Formation and Evolution of Cosmological Strings

1. Introduction

Grand unified gauge theories imply the existence of phase transitions in the very early universe, at which symmetries are spontaneously broken and Higgs condensates are formed. Topologically-stable defects may in some cases be created in these transitions, and may survive long after the transition, and perhaps even to the present. The presence of such defects may have observable consequences for the present universe.

If G and H are respectively the symmetry groups before and after a transition, then the possible topologically-stable defects generated in the transition are determined from the homotopy of G/H. When G is a simple group, $\pi_k(G/H) = \pi_{k-1}(H)$. Nontrivial $\pi_0(G/H)$ allows the formation of two-dimensional "domain walls" separating different components of the low-temperature phase. Such domain walls would involve immense energies, presumably entirely inconsistent with the energy content of the present universe, and would be unstable to gravitational collapse. A nontrivial $\pi_2(G/H)$ implies topologically-stable pointlike "magnetic monopole" configurations, expected to be completely stable, except for pair annihilation. The consequences of such objects have been considered at length elsewhere. In these notes we consider cases in which $\pi_1(G/H)$ is nontrivial, so that one-dimensional "strings" may be created. In sect. 2 we discuss the configuration of strings generated at the phase transition, and in sect. 3 we consider the dynamical evolution of these strings in the expanding universe. Sect. 4 discusses the significance (if any) of the results for theories of galaxy formation. For simplicity, we shall usually consider the simplest model, in which $G/H = U(1)$, and $\pi_1(G/H) = \mathbf{Z}$. The group orientation of the relevant Higgs field φ may then be specified by a single phase angle. More complicated and perhaps more realistic models introduce presumably inessential complications.

2. String formation

Above the transition temperature the phase angle in the Higgs field φ is presumably uncorrelated between different points in the universe, except in so far as kinetic energy terms in its Lagrangian imply a maximum spatial gradient of the same order as the ambient temperature. At the phase transition, correlations in the field develop over some characteristic distance ξ. For a static system, ξ is finite in a first-order phase transition, and infinite in a second-order one. However, in the expanding early universe, the correlation length can never exceed the horizon size, since correlated regions can presumably grow only at the speed of sound. As a simple approximation, one may divide the universe into many cubes with side length ξ, and take the direction of the Higgs field to be constant throughout each cube. A string passes through the centre of a square formed by the faces of four adjacent cubes if the direction of the Higgs field in the internal $U(1)$ space rotates through 2π when traced around the square in real space. The sense of rotation determines the direction of the string. As a further approximation, it is convenient to take the internal symmetry group for the Higgs field to be a discrete group Z_k, rather than the continuous group $U(1)$. The Higgs field in each cube is then randomly assigned a direction specified by an integer between 0 and $k-1$. Fig. 1 illustrates typical squares formed by faces of four adjacent cubes with $k=3$. The second column in fig. 1 shows the mapping from real space to group space inferred from the Higgs field directions in the first column. A continuous dependence of the Higgs field on position is mimiced by assuming that at each angular step in real space, the Higgs field direction rotates to its new direction in group space in the shorter of the two possible senses. Thus a rotation from direction 0 to direction 1 is in the clockwise sense (with the conventions of fig. 1), while a rotation from 0 to 3 is in the anticlockwise sense. For odd k, the sense thus deduced is always unique. As illustrated in fig. 1, a string is present if the mapping from real space to group space has nonzero winding number. If the Higgs field directions are assigned randomly with equal probabilities, one finds for $k=3$ that the probability of a string with winding number +1 to occur in a particular square is $p_+ \simeq 0.148$. Strings with winding number -1 occur by symmetry with an equal probability $p_- = p_+$. The winding number of a string determines its direction: a left-hand rule is taken by convention. With $k=5$, the probability for a string to occur increases to 0.160, while in the limit $k \to \infty$, the probability tends to 1/6. For simplicity, we shall usually assume that $k=3$.

The characteristics of strings are conveniently investigated by Monte Carlo simulation. With the universe divided into a regular lattice of size ξ cubes, one first assigns randomly to each cube a Higgs field direction specified by an integer between 0 and $k-1$. Then for each side at which four cubes meet, the prescription described above is applied to the values on the surrounding four faces to determine whether a string is present. In this way, each segment of each string is identified. The construction is such that a particular string must either be closed, forming a loop, or must terminate on the boundary of the system. An ambiguity arises when multiple string segments are found to pass through a particular vertex. We assume that each incoming string is assigned randomly to an outgoing string, mimicing the behaviour expected in the absence of a regular cubical lattice. With $k=3$, the probability for no strings to pass through a particular vertex is $\simeq 0.16$. The probability for a single string to enter (and leave) the vertex is $\simeq 0.48$; and the probabilities for two or three strings to pass through the vertex are respectively $\simeq 0.31$ and 0.06. The average number of strings at a given vertex is thus $\simeq 0.42$. Notice, however, that the probabilities for multiple strings at a vertex are not independent: the total probabilities for different numbers of strings do not therefore follow a simple binomial distribution.

In the simplest approximation, each string might be taken as a random walk with step length ξ. In practice, some correlation effects are present. When a string passes a particular vertex one finds that the probability for it continue in a straight line without deflection is $\simeq 0.093$, while the probability for it to be deflected to one of the four orthogonal directions is $\simeq 0.23$. The ratio of these probabilities is $\simeq 2.5$, while for a true random walk they would be equal.

In actual simulations, a cubical lattice with side length up to about 50ξ is used. Only about 13% of the cubes then lie on the surface. Nevertheless, many strings terminate at the surface, rather than forming closed loop in the interior. However, such strings only very rarely penetrate more than about 5ξ from the surface, and their effects are thus usually unimportant. Figure 2 shows the distribution of loop lengths for closed strings.

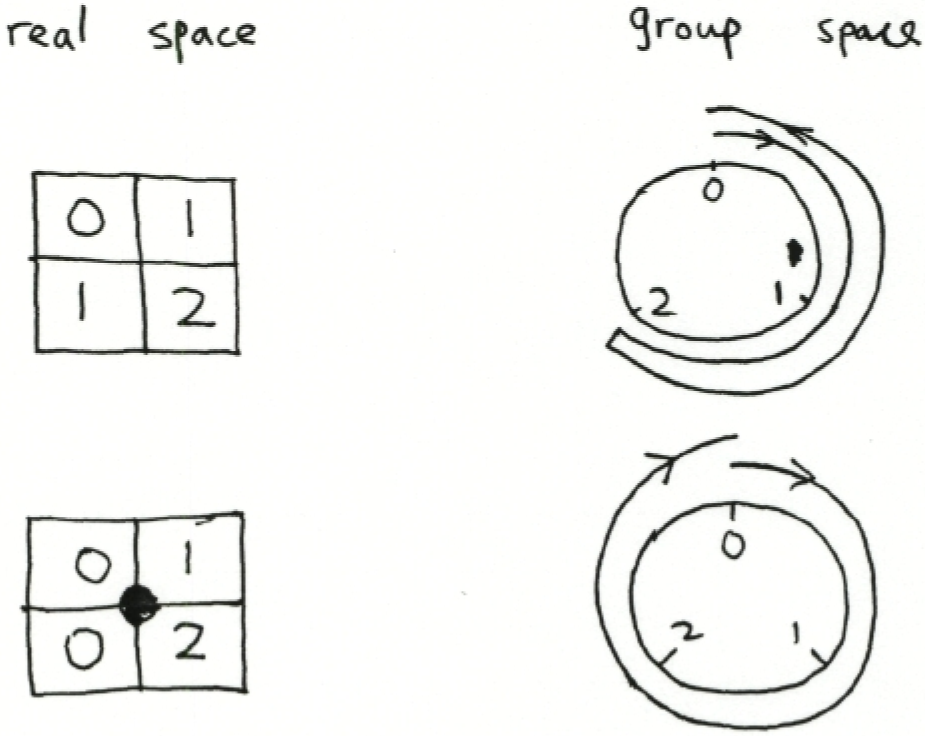
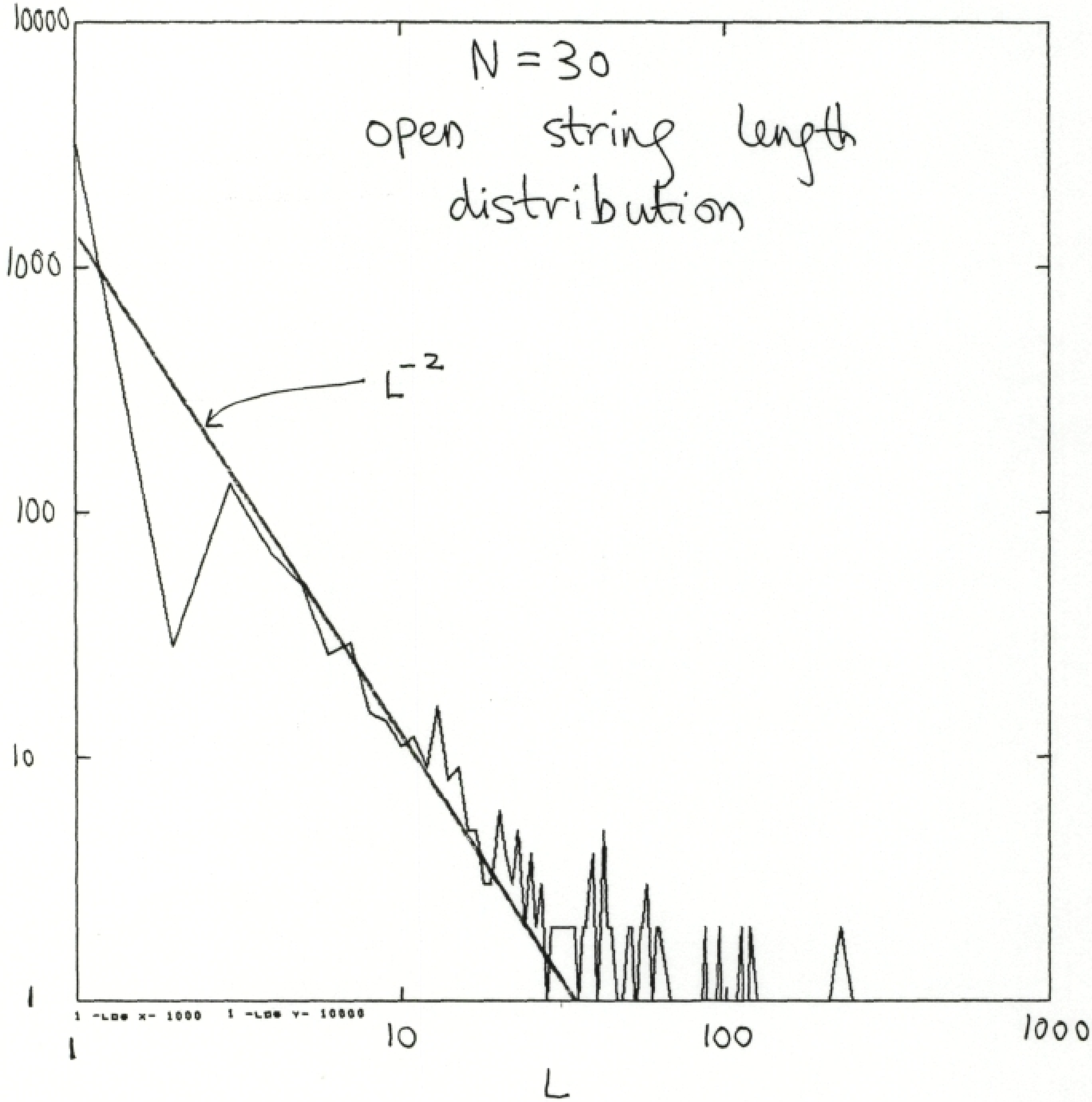
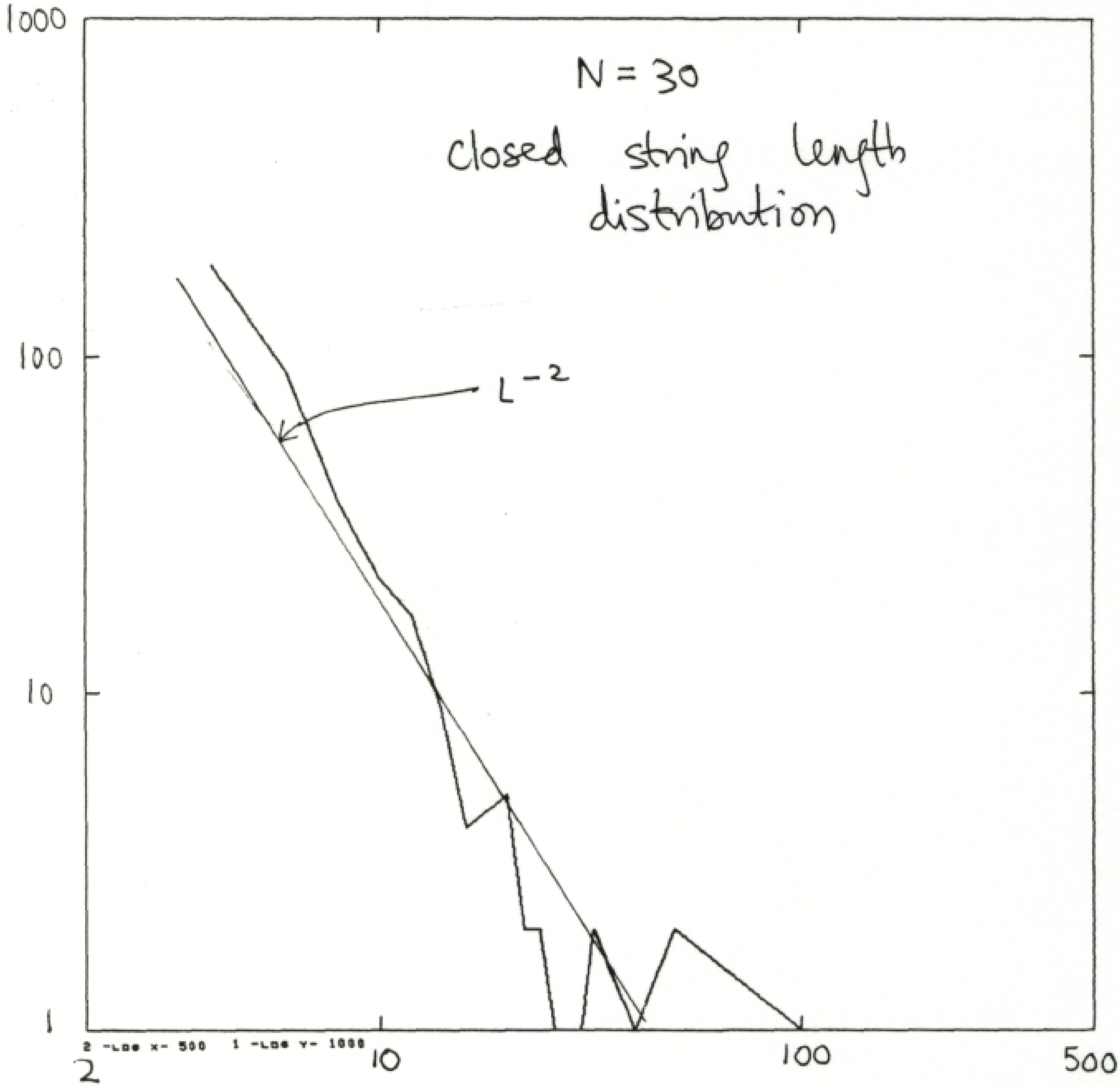


Fig. 1

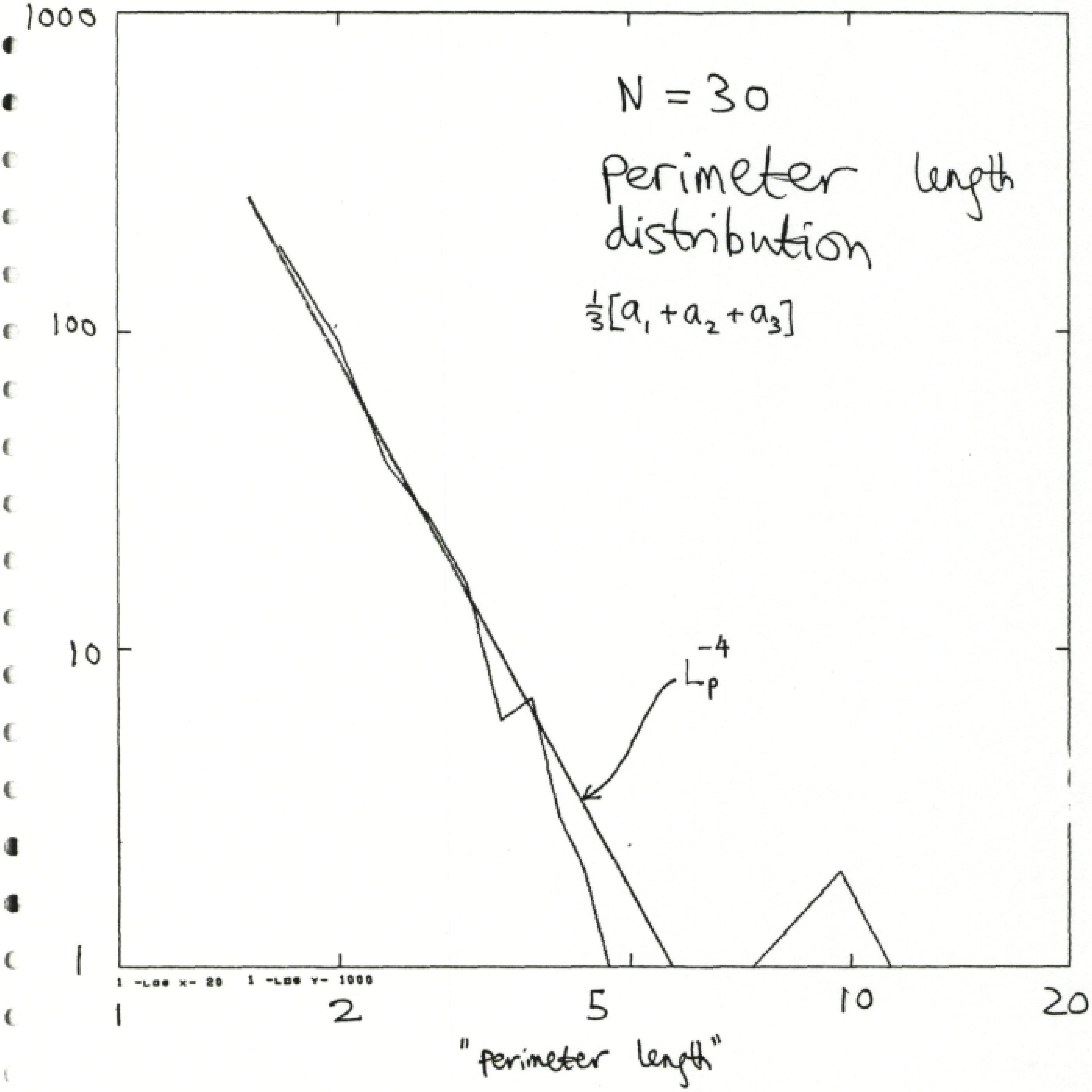


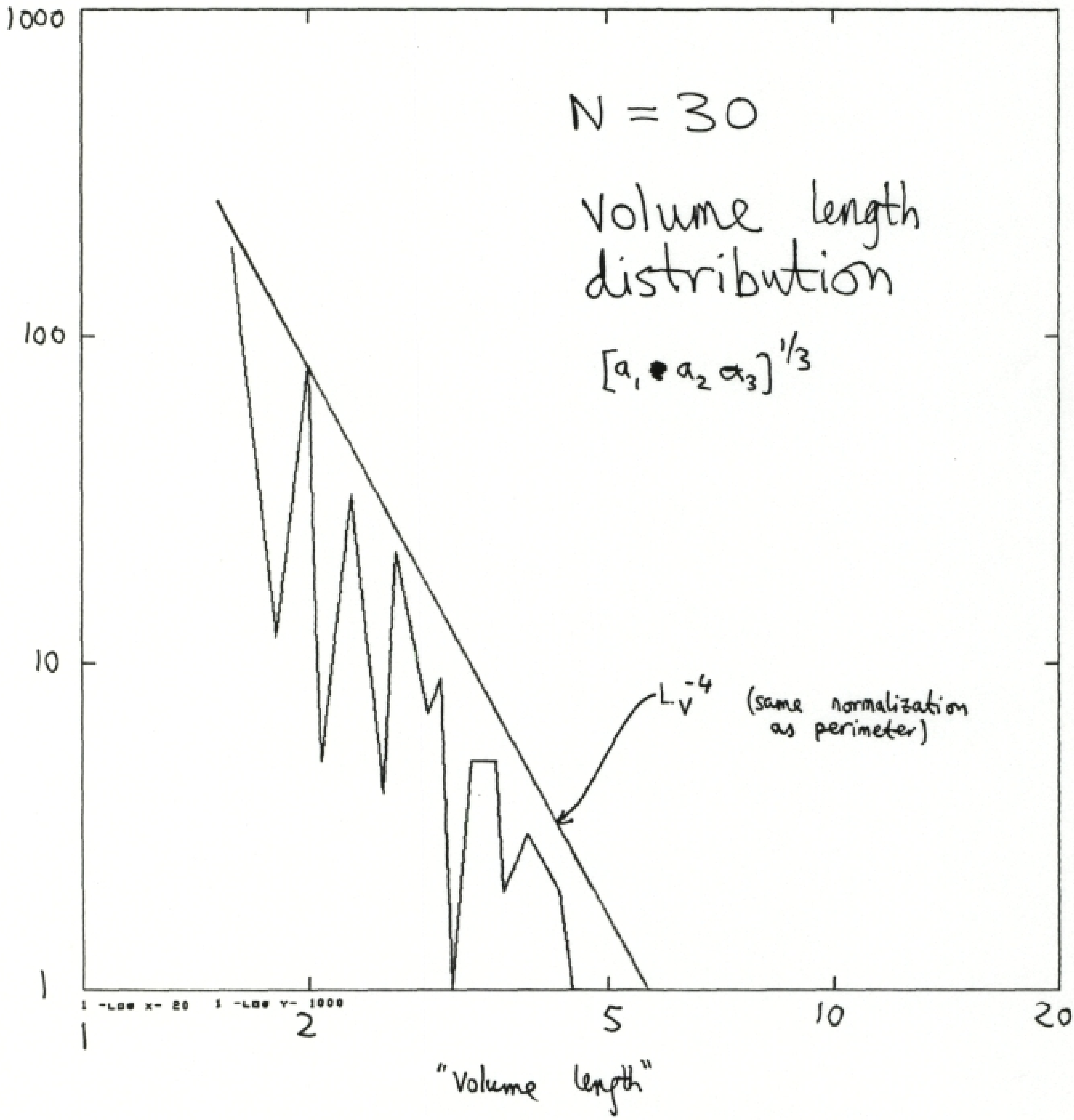
N = 30
open string length
distribution
L^{-2}
10000
1000
100
10
1
1 -LOG X- 1000 1 -LOG Y- 10000
1
10
100
1000
L



N = 30
Closed string length distribution
L^{-2}
1000
100
10
1
2 -LOG X- 500 1 -LOG Y- 1000
2
10
100
500

(should be normalized)




N = 30
volume length distribution
$[a_1 a_2 a_3]^{1/3}$
L_V^{-4} (same normalization as perimeter)
1000
100
10
1
1 -LOG X- 20 1 -LOG Y- 1000
1
2
5
10
20
"Volume length"