this awkward mess of three different rotation-like groups, all kind of plumbed together. And so people thought, back in the 1970s and early 1980s that, gosh, it would be much more elegant

If all those three, sort of, pieces of rotation-like stuff

All came from a single group that was a single, grand, unified kind of theory that was associated with one group.

So, when you do that, if you have the group SU5, or SO10, or these bigger groups, that implies the existence of some additional gauge bosons that, one could imagine have very high mass, so they haven't been observed yet.

But those additional gauge bosons will pretty much necessarily lead to proton decay.

Now, the heavier they are, the lower the probability of proton decay. And so, for example, I think, if I remember correctly, 10 to the... let's see, 10 to the 15 times the mass of the proton.

So that was, that's a thousand trillion times the mass of a proton. I think at that...

mass, one's kind of safe from the limits on proton decay. Protons would decay, but at a rate that is lower than the rate at which they know not to decay. So in these grand unified theories, proton decay is kind of inevitable, but very, very, very small.

And...

It's small now, but in the early universe, when the temperature of the universe was so high that these very high masses didn't matter. Things were bouncing around with kinetic energy that overcame these kind of rest energies, and at that point, it's as if these particles had very low mass, because there's just so much

kinetic energy associated with temperature. So in the very, very, very early universe, the... these particles that are very rare now, because they have such high mass, they're very... the uncertainty principle says they occur very rarely. In the early universe, they would occur a lot. And so that means in the early universe, there would be lots of proton decay, or more to the point, there'd be lots of transformations between protons and positrons.

Things which, where the,

the usual sort of conservation of proton number doesn't apply. So that might have happened in the very, very early universe.

So what

Well, a big so what that I worked out back around 1980 is, 1979, 1980-ish, is one big so what of that.

Is, with one more assumption.

you can explain why there's more matter than antimatter in the universe. It's a big mystery why there are protons, but there aren't antiprotons much in the universe. In the very early universe, there will have been antiprotons, but the mismatch between protons and antiprotons is only one part in 100 million. So, it's very, very close between protons and antiprotons. There's just a tiny excess of protons over antiprotons that leads to all the matter that we see today.

And explaining why that happens, why that happened in the early universe, was kind of a high-wire act of connections between, kind of, so-called baryon number violating, proton number changing interactions, and time reversal violation, and some other things.

that all sort of put together into a rather nice piece of physics that shows why there would be expected to be a difference between the amount of matter and antimatter in the current universe. But since we haven't observed proton decay, we can't kind of home in on that theory. It's not clear it's the right theory, but that's kind of how that works. So, in a sense, the possibility of proton decay

Which might seem bad today.

makes for any days at all, because if there was perfect symmetry between matter and antimatter in the universe, there would be no matter of the kind that makes stars and galaxies and things like this. So we kind of, we kind of,

It's a good thing. If proton decay might be a little teeny-weeny bit bad, but not really very bad if the lifetime is so long. I mean, and if, you know, just to be clear, let's see, I think there are... trying to remember the numbers. I think there are 10^{57} protons in the Sun, and so if the lifetime of a proton is 10^{32} years, that means, oh gosh, this is now arithmetic, I'm not so good at this, Let's see, we're $37 - 37 = 0$. So, 10^{25} .

So that means that, in a year, if there are 10^{57} protons in the sun, and the lifetime of a, of the,

the proton is 10^{32} years. That means in the Sun, about, 10^{30} ... 10^{24} , protons will decay in a year inside the Sun.

It's kind of an interesting number, because that's about a mole of protons. That means that's... that's what you get in, you know, where a mole of water is,

Let's see... 18 grams of water.

is a... yeah, that's right. So not very much water is a... is a mole of water, but that amount of material from the sun would go missing every year if the lifetime of the proton was 10^{32} years. I don't think we would miss it.

But it would go missing, it would just evaporate into... the protons would decay.

The,

let's see, I should probably wrap up here. I think, actually, I'm doing another live stream that's part of my day job, but, you guys are asking all kinds of interesting questions. I'd love to go on yakking on here.

But, I think we have to leave them for another time.

So, this was some quite advanced questions today, but, but,

I... I did my best here to try and explain them. Actually, I was... I was,

I did okay at some concepts that you usually only get in graduate-level physics and so on. I was doing my best to explain them without requiring the whole tower of what you have to get to with graduate-level physics.

Anyway...

Well, thanks for joining me. As I say, I'm about to go do a day job livestream, but, look forward to chatting with you another time on this... in this series.

So, bye for now.