

## SPONTANEOUS SYMMETRY BREAKING AND THE EXPANSION RATE OF THE EARLY UNIVERSE

EDWARD W. KOLB<sup>1</sup>

W. K. Kellogg Radiation Laboratory, California Institute of Technology

AND

STEPHEN WOLFRAM<sup>2</sup>

High Energy Physics Laboratory, California Institute of Technology

Received 1979 December 3; accepted 1980 January 30

### ABSTRACT

Gauge theories for weak interactions which employ the Higgs mechanism for spontaneous symmetry breakdown imply that there should exist a large vacuum energy associated with the Higgs scalar field condensate. A cosmological term in Einstein's field equations can be arranged to remove the unobserved gravitational effect of this vacuum energy in the present universe. However, in the early universe, the spontaneously broken symmetry should have been restored, leaving the cosmological term uncanceled. In this paper we investigate the conditions necessary for the uncanceled cosmological term to be dynamically important in the early universe. We find that if certain mass relations are satisfied (in particular if the physical Higgs boson is significantly lighter than the gauge boson), then for a brief period, the expansion rate of the universe will be determined by the uncanceled cosmological term prior to symmetry breaking. For example, in the Weinberg-Salam model with  $\sin^2 \theta_w = 0.23$ , if the mass of the physical Higgs boson is less than 11 GeV, the universe would have undergone a period of nonadiabatic expansion prior to the temperature at which the symmetry is broken.

*Subject headings:* cosmology — elementary particles

### 1. INTRODUCTION

It appears very likely that the basic Lagrangian for weak interactions is invariant under a local gauge symmetry (see, e.g., Taylor 1976). However, the observation of masses for bosons and fermions participating in weak interactions shows that such a symmetry is not manifest, at least under the conditions of present experiments. In models for weak interactions, the breaking of the symmetry is usually achieved by the Higgs mechanism. According to this, all massive particles are coupled to a scalar (Higgs) field whose self-couplings are such that it attains a suitable non-zero expectation value in the "vacuum" (lowest energy) state. However, there is thus far no direct experimental evidence for or against the existence of Higgs scalar fields (Gaillard 1978). Certainly most of the predictions of models for weak interactions of relevance at presently accessible energies are entirely independent of the mechanism of mass generation. There exist some requirements on the spectrum of masses necessary to maintain the self-consistency of the models (Dicus and Mathur 1973; Lee, Quigg, and Thacker 1977; Politzer and Wolfram 1979; Huang 1979), but none are amenable to immediate experimen-

tal investigation. Of course, the observation of a Higgs particle would confirm that the Higgs mechanism is operative, but the prospects for such an observation are probably somewhat remote (Gaillard 1978). In the attempt to investigate the Higgs mechanism one is therefore led to consider its effects in cosmology. The existence of a vacuum expectation value for the Higgs field apparently leads to a very large energy density in the universe (Linde 1974), whose presence is completely contradicted by observation (Weinberg 1972; Misner, Thorne, and Wheeler 1973). However, the introduction of a compensating cosmological constant term into Einstein's equations can be arranged to remove this contradiction (Linde 1974) by providing an effective vacuum energy density which cancels the large vacuum energy density associated with the Higgs field. However, such a cancellation may not have been maintained throughout the evolution of the universe.

There are indications that, at the high temperatures of the early universe, symmetries which are presently "spontaneously broken" by the Higgs mechanism should have been restored and that the vacuum expectation value of the Higgs field should then vanish (Kirzhnits and Linde 1972; Kirzhnits 1972). In that case, the large energy density contributed by the Higgs field should have disappeared, leaving uncanceled the cosmological term which must be arranged to remove its effects at lower temperatures. In this paper, we

<sup>1</sup> Work supported in part by the National Science Foundation (PHY76-83685).

<sup>2</sup> Work supported in part by the Department of Energy (DE-AC-03-79ER0068) and by a Feynman Fellowship.

report on the consequences of this phenomenon for the development of the early universe. We find (despite previous claims to the contrary [Bludman and Ruderman 1977]) that it can determine the expansion rate of the universe for a short period, but unfortunately this probably does not result in sufficient modification of the evolution of the universe to lead to presently observable effects.

This paper is organized as follows. In § II we discuss the gravitational effects of a Higgs condensate (Higgs field with a classical vacuum expectation value). In § III we consider the restoration of spontaneously broken symmetries in the early universe and its consequences for the evolution of the universe.

## II. GRAVITATIONAL EFFECTS OF SPONTANEOUS SYMMETRY BREAKDOWN

An inevitable consequence of *quantized* field theories is the existence of zero-point quantum fluctuations in the fields, usually leading to a nonzero energy density for the "vacuum." These effects may be removed by considering only normal-ordered products of field operators. In unbroken supersymmetric theories they cancel between fluctuations in boson and fermion fields and so are exactly zero. It therefore appears possible that these processes should have no gravitational effects, at least when gravitation is treated classically. In the Higgs mechanism, however, there must exist a field whose vacuum expectation value is purely *classical*. The resulting vacuum energy density is of a somewhat different character than that associated with zero-point fluctuations, and it appears likely that its gravitational effects cannot be neglected. (To determine whether this is indeed correct, one would have to investigate the Higgs mechanism in a quantized theory of gravitation.)

We begin from Einstein's field equations with a cosmological term (we choose units such that  $G = c = \hbar = 1$ ):

$$R_{\mu\nu} = -\frac{1}{2}Rg_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}. \quad (2.1)$$

For a perfect relativistic fluid the energy-momentum  $T_{\mu\nu}$  is given by (our metric has signature  $+- - -$ ):

$$T_{\mu\nu} = -pg_{\mu\nu} + (p + \rho)U_\mu U_\nu, \quad (2.2)$$

where  $p$  is the pressure,  $\rho$  the energy density, and  $U$  the velocity of the fluid in a comoving frame. We may absorb the cosmological term in equation (2.1) by defining a generalized energy-momentum tensor (see, e.g., Zel'dovich and Novikov 1971):

$$T_{\mu\nu}^* = (\Lambda/8\pi)g_{\mu\nu} + T_{\mu\nu}, \quad (2.3)$$

so that Einstein's equations (2.1) become

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu}^*. \quad (2.4)$$

Introducing a "vacuum energy density,"

$$\epsilon_\Lambda = \Lambda/8\pi, \quad (2.5)$$

the generalized energy-momentum tensor for a perfect

fluid may be written in the form

$$T_{\mu\nu}^* = -p^*g_{\mu\nu} + (p^* + \rho^*)U_\mu U_\nu, \quad (2.6)$$

where

$$p^* = p - \epsilon_\Lambda, \quad (2.7a)$$

$$\rho^* = \rho + \epsilon_\Lambda. \quad (2.7b)$$

For a field theory with spontaneous symmetry breaking (SSB), extra terms must be added to the energy-momentum tensor (2.2). We consider the complex scalar field ( $\phi$ ) theory with Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi),$$

$$V(\phi) = -(\mu^2/2)\phi^2 + (\lambda/4)\phi^4 \quad (\mu^2 > 0). \quad (2.8)$$

The  $\phi$  field then has a classical vacuum expectation when  $\delta V/\delta\phi = 0$ , so that

$$\phi_0 \equiv \langle \phi \rangle = (\mu^2/\lambda)^{1/2}. \quad (2.9)$$

A form of equation (2.8) is necessary in the Lagrangian for the Weinberg-Salam model. In that model

$$\phi_0^2 = (\sqrt{2}G_F)^{-1} = 4m_W^2/g^2, \quad (2.10)$$

where  $G_F$  is the Fermi coupling constant,  $m_W$  is the mass of the charged vector boson, and  $g$  is the  $SU(2)_L$  coupling constant:

$$(4\pi\alpha)^{1/2} = e = g' \cos \theta_W = g \sin \theta_W. \quad (2.11)$$

The classical expectation value of the energy-momentum tensor for the  $\phi$  field is simply

$$\langle T_{\mu\nu} \rangle = V(\phi_0)g_{\mu\nu} \equiv \epsilon_{SSB}g_{\mu\nu}. \quad (2.12)$$

For spontaneous symmetry breaking to occur,  $\phi = \phi_0$  must correspond to the absolute minimum of  $V(\phi)$ , so that  $V(\phi) < V(0) = 0$ . One then finds ( $\Xi$  represents the effect of higher order terms in the effective potential):

$$\begin{aligned} V(\phi_0) &= -(\mu^4/4\lambda)(1 - \Xi) \\ &= -(m_H^2 m_W^2/2g^2)(1 - \Xi). \end{aligned} \quad (2.13)$$

In equation (2.13)  $m_H$  is the mass of the Higgs particle fluctuations of the  $\phi$  field about its classical vacuum expectation value,  $\phi_0$ :

$$m_H^2 = (\delta^2 V/\delta\phi^2)|_{\phi=\phi_0} = 2\mu^2. \quad (2.14)$$

To the one-loop level, ignoring contributions from Higgs scalar loops (Coleman and Weinberg 1973),

$$\Xi = \frac{4\phi_0^2}{m_H^2} \frac{1}{1024\pi^2} \left[ \sum_{\text{gauge bosons}} 3g_i^4 - \sum_{\text{fermions}} f_i^4 \right], \quad (2.15)$$

where the  $f_i(g_i)$  are the couplings of the fermions (gauge bosons) to the Higgs. Note that because of Fermi statistics (closed fermion loops have a relative minus sign) the fermion contribution to  $\Xi$  is negative. From equations (2.12) and (2.13) it is clear that the

spontaneous symmetry breaking vacuum energy density is

$$\begin{aligned}\epsilon_{\text{SSB}} &= -\frac{m_H^2 m_W^2}{2g^2} (1 - \Xi) \\ &\approx -8 \times 10^3 (m_H [\text{GeV}])^2 \\ &\quad \times \left[ 1 - \frac{21}{(m_H [\text{GeV}])^2} \right] \text{GeV}^4 \\ &\approx -2 \times 10^{21} (m_H [\text{GeV}])^2 \\ &\quad \times \left[ 1 - \frac{21}{(m_H [\text{GeV}])^2} \right] \text{g cm}^{-3}. \quad (2.16)\end{aligned}$$

For numerical estimates of  $\Xi$  see, e.g., Politzer and Wolfram (1979); here we ignore the fermion contribution. In this model,  $m_H \gtrsim 4.5 \text{ GeV}$  (Linde 1976a; Weinberg 1976), since  $\epsilon_{\text{SSB}} \geq 0$  for spontaneous symmetry breakdown to occur. There is some theoretical incentive for the guess  $m_H \approx 9 \text{ GeV}$  (Coleman and Weinberg 1973).

Observations of the present rate of expansion of the universe show that its average mass density is less than  $10^{-29} \text{ g cm}^{-3}$ . According to equation (2.16) the vacuum energy density contributed by the Higgs field is typically more than a factor of  $10^{50}$  too large. (The possibility that  $m_H \lesssim 10^{-16} \text{ eV}$  is experimentally excluded.) This discrepancy could be taken to show that the Higgs mechanism is not operative and would suggest that the symmetry breakdown is dynamical in origin. We shall, however, not take this point of view, but rather assume the Higgs mechanism and investigate a possible cosmological resolution of the discrepancy. Addition of further Higgs fields cannot remove the vacuum energy, since each contributes a negative amount to  $\langle T_{\mu\nu} \rangle$ , as they must all have  $V(\phi_0) < 0$  for spontaneous symmetry breakdown to have occurred.

One possible mechanism (Linde 1974) which may remove the effects of the vacuum energy arising from the Higgs field is the presence of a large compensating cosmological term in Einstein's equations (2.1) which leads to a canceling effective vacuum energy density (see eq. [2.5]). To avoid contradictions with observation we must then demand that in the present universe

$$\epsilon_{\text{TOT}} = \epsilon_{\text{SSB}} + \epsilon_{\Lambda} \lesssim 10^{-29} \text{ g cm}^{-3}. \quad (2.17)$$

Of course, it would be very surprising if these two contributions to the effective energy density of the universe, having such different origins, should cancel to the required accuracy of better than about one part in  $10^{50}$ . Our purpose is to investigate whether such a delicate cancellation could be maintained throughout the history of the universe. We find that, while it is probable that the cancellation failed under certain conditions, its failure does not appear to result in observable consequences for the present universe. However, we do find that it is quite possible that the expansion rate of the universe, i.e.,  $\dot{a}/a$  (where  $a$  is the

Robertson-Walker scale parameter) may for a period have been determined by  $\epsilon_{\Lambda}$ , rather than the temperature of the relativistic particles.

### III. THE RESTORATION OF SPONTANEOUSLY BROKEN SYMMETRIES IN THE EARLY UNIVERSE AND ITS CONSEQUENCES

The conventional results (eq. [2.9]) on spontaneous symmetry breakdown are all obtained in the approximation that fluctuations in the Higgs field are unimportant in determining the symmetry of the "vacuum." However, when the ambient temperature ( $T$ ) is sufficiently high, this approximation will inevitably break down, as thermal fluctuations in the Higgs field strength become comparable to the difference between its zero temperature value ( $\phi_0$ ) in the "ordered phase" (in which symmetry breakdown occurs) and in the "disordered phase," or ordinary vacuum state ( $\phi = 0$ ) (Kirzhnits and Linde 1972; Kirzhnits 1972). At zero temperature the "order parameter"  $\langle \phi \rangle$  is given simply by equation (2.9):  $\langle \phi \rangle^2 = \mu^2/\lambda$ . However, at finite temperature a simple argument suffices to show that fluctuations in the  $\phi$  field about  $\phi = \phi_0 \approx (\mu/\sqrt{\lambda})$  lead to (Kirzhnits and Linde 1972; Weinberg 1974; Dolan and Jaciw 1974):

$$\langle \phi \rangle^2 = \phi_0^2 - \langle (\phi - \phi_0)^2 \rangle \approx \mu^2/\lambda - T^2/3. \quad (3.1)$$

It is therefore clear that a phase transition to the disordered phase  $\phi = 0$  occurs when

$$T = [3\mu^2/\lambda]^{1/2}. \quad (3.2)$$

If now we include the coupling of the Higgs field to the gauge bosons (which is essential in order to generate the gauge boson masses), then the order parameter becomes (Kirzhnits and Linde 1972; Weinberg 1974; Dolan and Jaciw 1974):

$$\langle \phi \rangle^2 \approx \mu^2/\lambda - T^2/3 - T^2/4\lambda \sum_{\text{gauge bosons}} g_i^2, \quad (3.3)$$

where the  $g_i^2$  are the couplings of the gauge bosons ( $W^+$ ,  $W^-$ ,  $Z^0$ ) to  $\phi$ . The difference in the coefficients of the terms in equation (3.3) corresponding to  $\phi$  field and gauge boson fluctuations is a consequence simply of the different numbers of spin states for the fields. In the Weinberg-Salam model, therefore, the critical temperature is given by<sup>3</sup>

$$T_c^2 \approx 12\mu^2/[3(3g^2 + g'^2) + 4\lambda]. \quad (3.4)$$

For  $\lambda = g^2 m_H^2/4m_W^2 \ll \frac{3}{4}(3g^2 + g'^2) \approx 1$ , the term associated with gauge boson fluctuations dominates, so that using  $\sin^2 \theta_W = 0.23$ ,

$$T_c \approx 1.2 m_H. \quad (3.5)$$

<sup>3</sup> This calculation is valid to all orders in  $g^2 T^2$ , but only to lowest order in  $g^2$  (here  $g$  represents either gauge or scalar coupling). Perturbation theory breaks down when  $T \approx T_c$ , since powers of  $g$  can be canceled by factors that become infrared divergent when the mass vanishes at  $T \approx T_c$ . Therefore, equation (3.4) should only be regarded as an estimate. However, we can expect the true critical temperature  $T_c^0$  to be in the range  $|T_c^0 - T_c| \lesssim g^2 T_c$ , where  $T_c$  is given by equation (3.4). See, e.g., Weinberg (1974) for details.



If the gauge boson term dominates,  $m_H$  is the natural scale for  $T_c$ , rather than  $m_W$  as assumed by Bludman and Ruderman (1977). (For  $m_H \gg 200$  GeV,  $T_c \approx \sqrt{(12)m_W/g} \approx 400$  GeV.) High fermion densities can influence  $\langle \phi \rangle^2$  only through the strong gauge boson fields which result from them. If  $J_\mu$  is the expectation value of the fermion four-current ( $J_\mu = \langle \bar{\psi} \gamma_\mu \psi \rangle$ ), then typically in the ordered phase

$$\langle \phi \rangle^4 (\lambda \langle \phi \rangle^2 - \mu^2) - J^2 = 0, \quad (3.6)$$

so that a large fermion charge density can serve to prevent symmetry restoration (Linde 1976b).

In the standard Friedmann model for the evolution of the universe, the expansion scale factor  $a(t)$  is determined from the equations ( $\dot{\phantom{x}}$  denotes time derivative):

$$\begin{aligned} (\dot{a})^2 &= -k + a^2 [8\pi\rho^*(a)/3], \\ 2\ddot{a} &= -[(\dot{a})^2/a] - k/a - 8\pi\rho^*(a), \end{aligned} \quad (3.7)$$

where account has been taken of a possible cosmological term in Einstein's equations by using the "generalized" density ( $\rho^*$ ) and pressure ( $p^*$ ) defined in equation (2.7). Since we shall consider the very early universe, the terms proportional to the intrinsic curvature  $k$  in equation (3.7) may be neglected.

We must now assume an equation of state for  $\rho(T)$  and  $p(T)$  in order to compute the evolution of the universe using equation (3.7). Quantum chromodynamics (QCD) has been found to provide an excellent theory for strong interactions, and it agrees with experiment in all cases where a comparison has been made (see, e.g., Field 1978). In particular, its property of asymptotic freedom has been tested in deep-inelastic lepton-nucleon interactions. This property leads to the prediction that at temperatures exceeding about  $2 \times 10^{12}$  K and/or densities above about  $10^{15}$  g cm $^{-3}$  or  $10^{16}$  g cm $^{-3}$ , assemblies of hadrons should behave like an ideal gas of quarks and gluons (Collins and Perry 1975). (Note that models [Hagedorn 1965; Harrison 1972] predicting a maximum temperature for hadronic matter are disfavored by recent experimental results indicating the presence of pointlike weakly interacting constituents within hadrons at short distances, in agreement with QCD.) In this case the energy density of the universe at high temperatures will be

$$\rho(T) \approx (\pi^2/15) T^4 N_{\text{eff}}(T), \quad (3.8)$$

where  $N_{\text{eff}}$  is the effective number of particle species in thermal equilibrium at a temperature  $T$ . Ultrarelativistic boson (fermion) spin states contribute  $1/2(7/16)$  to  $N_{\text{eff}}$ . For  $m_e \lesssim kT \lesssim m_\mu$ ,  $N_{\text{eff}} \approx 5$  due to contributions from  $\gamma$ ,  $e^\pm$ ,  $\nu_i$ ,  $\bar{\nu}_i$  ( $i = e, \mu, \tau$ ), while above  $kT \approx m_\mu$ , it receives a contribution 1.75 from  $\mu^\pm$ . According to QCD, above  $kT \approx 0.5$  GeV it should be the quarks and gluons rather than the hadrons into which they are seen to be combined at lower temperatures, which contribute to  $N_{\text{eff}}$ , giving  $N_{\text{eff}} \approx 45$ . In deriving  $N_{\text{eff}}$  we have also included a massless  $W^\pm$  and

$Z$ , since above  $T_c$  the symmetry is restored and they are massless. The pressure should be related to the energy density by the ideal gas law for ultrarelativistic particles:

$$p = \frac{1}{3}\rho. \quad (3.9)$$

Using the result (3.9), equations (3.7) become

$$(\dot{a})^2 = (8\pi/3)(\rho + \epsilon_{\text{TOT}})a^2, \quad (3.10a)$$

$$\ddot{a} = -(8\pi/3)(\rho - \epsilon_{\text{TOT}})a, \quad (3.10b)$$

where  $\epsilon_{\text{TOT}}$  was defined in equation (2.15) as  $\epsilon_{\text{SSB}} + \epsilon_\Lambda$ . Below  $T = T_c$ , the symmetry is spontaneously broken, and one can arrange  $\epsilon_{\text{TOT}} \approx 0$ . Above  $T = T_c$ , the symmetry should be restored, so that  $\epsilon_{\text{SSB}} = 0$ , leaving uncanceled the large cosmological term  $\epsilon_\Lambda$ , which contributes a constant energy density, independent of temperature. At early times, the radiation energy density, which grows like  $T^4$ , should have been entirely dominant. However, as  $T$  decreased,  $\rho$  may have fallen below  $\epsilon_\Lambda$ . At  $T = T_c$ ,

$$\rho \approx (45\pi^2/15)T_c^4 \approx 60 m_H^4, \quad (3.11)$$

where the second equality follows as long as  $m_H \lesssim 300$  GeV. On the other hand, the effective vacuum energy density contributed by the cosmological term is (minus the Higgs condensate energy density for  $T < T_c$ ):

$$\epsilon_\Lambda \approx \frac{m_W^2 m_H^2}{2g^2} \approx 8 \times 10^3 (m_H [\text{GeV}])^2 \text{GeV}^4. \quad (3.12)$$

The condition for  $\epsilon_{\text{TOT}}$  to be greater than  $\rho$  prior to symmetry breaking (when  $\epsilon_{\text{TOT}} = 0$ ) is

$$\begin{aligned} 8 \times 10^3 (m_H [\text{GeV}])^2 &\geq 60 (m_H [\text{GeV}])^4, \\ 11.5 \text{ GeV} &\geq m_H. \end{aligned} \quad (3.13)$$

The limit in inequality (3.13) includes  $m_H \approx 9$  GeV, which is favored from other considerations (Coleman and Weinberg 1973; Weinberg 1979).

If  $m_H$  satisfies the inequality (3.13) the following scenario can be imagined.

1.  $T \gg T_c$ . In this region  $\rho \gg \epsilon_{\text{TOT}} = \epsilon_\Lambda$ . The generalized energy density is dominated by the contribution of relativistic particles, and equation (3.10a) becomes ( $G^{-1/2} = m_p = 1.22 \times 10^{-22}$  GeV):

$$\frac{\dot{a}}{a} = \left[ \frac{8\pi}{3} G \frac{\pi^2}{15} T^4 N_{\text{eff}}(T) \right]^{1/2} \approx [N_{\text{eff}}(T)]^{1/2} \frac{T^2}{0.43 m_p}. \quad (3.14)$$

Since  $T \approx a^{-1}$ , the solution to equation (3.14) is  $a \approx t^{1/2}$ . This is the standard hot big-bang expansion rate.

2.  $T \gtrsim T_c$ . If inequality (3.13) is satisfied, then  $\epsilon_{\text{TOT}} = \epsilon_{\text{SSB}} > \rho$ . The generalized energy density is domi-

nated by  $\epsilon_{\text{TOT}} = \epsilon_\Lambda$ , and equation (3.10a) becomes

$$\frac{\dot{a}}{a} = \left( \frac{8\pi G}{3} \epsilon_\Lambda \right)^{1/2} = \left( \frac{8\pi G}{3} \frac{m_W^2 m_H^2}{2g^2} \right)^{1/2} \\ \approx \frac{2.6 \times 10^2}{m_p} (m_H [\text{GeV}]) \text{ GeV}^2. \quad (3.15)$$

The solution to equation (3.15) is  $a \approx \exp(t)$ . This expansion rate differs from the standard expansion rate.

3.  $T \lesssim T_c$ . Below  $T_c$ , the symmetry is broken,  $\epsilon_{\text{TOT}} = \epsilon_{\text{SSB}} + \epsilon_\Lambda = 0$ , and the expansion rate is once again the standard form (3.14).

Our conclusion is that the vacuum energy density of the Higgs scalar field could have had important dynamical effects in the early universe; it could have for a brief period dominated the expansion rate. The requirement for this to happen is that the Higgs be much lighter than the vector bosons (but as shown

above, still allowed). In this case the Higgs mass sets the scale for the critical temperature, rather than the vector mass.

A glitch in the expansion rate at  $T \approx T_c$  may not have direct observational consequences. However, there are several interesting phenomena that may occur. If  $T_c \ll m_W$ , there is likely to be a first order phase transition, generating entropy and perhaps causing inhomogeneities to develop. If for any period  $\epsilon_\Lambda > \rho$ , then the effective pressure of the universe (2.7a) is negative.

Although we have considered only the Weinberg-Salam model, our results could apply in any spontaneously broken gauge theory, in which the Higgs mass is much less than the gauge field mass. If the Lagrangian for the scalar field is not incorporated into a gauge theory, the results of Bludman and Ruderman (1977) obtain, and the vacuum energy can never be important in determining the expansion rate.

#### REFERENCES

- Bludman, S. A., and Ruderman, M. A. 1977, *Phys. Rev. Letters*, **38**, 255.  
 Coleman, S., and Weinberg, E. 1973, *Phys. Rev. D*, **7**, 1888.  
 Collins, J. C., and Perry, M. J. 1975, *Phys. Rev. Letters*, **34**, 1353.  
 Dicus, D. A., and Mathur, V. S. 1973, *Phys. Rev. D*, **7**, 3111.  
 Dolan, L., and Jaciw, R. 1974, *Phys. Rev. D*, **9**, 3320.  
 Field, R. D. 1978, "Dynamics of High Energy Reactions," CALT-68-683, presented at 19th Internat. Conf. on Particle Physics, Tokyo, 1978.  
 Gaillard, M. K. 1978, *Comm. Nucl. Part. Phys.*, **8A**, 31.  
 Hagedorn, R. 1965, *Nuovo Cimento Suppl.*, **3**, 147.  
 Harrison, E. R. 1972, *Comm. Ap. Space Phys.*, **4**, 187.  
 Huang, P. Q. 1979, *Phys. Rev. Letters*, **42**, 873.  
 Kirzhnits, D. A. 1972, *JETP Letters*, **15**, 529.  
 Kirzhnits, D. A., and Linde, A. D. 1972, *Phys. Letters*, **42B**, 471.  
 Lee, B. W., Quigg, C., and Thacker, H. B. 1977, *Phys. Rev. Letters*, **38**, 883.  
 Linde, A. D. 1974, *JETP Letters*, **19**, 183.  
 ———. 1976a, *JETP Letters*, **23**, 73.  
 ———. 1976b, *Phys. Rev. D*, **14**, 3345.  
 Misner, C. W., Thorne, K. S., and Wheeler, J. A. 1973, *Gravitation* (New York: Wiley).  
 Politzer, H. D., and Wolfram, S., 1979, *Phys. Letters*, **82B**, 242.  
 Taylor, J. C. 1976, *Gauge Theories of Weak Interactions* (Cambridge: Cambridge University Press).  
 Weinberg, S. 1972, *Gravitation and Cosmology* (New York: Wiley).  
 ———. 1974, *Phys. Rev. D*, **9**, 3357.  
 ———. 1976, *Phys. Rev. Letters*, **36**, 294.  
 ———. 1979, *Phys. Letters*, **82B**, 297.  
 Zel'dovich, Ya. B., and Novikov, I. D. 1971, *Relativistic Astrophysics* (Chicago: University of Chicago Press).

EDWARD W. KOLB: W. K. Kellogg Radiation Laboratory 106-38, California Institute of Technology, Pasadena, CA 91125

STEPHEN WOLFRAM: Charles C. Lauritsen Laboratory of Physics 452-48, California Institute of Technology, Pasadena, CA 91125