

Transverse-momentum and angular distributions of hadroproduced muon pairs

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We study the angular distribution of muons in the dimuon rest frame from $pp \rightarrow \mu\bar{\mu}X$ at high energy and large dimuon mass. Including smearing due to quark transverse momenta, we show that the Drell-Yan model predicts a polar angular distribution in the t -channel helicity frame of $1 + \alpha \cos^2\theta$ with $\alpha \gtrsim 0.8$. Experimental deviations from this prediction would cast serious doubt on the Drell-Yan picture.

Data¹ on the cross section for $pp \rightarrow \mu\bar{\mu}X$ as a function of the $\mu\bar{\mu}$ invariant mass suggest that the μ -pair spectrum is dominated by a few narrow resonances (ψ, ψ', Y, \dots), superimposed on a significant continuum. The normalization, mass dependence, and scaling properties of the continuum were predicted rather accurately on the basis of the Drell-Yan model² illustrated in Fig. 1. The quark x distributions used in the model coincide with those deduced from studies of lepton-hadron interactions³; x is the fraction of proton longitudinal momentum carried by a quark. Further tests of the Drell-Yan model are imperative. In this article we provide predictions for the angular distribution of the muons in their center-of-mass system with respect to both t - and s -channel reference axes. For this it is necessary to make an explicit quantitative study of the effects of allowing the quarks to carry finite momenta transverse to the direction of the incident hadron momentum. Transverse momenta distort the angular distributions of the muons.

In computations with quark-parton models, it has often been assumed that only the quark momentum components parallel to the incident hadron momentum direction need be considered. However, it is clear that the finite size of the proton

implies that the quarks within have finite components of momentum \vec{k} in all directions. As is evident from Fig. 1, the dimuon system is produced with transverse momentum $P_T = 0$ unless the quarks themselves have finite k_T . By contrast, the data⁴ exhibit a $\langle |P_T| \rangle_{\mu\bar{\mu}}$ which increases with $Q^2 = M_{\mu\bar{\mu}}^2$ up to $M_{\mu\bar{\mu}} \approx 3 \text{ GeV}^2$, and then levels off at $\langle |P_T| \rangle_{\mu\bar{\mu}} \approx 1.16 \pm 0.12 \text{ GeV}$ up to the largest masses measured ($M_{\mu\bar{\mu}} \sim 15 \text{ GeV}$). In the Drell-Yan model, $\langle P_T^2 \rangle_{\mu\bar{\mu}} = 2\langle k_T^2 \rangle_q$ for $s \rightarrow \infty$ and $M_{\mu\bar{\mu}} \rightarrow \infty$. Quark transverse momenta are deduced also from other processes,⁵ with $\langle |k_T| \rangle$ in the range 0.5 to 1 GeV.⁶

We suppose therefore that there exists a probability distribution $G_{i/p}(x_j, \vec{k}_{Tj})$ for finding in a proton a quark of type i with momentum $\vec{k}_j = ((\sqrt{s}/2)x_j, \vec{k}_{Tj})$. According to the Drell-Yan model, the differential cross section for the process $pp \rightarrow \mu\bar{\mu}X$ is

$$\begin{aligned} \frac{d\sigma}{d^4P} &= \sum_i \int dx_1 dx_2 d^2\vec{k}_{T1} d^2\vec{k}_{T2} \delta^{(4)}(P - k_1 - k_2) \\ &\quad \times G_{i/p}(x_1, \vec{k}_{T1}) G_{\bar{i}/p}(x_2, \vec{k}_{T2}) \hat{\sigma}(q_i \bar{q}_i \rightarrow \mu\bar{\mu}) \\ &= 2 \frac{d\sigma}{dM_{\mu\bar{\mu}}^2 dy d^2P_T}. \end{aligned} \quad (1)$$

Here \bar{i} denotes an antiquark. We take the quarks to be massless⁷ so that the muons from $q\bar{q} \rightarrow \mu\bar{\mu}$ have an angular distribution⁸ $d\hat{\sigma}/d\cos\theta^* \propto (1 + \cos^2\theta^*)$ in their center-of-mass system, with θ^* measured relative to the quark momentum direction.

As in the work of Feynman, Field, and Fox,⁶ for example, we assume that the quark distribution functions may be factored into x -dependent and $|\vec{k}_T|$ -dependent parts,

$$G(x, \vec{k}_T) = x_R^{-1} x G(x) f(|\vec{k}_T|), \quad (2)$$

with $x_R^2 = [x^2 + 4k_T^2/s]$. For $xG(x)$, we employ the

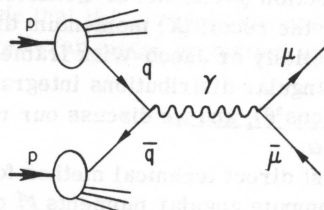


FIG. 1. Drell-Yan diagram for the production of massive dimuon pairs in pp collisions. q denotes a quark and \bar{q} an antiquark.

TABLE I. Possible choices for the quark transverse-momentum distributions. The value of λ is determined from fits to $\langle |P_T| \rangle_{\mu\bar{\mu}}$.

Model	$f(k_T)$	$\langle k_T^2 \rangle_q$	$\langle k_T \rangle_q$	λ	$\langle k_T^2 \rangle_q$ (GeV ²)	$\langle k_T \rangle_q$ (GeV)
Exponential	$e^{-\lambda k_T }$	$6/\lambda^2$	$2/\lambda$	$\lambda = 2.7 \text{ GeV}^{-1}$	0.8	0.7
Gaussian	$e^{-\lambda k_T^2}$	$1/\lambda$	$\frac{1}{2}(\pi/\lambda)^{1/2}$	$\lambda = 1.2 \text{ GeV}^{-2}$	0.8	0.8
Inverse power	$(k_T^2 + \lambda)^{-n}$	$\lambda/(n-2)$	$(n-1)\sqrt{\lambda} B(\frac{3}{2}, n-\frac{3}{2})$			
	$n=4$	$\lambda/2$	$\approx 0.6\sqrt{\lambda}$			
	$n=8$	$\lambda/6$	$\approx 0.35\sqrt{\lambda}$			
	$n=4 \text{ for } q;$ $n=8 \text{ for } \bar{q}$			$\lambda = 2.5 \text{ GeV}^2$	1.25 (q) 0.40 (\bar{q})	0.95 (q) 0.55 (\bar{q})

Field-Feynman quark distributions,³ while we investigate three different forms for the k_T dependence. Our choices are summarized in Table I. The values for n in our function $(k_T^2 + \lambda)^{-n}$ are motivated by constituent-counting rules.⁹ The only free parameter in each case is λ . We determine it by requiring that Eq. (1) reproduce the result $\langle P_T \rangle_{\mu\bar{\mu}} = 1.16 \pm 0.12 \text{ GeV}$ for large $M_{\mu\bar{\mu}}$. In Fig. 2, we show the transverse-momentum distribution provided by our models, at $y \approx 0$ and $M_{\mu\bar{\mu}} = 5 \text{ GeV}$. Similar results are obtained for other values of $M_{\mu\bar{\mu}}$. Although all our models yield the same P_T -integrated cross section and $\langle |P_T| \rangle_{\mu\bar{\mu}}$, some differences are apparent in Fig. 2, particularly the value of the cross section at $P_T = 0$. Changes in the quark x distributions do not appreciably alter any of our results. The curves in

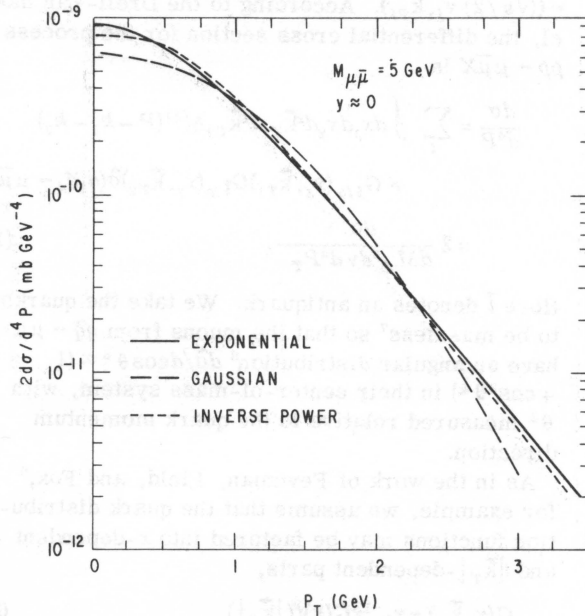


FIG. 2. Differential cross section $d\sigma/d^4P = 2d\sigma/dM_{\mu\bar{\mu}} dy dP_T^2$ versus P_T at $y \approx 0$, $M_{\mu\bar{\mu}} = 5 \text{ GeV}$ and $p_{\text{lab}} = 400 \text{ GeV}/c$. y is the c.m. rapidity of the dimuon. Results obtained from our three models are shown.

Fig. 2 are absolutely normalized inasmuch as the Field-Feynman x distributions are themselves normalized. After integrating over P_T , we obtain distributions $d^2\sigma/dM dy$ at $y \approx 0$ in good agreement with those of previous computations² and with recent data.¹

Examining the variation of $\langle |P_T| \rangle_{\mu\bar{\mu}}$ as $M_{\mu\bar{\mu}}$ changes, we find that in all cases $\langle |P_T| \rangle_{\mu\bar{\mu}}$ is nearly independent of $M_{\mu\bar{\mu}}$ for $M_{\mu\bar{\mu}} \gtrsim 3.5 \text{ GeV}$. Below 3.5 GeV, all but the Gaussian show a fall of $\langle |P_T| \rangle_{\mu\bar{\mu}}$ from $\sim 1.1 \text{ GeV}$ to ~ 0.8 for $M_{\mu\bar{\mu}} \sim 0.5 \text{ GeV}$, in qualitative agreement with the data.⁴ While pleasing, the agreement at low mass should perhaps be regarded as fortuitous since there are theoretical and experimental reasons for lack of confidence in the model for $M \lesssim 4 \text{ GeV}$. We are mostly concerned here with large $M_{\mu\bar{\mu}}$.

Having determined acceptable forms for $f(\vec{k}_T)$, we turn to a discussion of the angular distribution of muons in the dimuon center-of-mass frame. With the advent of large angular acceptance experiments on $pp \rightarrow \mu\bar{\mu}X$, a measurement of this important distribution becomes possible. If the quarks have no momentum transverse to the momenta of their parent hadrons, then the muons should exhibit the same (approximately $1 + \cos^2\theta^*$) distribution with respect to the incident hadron axis as they must with respect to the quark axis. In the $\mu\bar{\mu}$ rest frame, convenient polar axes for the discussion of this angular distribution are the beam direction (t -channel or Gottfried-Jackson frame) or the recoil (X) momentum direction (s -channel helicity or Jacob-Wick frame). We write the final angular distributions integrated over ϕ as⁸ $(1 + \alpha \cos^2\theta)$, and we discuss our results in terms of α .

The most direct technical method for obtaining α is to compute angular moments t_L^M of the Drell-Yan cross section. For the moment t_0^2 , we replace $\hat{\sigma}$ in Eq. (1) by $D_{00}^2(\phi', \theta', 0)\hat{\sigma}$, with $D_{00}^2 = \frac{1}{2}(3 \cos^2\theta' - 1)$. Here (θ', ϕ') are angles which define the quark direction relative to our chosen system of reference axes. Noting further that

$$\int dx_1 dx_2 d^2\vec{k}_{T1} d^2\vec{k}_{T2} \delta^4(P - k_1 - k_2) = \frac{1}{2} \int d\Omega' x_{R1} x_{R2}, \quad (3)$$

we derive

$$\sqrt{10} t_2^0 d\sigma/d^4P = \frac{1}{2} \sum_i \int d\Omega' x_i G_{i/P}(x_1) x_2 G_{i/P}(x_2) \times f_i(|\vec{k}_{T1}|) f_i(|\vec{k}_{T2}|) D_{00}^2 \hat{\sigma}. \quad (4)$$

In Eq. (4), the x_i and \vec{k}_{Ti} must be expressed in terms of θ' and ϕ' . Finally,

$$\alpha = \frac{3\sqrt{10} t_2^0}{4 - \sqrt{10} t_2^0}. \quad (5)$$

We also evaluated the moments t_2^1 and t_2^2 , connected with ϕ dependence, and we shall discuss their values elsewhere. A summary of our results for α is presented in Fig. 3. All our computations are done at $p_{lab} = 400$ GeV/c, but little change is observed in the values of α for $p_{lab} = 1200$ GeV/c. Again taking $M_{\mu\bar{\mu}} = 5$ GeV as a typical mass in the Drell-Yan continuum region, we show in Fig. 3(a) the variation of α versus x_F for the dimuon. In the t -channel frame, α is nearly independent of x_F , but α varies considerably if s -channel axes are used. The rapid variation of α in the s -channel frame at small P_T and small x_F is due simply to the fact that the s -channel axes are ill-defined in this kinematic regime. (The recoil system is nearly motionless.) In the small P_T , small x_F region which contributes most to the cross section, the t -channel frame is therefore preferable.

In Fig. 3(b), we present the variation of α_t with $M_{\mu\bar{\mu}}$ at $x_F \approx 0$ for our three models. After a rapid rise at small $M_{\mu\bar{\mu}}$, α_t becomes roughly constant, taking on values $\alpha_t \approx 0.8$ for $M_{\mu\bar{\mu}} \gtrsim 5$ GeV. Finally, we comment on the P_T dependence of α , not shown here. For $M_{\mu\bar{\mu}}$ and x_F , our results exhibit a systematic decrease of α as P_T is increased. For large enough P_T , α_t becomes negative, corresponding to preferential muon emission transverse to the beam axis.

We conclude that if t -channel axes are used, the naive Drell-Yan model prediction $d\sigma/d\cos\theta$

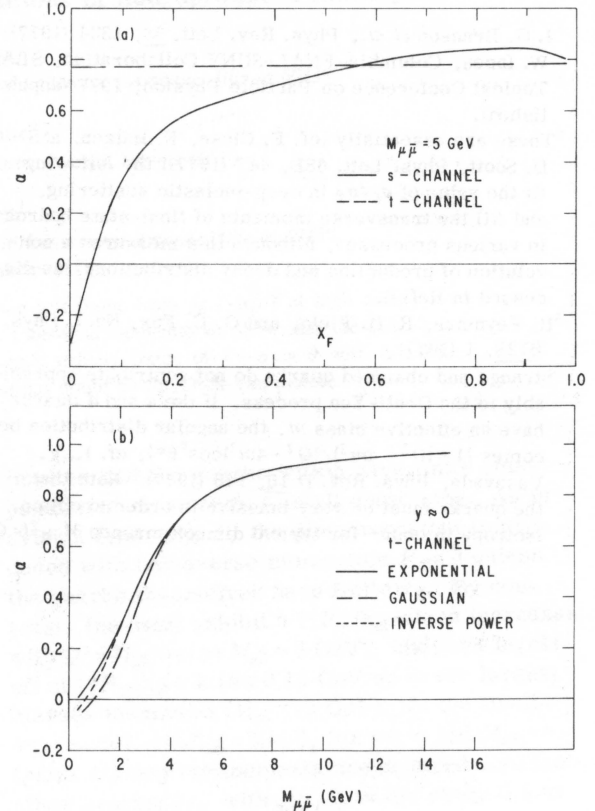


FIG. 3. (a) α versus $x_F = 2P_L/\sqrt{s}$ at $M_{\mu\bar{\mu}} = 5$ GeV for our exponential model. P_L is the c.m. longitudinal momentum of the dimuon. The solid (dashed) curve shows values of α determined with respect to the s -channel (t -channel) polar axis. (b) α versus $M_{\mu\bar{\mu}}$ at $x_F \approx 0$ for all three models. The t -channel axes are used. In (a) and (b), results are averaged over P_T and are for $p_{lab} = 400$ GeV/c.

$\propto (1 + \alpha \cos^2 \theta)$, with $\alpha = 1$, is nearly preserved even after integration over quark transverse momenta. Measurement of large deviations of α from unity (i.e., $\alpha \lesssim 0.8$) at large $M_{\mu\bar{\mu}} (> 4$ GeV) and modest P_T would cast serious doubt on the validity of the naive Drell-Yan picture.¹⁰

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- ⁸ CP -violating couplings could give rise to $\cos\theta$ terms (for $\bar{p}p$ rather than pp as the external hadrons), but spin-2 exchange is necessary for a $\cos^4\theta$ term.
- ⁹M. Duong-Van, SLAC Report No. SLAC-Pub-1819, 1976 (unpublished); J. F. Gunion, Phys. Rev. D 14, 1400 (1976).
- ¹⁰The Drell-Yan model and our analysis are not applicable for $M_{\mu\mu}$ in the region of obvious resonances ($\psi, T(9.5), \dots$). For example, in one model of ψ hadroproduction (Ref. 11), two gluons fuse to produce a χ state, which then decays, $\chi \rightarrow \psi\gamma$. If the ψ - χ mass difference is ignored, then muons from ψ 's arising from spin-0 χ 's will be isotropic ($\alpha=0$). For higher spin χ 's, a set of χ density matrix elements leading to $\alpha=1$ can always be chosen. We shall make further statements about the resonance regions in another article.
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