# INTRODUCTION TO THE MEAK INTERACTION: 

VOLUME ONE

STEPMEN MOLFRAM

# INTRODUCTION TO THE WEAK INTERACTION 

Stephen Wolfram

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### 1.1 Natter Waves.

In 1923 L.de Broglie suggested (1) that radiation and matter might in some way be both wave-like and corpuscular. He was led to this idea by the fact that in some experiments, such as that of Young's slits (2), light behaved as if it were a wave, whereas in others, such as compton scattering (3), it behaved like a corpuscle. He postulated that electrons might also possess a dual nature, and should thus be able to display wave-like characteristics.

Let us now assume that matter or de Broglie waves exist, and attempt to find their form for a single particle moving uniformly in the absence of any external force ${ }^{l}(4)$. Let the mass of the particle be $m$, its momentum $\underline{p}$ and its energy be E. We should expect the de Broglie wave of the particle to be longditudinal and hence we represent it in the standard manner by the wave function (5):
$\psi(\underline{x}, t)=A \exp (j \underline{x} \cdot \underline{k}-j f t)$,
where $\underline{x}$ and $t$ are the position and time co-ordinates of points on the wave, $A$ is the amplitude of the wave, $f$ is its frequency, and $\underline{k}$ is its wave or propagation vector. De Broglie's problem was to find a formula for $k$ in terms of the kinematical and dynamical variables of the particle. The wave described by (1.1.1) is a plane wave, whose planes of constant phase, $\phi$, are given by $(\underline{x} \cdot \underline{k}-f t)=\phi$.
These planes, and hence the whole wave, propagate with the phase velocity $\mathrm{v}_{\mathrm{f}}=\mathrm{fk} / \mathrm{k}^{2}$.
However, in the light of later developments, we find that we must not equate the phase, but the group velocity of the de Broglie wave to the velocity of the particle. Group velocity is the velocity with which a signal or 'packet' of energy may be propagated on the wave in a dispersive medium, and it is given by the formula (6):
$v=(d f / d v)(d v / d k)$.
A further postulate is that the relation
which Planck had suggested for photons in $1900(7)$, and which had been verified experimentally by Lenard (8) and Willikan (9), also holds for de Broglie waves. $X$ is a constant known as Dirac's constant, defined $\#=h / 2 \pi$,
where $h$ is Planck's constant, the value of which is currently acknowledged to be (10):
$h=6.6219620(10) \times 10^{-34} \mathrm{~J}$.
Using relativity (see Appendix A), we obtain
$\left.E=j e=\left(m c^{2}\right) /\left(\sqrt{\left(1-(v / c)^{2}\right.}\right)\right)$,
and substituting in (1.1.4), we have
$\frac{d k}{d v}=\frac{1}{v} \frac{d f}{d v}=\left(\frac{n}{\bar{Z}}\right)\left(1-\frac{v^{2}}{c^{2}}\right)^{-3 / 2}$.
Since
$k=1 / \lambda$,
it is reasonable to assume that the boundary condition
$k=0, \quad v=0$
pertains, and hence, by integrating (1.1.9)
$\mathrm{j} k=\frac{\mathrm{mv}}{\sqrt{\left(1-(\mathrm{V} / \mathrm{c})^{2}\right)}}=\mathrm{p}$,
which is known as de Broglie's relation. Since the direction of $k$ must always be the same as that of $p$ in (1.1.12) it is often easier to express this formula:谈 $=2$,
in vector notation. De Broglie's relation may be obtained wi thout assuming (1.1.5) using relativity, and this method is used in Wichmann: Quentum Physics, MGGraw-Hill 1971, pp. 183-186.

### 1.2 Experimental Verification of De Broslie's Hypothesis. (11)

In 1927 C.Davisson and L.Germer (12) attempted to detect and record electron diffraction patterns. An electron bean emitted from a heated tungsten filament was focused and accelerated to an energy of between 15 and 350 oV by a charged slit. The beam was deflected by a nickel crystal upon which it was normally incident and was detected by a sensitive electrometer which could make an angle of between $20^{\circ}$ and $90^{\circ}$ with the original beam. The interference patterns
observed were very similar to those produced by 'sof't' Laue-Bragg x-rays (13), which was satisfactory, since the electrons' wavelength as predicted by de Broglie's relation was very similar to that of these x-rays.

Thomson performed a similar experiment of a more spectacular nature using the Debye-Scherrer method ${ }^{2}$ in x-ray diffraction work. He scattered an undirectional monochromatic beam of cathode rays with a very thin film containing a large number of randomily orientated crystals of white tin. Experiments with $x$-rays had show that the diffracted beam should emerge from the group of crystals along the surfaces of concentric cones centred about the incident direction. In Thomson's experiment (14) a photographic plate was placed 32.5 cm from the group of crystals at a normal to the beam. The image on this plate was found to consist of a series of concentric circles similar to those obtained with x-rays. In order to prove that these were caused by the electrons themselves and not by secondary electromagnetic radiation, a magnetic field was applied to the diffracted bean, causing the image to move. Ponte (15) developed Thomson's technique by using metallic oxides deposited on a thin metal wire instead of delicate crystalline films. In 1920 Kupp and Worsnop (16) showed that electrons were also diffracted by ruled gratings.

Johnson investigated the wave-like nature of hydrogen (17) by reflecting the gas from crystal surfaces. His detector was a plate smoked with molybdenum trioxide, which becomes blackened when it reacts with hydrogen. Stern, Knauer, and Estermann found that the wavelength of molecules in hydrogen were in exact agreement with de Broglie's relation (18). Bllett, 0lson and Zahl (19) then showed that mercury, cadmium and arsenic ion beans could be diffracted by crystals, using rock salt as a detector. Some years later Zinn (20) demonstrnted that neutrons also displayed wave characteristics. Neutrons from a chain-reacting pile were slowed down by graphite blocks and collimated by a series of cadmium slits. They were reflected from the face of a calcite crystal and detected by means of a boron trifluoride counter.

## 1.3 The Sohrodingor Equation. (21)

In order to progress beyond problems concerning purely a continuous hamonic de Broglie wave, it is useful to have an equation for which both this wave and more complicated waves are solutions. This equation must obey two fundamental requirements. First, it must be linear, so that its solutions may be superposed to produce effects such as interference, and second, it must contain only such constants as $\bar{K}$ and the mass of the particle, and it must be independant of all kinematic variables of the particle. An equation of this type may be obtained directly from the wave equation (1.1.1.). Differentiating this partially with respect to once and with respect to $x$ twice, we obtain $\partial \psi / \partial t=-2 \pi j \rho \exp 2 \pi j(k x-f t)$
and
$\partial^{2} V / \partial x^{2}=-4 \pi^{2} k^{2} \exp 2 \pi j(k x-i t)$.
Substituting with the original function $\psi$, we find that
$\partial \psi / \partial t=-2 \pi j P$
and
$\partial^{2} V / \partial x^{2}=-4 \pi^{2} k^{2}$.
Solving for $f$ and $k$ in (1.3.3) and (1.3.4) and substituting these solutions in (1.1.13)

$$
\begin{equation*}
j \neq-\frac{\partial \psi}{\partial t}=\frac{x^{2}}{2 m} \frac{\partial^{2} \psi}{\partial x^{-}} \tag{1.3.5}
\end{equation*}
$$

which is known as the Schredinger equation for a free particle ${ }^{1}$.
The above treatment may readily be extended into three dimensions. We use the vector form of the de Broglie relation (1.1.13), and assume that $k=|\underline{k}|$,
so that the Schrddinger equation in three dimensions becomes
$j M \frac{\partial y}{\partial t}=-\frac{x^{2}}{2 m} \nabla^{2}$,
where $\nabla^{2}$ is the Laplacian operator
$\nabla^{2}=\frac{\partial^{2}}{\partial x_{1}}{ }^{2}+\frac{\partial^{2}}{\partial x_{2}^{2}}+\frac{\partial^{2}}{\partial x_{3}} 2$.
By comparing (2.3.7) and (1.1.13), we can deduce that the energy and momontum of a free particle may be represented by differential operators acting on its
wave function
$\mathrm{E} \rightarrow \mathrm{jH} \frac{\partial}{\partial t}$
$p \rightarrow 3 K \operatorname{grad} 3$
We now write dow the four so-called 'hamonic' or plane wave solutions for the Schredinger wave function $\Psi(22)$ :
$\cos (k x-I t)$,
$\sin (k x-f t)$,
$e^{j(k x-f t)}$,
$e^{-j(k x-f t)}$.

### 1.4 Interpretation of the Wave Function. (23)

In classical physics, the value of a wave function represents an associated physical parameter. For example, in the case of waves in air, it represents the displacement of the air particies. In 1926 Born suggested (24) that the value of $\psi(x, t)$ for the de Broglie wave of a particle corresponds to the probability that the particle will be at a point $x$ on the $x$-axis at a given time $t$. However, a probability must be real and positive, but values of $\psi(x, t)$ are, in general, complex. Hence Born suggested that the probability should be the product of the value of the wave function and its complex conjugate, so that the probability of finding the particle between $x$ and $x+d x$ is given by $P(x, t) d x=\psi^{*}(x, t) \psi(x, t) d x$. This so-called 'position probability density' may readily be extended into three dimensions:
$P(x, y, z, t) d x d y d z=\Psi^{-4}(x, y, z, t) \psi(x, y, z, t) d x d y d z$. (1.4.2) Let $\in$ be the volume of the infinitesimal element of space $d x d y d z$. Since it is a certainty that the particle must exist somewhere in space, $_{+\infty}$ $\int_{\substack{-\infty \\ \text { hence } \\+\infty}}^{+\infty} P(\varepsilon, t) d \epsilon=1$,
$\int_{-\infty}^{+\infty} v \psi^{*}(\epsilon, t) \psi(\epsilon, t) d \epsilon=1$.
Thus, in order to normalize a given solution to the Schrbdinger equation, we multiply it by its complex conjugate and integrate over all space, obtaining a real number $N$. By dividing both the wave function and its complex conjugate by
$\sqrt{\mathrm{N}}$, we have
$\int_{V} \frac{\psi^{*}(\epsilon, t)}{\sqrt{N}} \frac{\psi(\epsilon, t)}{\sqrt{N}} d \epsilon=\mathbb{N} / \mathbb{N}=1$.
Hence $\Psi$ is said to have been normalized, and $\sqrt{N}$ is said to be its normalization factor.

If the function $\psi$ is replaced by its complex conjugate $\psi^{*}$ in the Schrodinger equation (1.3.5) we have
$j 2 \frac{\partial \psi^{*}}{\partial t}=\frac{\psi^{2}}{2 \pi} \frac{\partial^{2} \psi^{*}}{\partial x^{2}}$.
Multiplying (1.3.5) by $\psi^{*}$ and (1.4.6) by $\psi$, and adding, we obtain
$\psi^{*} \frac{\partial^{2} \psi}{\partial x^{2}}-\psi \frac{\partial^{2} \psi}{\partial x^{2}}+\frac{4 \pi i m}{h}\left(\frac{\partial \psi}{\partial t} \psi^{*}+\frac{\partial \psi^{*}}{\partial t} \psi\right)=0$, (1.4.7)
$\frac{\partial}{\partial t}\left(\psi^{*} \psi\right)+\frac{h}{4 \pi j \text { or }} \frac{\partial}{\partial x}\left(\psi^{*} \frac{\partial \psi}{\partial \pi}-\psi \frac{\partial \psi^{*}}{\partial x}\right)=0$.
We adopt Born's suggestion (24) and define the probability density
$P=\Psi^{*} \psi$
and the probability current ${ }^{4}$
$S_{x}=\frac{h}{2 \pi j m}\left(\psi^{*} \frac{\partial \psi}{\partial x}-\psi \frac{\partial \psi^{*}}{\partial x}\right)$,
so that (1.4.8) now becomes
$\frac{\partial P}{\partial t}+\frac{\partial S}{\partial x^{x}}=0$.
This equation may be extended to include the three dimensional case by writing $\frac{\partial P}{\partial t}+\operatorname{div} \underline{S}=0$.
Equations of this type are common throughout physics, and represent the conservation of a fluid (25). For example, it shows that for a liquid of density $P$ and with rate of flow $\underline{S}$, the rate of increase of liquid per unit volume is equal to the rate of flow into that volume. (1.4.12) is known as the continuity equation of probability. It may also be obtained by showing that the normalization constant of a wave function is independent of time, and this method is used in Schiff: Quantum Mechanics, NoGraw-Hill, 1955, p. 23.

The Born interpretation makes it possible to write an expression for the expectation value of a physical parameter associated with a particle. The expectation value of a given measurement is defined as the most probable result
of that measurement. Hence we may write the expectation value of the position vector $\underline{\underline{r}}$ of the particle as

$$
\begin{equation*}
\langle\underline{\underline{r}}\rangle=\int_{V} \underline{\underline{r}}(\underline{\underline{r}}, t) d \epsilon=\int_{V} \psi^{*}(\underline{r}, t) \underline{\underline{r}} \psi(\underline{\underline{v}}, t) d \epsilon, \tag{1.4.13}
\end{equation*}
$$

$\psi$ being normalized. Parameters which do not depend directly upon $\leq$ and $t$ must be converted into operators before their expectation values may be computed. The question of how these operators may be combined with the probability density may be solved by assuming that
$\langle E\rangle=\left\langle\underline{\underline{D}}^{2} / 2 m\right\rangle+\langle V\rangle$,
$V$ being the potential of some external force. In terms of the differential operators (1.3.9) and (1.3.10), (1.4.14) may be rewritten
$\left\langle j \neq \frac{\partial}{\partial t}\right\rangle=\left\langle-\frac{k^{2}}{2 m} \nabla^{2}\right\rangle+\langle v\rangle$.
This equation is only consistent with (1.3.5) if we say that
$\langle p\rangle=\int_{v} \psi^{*}(\psi) d e$,
where F is a parameter and $f$ is its corresponding operator. Thus

$$
\begin{equation*}
\langle E\rangle=\int_{v} \psi^{*} \mathcal{H} \frac{\partial}{\partial t} d \in, \tag{1.4.17}
\end{equation*}
$$

and
$\langle\underline{p}\rangle=\int_{V} \psi^{\prime \prime}\left(-j h^{\prime}\right) \operatorname{grad} \psi d \epsilon$
1.5 Quantization.

Let us consider a particular solution to (1.3.5) of the form
$\psi(\underline{I}, t)=f(\underline{r}) \cdot g(t)$.
Substituting for $\psi$ in (1.3.5) and dividing both sides by the right-hand side of (1.5.1), we obtain
$\frac{j h 1}{6} \frac{d g}{d t}=\frac{1}{f}\left(-\frac{k^{2}}{2 m} \nabla^{2} \hat{f}+\nabla(\underline{x}) \rho\right)$.
Since the left-hand side of (1.5.2) is dependant purely on $g$ and the right-hand side purely on $f$, both sides must equal a constant, E. Thus we obtain the expression
$g(t)=C e^{-j E t / \nmid}$
for $g$, and
$\left(-\frac{\underline{z}^{2}}{2 i n} \nabla^{2}+V(\underline{x})\right) f(\underline{x})=E f(\underline{x})$
for $f$. Thus a particular solution to the wave equation is
$\psi(\underline{x}, t)=f(\underline{z}) \mathrm{e}^{-j B t / x}$.
We now apply the function (1.3.9) to (1.5.5) to obtain
$J K \frac{\partial \psi}{\partial t}=E V$.
Thus $\mathbb{E}$ is an eigenvalue ${ }^{5}$ of the energy operator, and $\psi$ is an eigenfunction of it. An energy eigenfunction is said to represent a 'stationary state' of a particle, since $\Psi^{*} \psi$ is constant in time. Similarly, $E$ is an eigenvalue in (1.5.4). Thus we see that only an eigenvalue $E$ is a possible energy for the particle, and hence we have quantized the Schrodinger equation (1.3.5).

As an example of the process of quantization, we shall nov quantize the harmonic oscillator ${ }^{6}$. The potential energy of a particle of mass 4 which has been displaced $x$ units from its equilibrium position is given by (26) $V(x)=2 \pi^{2} m v_{0}^{2} x^{2}$.
Substituting with this in the one-dimensional Schrodinger equation (1.3.5) we have $\frac{\partial^{2} \psi}{\partial x^{2}}+\frac{8 \pi^{2} \pi}{h^{2}}\left(w-2 \pi^{2} m v_{0}^{2} x^{2}\right)=0$.
We now introduce the variables (27)

$$
\begin{equation*}
\lambda=8 \pi^{2} m W / h^{2} \tag{1.5.9}
\end{equation*}
$$

and
$\alpha=4 \pi^{2} m v_{0} / h$,
so that (1.5.8) becomes
$\frac{\partial^{2} \psi}{\partial x^{2}}+\left(\lambda-x^{2} x^{2}\right)=0$.
Following Sommerfeld: Wave Mechanics, Dutton, 1929, p.11, we now use the polynomial method for finding a solution for $\psi$. First we find the asymptotic solution for $\psi$. Since for large $|x|, \lambda$ is negligible compared with $x^{2} x^{2}$, the asymptotic wave equation becomes
$\frac{\partial^{2} \psi}{\partial x^{2}}=\alpha^{2} x^{2} \psi$
whose only physically meaningful solution is

$$
\begin{equation*}
\psi(x)=e^{-(\alpha / 2) x^{2}} \tag{1.5.13}
\end{equation*}
$$

Now we let
$\psi(x)=e^{-(\alpha / 2) x^{2}} f(x)$
for finite $x$. Obtaining a power series, differentiating, and solving for $\psi$, we obtain
$\psi_{n}(x)=N_{n} e^{-\rho^{2} / 2} H_{n}(\rho)$,
where
$\rho=\sqrt{\alpha} x$.
$H_{n}(\rho)$ is a polynomial of degree $n$ in $\rho$, and $N_{n}$ is the nomalization constant for $\psi_{n}$. (1.5.15) is analogous to (1.5.5) for a free particle, and may be quantized in a similar manner. The implications of the quantization of (1.5.15) may be found in Schiff: cuantum Kechanics, KcGraw-Hill 1935, pp. 73-82, or in Pauling and willson, Introduction to Quentum Mechanics, McGraw-\#ill 1955, pp. 60-69.

### 1.6 The Hanilton Equations. (28)

We define a function L, known as the Lagrangian (29), such that
$\mathrm{L}=\mathrm{T}-\mathrm{V}$.
Thus, in one dimension, for a single particle
$I=\frac{1}{2} m \dot{x}^{2}-V$,
and since $V$ is a function of $x$ but not of $\dot{x}$,
$\partial L / \partial \dot{x}=m \dot{x}$,
and
$\frac{d}{d t}\left(\frac{\partial L}{\partial \dot{x}}\right)=\mathrm{m} 2=\frac{d p}{d t}$.
The partial derivative with respect to $x$ is
$\partial L / \partial x=-\partial V / \partial x=m 2$,
and hence the equation of motion in one dimension for the particle is
$\frac{d}{d t}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=0$.
This equation of motion may readily be extended into three dimensions by writing three separate equations, one in $x$ and $\dot{x}$, one in $y$ and $\dot{y}$, and a thini in $z$ and $\dot{z}$. Often the co-ordinates of a system, each corresponding to one of its degrees of freedom, are written $q_{T}$, so that the Lagrange equation (1.6.6) should be solved for all permitted $r$. From (1.6.6) we see that
$p_{T}=\partial L / \partial \dot{q}_{r}$.
We now define $H$, the Bamiltonian function (30), such that
$\mathrm{H}=\mathrm{T}+\mathrm{V}$.
For a single particle in one dimension, where V is assumed to be time-independant,
$H=\frac{1}{2} m \dot{x}^{2}+V$.
Thus
$H=(1 / 20) p^{2}+V$
and
$\delta_{H}=(p / m) \delta_{p}+(\partial V / \partial x) \delta x$.
Hence
$\frac{\partial H}{\partial \mathrm{P}}=\frac{\mathrm{P}}{\mathrm{m}}=\dot{x}$
and
$\frac{\partial H}{\partial x}=\frac{\partial V}{\partial x}=-p$.
Equations (1.6.12) and (1.6.13) whitten in the more general form
$\frac{\partial H}{\partial p_{r}}=\dot{q}_{r}$,
$\frac{\partial H}{\partial q_{r}}=-\dot{p}_{r}$,
consitute the Hamilton or canonical equations, which are true for any mechanical system. A complete proof of these relations may be found in Jeffreys:
Wathematical Physics, C.U.P., 1956, pp. 325-327.

### 1.7 The Dirac Equation. (31)

The non-relativistic relation
$E=p^{2} / 2 m$
corresponds to the relativistic (32) relation
$E^{2}=c^{2} p^{2}+m^{2} c^{4}$.
Substituting in (1.7.2) with the operators (1.3.9) and (1.3.10) we have
$-\not \eta^{2} \frac{\partial^{2} \psi}{\partial t^{2}}=-\not z^{2} c^{2} \nabla^{2} \psi+m^{2} c^{4} \psi$
which is the relativistic fom of the Schrodinger equation. It has solutions of the type
$\exp j(\underline{k} \cdot \underline{r}-f t)$,
where
准 $= \pm\left(a^{2} c^{2} \underline{k}^{2}+m^{2} c^{4}\right)^{\frac{1}{2}}$.
The possibility of the right-hand side of (1.7.5) being negative corresponds to the existence of a negative energy or anti-matter state ${ }^{6}$ of any particle.

By substituting the relativistic formula
$\mathrm{E}^{2}=\mathrm{p}^{2}+\mathrm{m}^{2}$
into the wave equation (1.1.1) and differentiating, we obtain
$\partial^{2} \psi / \partial t^{2}=\left(\nabla^{2}-m^{2}\right) \psi$, (32)
which is known as the Klein-Gordon equation, and which is suitable for describing free spinless relativistic particles.

Due to certain difficuities which arose in the interpretation of (1.7.7), due to the existence of second-order differentials in the relation, Dirac suggested in 1928 (33) that both space and time derivatives should occur to first onder in a relativistic wave equation. He used the operator $\partial / \partial x_{r} \quad(r=1,4)$
where $\underline{X}$ is a four vector containing the three space components and an imaginary fourth component of time, accoraing to the Minkowski convention (34), and combined it with another four vector in order to obtain a scalar product which was Lorentz invariant. Thus he obtained

$$
\begin{equation*}
Y_{r} \frac{\partial}{\partial x_{r}}=\sum_{r=1}^{4} \gamma_{r} \frac{\partial}{\partial x_{r}} \tag{1.7.9}
\end{equation*}
$$

which is a Lorentz invariant scalar operator. However, it is possible that the wave equation for a particle contains another scalar operator not involving the term (1.7.8), and thus we write the equation in its most general form:
$\left(Y_{I} \frac{\partial}{\partial x_{r}}+c\right) \Psi=0$.
This equation must satisfy Einstein's relation (35):
$p_{r}^{2}=-m^{2}=\left(\frac{\partial^{2}}{\partial x_{r}}-m^{2}\right) \psi=0$,
and operating on the left by
$\left(\gamma_{s} \frac{\partial}{\partial x_{s}}-c\right)$,
we have
$\left(\gamma_{s} Y_{I} \frac{\partial}{\partial x_{s}} \frac{\partial}{\partial x_{x}}-c^{2}\right) \Psi=0$.
Thus we see that (1.7.11) is satisfied by (1.7.10) if and only if $\gamma_{s} Y_{I}+Y_{I} Y_{s}=2 \delta_{s r}$,
where $\delta$ is the Kronecker delta function (36), such that

$$
\begin{array}{ll}
\delta_{r s}=1 & (r \neq s) \\
\delta_{r s}=0 & (r=s) \\
\text { and } & \\
0=\text { III } & \tag{1.7.16}
\end{array}
$$

The comutation relation (1.7.14) implies that $\gamma$ is not a number, since it does not commute, but no unique solution is possible, since (1.7.14) is the only defining relation. $\gamma$ is usually identified with the set of $4 \times 4$ Dirac-Pauli (37) matrices, defined

$$
\begin{array}{ll}
Y_{k}=\left[\begin{array}{cc}
0 & -j \sigma_{k} \\
j \sigma_{k} & 0
\end{array}\right] & (k=1,2,3)  \tag{1.7.17}\\
Y_{4}=\left[\begin{array}{cc}
1 & 0 \\
0 & -1
\end{array}\right]
\end{array}
$$

each element standing for a $2 \times 2$ matrix. $\sigma_{k}$ are the Pauli spin matrices:
$\sigma_{1}=\left[\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right] \quad \sigma_{2}=\left[\begin{array}{rr}0 & -j \\ j & 0\end{array}\right] \quad \sigma_{3}=\left[\begin{array}{rr}1 & 0 \\ 0 & -1\end{array}\right]$
and
$I=\left[\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right] \quad 0 \quad=\left[\begin{array}{ll}0 & 0 \\ 0 & 0\end{array}\right]$.
The multiplication of the $Y$ matrices among themselves yields sixteen further independent matrices:
product number of matrices
i (unit matrix)
$\gamma_{r} \quad 4$
$\gamma_{r} \gamma_{s} \quad(r<s) \quad 6$
$\gamma_{r} \gamma_{s} \gamma_{t} \quad(r<s<t) \quad 4$
$\gamma_{1} \gamma_{2} \gamma_{3} \gamma_{4}\left(=\gamma_{5}\right) \quad 2$
It is useful also to define a matrix $\gamma_{5}$

$$
Y_{5}=\left[\begin{array}{rr}
0 & -1  \tag{1.7.20}\\
-1 & 0
\end{array}\right] \quad, \quad Y_{5}^{2}=1
$$

and to define the other products in terms of it:
1
$\gamma_{r}$

| j $Y_{F} Y_{s}$ | $(r<s)$ | $T$ |
| :--- | :--- | :--- |
| $j Y_{F} Y_{5}$ |  | A |
| $Y_{5}$ |  |  |

The letters on the right-hand side imply that the products behave as scalars, vectors, tensors, axial vectors, and pseudoscalars respectively under the Lorentz transformation. With the Dirac-Pauli matrix choice of representation, we may write
$\gamma_{r}=\gamma_{r}^{\dagger}$,
where $\gamma^{+}$is the complex conjugate transpose or Hermitean adjoint of $\gamma$.
From (1.7.21) we see that $\gamma$ is Hemitean or self-adjoint. A useful relation between the $\gamma$ matrices is
$Y_{5} Y_{2}+Y_{T} Y_{5}=2 \delta_{r 5}$.
The Dirac equation mey also be obtained by means of group theory, and this approach is used in Ownees: Introduction to Particle Physics, Wiley 1970, pp. 191-207, and Weyl: The Theory of Groups and Quentum Nechanics, Dover 1928, pp. 202-218.

In the Dirac equation (1.7.10), $\gamma_{Y}$ is a matrix, and so also is $\psi$.
Since
$(A B)^{\dagger}=B^{+} A^{\dagger}$,
$\left(\frac{\partial \psi}{\partial x_{r}}\right)^{+} \gamma_{r}^{+}+m \psi^{+}=\left(\frac{\partial \psi}{\partial x_{r}}\right)^{\dagger} \gamma_{r}+m \psi^{+}=0$,
and hence, multiplying from the right by $\gamma_{4}$ and introducing the definition
$\widetilde{\psi}=\psi^{\top} \gamma_{4}$,
we obtain the adjoint form of the Dirac equation:
$\left(\frac{\partial \widetilde{\psi}}{\partial x_{r}} \gamma_{r}-m\right) \tilde{\psi}=0$

### 1.8 The Solution of the Dirac Equation.

In order to obtain an expression for the current of a Dirac particle, we first reduce the continuity equation (1.4.12) to four-vector notation
$\partial s_{T} / \partial x_{r}=0$,
and this is satisfied if we define
$S_{r}=j \tilde{\psi} \gamma_{r} \psi$,
and thus, from (1.7.10) and (1.7.26), we have
$j\left(\partial / \partial x_{r}\right)\left(\tilde{\psi} Y_{r} \psi\right)=j(m \tilde{\psi} \psi-m \tilde{\psi} \psi)=0$,
and
$P=(1 / j) s_{4}=\tilde{\psi} Y_{4} \psi=\psi^{*} Y_{4} Y_{4} \psi=\psi^{*} \psi$, (1.8.4) which is the conventional quantum mechanical expression for probability density, (1.4.9).

The Dirac equation may be proved to describe the wave functions of massive particles of spin or total angular momentum $\frac{1}{2}$, such as the electron or proton, and this proof is given in Nuirhead: Elementary Particle Physics, Pergamon 1971, pp. 50-52. We now examine a few of the simpler solutions of the Dirac equation, which represent possible wave functions for spin $\frac{1}{2}$ particles. Obviously, since it must be compatible with the $\gamma$ matrix in (1.7.10), any solution must be a four-component wave function, known as a spinor. We write the plane wave solution as

$$
\begin{equation*}
\psi(\underline{x}, t) \quad=u_{j} e^{j p_{j} x_{j}} \quad(j=1,4) \tag{1.8.5}
\end{equation*}
$$

where $u$ is a spinor. Now we set
$\underline{D}=p_{3}$,
and thus we obtain, from (1.7.10),
$(-E+m) u_{1}+p u_{3}=0$
$(-E+m) u_{2}-p u_{4}=0$
$(\mathrm{B}+\mathrm{m}) u_{3}-p u_{1}=0$
$(E+m) u_{4}+p u_{2}=0$.
We know that
$p^{2}=E^{2}-m^{2}$
and that
$\sum u_{j}^{2}=1$,
since $u$ is normalized. Hence, from (1.8.7)
$u_{3} / u_{1}=p /(E+m)$,
and writing
$u_{1}=1$,
we deduce that
$u_{3}=p /(E+m)$,
$u_{2}=u_{4}=0$.

Similarly, from (1.8.8), setting
$u_{1}=u_{3}=0$,
we obtain
$u_{2}=1$,
$u_{4}=-p /(B+m)$.
Thus two of our four possible solutions may be written
$u_{++}=\left[\begin{array}{c}1 \\ 0 \\ p /(E+m) \\ 0\end{array}\right] \quad u_{+-}=\left[\begin{array}{c}0 \\ 1 \\ 0 \\ -p /(E+m)\end{array}\right]$
From (1.8.9) and (1.8.10), we obtain the other two solutions:
$u_{-+}=\left[\begin{array}{c}-p /(|E|+m) \\ 0 \\ 1 \\ 0\end{array}\right] \quad u_{--}=\left[\begin{array}{c}0 \\ p /(|E|+m) \\ 0 \\ 1\end{array}\right]$
The solutions (1.8.21) may also be found from the alternative plane wave
$\psi(\underline{I}, t)=u_{j} e^{-j p j x_{j}}$,
which has momentum $-\underline{D}$ and energy -E . These assignments are permitted since $\mathrm{E}=\sqrt{\mathrm{p}^{2}+\mathrm{m}^{2}}$
does not determine the sign of E . Thus the solutions (1.8.21) are negativeenergy ones, corresponding to the antiparticles of spin $\frac{1}{2}$ particles. Naturally, any combination of the solutions (1.8.20) and (1.8.21) is also permitted. If we consider the rest frame of the particle, where
$p=0$,
then all terms in $p$ dissappear, and from $(1.8 .20)$ we are left with
$u_{+-}=\left[\begin{array}{l}1 \\ 0\end{array}\right] \quad u_{++}=\left[\begin{array}{l}0 \\ 1\end{array}\right]$
which are the two-component non-relativistic Pauli spinors (38) and which correspond to the two possible spin projections for an electron.

### 2.1 The Operatore PCT.

The parity or space reflection operator, $P$, reverses the sign of the $x$-co-ordinates of a wave function, which is equivalent to reflecting it in the plane

$$
\begin{equation*}
x=0 . \tag{2.1.1}
\end{equation*}
$$

Most physical systems can be described by wave functions which are eigenfunctions of the $P$ operator. For these, we may write
$P \psi(\underline{x})=\psi(-\underline{x})=\epsilon \psi(\underline{z})$.
From geometrical considerations, it is obvious that
$P^{2} \psi(\underline{x})=\psi(\underline{x})$
and hence

$$
\begin{equation*}
\epsilon^{2}=1, \quad \epsilon= \pm 1 \tag{2.2.4}
\end{equation*}
$$

Thus, for systems with only one linearly independent eigenfunction corresponding to a particular eigenvalue, the wave function has a definite intrinsic parity, $p$, which may be even $(P=1)$ or odd ( $P=-1$ ).

We assume that we may write a Hamiltonian in terma of the three-vector $\underline{x}$ and that
$H(\underline{x})=H(-\underline{x})$,
i.e. the Hamiltonian is invariant under the $P$ operator. The time-dependent Schrobdinger equation (1.3.7) written in Hamiltonian form is
$H(\underline{x}) \psi(\underline{x}, t)=j \frac{\partial \psi(x, t)}{\partial t}$
in natural units $(y=c=1)$. Replacing $x$ by $-x$ in (2.1.6) and defining
$\Psi^{\prime}(\underline{x}, t)=\Psi(-\underline{x}, t)$,
we have, using (2.1.5),
$H(\underline{x}) \psi^{\prime}(\underline{x}, t)={ }_{j} \frac{\partial \psi^{2}(x, t)}{\partial t}$.
Thus the function $\Psi^{\Gamma}(\underline{x}, t)$ is a solution to the same differential equation as $\psi(\underline{x}, t)$.

For the Dirac equation, we write the equation of motion of a particle
in two parts (1):
$H(\underline{X}) \psi(\underline{x}, t)=j \frac{\partial \psi(x, t)}{\partial t}$
$H(\underline{x})=\underline{\alpha} \underline{p}+\gamma_{4} m+V(\underline{x})=-j x_{k}\left(\partial / \partial \underline{x}_{k}\right)+\gamma_{4} \underline{\underline{I}}+V(\underline{x})(2.1 .10)$ We assume a similar symmetry for the potential $V(\underline{x})$ as (2.1.5) for the Hamiltonian, and write
$\mathrm{V}(-\underline{\mathrm{x}})=\mathrm{V}(\underline{\mathrm{x}})$.
Using the anticomutation relations of the $\gamma$ matrices, we obtain the property
$H(-\underline{x})=Y_{4} H(\underline{x}) Y_{4}$
for the Hamiltonian in (2.1.10). Applying the P operator and using (2.1.12), (2.1.9) becomes

$$
\begin{equation*}
\gamma_{4} H(\underline{z}) \gamma_{4} \psi(-\underline{z}, t)=j \frac{\partial \Psi(-z, t)}{\partial t} . \tag{2.1.13}
\end{equation*}
$$

Introducing
$\psi^{\prime}(\underline{x}, t) \quad Y_{4} \psi(-\underline{x}, t)$,
and multiplying by $\gamma_{4},(2.1 .13)$ simplifies to
$H(\underline{x}) \psi^{\prime}(\underline{x}, t)=j \frac{\partial \psi^{\prime}(x, t)}{\partial t}$.
(2.1.15) is identical with the original equation (2.1.9), and hence this equation is invariant under the P operator, implying that in all physical processes which are describable by the Dirac equation, the quantum number of intrinsic parity is conserved.

The charge conjugation operator, $C$, transforms a given particle into its antiparticle. Prom similar considerations as those which we employed for parity, we may see that many physical systems will have a definite eigenvalue with respect to the C operator, or C parity. Symmetry under the C operator in classical physica is shown by the invariance of Naxwell's equations under a change in the sign of the charge and current densities. Only a few particles have even $C$ parity, since all baryons (mucleons and hyperons) have $B=1$, while antibaryons have $B=-1$, and similarly leptons have $L=1$, whereas antileptons have $L=-1$. Since both baryon number, $B$, and lepton number, L, are thought to be conserved in all reactions, no baryon or lepton may ever commute with its antiparticle. The only stable particles with neither charge nor lepton or baryon number are the photon, the $\pi^{\circ}$, and the $K^{\circ}$. Of these, only the $\pi^{\circ}$ has even $C$ parity ${ }^{1}$ (2).

In 1939, Wigner introduced the time reversal operator, 7 , (3) whereby all time variables in an expression change sign. We take the time-dependent non-relativistic Schr8dinger equation, stated in Hamiltonian form (2.1.6). We define $\psi_{0}:$
$\psi(\underline{x}, 0)=\psi \psi_{0}(\underline{x})$.
and
$\psi(\underline{x}, T)=\psi_{1}(\underline{x})$.
In the original system, the wave function or state vector ${ }^{2} \psi$ evolves from $\psi_{0}$ into $\psi_{1}$ after a time $T$, but in the time-reflected system, the state $\psi_{1}$ develops into $\Psi_{0}$ after the same time I. If we were simply to negate the $t$ variable in (2.1.6), then the expression would change sign, since the righthand side of the equation is linear in the first derivative with respect to $t$. In order to compensate for this, wo take the complex conjugate of our expression, so that the $j$ on the right-hand side introduces a further change of sign. Thus we define the time-reflected wave function by

$$
\begin{equation*}
\psi^{\prime}(\underline{x}, t)=\Psi^{*}(\underline{x}, T-t) \tag{2.1.18}
\end{equation*}
$$

which fulfills the boundary conditions (2.1.16) and (2.1.17) after complex conjugation. The state vector $\mathcal{\psi}^{\prime}$ is found to obey an equation which is obtained from (2.1.6) by complex conjugation of each term:
$H^{*} \psi^{\prime}(\underline{x}, t)=j \frac{\partial \psi^{\prime}(x, t)}{\partial t}$.
Since it is only the absolute square of the Schrydinger wave function which is of any physical significance, the operation of complex conjugation does not affect the validity of the Schrvidinger equation, and hence the two equations (2.1.6) and (2.1.19) are essentially congruent. Thus, any particle or systom which is describable by the Schrddinger equation must be invariant under the operation of time reversal or reflection.

### 2.2 The Free Scalar Field.

The free scalar Hermitean (real) field fulfills the Klein-Gorion equation $\left(\square-m^{2}\right) \phi(x)=0$,
where $\square$ is the d'Alembertian operator (4) given by
$\square=\frac{\partial^{2}}{\partial x_{1}^{2}}+\frac{\partial^{2}}{\partial x_{2}^{2}}+\frac{\partial^{2}}{\partial x_{3}^{2}}+\frac{\partial^{2}}{\partial x_{4}^{2}}$.

In order to discover more about the degrees of freedom and nature of the field, it is useful to perform a Fourier decomposition (5) on it, or to resolve it into a series of hamonic components. Thus we obtain.

$$
\begin{equation*}
\phi(x)=\frac{1}{\sqrt{V}} \sum_{k} \frac{1}{\sqrt{2 \omega}}\left(e^{j k z} a(\underline{k})+e^{-j k z} a^{+}(\underline{k})\right) . \tag{2.2.3}
\end{equation*}
$$

The nomalization constant $(1 / \sqrt{2 \omega})$ is so chosen so that the operators $a(\underline{k})$ and $a^{+}(\underline{k})$ fulfill the commutation relations (6)

$$
\begin{align*}
& {\left[a(\underline{k}), a\left(\underline{k^{\prime}}\right)\right]=\left[a^{\top}(\underline{k}), a^{\top}\left(\underline{k}^{\prime}\right)\right]=0}  \tag{2.2.4}\\
& {\left[a(\underline{k}), a^{+}\left(\underline{k}^{\prime}\right)\right]=\delta_{k k^{\prime}}}
\end{align*}
$$

where
$[\mathrm{X}, \mathrm{Y}]=\mathrm{X} \cdot \mathrm{Y}-\mathrm{Y} . \mathrm{X} \quad$.
$V$ represents a large normalization volume, which is usually considered to be a rectangular box with sides $L_{x}, L_{y}$ and $L_{z}$. $\mathbb{E}$ is an energy-momentum fourvector ${ }^{3}$, which, because of (2.2.1), obeys the relation
$k^{2}+$ mim $^{2}=0$,
and, by nomalization,
$k_{i} L_{i}=n_{i} 2 \pi \quad(i=x, y, z)$.
Thus the total number of state vectors $\underline{k}, \Delta n(7)$, in the interval $d^{3} k$ is $\Delta_{n}=\left(1 /(2 \pi)^{3}\right) \nabla d^{3} k$.
When $V$ is large, it is of convenient to replace the summation over allowed values of the momentum by the formula
$\frac{2}{V} \sum_{k} f(\underline{k})=\frac{1}{(2 \pi)^{3}} \int f(\underline{k}) d^{3} k$,
$f(\underline{k})$ being a slowly-varying function giving all allowed momenta. The equating of the two sides of (2.2.10) involves a slight error, but this tends to zero as the nomalization volune $V$ becomes very large.

We now consider the operator $\mathrm{a}(\mathrm{k})$ and its Hermitean adjoint $\mathrm{a}^{\dagger}(\underline{k})$. From the computation relations (2.2.4) and (2.2.5) we may guess that $a(\underline{k})$ and $\mathrm{a}^{\dagger}(\underline{k})$ are the destruction (or annihilation) and the creation operators respectively. We begin with a nomalized state
|nk
containing $n$ bosons, each with a nomentum $\underline{k}$. Here we employ the Dirac notation (8): $|x\rangle$ is a ket, and represents the wave function or state vector of the particle or systen of particles $X .\langle X|$ is a bra, and represents the complex
conjugate of the ket. We operate on the state (2.2.11):
$a_{k}^{+}\left|n_{k}\right\rangle=\sqrt{(n+1)_{k}}\left|(n+1)_{k}\right\rangle$,
thus increasing the number of bosons by one, and creating a new particle with momentum $k$. $\sqrt{(\mathrm{n} \text { 1) }}$ is added as a nomalization factor, to aid the physical interpretation of the operator. Similarly
$a_{k}\left|n_{k}\right\rangle=\sqrt{n_{k}}\left|(n-1)_{k}\right\rangle$,
so that in this case, we have destroyed a particle. We note that we can create n particles by operating on the vacuum state $|0\rangle$ :
$|n\rangle=(1 / \sqrt{n!})\left(a^{+}(\underline{k})\right)^{n}|0\rangle$.
Hence, the Ramiltonian for the system is given by
$H=\sum_{k} \omega a^{+}(\underline{k}) a(\underline{k})$,
where
$\omega=\sqrt{k^{2}+m^{2}}$.
By operating with H on a state $|\psi\rangle$ containing $n_{1}$ particles, each with momentum $k_{1}$ and energy $\omega_{1}$, then on one containing $n_{2}$ particles, and so on, we obtain $H|\psi\rangle=\sum_{i} n_{i} \omega_{i}|\psi\rangle$.
From (2.2.17), we may deduce that our states are eigenstates of $H$ with eigenvalues equal to the total energy of the state. Furthenmore, we observe that the operator
$a^{\dagger}(\underline{k}) a(\underline{k})$
corresponds to the total number of particles with momentum $\underline{k}$ present in the system.

We now consider the quantization of a free charged scalar field. Here the field is no longer Hermitean, and hence we must write two Fourier decompositions:

$$
\begin{align*}
& \phi(x)=\frac{1}{\sqrt{V}} \sum_{k} \frac{1}{\sqrt{2 \omega}}\left(e^{j k z} a(\underline{k})+e^{-j k z} b^{+}(\underline{k})\right)  \tag{2.2.19}\\
& \phi^{+}(x)=\frac{1}{\sqrt{V}} \sum_{k} \frac{1}{\sqrt{2 \omega}}\left(e^{j k z} b(\underline{k})+e^{-j k z} a^{+}(\underline{k})\right) . \tag{2.2.20}
\end{align*}
$$

Here the operators $a(k)$ and $b(k)$ are the destruction operators of two different types of particles with the same mass. They obey the commutation relations
$\left[a(\underline{k}), a\left(\underline{k}^{\prime}\right)\right]=\left[b(\underline{k}), b\left(\underline{k}^{\prime}\right)\right]=\delta_{k k^{\prime}}$.
Other commutators vanish. The neutral Hamiltonian (2.2.15) now becomes
$\mathrm{H}=\sum_{k} \omega\left(a^{\dagger}(\underline{k}) a(\underline{k})+b^{+}(\underline{k}) b(\underline{k})\right)$,
and thus the energies of the two kinds of particles must be added together to give the total energy of the system. The difference between our two types of
particles, $a$ and $b$, is that the signs of their charges are opposite. The total charge, $Q$, of the system is given by the operator $Q=e \sum_{k} \omega\left(a^{+}(\underline{k}) a(\underline{k})-b^{+}(\underline{k}) b(\underline{k})\right)$,
where $e$ is the electronic charge. It is possible to demonstrate that (2.2.23) corresponds (with the addition of a constant tems) to the integral over all space of the time component of the vector
$S_{r}(x)=j_{e}\left(-\phi(x) \frac{\partial \phi(x)}{\partial x_{r}}+\frac{\partial \phi^{+}(x)}{\partial x_{r}} \phi(x)\right)$,
which fulfills the contimuity condition
$\frac{\partial S_{I}(x)}{\partial x_{T}}=0$.
Thus (2.2.24) is ueually interpreted as the current density for charged particles. It vanishes for a neutral or Hemaitean field.

### 2.3 Transformational Properties of tho Scalar Pield.

Under the transformation $P$, we say that the four-vector $x_{r}$ transforms:
$P\left(x_{r}\right)=-x_{r} \quad(r=1,2,3)$
$P\left(x_{4}\right)=x_{4}$.
Thus
$P(\phi(P(x)))=\epsilon_{P} \phi(x)$.
Since
$\epsilon^{2}=+1$,
we find that, as in (2.1.4)

$$
\begin{equation*}
E= \pm 1 . \tag{2.3.6}
\end{equation*}
$$

The eigenvalue +1 corresponds to the scalar field, and that of -1 to the pseudoscalar one.

Under the operator T, a four-vector $x_{r}$ tranaforms:
$T\left(x_{r}\right)=x_{r} \quad(r=1,2,3)$
$T\left(x_{4}\right)=-x_{4}$.
Thus
$T(\phi(T(x)))=\epsilon_{T} \phi(x)$.
Here $\epsilon_{T}$ is an arbitrary phase factor of the form
$\epsilon_{T}=\exp (j \theta)$.
However, in (2.3.9) all numbers in the field $\phi(x)$ are transformed into their complex conjugates, since $T$ is an antiunitary operator. A unitary operator or
matrix is one such that
$\mathrm{UU}^{+}=\mathrm{U}^{+} \mathrm{J}=I$,
I being the identity element. In antiunitary operator transforms
$A A^{+}=A^{+} A=I^{*}$.
One interesting feature of time reversal is that it transfoms all outgoing states into incoming ones and vice-versa. Using the interacting field (Heisenberg definition) (9), we obtain
$a_{\text {in }}^{\dagger}\left(k, k_{0}\right) \xrightarrow{T} a_{\text {out }}^{\dagger}\left(-k, k_{0}\right)$,
where $a_{i n}^{+}$acting on the vacuum creates an incoming particle, and $a_{\text {out }}^{+}$an outgoing one.

The charge conjugation operator, $C$, acts on the scalar field:
$C(\phi(x))=\epsilon_{C} \phi^{+}(x)$,
which implies the transformation
$c|\phi, k\rangle=\epsilon_{c}^{*}|\bar{\phi}, k\rangle$,
where $\phi$ represents a particle in the field and $\bar{\phi}$ its corresponding antiparticle. If the transformations $C$ and $T$ commute, which is probable, since they are physically unconnected, we see that

$$
\begin{equation*}
\epsilon_{T} \epsilon_{C}= \pm 1 . \tag{2.3.16}
\end{equation*}
$$

If $\phi(x)$ is a Hermitean field, then
$\epsilon_{C}= \pm 1$.

### 2.4 The Free Spinor Field.

We recall the Dirac equation (1.7.10) and Fourier decompose the state vector $\psi(x)$ which we now reinterpret as a field operator (10). Thus

$$
\begin{align*}
& \Psi_{\alpha}(x)=\frac{1}{\sqrt{V}} \sum_{\underline{Q}, r}\left(e^{j q x} u_{\alpha}(+)(r)(\underline{q}) a^{(r)}(\underline{q})\right.  \tag{2.4.1}\\
&\left.+e^{-j q x} u_{\alpha}(-)(r)(-\underline{q}) b^{+(r)}(\underline{q})\right)
\end{align*}
$$

where
$u_{x}{ }^{(r)}$ (q)
is a Dirac spinor with polarization (spin) state $r$ and momentum $g$, and runs from 1 to 4 according to the gamma matrices.
$a^{(r)}(\mathrm{g})$
is the annihilation operator for a particle with polarization $r$ and momentum
q. Similarly we see that
of ( r )
is the annihilation operator for antiparticles with polarization $r$ and momentum g, and this will occur in place of $\mathrm{b}^{\dagger}$ in the adjoint field operator.
$\mathrm{b}^{+(\mathrm{r})}$ (q)
creates an antiparticle and
$a^{\dagger(r)}(q)$
creates a particle. For an electron, which has a spin of $\frac{1}{2}$, we can have two possible spin orientations, parallel to the momentum q, or antiparallel to it, corresponding to the polarizations $r=1$ and $r=2$ respectively. These states are known as left- and rigit-handed states of the electron. The operators (2.4.3), (2.4.4), (2.4.5) and (2.4.6) obey the anticommutation relations $\left[a^{(r)}(\underline{q}), a^{+\left(r^{\prime}\right)}\left(\underline{q}^{\prime}\right)\right]_{+}=\left[b^{(r)}(\underline{q}), b^{+\left(r^{\prime}\right)}\left(\underline{q}^{\prime}\right)\right]_{+}=\delta_{r r^{\prime}} \delta_{q q^{\prime}} \quad$ (2.4.7) and all other anticommtators vanish.
$H=\sum_{\underline{q}, r}^{\text {From }} E_{\left(a^{+(r)}(\underline{q}) a^{(r)}(\underline{q})+b^{T(r)}(\underline{q}) b^{(r)}(\underline{q})\right) .}$
Since we are now concerned with half-spin particles (fermions) which obey the
Pauli exclusion principle ${ }^{3}$ (10), the value of the occupation operator (2.2.18) can only be either one or zero. Frou (1.8.3) we may write the current density of a spinor field
$S_{r}(x)=(j e / 2)\left[\widetilde{\psi}(x), \gamma_{r} \psi(x)\right]$.
This current density is adjusted so that the vacuum expectation values of
all $S_{r}$ vanish:
$\left\langle\left. 0\right|^{r} \mathrm{~S}_{r}(\mathrm{x}) \mid 0\right\rangle=0$.
Purthermore, (2.4.9) implies
$\begin{aligned} Q & =-j \int_{\sum_{\underline{1}} S_{4}(x) d^{3} x}\left(a T^{(r)}(\underline{q}) a^{(r)}(\underline{q})-b^{T(r)}(\underline{q}) b^{(r)}(\underline{q})\right) \\ & =e\end{aligned}$

### 2.5 Transformational Properties of the Spinor Field.

Under space inversion the spinor field behaves
$P(\psi(P(x)))=\epsilon_{P} Y_{4} \psi(x)$
where

$$
\epsilon_{P}= \pm 1, \pm j
$$

and under time reversal
$T(\psi(T(x)))=\epsilon_{T} B \psi(x)$
where $B$ is a unitary matrix such that
${ }_{\mathrm{B}} Y_{\mathrm{B}}{ }^{\mathrm{B}^{-1}}=\tilde{\gamma}_{r}$
$=-\mathrm{B}$.
Under charge conjugation
$c(\psi(x))=\epsilon_{C} c^{-1} \widetilde{\ddot{Y}}(x)$
where $C$ is a unitary matrix fulfilling the condition (2.5.5) and
c $\gamma_{I} c^{-1}=-\gamma_{I}$.
If $T$ and $C$ commute, then (2.3.16) is valid again. If
$c(\psi(x))=\psi(x)$
then $\psi(x)$ is a Majorana field (11), and is said to be self-conjugate.
Thus, for a Majorana field $X(x)$,
$X(x)=c(X(x)) \quad$ (2.5.9)
All particles in a Kajorana field must be identical with their antiparticles, i.e. they must have even $C$ parity. An interesting consequence of (2.5.9) is that a Majorana field may not be subjected to a gauge transformation of the first type. This transformation has the form (12)
$\psi \longrightarrow \psi \mathrm{e}^{j \lambda G}$,
where $\lambda$ is a real arbitrary parameter and $G$ is the particular gauge. Bxamples of gauges in particle physics are charge, baryon gauge, lepton gauge, hypercharge gauge, and, in weak interaction theory, vector current gauge. Then we say that an interaction is invariant under a gauge transformation, we mean that it is invariant under the one-dimensional internal symmetry group $V_{1}^{G}$. One reason for which the Kajorana field may not be subjected to a gauge transformation is that the vector current associated with gauge invariance:
$\left(\bar{X}(x) \gamma_{r} X(x)\right)$
vanishes because of (2.5.9). However, the chiral gauge transformation
$X(x) \longrightarrow e^{j \alpha \gamma_{5}} \quad X(x)$
may still be applied to the Majorana field, and its associated current

$$
\begin{equation*}
\bar{X}(x) \gamma_{r} \gamma_{5} X(x) \tag{2.5.12}
\end{equation*}
$$

is nonvanishing.
From (1.7.10) we now write the Dirac equation for zero mass:
$Y_{r}\left(\partial / \partial x_{r}\right) \psi(x)=\left(Y_{i}\left(\partial / \partial x_{i}\right)+Y_{4}\left(\partial / \partial x_{4}\right) \psi(x)=0(2.5 .14)\right.$ Multiplying through by $\gamma_{4}$ and using the $\gamma$ matrix properties, we obtain $\left(j \gamma_{5} \sum_{i}\left(\partial / \partial x_{i}\right)+\left(\partial / \partial x_{4}\right) \psi(x)=0\right.$.
If we choose a representation such as the one discussed earlier for $\gamma_{5}$ so that it is diagonalized ie. all its nonzero components lie on its leading diagonal, and then write our field $\psi(x)$ in terms of it, we have
$\begin{aligned} \psi(x) & =\frac{1}{2}\left(1+Y_{5}\right) \psi(x)+\frac{1}{2}\left(1-Y_{5}\right) \psi(x) \\ & =\alpha(x)+\beta(x) .\end{aligned}$
(2.5.15) now resolves into two uncoupled relations
$\left(j \sigma_{r}\left(\partial / \partial x_{r}\right)+\left(\partial / \partial x_{4}\right) \propto(x)=0\right.$
$\left(-j \sigma_{r}\left(\partial / \partial x_{r}\right)+\left(\partial / \partial x_{4}\right) \beta(x)=0\right.$.
(2.5.17) and (2.5.18) are known as the Well equations for massless spin $\frac{1}{2}$ particles, and which describe the neutrinos. The spinors $\alpha(x)$ and $\beta(x)$ have only two components each, and hence the matrices $\sigma_{r}$ are $2 \times 2$ matrices. These are usually identified with the Pauli spin matrices (1.7.18). Wo find that the massless spinor $\psi(x)$ may be represented, according to the Well equations (13) $\psi(x)=\left[\begin{array}{r}\alpha(x) \\ \sigma_{2} \alpha^{*}(x)\end{array}\right]$
since if $\bar{\alpha}(x)$ is a solution to (2.5.17), then $\sigma_{2} \alpha(x)$ will be a solution to (2.5.18).

Let $w(p)$ be a tro-component spinor. We set
$\alpha(\mathrm{x})=w(p) e^{j p \mathrm{x}}$,
which is a plane wave solution to the Well equation, we find that $w(p)$ satisfies $((\underline{\sigma} \cdot \underline{\underline{L}})+\mathrm{E}) w(\mathrm{p})=0$,
and thus nontrivial solutions exist only when
$\mathrm{E}^{2}=\mathrm{p}^{2}$,
$\mathrm{E}= \pm|\mathrm{n}|$,
the latter possibility corresponding to the existence of an antiparticle state. We let $v(p)$ be the antiparticle solution, and write two relations concerning the orientation of the particle spin, $\underline{\sigma}$ :
$((\underline{\sigma} \cdot \underline{p}) /(|\underline{q}|)) u(p)=-u(p)$
$((\underline{\underline{\sigma}} \cdot \underline{\mathrm{p}}) /(|\underline{\mathrm{p}}|)) \mathrm{v}(\mathrm{p})=\mathrm{v}(\mathrm{p})$.
Thus the particle's spin is always aligned antiparallel to its momentum, and the
antiparticle's is always aligned parallel to it. We define an operator

$$
\begin{equation*}
\alpha(p)=(\underline{\sigma} \cdot \underline{p}) /|\underline{p}|, \tag{2.5.26}
\end{equation*}
$$

called helicity, which is negative for the solution $u(p)$ and positive for $v(p)$. These states are said to be left- and right-handed respectively. In general, helicity, H, is defined
$\mathrm{H}=\propto \mathrm{v} / \mathrm{c}$
and hence it is only constant for massless particles. The relation (2.5.27) may be obtained from the Dirac equation and this is done in Nuiriead:興ementary Particle Physics, Porgamon 1972, pp. 41-46.

### 2.6 Interacting Fields.

In the time-dependent Hamiltonian form of the Schrsdinger equation (2.1.6), we assume that the total Hamiltonian may be written as the sum of a free and an interacting Haniltonian, which we label $H_{0}$ and $H_{工}$ respectively. We now define the state vector $\phi(\underline{x}, t)$ :
$\phi(\underline{x}, t)=e^{j H_{0} t} \psi^{\prime}(\underline{z}, t)$
and the operator $O(\underline{x}, t)$ :
$O(\underline{x}, t)=e^{j H_{0} t} \psi e^{-j H_{0} t}$.
At
$t=0$
the total Hamiltonian, A , is identical to the free Hamiltonian, $\mathrm{H}_{0}$. From (2.6.1) , (2.6.2), and (2.1.6), we thus obtain
$j \frac{\partial \phi(x, t)}{\partial t}=H_{I} \phi(\underline{x}, t)$,
and hence we see that the state vectors in the 'interaction picture' (14) have the same dependence on the interacting Hamiltonian as those in the 'Schrodinger picture' have on the total Hamiltonian. Let us rewrite (2.6.2) as $O(\underline{z}, t)=e^{j H_{0} t} O(\underline{x}, 0) e^{-j H_{0} t}$.
We now interpret $O(t)$ defined in (2.6.5) as being an operator in the 'Heisenberg picture', ignoring the poaition vector ( $\Sigma$ ). In the 'Heisenberg picture' the free Hamiltonian in the 'interaction picture' acts as the total Hamiltonian, and hence the operator in the 'interaction picture' is identical to the one in the 'Heisenberg picture' for the case of a non-interacting system. Thus both equations of motion and comutation relations for operators from the
'Heisenberg picture' may be used in the 'interaction picture'. From (2.6.5) we may write the differential form of the Heisenberg equation of motion $j \frac{\partial o(t)}{\partial t}=\left[0(t), H_{0}\right]$.

We shall now transform some of the results which we obtained in 2.2 concerning the free scalar field into the 'interaction picture'. The destruction operator $a(k)$ defined in (2.2.13) is now redefined
$a(\underline{k}, t)=e^{j H_{0} t} a(\underline{k}) e^{-j H_{0} t}$,
and the creation operator is similarly redefined. Thus the Fourier decomposition of the field becomes
$\phi(x)=\frac{1}{\sqrt{V}} \sum_{\underline{k}} \frac{1}{\sqrt{2 \omega(k)}}\left(a(\underline{k}, t) e^{j k x}+a^{\top}(\underline{k}, t) e^{-j k z}\right)$.
We now wish to solve $(2.6 .7)$. The only temm in the scalar field Hamiltonian (2.2.15) which does not commute with $a(\underline{k})$ is

$$
\begin{equation*}
\omega(\underline{k}) a^{\top}(\underline{k}) a(\underline{k}) . \tag{2.6.9}
\end{equation*}
$$

Uaing the method outlined in Schwebor, Bethe, de Zoffman: Fields, in Wesons and Fields (Vol. I), Evanston 1955, section 15b, we obtain $a(\underline{k}, t)=a(\underline{k}) e^{-j \omega(\underline{k}) t}$
and
$a^{+}(\underline{k}, t)=a^{+}(\underline{k}) e^{-j \omega(\underline{k}) t}$,
where
$\omega(\underline{k})=k_{0}=\sqrt{\left(\bar{m}^{2}+\underline{k}^{2}\right)}$.
At this point, we may substitute with $(2.6 .10)$ and $(2.6 .11)$ in $(2.6 .8)$ and we obtain for the scalar interacting field

$$
\begin{equation*}
\phi(x)=\frac{1}{\sqrt{v}} \sum_{k_{0}=\omega(\underline{k})} \frac{1}{\sqrt{2 k_{0}}}\left(a(k) e^{j k x}+a^{\dagger}(\underline{k}) e^{-j k x}\right) \tag{2.6.13}
\end{equation*}
$$

summing over all allowed momentak. Ofton the field is decomposed
$\phi(x)=\phi^{+}(x)+\phi^{-}(x)$,
where $\phi^{ \pm}$are the parts of the field containing only destruction or creation operators respectively. The vacuum definition
$a(\underline{k})|0\rangle=0$,
which implies that it is not possible to remove a particle from the vacuum, may now be replaced by
$\phi^{+}|0\rangle=0$.
and from (2.6.13) and (2.6.14)
$\left(\phi^{+}\right)^{\dagger}=\phi^{-}$.
Now let us attempt to calculate the value of the commutator
$\left[\phi(x), \phi\left(x^{\prime}\right)\right]$,
where $x$ and $x^{\prime}$ are two position space-time four-vectors. Since two creation or two destruction operators commute, only cross-terms will contribute, and thus $\left[\phi(x), \phi\left(x^{\prime}\right)\right]$
$=\frac{1}{v} \sum_{k_{0}=\omega(k)} \sum_{k_{0}^{\prime}=\omega\left(k^{\prime}\right)}$
$\frac{1}{2 \sqrt{k_{0}^{k \prime}}}\left(\left[a(\underline{k}), a^{\dagger}(\underline{k})\right] e^{j k x-j k^{\prime} x^{\prime}}\right.$
$\left.\left[a^{\top}(k), a\left(k^{\prime}\right)\right] e^{j k x-j k^{\prime} x^{\prime}}\right)$
$=\frac{1}{(2 \pi)^{3}} 3-2 \int_{k_{0}=\omega(k)} \frac{d^{3} k}{k_{0}}\left(e^{j k\left(x-x^{\prime}\right)}-e^{-j k\left(x-x^{\prime}\right)}\right)$,
anticipating going to the limit of normalization and using the approximation (2.2.10). We define
$\Delta(x)=\frac{1}{(2 \pi)^{3}} \int_{k_{0}=\omega(\underline{k})} \frac{d^{3} k}{k_{0}} \sin k x$,
so that
$\left[\phi(x), \quad\left(x^{\prime}\right)\right]=j \Delta\left(x-x^{\prime}\right)$.
Prom (2.6.20) we see that $\Delta(x)$ is a real odd function of $x$, and is a Lorentz invariant scalar. Thus
$\Delta(\underline{x}, t=0)=\frac{1}{(2 \pi)^{3}} \int \frac{\frac{a}{}^{3} k}{\omega(\underline{x})} \sin \underline{k x}=0$,
so that the 'equal times' commutator
$\left[\phi(\underline{x}, t), \phi\left(\underline{x}^{\prime}, t\right)\right]=0$.
Since the $\triangle$ function is a Lorentz invariant
$\Delta(x)=0 \quad: \quad x^{2}=x^{2}-t^{2}>0$,
and thus the $\Delta$ function vanishes outside the 'light-cone'
$|\underline{x}|=|t|$,
and similarly the commutator in (2.6.21) vanishes for points with space-like separation:
$\left(x-x^{\prime}\right)^{2}>0$.
If the commutator (2.6.18) was non-vanishing for space-like separated points $x$ and $x^{\prime}$, then this would imply that the measurements of the field at two separated points in space interfered with each other, necessitating a signal travelling faster than the velocity of light. The vanishing commutator thus upholds the special theory of relativity.

The CPT theorem was first mentioned by Schwinger in 1951 (16), it was verified by Ludders (17) and was proved by Pauli in 1954 (18). Hence it is sometimes called the 'Luders-Pauli theorem'. It states that, if all physical systems can be described by relativistic field equations, then all systems should be invariant with respect to the combined transformation CPT or 'strong reflection', $S$ (19). Since it is thought that all fields can be constructed ultimately from the spin $\frac{1}{2}$ Dirac field, it is the CPM transformation properties of this field wiich we consider here. We tabulate the transforwation properties of the Dirac field operator $\psi(\underline{z})$ and its adjoint $\dddot{\psi}(\underline{z})(20)$ : $\begin{array}{cccc}\underline{\text { Operator }} & \underline{C} & \underline{P} & \underline{T} \\ \tilde{\psi}(\underline{x}) & c \tilde{\psi}(\underline{x}) & Y_{4} \psi\left(-\underline{z}, x_{0}\right) & T \tilde{\psi}\left(\underline{z},-x_{0}\right) \\ \tilde{\psi}(\underline{x}) & c^{-1} \tilde{\psi}(\underline{x}) & \tilde{\psi}\left(-\underline{z}, x_{0}\right) Y_{4} & -T^{-1} \psi\left(\underline{x},-x_{0}\right)\end{array}$
where X is a space-time four-vector. From this table, it is possible to calculate the properties of the Dirac field wien two or more operators are applied to it. Adopting
$\psi(\underline{x}) \xrightarrow{T} \tilde{\psi}\left(\underline{x},-x_{0}\right)$,
where $T$ is an operator acting on the spin indices of the field function, we may obtain an explicit expression for T. We assume that the Dirac equation (1.7.10) and its adjoint (1.7.26) are invariant under time reflection, and by considering these two expressions we see that the matrix T must obey the relations:
$T^{-1} \gamma_{r} T=T\left(-Y_{r}\right)=c^{-1} Y_{r} C \quad(r 1,2,3)$
$\mathrm{T}^{-1} Y_{4}^{T}=T\left(-Y_{4}^{T}\right)=c^{-1} Y_{4}^{T} \mathrm{C}$,
and thus
$T=\gamma_{4} Y_{5}^{c}$.
Hence we may write
$\operatorname{PT}(\psi(\underline{x}))=Y_{5} C \psi\left(-\underline{x},-x_{0}\right)=Y_{5} c(\psi)\left(-\underline{\underline{x}},-x_{0}\right)$
and similarly
$\operatorname{PT}(\tilde{\psi}(\underline{x}))=\gamma_{5} c(\bar{\psi})\left(-\underline{x}, x_{0}\right)$.
Thus
$\operatorname{CPI}(\psi(\underline{x}))=Y_{5} \psi(-\underline{x})$
and
$\operatorname{CPT}(\tilde{Y}(\underline{x}))=-\tilde{\psi}(-\underline{x}) Y_{5}$.
We know that the sixteen different field operators or bilinear covariants obtained by combining one Dirac field operator with the Hermitean conjugate of another, may be placed in five groups according to their properties under the Lorentz transformation. We tabulated these possibilities in 1.7. Under the combined CPT operation, we have
$\mathrm{F}_{\mathrm{S}}(\underline{\mathrm{x}}) \longrightarrow \mathrm{F}_{\mathrm{S}}(-\underline{x})$
$\mathrm{P}_{\mathrm{V}}(\underline{\mathrm{x}}) \longrightarrow-\mathrm{F}_{\mathrm{V}}(-\underline{x})$
$\mathrm{F}_{\mathrm{T}}(\underline{\mathrm{x}}) \longrightarrow \mathrm{F}_{\mathrm{T}}(-\underline{x})$
$\mathrm{P}_{\mathrm{A}}(\underline{\mathrm{x}}) \longrightarrow-\mathrm{F}_{\mathrm{A}}(-\underline{\mathrm{x}})$
$F_{P}(\underline{x}) \longrightarrow F_{P}(-\underline{x})$,
where $F_{S}$, for example, is the scalar bilinear covariant. We find that any interaction Hamiltonian which is Lorentz invariant and which we construct from the field operators $(2.7 .9),(2.7 .10),(2.7 .11),(2.7 .12)$ and (2.7.13) is invariant under the combined CPT transformation. As an example, we take the interaction Hamiltonian:
$H_{I}=g \int d^{3} x F_{S}(\underline{x}) F_{p}(\underline{x})+$ Herw. conj. ,
where $g$ is a constant dependent on the strength of the interaction, and is known as the coupling constant. This Hamiltonian is invariant under CPT but not under the three operators performed separately. Thus we have shown that any Lorentz invariant interaction Hamiltonian conatructed from the Dirac field operators is invariant under CPP, and that hence, if it is not invariant under one of the transfomations on its own, then it must also be not invariant under another in order to preserve invariance under the combined transformation.

The argument given above applies only to spin $\frac{1}{2}$ fields, since it is these which we shall consider primarily in later chapters. However, it is possible to prove the CPT theorem in general, assuming Lorentz invariance and the connection between spin and statistics ${ }^{4}$. This is done in Narshak, Riasruddin, Ryan: Weak Interactions of Blementary Particleg, Wiley 1969, pp. 74-75. in alternative proof, assuming only the axioms of field theory, was presented in Jost: Helv, Phys, Acta 30, 409, (2957).

### 3.1 Phenomenolory of Beta Decay.

In 1896 pocquerel (1) found an ionizing radiation issuing from uranium salts. In 1899 Giesel (2) demonstrated that it was deflected by a magnetic field, and it 1900 the Curies (3) showed that it consisted of a strean of negatively-charged particles. In the same year Becquerel (4) obtained a value for the charge-to-mass ratio, e/w, for the new radiation, and by 1903 wien had made a first estimate for the magnitude of their charge (5). Autherford numed this radiation 'beta radiation' (6) in 1902, and by 1904 the experiments of Kaufimam (7), Bestolnoyer (8) and Bucherer (9) had confirmed that it consiated of a stream of high-energy electrons whose mass varied in the manner prescribed by relativity.

At first, absorption experiments seemed to indicate that the electrons emitted in the beta decay of nuclei wero monoenergetic, but in 1909 Wilson (10) showed that they did, in fact, have a wide range of velocities. Von Baoyer, Hahn and Keitner (11) found that there existed several distinct velocity groups in the beta rays emitted from mesothorium tro. The much more accurate method of the Dempster mass spectrograph nas used by Rutherford and Robinson (12) in 1913, who found a number of well-defined linea in the beta ray opectrum of radium $B$ and $C$. The principle of the inass spectrograph was to place the radioactive source to be analysed, in the form of a thin wire, at the edge of a uniform magnetic field, so that the emitted beta rays would be bent round in semicircles whoso radii were invergely proportional to their velocity. The distribution of the deflected beta rays was recorded by means of a photographic plate. In 1914, Chadwick (13) showed that two types of beta ray spectra existed, continuous speotra and line spectra. The line spectra were explained as being due to the removal of extramclear atomic electrons from their nolls by high-energy gama rays from the disintegrating nucleus.

However, the existence of continuous beta ray spectra posed a problem. If the nuclei in the initial and final statea in the decay had unique characteristic
energies, then it was difficult to see how the transformation could occur by the emission of an electron which could have any energy lying within a large continuous range. The reaction appeared to violate the law of the conservation of energy. The first suggestion was that the electrons were initially ewitted with energies of the observed maxima for particular nuclei, and that then random energy losses occured, cauning a continuous energy spectrum. Hovever, this hypothesis was refuted by Ellis and Wooster in 1927 (14) who measured the rise in temperature of a lead vessel containing the beta emitter radium $E$. They found that the onergy absorbed by thoir 1.3 mm of lead corresponded to the average energy of the continuous spectrum, and not to the marimum energy, as the energy-loss hypothesis would have predicted.

Following the discovery of the neutron by Chadwick in 1932 (15), it was suggested that beta decay was cauced by the decay of a bound neutron into a proton and an electron. The proton would remain bound within the nucleus, but the electron would be emitted as a beta ray. Pollowing the precise measurement of the energy loss in beta decay by Ellis and Nott in 1933 (16), Pauli postulated (17) the existence of a new particle which would be emitted together with the electron in beta decay. This particle would have to be light and apparently undetectable, and, in order to conserve electric charge, it would also have to be neutral. In 1934, Femi (21) introduced the four-femmion interaction (see 3.3) and named the new particle the neutrino. Since total angular momentum or spin must be conserved in beta decay, the neutrino was assigned a spin of $\frac{1}{2}$. Thus in the neutron decay
$\mathrm{n} \longrightarrow \mathrm{p}+\mathrm{e}^{-}+\overline{\mathrm{v}}$,
the spins of the proton and election, for example, must be antiparallel, leaving the neutrino to have its apin parallel to that of the neutron. As we shall see later, it was advantageous to assume that it was not a neutrino but an antineutrino which was emitted in beta decay.

### 3.2 The Detection of the Noutrino.

Twenty-five yeare after the noutrino had first been suggested by Pauli (27), Cowan, Reines, Harrison, Kruse and KcGuire (18) attempted to detect it. Since the neutrino itself could not be induced to interact with any type of detector,
and did not appear to decay, it was necessary to use sowe inverse reaction in which the neutrino took part. Tho process

in which a proton oaptures an electron from the inner K shell and becomes a neutron with the ertission of a neutrino, was known to occur. Hence, by crossing symmetry (I invariance and the Peynman rules), $\overrightarrow{\mathrm{v}}+\mathrm{p} \longrightarrow \mathrm{e}+\mathrm{n} \quad$ (3.2.2) should also take place. It was this reaction which Cowan et al. attompted to detect. Their source of antineutrinos was the Savannah River nuclear power station, which produced an estimatod flux of $10^{17} \mathrm{~m}^{-2} 11 \mathrm{KeV}$ antineutrinos. If the process (3.2.2) occured, then the resultant positron would produce an ionization trail in a p-terphenyl liquid scintillator ${ }^{1}$, and, when it came to rest, would be annihilated by a negative electron, producing two gamina rays, each with an energy of about 0.5 KeV . Furthemore, it was discovered that the placing of a layer of $\mathrm{Cd} \mathrm{Cl}_{2}$ solution within the scintillator resulted in the probable capture of the slow noutron by an atomic nucleus, with the emisgion of three or four gamma rays carrying a total energy of around 9.1 NeV . These would be observed as coincident flashes in the surrounding scintillator.

Cowan et al. sandwiched a tank of water containing a low concentration of $\mathrm{Cd} \mathrm{Cl}_{2}$ between two liquid scintillator tanks, and enclosed the whole assembly within an array of about 500 phototultiplier tubes. They expected the antincutrino from the nuelear reactor to interact with a hydrogen proton in the layer of water according to (3.2.2). The flash of light from the neutron capture gania rays was estimated to occur about $10 \mu_{8}$ after that from the positron annihilation. There were about 10 tona of liquid in the sointillators, and hence a considerable number of spurious events took place, due to cosmic rays and fast neutrons direct from the reactor, but these were usually revealed either by an exror

1. Sointillators are substances winich euit pulses of light when charged particles pass through them. The resultant light is usually directed into photomultiplier tubos, where it is converted into electrical aignals so that it may be analysed by electronic equipment. See e.g. Birks: The Theory and Practice of Scintillation Counting, Pergamon 1964.
in timing or by a pulse from a further scintillator situated above the other two. The equipment was in operation for about 2085 hours, with an average event density of $3.0 \pm 0.2 \mathrm{hr}^{-1}$, so that, by the end of 1956 , it was possible to calculate the experimental cross-section for (3.2.2) as
$\left(\begin{array}{ll}1.2 & +0.7 \\ -0.4\end{array}\right) \times 10^{-43} \mathrm{~cm}^{2}$,
in agreement with the theoretical prediction of $(1.0 \pm 0.16) \times 10^{-43} \mathrm{~cm}^{2}$.
A number of tests were perfomed, such as increasing the amount of shielding surrounding the scintillators, demonstrating conciusively that the counting rate was associated with particles from the reactor. In 1957, the reaction
$\overrightarrow{\mathrm{v}}+\mathrm{d} \longrightarrow \mathrm{e}+\mathrm{n}+\mathrm{n}$ vas also definitely detected (19), and its crose-section was measured as $2 \times 10^{-45} \mathrm{~cm}^{2}$. (3.2.6)

At the same time as the work on antineutrinos described above was in progress, a search was also carried out for the neutrino. Davis, Harmer and Hoffmann (20) used the inverse of the $\overline{\mathrm{K}}$-electron capture process:
$\mathrm{v}+\mathrm{n} \longrightarrow \mathrm{e}^{-}+\mathrm{p}$,
and attempted to detect it in the case of
${ }_{17}^{37} \mathrm{Cl}+\mathrm{V} \longrightarrow{ }_{18^{A}}^{37}+e^{-}$,
with an expected cross-section of
(12 土 6) $\mathrm{x} \quad 10^{-44} \mathrm{~cm}^{2} /$ atom .
They set up a tank containing about 3900 litres of $\mathrm{C} \mathrm{Cl}_{4}$ in a heavilyshielded underground location near the Brookhaven nuclear pile, which produced neutrinos at a rate of about $4 \times 10^{18} \mathrm{~s}^{-1}$. Since the half-life of ${ }_{18}^{37}$ is in the order of 34 days, the tank was left untouched for between 36 and 75 days, after any argon initially present had been removed by bubbling gaseous holium through the tank. After irradiation, helium vas again bubbled through the tank, and the argon was separated from it by iractional distillation, using solid $\mathrm{CO}_{2}$ as a coolant. The sample was then tested with a Geiger counter, but was not found to exhibit detectable
traces of K-electron capture radiation, setting an upper limit on the cross-section for the reaction (3.2.8) of
$2 \times 10^{-42} \mathrm{~cm}^{2} /$ atom .

### 3.3 The Neutron Decay Hamiltonien.

In 1934, Ferni suggested (21) that a Hamiltonian for the neutron decay (3.1.1) might be constructed in direct analogy to that for the generation of electronagnetic radiation. The latter is proportional to the electric current four-vector
$J_{r}=j e \bar{\psi} Y_{r} \psi$,
and Permi postulated that the neutron should beta decay at a rate proportional to the 'current' $\widetilde{\psi}_{p} \gamma_{I} \psi_{\mathrm{n}}$ associated wi th the protonneutron transition. Since the complete Hamiltonian must be a Lorentz-invariant scalar, it must initially consist of the product of two four-vector currents. Thus, ascuming that no dorivatives of the basic fields are involved, we writo:
$H_{I}=\int G\left(\bar{\psi}_{p} Y_{I} \psi_{\mathrm{n}}\right)\left(\bar{\psi}_{e} Y_{\mathrm{I}} \psi_{\mathrm{v}}\right)+$ Herm. conje,
where $G$ is the beta decay coupling constant ${ }^{1}$, and we have transposed all the particles in (3.1.1) on to the leit hand side of the equation, since the Hamiltonian is sligitly coeior to handle in the more symmetric form (3.3.2). The latter is known as the 'four-fermion interaction', since it involves four fermion fields interacting at a point in spacetine. It is not, however, the most general neutron decay Hamiltonian, since it assumes that only a vector Dixac interaction occurs. Generalizing for all five bilinear covariants, and enforcing the condition that the Hamiltonian be a Lorontz-invariant scalar, we obtain (22)
$H_{I}^{I} \quad=\quad \sum_{i} c_{i} \int\left(\tilde{\psi}_{p} o_{i} \psi_{n}\right)\left(\tilde{\psi}_{e} o_{i} \psi_{v}\right)+$ Herm. conj.,
where the $O_{i}$ are the five different types of $Y$-watrix product. The $C_{i}$ are the coupling constants for each kind of interaction. However, with

1. Coupling constants are complex paramieters which characterize the strength of a particular interaction. For an interesting discussion of their underlying theory, see Heisemberg: Introduction to the Unified Field Theory of Elementary Particles, wiley 1966, pp. 79-89.
less rigorous but still physically acceptable Lorentz invariance required ${ }^{1}$, we obtain another possible Hamiltonian (23):
$H_{I}^{2}=\sum_{i} C_{i} \int\left(\hat{\psi}_{p} o_{i} \psi_{n}\right)\left(\hat{\psi}_{e} o_{i} \gamma_{5} \psi_{v}\right)+$ Herm. conj.

We now assume that the total interaction Hamiltonian is given by
$H_{I}=H_{I}^{I}+H_{I}^{2}$,
or
$H_{I}=\sum_{i} \int\left(\hat{\psi}_{p} o_{i} \psi_{n}\right)\left(\tilde{\psi}_{e} o_{i}\left(c_{i}+c_{i}^{\prime} \gamma_{5}\right) \psi_{v}\right)+$

$$
\begin{equation*}
+ \text { Hezw. conj., } \tag{3.3.6}
\end{equation*}
$$

which is the most general local (point interaction) nonderivative Hamiltonian for neutron decay.

We now consider the errect on (3.3.6) of altering the order of the particles in the process (3.1.1). Cur Hamiltonian above directly describes the two reactions
$\mathrm{n} \longrightarrow \mathrm{p}+\mathrm{e}+\mathrm{v}$,
$\mathrm{n}+\mathrm{v} \longrightarrow \mathrm{p}+\mathrm{e}$.
However, the neutron decay could equally have been written
$\mathrm{n}+\mathrm{v} \longrightarrow \mathrm{e}+\mathrm{p}$
or
$\mathrm{n}+\overline{\mathrm{p}} \longrightarrow \overline{\mathrm{v}}+\mathrm{e}$,
requiring a rearrangement in the order of the fiold operators in the Hamiltonian. Let us write the particles of (3.1.1) in the form of numbers:
$\mathrm{p}: 1 \quad \mathrm{n}: 2 \quad \mathrm{e}: 3 \quad \mathrm{~F}: 4$.
$A_{i}\left(\begin{array}{lll}1 & 2 & 3\end{array}\right)=\left(\hat{\psi}_{1} 0_{i} \psi_{2}\right)\left(\hat{\psi}_{3} 0_{i} \psi_{4}\right)$,
summing over all pomitted $i$, as before. Since there are only five
Lorentz invariants of the form (3.3.12), the coupling constants for
$A_{j}\left(\begin{array}{llll}3 & 2 & 1 & 4\end{array}\right)$,
for example, must be a linear combination of those for the originel Hamiltonian (3.3.12). The relation is given by the Fierz reoriering theorem (24):
$A_{i}\left(\begin{array}{lll}3 & 2 & 1\end{array} 4\right)=\sum_{j=1}^{5} \lambda_{i j} A_{j}\left(\begin{array}{llll}1 & 2 & 3 & 4\end{array}\right)$,

1. In fact, under only the proper orthochronous Lorentz group.
where
$\lambda_{i, j}=\frac{1}{4} \cdot\left[\begin{array}{rrrrr}1 & 1 & 1 & 1 & 1 \\ 4 & -2 & 0 & 2 & -4 \\ 6 & 0 & -2 & 0 & 6 \\ 4 & 2 & 0 & -2 & -4 \\ 1 & -1 & 1 & -1 & 1\end{array}\right]$
Denoting the five coupling congtants by $S, V, T, A$ and $P$ when they occur in the order ( 12334 ) and by $S^{\prime}, V^{\prime}, T^{\prime}, A^{\prime}$ and $P^{\prime}$ when they oocur ( 3214 ), we have

| $S^{\prime}$ | $=$ | $-\frac{1}{4}(S+V+T+A+P)$, |
| :--- | :--- | :--- |
| $V^{\prime}$ | $=$ | $-\frac{1}{4}(4 S-2 V+2 A-4 P)$, |
| $T^{\prime}$ | $=-\frac{1}{4}(6 S-2 T+6 P)$, |  |
| $A^{\prime}$ | $=-\frac{1}{4}(4 S+2 V-2 A-4 P)$, |  |
| $P^{\prime}$ | $=-\frac{1}{4}(S-V+T-A+P)$. |  |

From the relations $(3.3 .16)$ we may deduce that

| $V^{\prime}-A^{\prime}=$ | $V-\mathrm{V}^{\prime}$, |
| :--- | :--- |
| $S^{\prime}-\mathrm{T}^{\prime}+\mathrm{P}^{\prime}=\mathrm{S}-\mathrm{T}+\mathrm{P}$, | $(3.27 \mathrm{a})$ |

and hence these combinations are invaxiant under Fierz reordering.
We now consider the properties of the neutron decay Hamiltonian
under the P, C and $P$ operatore. In 2.7 , we showed that any interaction, such as our neutron decay Haniltonian, constructed from Dirac ( $J=\frac{1}{2}$ ) fields must be invariant under the combined transformation CPP (IdderePauli Theorem), but not necessarily under the separated operators. Por the space reflection operator, $P$, we have $P\left(\bar{H}_{I}\left(\underline{x}, x_{0}\right)=\epsilon_{P} \sum_{i} \int\left(\tilde{\Psi}_{p}\left(-\underline{x}, x_{0}\right) o_{i} \psi_{n}\left(-\underline{x}, x_{0}\right)\right) x\right.$ $x\left(\widetilde{\psi}_{e}\left(-\underline{x}, x_{0}\right) o_{i}\left(c_{i}-c_{i} Y_{5}\right) \psi_{v}\left(-\underline{x}, x_{0}\right)\right)+$ + Hemp. conj.,
where the intrinsic parity factor $\epsilon_{p}$ is given by
$\epsilon_{P}=\epsilon_{\mathrm{P}}^{*}(\mathrm{p}) \cdot \epsilon_{\mathrm{P}}(\mathrm{n}) \cdot \epsilon_{\mathrm{P}}^{*}(\mathrm{e}) \cdot \epsilon_{\mathrm{P}}(\mathrm{v})$.
Thus $P$ invariance would require
$C_{i}=0 \quad(i=1,5)$,
and for this reason, the $\mathrm{C}_{\mathrm{i}}^{1}$ are known as the parity-violating coupling
constants. Noninvariance of the weak Hamiltonian under the parity operator would imply that the weak interaction differentiates between different directions in space, and hence that the multiplicative quantum number of intrinsic parity, which is assigned to all hadrons ${ }^{1}$, is not a 'good' or conserved quantum number in the weal interactions. For the time reversal operator:

$$
\begin{align*}
& T\left(H_{I}\left(\underline{x}, x_{0}\right)\right)= \epsilon_{T} \sum_{i} \int\left(\tilde{\psi}_{p}\left(\underline{x},-x_{0}\right) o_{i} \psi_{n}\left(\underline{x},-x_{0}\right)\right) x \\
& x \quad\left(\tilde{\psi}_{e}\left(\underline{\underline{x}},-x_{0}\right) o_{i}\left(c_{i}+c_{i}^{\prime} \gamma_{5}\right) \psi_{v}\left(\underline{x},-x_{0}\right)\right)+ \\
&+ \text { Hern. conj., } \tag{3.3.21}
\end{align*}
$$

where
$\epsilon_{T}=\epsilon_{T}^{*}(\mathrm{p}) \cdot \epsilon_{T}(\mathrm{n}) \cdot \epsilon_{\mathrm{T}}^{*}(\mathrm{e}) \cdot \epsilon_{\mathrm{T}}(\mathrm{v})$.
If the neutron decay Hamiltonian ia to be invariant under $T$, then it must act as a pure scalar under this transformation, so that

$$
\begin{align*}
& \epsilon_{T} c_{i}^{*}=c_{i} \\
& \epsilon_{T} c_{i}^{\prime *}=c_{i}^{\prime}
\end{align*}
$$

With respect to the charge conjugation operator, the Hamiltonian behaves:
$c\left(H_{I}\left(\underline{x}, x_{0}\right)\right)=\epsilon_{C} \sum_{i} \int\left(\widetilde{\psi}_{n}\left(\underline{x}, x_{0}\right) o_{i} \psi_{p}\left(\underline{z}, x_{0}\right)\right) x$

$$
\mathrm{x} \quad\left(\tilde{\psi}_{v}\left(\underline{x}, x_{0}\right) o_{i}\left(c_{i}+c_{i} \gamma_{5}\right) \psi_{e}\left(\underline{x}, x_{0}\right)\right)+
$$

$$
\begin{equation*}
+ \text { Hera. conj., } \tag{3.3.24}
\end{equation*}
$$

where

$$
\begin{equation*}
\epsilon_{C}=\epsilon_{C}^{*}(p) \cdot \epsilon_{C}(n) \cdot \epsilon_{C}^{*}(e) \cdot \epsilon_{C}(v) . \tag{3.3.25}
\end{equation*}
$$

Thus we may deduce that $C$ invariance demands

$$
\begin{align*}
\epsilon_{C}^{*} c_{i} & =C_{i}  \tag{3.3.26a}\\
\epsilon_{C}^{*} C_{i}^{\prime} & =-c_{i}^{\prime}
\end{align*}
$$

Finally, under the combined transformation CPT:
$\operatorname{CPT}\left(H_{I}\left(\underline{x}, x_{0}\right)\right)=\epsilon_{C} \epsilon_{P} \epsilon_{T} \sum_{i} \int\left(\widetilde{\psi}_{\mathrm{n}}\left(-\underline{\underline{x}},-x_{0}\right) o_{i} \psi_{p}\left(-\underline{x},-x_{0}\right)\right) x$

$$
\begin{gather*}
x\left(\tilde{\psi}_{v}\left(-\underline{x},-x_{0}\right) o_{i}\left(c_{i}-c_{i} \gamma_{5}\right) \psi_{\theta}\left(-\underline{x},-x_{0}\right)\right)+ \\
+ \text { Herm. conj. } \tag{3.3.27}
\end{gather*}
$$

1. As we shall see in 3.7 , the weak Hamiltonian is indeed not invariant under the parity operator, and hence those particles winch only take part in weak interactions have undefined parity. The photon's parity has been established as -I by observing electromagnetic interactions.
so that the condition for CPT invariance is simply
$\epsilon_{\mathrm{C}} \epsilon_{\mathrm{P}} \epsilon_{\mathrm{T}}=+1$,
and this, it is thought, can always be arranged.
Since there exists good evidence that the neutrino is massless ${ }^{1}$, we now assume that the free neutrino field is invariant under the so-called 'gauge' transformations (24)
$\exp \left(j a Y_{5}+j b\right)$,
as a consequence of lepton conservation (see 4.5). Since the weak interaction coupling constant is very small, we see that, so long as we only calculate to first order in the weak interaction, the interacting neutrino field may be represented just as well by ${ }^{2}$
$\exp \left(j a Y_{5}+j b\right) \psi_{v}\left(\underline{z}, x_{0}\right)$
as by
$\psi_{\mathrm{V}}\left(\underline{x}, x_{0}\right)$.
Replacing (3.3.31) by (3.3.30) in (3.3.6), we write
$H_{I}=\int \sum_{i}\left(\tilde{\psi}_{p} o_{i} \psi_{n}\right)\left(\tilde{\psi}_{\theta} o_{i}\left(B_{i}+B_{i}^{\prime} \gamma_{5}\right) \psi_{v}\right)+$ + Herm. conj.,
where
$B_{i}=e^{j o}\left(C_{i} \cos a+j C_{i}^{\prime} \sin a\right)$,
$B_{i}^{\prime}=e^{j b}\left(C_{i}^{\prime} \cos a+j C_{i} \sin a\right)$.
We note that, on transforming from $\mathrm{H}^{2}$ to H ', some bilinear combinations of the coupling constants remain unchanged:
$\mathrm{C}_{i} \mathrm{C}_{j}^{\text {棌 }}+\mathrm{C}_{i}^{\prime} \mathrm{C}_{j}^{*}=\mathrm{B}_{i} \mathrm{~B}_{j}^{\prime+}+\mathrm{B}_{i}^{\prime} \mathrm{B}_{j}^{*}, \quad(3.3 .3 \mathrm{Aa})$
$C_{i} C_{j}^{*}+C_{i}^{\prime} C_{j}^{\prime *} \Rightarrow B_{i} B_{j}{ }^{*}+B_{i}^{\prime} B_{j}^{\prime *} \cdot(3.3 .34 \mathrm{~b})$
Obviously the interactions represented by $H_{I}^{\prime}(3.3 .32)$ and by $H_{I}$ are physically indistinguishable, provided that we calculate them only to first order, and hence we may state the conditions for invariance under the operators $P, C$, and $T$ in terms of our new coupling constants $B_{i}$ and
2. The upper limit on the neutrino mass is usually thought to be about $60 \mathrm{eV} / \mathrm{c}^{2}$ (Rarash-Schmit et al., Phys. Lett. 50B, I (1974)).
3. Obviously, this is only correct if the neutrino takes part purely in weak interactions. Neutrino-photon interactions are sometimes postulated, but these would have too low an amplitude to affect our calculations above.
$B_{i}^{\prime}$ rather than of the $C_{i}$ and $C_{i}^{\prime}$. The condition for $P$ invariance now reads: $B_{i}^{\prime}=0$;
$T$ invariance demands that the coupling constants $B_{i}$ and $B_{i}^{\prime}$ must all be relatively real ${ }^{1}$, and $C$ invariance that all the $B_{i}$ must be real, and all the $B_{i}^{\prime}$ pure imaginary. Using (3.3.33), we may now translate these conditions into statements involving the $C_{i}$ and $C_{i}^{\prime} . P$ invariance:
$C_{i} C_{j}^{\prime}+C_{i}^{\prime} C_{j}=0$;
Tinvariance:
$\operatorname{Im}\left(c_{i} c_{j}^{\prime}+c_{i}^{\prime} c_{j}\right)=0$,
$\operatorname{Im}\left(C_{i} C_{j}+C_{i}^{\prime} C_{j}^{\prime}\right)=0 ;$
and C invariance:
$\operatorname{Re}\left(C_{i} C_{j}^{\prime}+C_{i}^{\prime} C_{j}\right)=0$,
$\operatorname{Im}\left(C_{i} C_{j}+C_{i}^{\prime} C_{j}^{\prime}\right)=0$.

### 3.4 The Kinematica of Beta Decay.

In beta disintegration, momentum must obviously be conserved in the three-body system of the electron, antineutrino, and residual nucleus, but not necessarily between the electron and antineutrino alone. Thus, if E is the kinetic energy of the electron and $E_{0}$ is the total energy available in the decay, then the antineutrino energy may be fixed as $\left(E_{0}-E\right)$, and since the neutrino is massless, this is also its momentum ${ }^{2}$. Hence we may now write the electron momenturn: (25)
$p_{e}=\sqrt{E\left(B+2 \mathrm{Im}_{0}\right)}$.
The statistical distribution of electron monenta may be obtained by considering the phase-space volume accessible to an electron with a momentum between $p_{e}$ and $p_{e}+d p_{e}{ }^{3}$, and to an associated antineutrino 1. i.e. the ratios must be real.
2. As usual, we employ natural units $\left(k^{\prime}=c=1\right)$. In non-natural units, the relation between the energy and momentum of a zero mass particle is $\mathrm{E}=\mathrm{pc}$.
3. The reason for this finite momenturn range is that the neutron has a finite lifetime ( $\sim 918 \mathrm{~s}$ ) and hence the uncertainty relation
$\Delta_{p} \Delta x \geqslant$ I forbids the precise measurement of the electron energy.
wi th momentum $p_{v} \longrightarrow p_{v}+d p_{v}$. The nurber of available electron states in the phase space element of volume $4 \pi p_{e}^{2} d p_{e}$ is
$p_{e}^{2} d p_{e} /\left(2 \pi^{2}\right)$,
assuming normalization to unit volume. Similarly, the number of possible antineutrino energy assignments is
$p_{v}^{2} d p_{v} /\left(2 \pi^{2}\right)$.
For given values of $p_{e}$ and $E_{0}$, the antineutrino energy is fixed at
$\left(E_{0}-E\right)$ with an uncertainty
$d p_{v}=d E_{0}$.
Assembling the factors (3.4.2) and (3.4.3), and substituting with (3.4.4), we find that the density of states or number of possible final states per unit energy range is given by
$\mathrm{dN} / \mathrm{dE} E_{0}=\left(p_{e}^{2} d p_{e}\right) /\left(2 \pi^{2}\right)\left(p_{v}^{2} d p_{v}\right) /\left(2 \pi^{2}\right)\left(1 / d E_{0}\right)$ $=\left(p_{e}^{2} d_{p_{e}}\right) /\left(2 \pi^{2}\right)\left(1 / 2 \pi^{2}\right)\left(E_{0}-E\right)^{2}$ $=\left(1 / 4 \pi^{4}\right) p_{e}^{2}\left(E_{0}-E\right)^{2} d p_{e}$
We know the transition probability fomula (the 'Golden Rule Number Tho'):

$$
W=2 \pi\left|M_{i f}\right|^{2}\left(d W / d E_{0}\right) \text {, }
$$

and since the matrix element $H_{i f}$ is approximately constant for all electron momenta in the so-called 'allowed' or 'favoured' transitions (see below), we may deduce that
$N\left(p_{e}\right) d p_{e} \quad \propto \quad p_{e}^{2}\left(E_{0}-B\right)^{2} d p_{e}$.
This formula gives an excellent description of the beta-ray spectrun for low-mass nuclides, but tends to become less accurate forhigher values of z. In order to correct this failing, we introduce the so-called 'nuclear coulomb factor', which compensates for the deceleration effect in the emitted electrons produced by electromagnetic attraction to the positivelycharged nucleus. For nonrelativistic electrons, we may write this factor as (26)
$P(Z, B)=2 \pi n\left((1-\exp (-2 n \pi))^{-1}\right.$,
where
$\mathrm{n} \sim \mathrm{ze}^{2} / \mathrm{v}_{\mathrm{e}}$,
$v_{e}$ being the velocity of the electron far from the nucleus, and e being the
universal electromagnetic coupling constant (the 'fine structure constant')' . Thus, for relativistic electrons, this factor tends rapidly to unity.

Prior to Fermi's theory of beta decay, all attempts at empirical
curve-fitting to the beta decay electron spectrum were unsucessful, but with the introduction of the Coulomb-corrected relation $N\left(p_{e}\right) d p_{e}=\left(E^{2} m_{e}^{5}\right)\left(64 \pi^{4}\right)\left|N_{i f}\right|^{2} F\left(z, p_{e}\right)\left(E_{0}-E_{e}\right)^{2} p_{e}^{2} d p_{e}$,

Nordsieck, Kurie, Richardson, and Paxton pointed out that a straight line (48) should be obtained by plotting $\left(N\left(p_{e}\right) / p_{e}^{2}\right)^{2}$ against E. This type of graph is known as a Kurie or Peri plot. In deriving the formula (3.4.10) we have made two major assumptions: first, that the matrix element is independent of $p_{e}$, and second, that the neutrino has zero rest mass. For nonzero antineutrino mass, the Kurie plot should bend and cut the axis at $E^{\prime}=E_{0} \quad-{ }_{\mathrm{I}}^{\mathrm{V}}$.
Experiments on the parameter $E^{\prime}$ indicate (28) that 'allowed' Kurie plots, such as that obtained from ${ }^{3}{ }_{\mathrm{H}}$ decay, are linear dow to an antineutrino mass of less than 60 eV . Nitematively, it is possible to measure the maximum energy of the electron emitted during a nuclear transition in which the energies of both states have been accurately determined. Investigations of this kind (29) have also failed to establish a nonzero antineutrino mass.

The best method of calculating the matrix elements involved in nuclear beta decays is to measure the total decay rates, R. Defining
$Q=\mathrm{E}_{0} / \mathrm{m}_{\mathrm{e}}$
and assuming that $P(Z, p)=1$, we have
$R=\left(\varepsilon^{2} m_{e}^{5}\right)\left(64 \pi^{4}\right)\left|N_{i f}\right|^{2} h(Q)$,
where
$h(Q)=\int_{0}^{Q}\left(\left(1+Q^{2}\right)^{\frac{1}{2}}-(1+Q)^{\frac{1}{2}}\right)^{2} Q^{2} d Q$

$$
\begin{equation*}
=-\frac{1}{4} Q-\frac{1}{12} Q^{3}-\frac{1}{30} Q^{5}+\frac{1}{4}\left(1+Q^{2}\right)^{\frac{1}{2}} \log \left(Q+\left(1+Q^{2}\right)^{\frac{1}{2}}\right), \tag{3.4.14}
\end{equation*}
$$

1. The currently acknowledged value for the electromagnetic coupling consent is $\alpha=\frac{e^{2} / h c}{}=1 / 137.03604(11)$ (Cohen, Taylor: J. Phys. Chem.
Ref. Data $2,663(1973)$ ).
which has the limiting forms
$h(Q) \sim(1 / 30) Q^{5}, \quad Q \gg 5$
(the Sargent rule), and
$h(Q) \sim(2 / 105) Q^{7}, \quad Q \ll 0.5$,
the logarithmic term being unimportant at high energies. Experimentally, the 'comparative half-life',
ht $=(h(Q)) T_{\frac{1}{2}}$,
of beta-emitters is often measured, and, by substituting values for $E_{0}$ in (3.4.14), this may be used to give imformation concerning matrix elements. We now append a table giving the values of $\log _{10}$ ht for a few typical beta decays: (30)


Table 3.1
We see that the beta decay matrix elements vary over a wide range of values, but that transitions with large amplitudes all involve

$$
\begin{equation*}
\Delta J= \pm 1,0 . \tag{3.4.18}
\end{equation*}
$$

Obviously the electron and antineutrino carry away zero angular momentum if they are emitted with their spins antiparallel, corresponding to the Permi selection rule

$$
\begin{equation*}
\Delta J=0, \tag{3.4.19a}
\end{equation*}
$$

and unit angular momentum if with their spins parallel, according to the Gamow-Teller rule

$$
\begin{equation*}
\Delta J= \pm 1 \tag{3.4.19b}
\end{equation*}
$$

In neither of the cases (3.4.19) does the decay involve a change in intrinsic parity. When beta decay matrix elements were first measured, it was thought that only transitions of the pure Fermi type (3.4.19a)
occured, but the observation of the Gamow-Teller decay ${ }_{\mathrm{He}} \longrightarrow 6^{6} \mathrm{Li}+\mathrm{e}^{-}+\overline{\mathrm{v}}$
invalidated this hypothesis. We note that decays need not be either pure Fermi or pure Gamow-Teller, but may be mixed. Since the spin of both initial and final states in the neutron decay is $\pm \frac{1}{2}$, this is an exmple of a mixed transition ${ }^{2}$. Denoting the Fermi-transition $(\Delta J=0)$ coupling constant by $C_{F}$, and the Gamow-Teller $(\Delta J=1)$ one by $C_{G T}$, we find that the neutron decay rate is proportional to $c_{p}^{2}+3 c_{G T}^{2}$, the factor of three entering because there are $(2 J+1)=3$ possible spin orientations for the lepton pair when its total spin is one.

We now attempt to calculate the values of our coupling constents $\mathrm{C}_{\mathrm{F}}$ and $\mathrm{C}_{\mathrm{GT}}$. We assume $\left|\hat{H}_{\text {if }}\right|^{2}=1$, and consider the decay ${ }^{14} \mathrm{O} \longrightarrow{ }^{14} \mathrm{~N}+\mathrm{e}+\mathrm{v}$,
which is a pure Fermi transition, since both initial and final states have zero spin. In fact, it is a 'superalloved' decay, since it simply involves the transformation of a proton into a neutron, and requires no rearrangement of the nucleus ${ }^{2}$. Hendrie and Gerhart measured the half-life of this decay, and found (31)
$t=70.91 \pm 0.04 \mathrm{~s}$,
while Bardin et al. found (32)
$t=71.00 \pm 0.13 \mathrm{~s}$,
in excellent agreement. By listing and evaluating correction factors, such as nuclear form factora ${ }^{3}$, screening of the electron wave function, and competition from K-electron capture decay, Durant et al, have estimated (33) the total correction factor as $+0.289 \%$. The necessary radiative corrections have also been calculated to greater accuracy by Kinoshita and Sirlin (34). Thus, by measuring the maximum electron energy in the decay (3.4.21), it has been deduced that the corrected value of ht is (33)
3043 s ,

1. Since $\left|\frac{1}{2}-1\right|=\left|\frac{1}{2}\right|=\left|-\frac{1}{2}+1\right|=\left|-\frac{1}{2}\right|$.
2. This may be expressed alternatively by saying that the isobars ${ }^{14} 0$ and
' N belong to the same isospin multiplet (see 5.1), i.e. they have the same mass except for slight electromagnetic self-mass corrections due to their differing charges.
3. i.e. the finite opatial extent of the nucleus.
implying that
$\mathrm{C}_{\mathrm{F}} \sim 1.37 \times 10^{-65} \mathrm{Jm}^{3}$.
The measured value of int for the neutron decay is (35)
$1170 \pm 35 \mathrm{~s}$,
so that
$(\mathrm{ht})_{\mathrm{n}} /(\mathrm{ht})_{0^{4}}=\left(\left|\mathrm{c}_{\mathrm{P}}\right|^{2}+3\left|\mathrm{c}_{\mathrm{G} \mid}\right|^{2}\right) /\left(2\left|\mathrm{c}_{\mathrm{F}}\right|^{2}\right)=(3043 \pm 10) /(1170 \pm 35)$,
yielding the importont ratios (36)

$$
\begin{align*}
\left|C_{G T}\right|^{2} /\left|C_{P}\right|^{2} & =1.563 \pm 0.008  \tag{3.4.26a}\\
\left|C_{G T}\right| /\left|C_{F}\right| & =1.250 \pm 0.09 \tag{3.4.26b}
\end{align*}
$$

We note that we have not yet establiahed the sign of this quantity.
We observe that in Table 3.1, we may divide the beta decays into two basic groups: those with $\log _{10}$ ht $\sim 3$, and those with $\log _{10}$ ht $>7$. The fomer are known as the 'allowed' or 'favoured' transitions, and the latter as the 'forbidden' ones. Porbidden transitions basically correspond to those in which the electron and antineutrino carry off orbital angular momentum from the decaying nucleus (37). Since (3.4.24) is comparatively enall, we are usually justified in making a nonrelativistic approximation ${ }^{2}$ and hence we write the lepton spinors as plane wave solutions to the Dirac equation: (26)
$u_{e}(\underline{\underline{I}})=\exp \left(3 \underline{p}_{e} \cdot \underline{r}\right)$,
$u_{v}(\underline{r})=\exp \left(-j \underline{p}_{v} \cdot \underline{r}\right)$,
assuming that no interaction occurs botween the daughter nucleus and the emitted leptons. For the neutrino, this is a very good approximation, but for the electron, only a fair one, since it is, in fact, subject to electromagnetic forces from the nuclear charge. We find that, for the momenta usually occuring in nuclear beta decay, the electron spinor dominates the matrix element. Taking the electron wave function (3.4.27a), we now perform a multipole expansion ${ }^{2}$, obtaining

1. Obviously this does not apply to the neutrino spinor, but the latter does not make a significant contributuion to the matrix element (since the neutrino mass is zero), and our predictions would not be changed, within our large margin of error, by a relativistic neutrino wave function.
2. Mathematically, a Maclaurin expansion.
$u_{e}=e^{j \underline{p} \cdot \underline{r}}=1+j \underline{p} \cdot \underline{r}+\frac{(j p \cdot r)^{2}}{2!} \cdots$
It may be shown that the first tern in this expansion corresponds to the form of the electron spinor occuring in the matrix elements of decays involving no change in orbital angular momentum, the second term to those in which it is altered by one unit, and so on. These transitions are known as $S$-wave $(\Delta l=0), \mathrm{P}$-wave $(\Delta l=1), D$-wave, P -wave, G -wave, etc. In allowed tranaitions, the S-wave term will dominate the decay rate, and hence the matrix element will be of order unity. However, in 'first-forbidden' transitions, the first term vanishee, and hence the major contribution comes from the second term of the multipole expansion. When $\underline{\underline{x}}$ is around the nuclear radius ${ }^{1}$, this will be $\sim 10^{-1}$. Taking into account the fact that a first-forbidien decay necessitates a change in nuclear configuration and parity, which decreases the matrix element, we find that our rough approximation yields values for ht of the correct orier of magnitude. 'Second-forbidden' transitions effectively consist of two consecutive first-forbidden ones, and have corresponding small matrix elements.

### 3.5 The Beta Decay of Unpolarized Nuclei.

The matrix element for noutron decay may be calculated directly from (3.3.6): (38)
$\langle f| H_{i}|i\rangle=(1 / \sqrt{2}) \sum_{i} \int \bar{u}_{p}^{(+)}\left(q_{p}\right) o_{i} u_{n}{ }^{(+)}\left(q_{n}\right)\left(c_{i} \bar{u}_{e}^{(+)}\left(q_{e}\right) o_{i} X\right.$ $\left.\chi \quad u_{v}^{(-)}\left(-q_{v}\right)+c_{i}^{\prime} \bar{u}_{e}^{(+)}\left(\underline{q}_{e}\right) o_{i} \gamma_{5} u_{v}^{(-)}\left(-\underline{q}_{v}\right)\right) \quad x$

$$
\begin{equation*}
x \exp \left(j x\left(q_{n}-q_{p}-q_{e}-q_{v}\right),\right. \tag{3.5.1}
\end{equation*}
$$

assuming that all particles involved in the interaction may be described by plane waves (i.e. they are nonrelativistic). However, in true nuclear beta decay, the decaying nucleon io bound within a nucleus, and hence it may not be described by a simple plane wave solution of the Dirac equation. Furthermore, the nucleus will usually contain many nucleons, any of which may undergo beta decay. The best approximation (38) is probably obtained by Fourier analyaing the plane wave nucleon spinor,

1. Since this is the domain of the integral appearing in the matrix element.
and then applying the matrix element (3.5.1) separately to each term of the decomposition. We denote the nonrelativistic Schrodinger wave function of the initial nucleon by $\psi_{a}$ and that of the final one by $\psi_{b}$, and we Fourier decompose each of these according to the standard formulae:
$\psi(\underline{x})=\sum_{\underline{p}} \psi(\underline{p}) e^{j \underline{p} \underline{x}}$,
$\psi(\underline{p})=\int e^{-j \underline{p} \underline{x}} \psi(\underline{x})$.
Thus the matrix element between the nuclear states $|a\rangle$ and $|b\rangle$, each of which contain several nucleons labelled by the index $r$, becomes:

$$
\begin{equation*}
\langle b| H_{I}|a\rangle=\sum_{r} \sum_{q_{n}, q_{p}} \psi_{b}^{(r)}\left(q_{n}\right) \psi_{a}^{(r)}\left(q_{p}\right)\left\langle q_{p}, q_{e}, q_{v}\right| H_{I}\left|q_{n}\right\rangle . \tag{3.5.3}
\end{equation*}
$$

Dropping the phase factor $\left(\exp \left(-j x_{0}\left(E_{n}-E_{p}-E_{\theta}-E_{v}\right)\right)\right.$ ), we may evaluate this matrix element explicitly, using nonrelativistic approximations (38):

$$
\begin{align*}
\langle\mathrm{b}| \mathrm{H}_{\mathrm{I}}|\mathrm{a}\rangle= & \sum_{i} \int \sum_{f} \psi_{\mathrm{b}}{ }^{(r)}(\underline{x}) o_{i} \psi_{a}^{(r)}(\underline{x}) \exp \left(-j x\left(q_{e}+q_{v}\right) \mathrm{x}\right. \\
& X \quad(1 / \sqrt{2}) \bar{u}_{e}^{(+)}\left(q_{e}\right) o_{i}\left(c_{i}+c_{i}^{\prime} \gamma_{5}\right) u_{v}^{(-)}\left(-\underline{q}_{v}\right) . \tag{3.5.4}
\end{align*}
$$

Because of the comparative smallness of the nuclear radius, we find (38) that we may replace the exponential factor in this formula by unity, and hence we may write

$$
\begin{align*}
& \langle b| H_{I}|a\rangle=\sum_{i} X_{i} \bar{u}_{e}^{(+)}\left(\underline{q}_{e}\right) p_{i} u_{v}^{(-)}\left(-q_{v}\right) \text {, }  \tag{3.5.5a}\\
& F_{i}=(1 / \sqrt{2}) o_{i}\left(c_{i}+c_{i}^{\prime} r_{5}\right) \text {, }  \tag{3.5.5b}\\
& y_{i}=\sum_{x} \int \psi_{b}^{*(r)}(\underline{x}) o_{i} \psi_{a}^{(r)}(\underline{x}) \text {. } \tag{3.5.5c}
\end{align*}
$$

Using standard nonrelativistic approximations for the $0_{i}$, we find that
$n_{i}=\langle\hat{i}\rangle, \quad i=5, V$,
$M_{i}= \pm\langle\sigma\rangle, \quad i=T, A$,
$N_{i}=0, \quad i=P$.
In order to compare our predictions with experiment, we use the usual formula for the intensity of particles with a given momentum: $I(q)=(1 / 2 \pi)^{5} q^{2}\left(E_{\max }-E\right)^{2} X d \Omega_{e} d \Omega_{v}$,
where

$$
\begin{align*}
x & =\mid \sum_{i}{\left.n_{i} \bar{u}_{e}^{(+)}\left(q_{e}\right) p_{i} u_{v}^{(-)}\left(-q_{v}\right)\right|^{2}}=\sum_{i, j} n_{i} n_{j}^{*} \bar{u}_{e}^{(+)}\left(\underline{q}_{e}\right) p_{i} u_{v}^{(-)}\left(-q_{v}\right) \bar{u}_{v}^{(-)}\left(-\underline{q}_{v}\right) \gamma_{4} F_{j}^{*} \gamma_{4} u_{e}^{(+)}\left(q_{e}\right) .
\end{align*}
$$

Since, in the majority of experiments, the initial nucleus is unpolarized, and the polarization of the enitted particles is not observed, we now sum over their polarization directions and take the average of the nuclear polarization: (38)

$$
\begin{align*}
x_{\text {unpol }}= & \sum_{i, j}\left(n_{i} n_{j}^{*}\right)_{\substack{\text { av } \\
\text { Nol }}}\left(1 / 4 E E_{v}\right) \operatorname{Tr}\left(\left(j \gamma_{k}^{q} e(k)-m\right) x\right. \\
& \left.x F_{i} j \gamma_{k} q_{v(k)} \gamma_{4} F_{j}^{* *} \gamma_{4}\right) \tag{3.5.8}
\end{align*}
$$

where we have used a number of standard fomulae associated with the Dirac equation. Since scalar and vector operators are obviously incapable of causing nuclear spin changes (39) because their products are independent of the Pauli spin matrices, these interactions must be associated only with Fermi transitions. Similarly, since the tensor and axial vector couplings can induce spin changes, these are taken as the terms responaible for Gamow-Teller decays. Formally, we now define
$\mathrm{N}_{\mathrm{F}}=\sum_{r} \int \psi_{b}^{*(r)}(\underline{x}) \psi_{a}^{(r)}(\underline{x})$,
$\left|n_{G I}\right|^{2}=\left(\sum_{k}\left|\sum_{r} \int \psi_{b}^{r(r)}(\underline{x}) \underline{\sigma}_{k} \psi_{a}^{(r)}(\underline{x})\right|^{2}\right)$ av ,
and in terms of these new matrix elements we have (38)
$\left(N_{i} N_{j}^{*}\right)_{a v}=\left|N_{F}\right|^{2} \quad i, j=S, V$
since both the scalar and vector couplings are independent of the initial nucleon spin;
$\left(K_{i} M_{j}^{*}\right)_{a v}=0 \quad i, j=S, V ; j, i=T, A$,
implying that no cross-terms of the foxv $\left(\mathrm{N}_{\mathrm{F}}\right) \cdot\left(\mathrm{H}_{\mathrm{GT}}\right)$ occur in the complete matrix element. We may alternatively reach this conclusion by writing (39)
$\left(\mathrm{N}_{\mathrm{F}}\right) \cdot\left(\mathrm{N}_{\mathrm{GIT}}\right)$
$\langle\hat{i}\rangle .\langle\underline{\sigma}$,
which vanishes upon averaging over all possible nuclear spin orientations. However, (3.5.10b) must obviously be modified if we take into account the
final state coulomb interaction between the daughter nucleus and the outgoing electron, but, as we saw in the preceeding section, this is slight for light nuclei. $(3.5 .8)$ and (3.5.9) yield three further relations concerning purely the axial vector and tensor couplings:

$$
\begin{align*}
& \left(M_{T}, M_{T}^{*}\right)_{a v}=\left(\left\langle\underline{\sigma}_{j}\right\rangle\left\langle\underline{\sigma}_{j},\right\rangle\right)_{a v}=\delta_{j, j},(1 / 3)\left(|\langle 6\rangle|^{2}\right)_{a v}= \\
& =\delta_{j, j},(1 / 3)\left|N_{G I}\right|^{2} \text {, }  \tag{3.5.10d}\\
& \left(\mathrm{K}_{\mathrm{A}} \mathrm{~N}_{\mathrm{A}}^{*}\right)_{\mathrm{av}}=\delta_{k, k^{\prime}}(1 / 3)\left|\mathrm{H}_{\mathrm{GI}}\right|^{2} \text {, }  \tag{3.5.10e}\\
& \left(N_{A} N_{T}^{*}\right)_{\mathrm{av}}=\left(K_{T} N_{A}^{*}\right)_{\mathrm{av}}=-\delta_{j, k}(1 / 3)\left|N_{G T}\right|^{2} \cdot(3.5 .10 \mathrm{f})
\end{align*}
$$ We now write

$X_{\text {unpol }}=(1 / 2)\left|\mathrm{K}_{\mathrm{P}}\right|^{2} A+(1 / 6)\left|\mathrm{K}_{\mathrm{GT}}\right|^{2} \mathrm{~B}$,
where, by trace evaluation (38),

$$
\begin{align*}
A= & \left(\left|c_{S}\right|^{2}+\left|C_{S}^{\prime}\right|^{2}\right)(1-v \cos \theta)+\left(\left|c_{V}\right|^{2}+\left|C_{V}^{\prime}\right|^{2}\right)(1+v \cos \theta)+ \\
& +(2 \pi / \widehat{B}) \operatorname{Re}\left(c_{S} C_{V}^{*}+C_{S}^{\prime} C_{V}^{\prime *}\right), \tag{3.5.12a}
\end{align*}
$$

$B=3\left(\left|C_{T}\right|^{2}+\left|C_{T}^{\prime}\right|^{2}\right)(1+(1 / 3) v \cos \theta)+3\left(\left|C_{A}\right|^{2}+\left|C_{A}^{\prime}\right|^{2}\right) \quad x$ $X(1-(1 / 3) v \cos \theta)+(6 m / E) R e\left(C_{T} C_{A}^{*}+C_{T}^{\prime} C_{A}^{*}\right)$,
introducing the variables $\theta$, the angle between the electron and neutrino momentum vectors, and $v$, the electron velocity. Pinally, integrating over all plane angles except $\theta$, and using the intensity formula (3.5.7a), we obtain

$$
\begin{align*}
I(q)= & \left(\xi / 4 \pi^{3}\right) q^{2}\left(E_{\max }-E\right)^{2}(1+a v \cos \theta+b(2 m / E)) \text { ane de, } \\
\xi= & \left.\frac{1}{2}\left(\left|C_{S}\right|^{2}+\left|C_{S}^{\prime}\right|^{2}+\left|C_{V}\right|^{2}+\left|C_{V}^{\prime}\right|^{2}\right)\left|{N_{P}}\right|^{2}+5.13 \mathrm{a}\right)  \tag{3.5.13a}\\
& +\frac{1}{2}\left(\left|C_{T}\right|^{2}+\left|C_{T}\right|^{2}+\left|C_{A}\right|^{2}+\left|C_{A}^{\prime}\right|^{2}\right)\left|N_{G T}\right|^{2}, \quad(3.5 .13 b)  \tag{3.5.13b}\\
a \xi= & \frac{1}{2}\left(\left|C_{V}\right|^{2}+\left|C_{V}^{\prime}\right|^{2}-\left|C_{S}\right|^{2}-\left|C_{S}^{1}\right|^{2}\right)\left|N_{F}\right|^{2} \\
& +1 / 6\left(\left|C_{T}\right|^{2}+\left|C_{T}^{\prime}\right|^{2}-\left|C_{A}\right|^{2}-\left|C_{A}^{\prime}\right|^{2}\right)\left|N_{G T}\right|^{2}, \quad \text { (3.5.13c) } \tag{3.5.13c}
\end{align*}
$$

$$
\begin{align*}
\quad b \xi & \frac{1}{2} \operatorname{Re}\left(c_{S} c_{V}^{*}+c_{S}^{\prime} c_{V}^{\prime *}\right)\left|n_{P}\right|^{2}+ \\
& +\frac{1}{2} \operatorname{Re}\left(c_{T} c_{A}^{*}+c_{T}^{\prime} c_{A}^{\prime *}\right)\left|n_{G T}\right|^{2} . \tag{3.5.13d}
\end{align*}
$$

b is known as the 'Fierz interference term' (40). We note that the coupling constants only appear in such combinations as $\left|c_{i}\right|^{2}+\left|c_{i}^{\prime}\right|^{2}$ and $C_{i} C_{j}^{*}+C_{i}^{\prime} C_{j}{ }^{*}$ in both decay-probability and angular-distribution formulae for unpolarized nuclai, and hence no experiment on the latter may ever provide information concerning the possible appearance of the terns $C_{i}$ and $C_{i}$ in the weak Hamiltonian (i.e. whether the weak interaction is parity invariant or not).

### 3.6 The V - A Theory.

Since there exist two distinct types of beta decay (Fermi and GemowTeller (41)) with different transformational properties, we are forced to conclude that at least two of the five possible Dirac interactions occur in the weak Haniltonian. As the pseudoacalar coupling is energydependent, it is undetectable in nomal beta decay, since the disintegration energy rarely exceeds 2 KeV . All the beta decays winich we consider here are 'allowed' and hence yield linear Kurie plots, implying no secondary energy dependence, and thus $b=0$. From our definition of $b(3.5 .13 \mathrm{~d})$, we may immediately deduce:
$\operatorname{Re}\left(C_{S} C_{V}^{*}+C_{S}^{\prime} C_{V}^{\prime *}\right)=0$,
$\operatorname{Re}\left(C_{T} C_{A}^{*}+C_{T}^{\prime} C_{A}^{\prime *}\right)=0$.
Assuming precise time-reversal invariance ${ }^{1}$, the relations (3.3.23) reduce to $\left(C_{S} C_{V}+c_{S}^{\prime} C_{V}^{\prime}\right)=0$,
$\left(C_{T} C_{h}+c_{T}^{\prime} C_{A}^{\prime}\right)=0$.

One simple hypothesis which explains the vanishing of these expressions is that the Femi interaction is either pure S or pure V and that the Ganow-Teller one is either pure T or pure A. Experimentally, we may ascertain which coupling is responsible for which type of transition by

1. As we shall see in chapter 8, this is not entirely justified. However, the experimental T -violating amplitude $\sim 10^{-3}$.
measuring the value of the coefficient a (3.5.13c) for pure decays. In a pure Fermi decay,
$a=\left(\left|G_{V}\right|^{2}-\left|G_{S}\right|^{2}\right) /\left(\left|G_{V}\right|^{2}+\left|G_{S}\right|^{2}\right)$
where
$\left|G_{i}\right|^{2}=\frac{1}{2}\left(\left|c_{i}\right|^{2}+\left|c_{i}\right|^{2}\right)$.
Experiments yield (42)
$a=0.97 \pm 0.14$,
definitely favouring a vector coupling. Experiments on the pure GamowTeller process
${ }^{6} \mathrm{He} \longrightarrow{ }^{6} \mathrm{Li}+e+\overrightarrow{\mathrm{v}}$
imply ( 42,43 )

$$
\begin{align*}
& a-\left(\left|G_{T}\right|^{2}-\left|G_{A}\right|^{2}\right) / 3\left(\left|G_{T}\right|^{2}+\left|G_{A}\right|^{2}\right)= \\
&=-0.3343 \pm 0.0030 . \tag{3.6.4~b}
\end{align*}
$$

suggesting an axial vector interaction in Gamow-Teller transitions. For some years, ${ }^{6}$ He e-v angular correlation experiments tended to favour the ST combination, but a critical analysis of these results (44) demonstrated that the experiments were, in fact, inconclusive.

There have been basically three attempts to predict the V-A structure of the weak interaction theoretically (45). The first method was suggested by Narghak and Sudarshan (46) in 1958. We define the so-called 'chirality' transformation
$\psi(x) \longrightarrow \psi^{\prime}(x)=\gamma_{5} \psi(x)$.
However, the Dirac equation yields
$Y_{I}\left(\partial / \partial x_{r}\right) \psi=-m \psi$,
and thus we obtain
$Y_{r}\left(\partial / \partial x_{Y}\right)\left(Y_{5} \psi\right)=-Y_{5}\left(Y_{T}\left(\partial / \partial x_{r}\right) \psi\right)=m\left(\gamma_{5} \psi\right)$.

Combining the equations $(4.6 .6)$ and $(4.6 .7)$ we may now write

$$
\begin{equation*}
\gamma_{T}\left(\partial / \partial x_{T}\right) \psi_{ \pm}=m \psi_{ \pm} \tag{3.6.8a}
\end{equation*}
$$

where
$\psi_{ \pm}=\frac{1}{2}\left(1+\gamma_{5}\right) \psi$,
which are eigenstates of $\gamma_{5}$ :

$$
\begin{equation*}
\gamma_{5} \psi_{ \pm}= \pm \psi_{ \pm} \tag{3.6.9}
\end{equation*}
$$

We usually define $\psi_{+}$to have positive chirality and $\psi_{-}$negative chirality. Writing $(45)^{1}$
$\phi=\frac{1}{2}\left(1+\gamma_{4}\right) \psi$,
$X=\frac{1}{2}\left(1-\gamma_{4}\right) \psi$,
we have

$$
\psi_{+}=\frac{1}{2}\left(1+Y_{5}\right) \psi=\frac{1}{2}\left[\begin{array}{c}
\phi-x  \tag{3.6.11a}\\
-(\phi-x)
\end{array}\right]
$$

where
$\psi=\left[\begin{array}{l}\phi \\ x\end{array}\right]$.
We note that, if we project with positive chirality, we obtain the two-component spinor $(\phi-X)$ and if with negative chirality $(\phi+x)$, and thus two of the components of the original four-component spinor are now redundant.

We observe that, in the special case of a masbless Dirac particle, the Dirac equation is invariant under the chirality tranafomations, since (4.6.8a) reduces to
$\gamma_{r}\left(\partial / \partial x_{r}\right) \psi_{ \pm}=0$.
Evidently the wave functions of free particles with finite rest mass chirality noninvariant. Nevertheless, Marshak and Sudarshan (46) suggested that the complete four-fermion interaction might be invariant under $\gamma_{5}$ applied to each field separately ${ }^{2}$. This implies that the Dirac operators appearing in the weak Haniltonian obey the relations $\left[0_{i}, r_{5}\right]_{+}=0$,
$0_{i}=0_{i} \gamma_{5}$.

1. These are known as 'non-chiral-projected' spinors.
2. This is obviously only plausible if the weak fiamiltonian is parity noninvariant.

The only two gamma-matrix products which anticommute with $Y_{5}$ are $Y_{r}(V)$ and $Y_{5} Y_{I}(A)$, and the condition (4.6.13b) demands that there must be an equal admixture of these two operators in the Hamiltonian. Thus the complete four-fermion interaction must be of the form $(G / \sqrt{2})\left(\tilde{\psi}_{n} \gamma_{r}\left(1+\gamma_{5}\right) \psi_{p}\right)\left(\tilde{\psi}_{e} Y_{r}\left(1+\gamma_{5}\right) \psi_{\mathrm{v}}\right)+$ Herm. conje, (3.6.14a)
where $G$ is a suitable coupling constant with the dimensions of (length) ${ }^{2}$. We now rewrite ( $3 \cdot 6.14 \mathrm{a}$ ):
$(G / \sqrt{2})\left(\left(\tilde{\psi}_{n} Y_{T} \psi_{p}\right)\left(\tilde{\psi}_{e} \gamma_{r}\left(1+Y_{5}\right) \psi_{V}\right)-\left(\tilde{\psi}_{n} j Y_{T} Y_{5} \psi_{p}\right) \quad x\right.$

$$
\begin{equation*}
\left.X \quad\left(\widetilde{\psi}_{e} j Y_{r} Y_{5}\left(1+Y_{5}\right) \psi_{\mathrm{v}}\right)+\text { Herm. conj. }\right) \tag{3.6.14~b}
\end{equation*}
$$

Comparing (3.6.14b) with our original Hamiltonian (3.3.6), we obtain:
$C_{i}=C_{i}^{\prime}=0 \quad(i=S, T, P)$,
$C_{V}=C \dot{V}$,
$C_{A}=C_{A}^{\prime}$,
$C_{V}=-C_{A}=(G / \sqrt{2})$.
We note that the result $(3.6 .15 \mathrm{~d}) \mathrm{is}$, at this point, purely arbitrary: if we had demanded invariance under negative rather than positive chiral projection, we would have obtained
$C_{V}=+C_{A}$.
A second method for deriving the correct V-A structure of the weak interactions was sugeested by Peynman and Gell-Nann in 1957 (47). They realized that a wave function with only two components could be made to satisfy the Dirac equation. Let $X(x)$ be a two-component spinor defined:
$\psi(x)=\left(1-(1 / \pi) Y_{r^{\prime}} \cdot\left(\partial / \partial x_{r}\right)\right) X(x)$,
which satisfies the Klein-Cordon equation
$\left(\square-m^{2}\right) \times(x)=0$
and the subsiduary condition
$\gamma_{5} X(x)=X(x)$.

We find that any wave function $\psi(x)$ satisfies both the free Dirac equation $\left(\gamma_{r}\left(\partial / \partial x_{r}\right)+m\right) \psi(x)=0$,
and also the 'chirality' relation
$\frac{1}{2}\left(1+Y_{5}\right) \psi(x)=X(x)$.
We now postulate that all spin $\frac{1}{2}$ particles may be described by two-component spinors of the form $X(x)$, and from our definition (3.6.16), (3.6.17), we may immediately deduce that ${ }^{1}$
$\psi_{e}=\frac{1}{2}\left(1+Y_{5}\right) \psi_{e}$.
This now implies a V - A structure for beta decay in the same manner as did the Marshak-Sudarshan formulation (46).

The third theoretical justification for a V - A interaction was proposed by Sakurai in 1958 (49). He noticed that the Dirac equation was invariant under the two transformations
$\psi \longrightarrow \eta Y_{5} \psi$,
$\mathrm{m} \longrightarrow$ - m
applied simultaneously, where
$\eta^{2}=1$.
This is known as 'mass-reversal' invariance. We now observe that the relativistic requirement
$m^{2}=p_{0}^{2}-|p|^{2}$
involves only $m^{2}$ and not $m$, and hence does not determine its sign. Thus we are forced to conclude that the relation

$$
\begin{equation*}
\left(\gamma_{r}\left(\partial / \partial x_{r}\right)-m\right) \gamma_{5} \psi(x)=0 \tag{3.6.23}
\end{equation*}
$$

is exactly equivalent to the usual Dirac equation (3,6.18). Using the argument outlined above, we see that the Sakurai formulation also predicts a V - A form for the beta decay interaction Hamiltonian. We note, however, that sofar, we have produced no theoretical or experimental evidence concerning the relative signs of the $V$ and $A$ couplings.

1. Assuming 'chirality invariance'.

### 3.7 The Nonconservation of Parity.

In 1956, Lee and Yang (50) suggested that the weak interaction might not be invariant under the parity operator, because of certain difficulties arising from the existence of two distinct decay modes for the $K^{\circ}$ with different intrinsic parities (see chapter 7). They realized that parity conservation in nuclear beta decay could be tested by measuring quantities which behaved as pseudoscalars under P. If parity were conserved, then these should vanish. In their original paper Lee and Yang (50) proposed that experiments should be performed to ascertain the values of the following parmeters:
$\underline{J} \cdot p_{e}$,
$\underline{\sigma} \cdot \underline{p}_{e}$,
J. $\left(\underline{6} \times \underline{p}_{e}\right)$,
where $I$ denotes the spin vector of the parent nucleus and $\underline{\sigma}$ that of the emitted electron. We now examine each of the quantities (3.7.1) in turn. Since $J$ is an axial vector, it is $P$ invariant, but $p_{\theta}$ is a pure vector, and hence it changes sign under P. A $0 \rightarrow 0$ beta decay obviously cannot be used to evaluate (3.7.1a) because the nuclear spin J is zero, ensuring that $\mathcal{J} \cdot \underline{p}_{e}$ is also zero, whether the weak interaction is parity invariant or not. As the testing of P invariance in forbidden transitions is very difficult because of insufficient theoretical information on their nature, we see that we must employ a Gamow-Teller decay of the form $J \longrightarrow J \pm 1$ for this purpose. We find (51) that the electron angular distribution, neglecting coulomb corrections, is given by $I(\theta)=4 \pi j\left(1+A p_{\theta} v_{\theta}\right)$,
where $\mathcal{F}$ was defined in ( $3 \cdot 5 \cdot 13 \mathrm{~b}$ ) and

$$
\begin{gathered}
\mathcal{F}_{A}=-\left|C_{A}\right|^{2}\left|M_{G T}\right|^{2} R\left(J_{a}, J_{b}\right)-2 \operatorname{Re}\left(C_{V} C_{A}^{*} M_{P} M_{G T} *\right) \quad X \\
\times \delta_{J_{a}, J_{b}} \sqrt{J_{a} /\left(J_{a}+1\right)}, \quad(3.7 .2 b) \\
R\left(J_{a}, J_{b}\right)=\frac{1}{2}\left(1 /\left(J_{a}+2\right)\right)\left(2+J_{a}\left(J_{a}+1\right)-J_{b}\left(J_{b}+1\right)\right.
\end{gathered}
$$

$$
\begin{array}{ll}
=-J_{a} /\left(J_{a}+1\right), & J_{b}=J_{a}+1 ; \\
=1, & J_{b}=J_{a}-1,
\end{array}
$$

$J_{a}$ and $J_{b}$ being the initial and final nuclear spins respectively. We note that, if we are considering positron rather than electron emission, then the sign of A is reversed. We have assumed a V - A structure. In 1957, Wu, Ambler, Hawyerd, Hoppes, and Hudson (52) set up an experiment to test for parity noninvariance in the pure Gamow-Teller decay ( $5 \longrightarrow 4$ ) ${ }^{60} \mathrm{Co} \longrightarrow{ }^{60} \mathrm{Ni}+e^{-}(0.312 \mathrm{MeV}) \longrightarrow{ }^{60} \mathrm{Ni}+\gamma(1.19 \mathrm{MeV})+\gamma(1.32 \mathrm{MeV})$
by observing spatial asymmetry in the decay electron distribution. In this case, (3.7.2a) reduces to
$I(\theta)=\left(1+A\langle\underline{J}\rangle \cdot p_{\theta} / E\right)$,
where
$A \xi=\left(2 C_{A}{ }^{*} C_{A}^{\prime}\right) /\left(\left|C_{A}\right|^{2}+\left|C_{A}^{\prime}\right|^{2}\right)$.
If parity were conserved, then $C_{A}^{\prime}=0$ and (3.7.4b) would vanish, so that the electron distribution would be isotropic.

In the experiment of the et al., the ${ }^{60}$ Co nuclei were polarized by placing them within a crystal of cerium magnesium (cobalt) nitrate, which exerts a strong internal magnetic field, and thermal motions in the sample were reduced to a minimum by cooling the whole assembly to 0.01 K using adiabatic demasnetization. Specimens for testing were made by selecting good cerium magnesium nitrate crystals and then growing a crystalline layer of ${ }^{60}$ Co, about $50 \mu \mathrm{~m}$ thick, on their surface. The degree of nuclear polarization effected by the magnetic field was monitored by observing the spatial anisotropy in the ${ }^{60}$ Co decay $\gamma$-rays with two NaI scintillation counters mounted in the polar and equatorial planes of the sample. The decay electrons were detected by means of a small anthracene scintillator crystal located about 2 cm above the ${ }^{60}$ Co source. Scintillations from this device were transmitted along a Lucite 'light pipe' about 1.2 m long to a photomultiplier. When the nuclei had been polarized, a pulse-height analyser connected to the electron-detection photomultiplier was activated, and the apparatus was left for about 15 min. ,
after which time the nuclei had become depolarized, and a heat-exchange gas was allowed to enter the sample chamber. The first few runs of the experiment were sufficient to demonstrate a definite asymmetry in the spatial distribution of decay electrons, and to indicate that A was negative, implying that beta particles were preferentially emitted in the direction opposite to that of the nuclear spin. Further experiments were performed to demonstrate that the anisotropy was not, for example, due to distortions in the magnetic field of cerium magnesium nitrate ${ }^{l}$. The variation in observed results for electron velocities between 0.4 and 0.8 was measured, and was found to agree with the formula (3.7.4). Since the ${ }^{60}$ Co decay (3.7.3) is a pure Gamow-Teller transition, we write $A=-2\left(\operatorname{Re}\left(C_{A}^{*} C_{A}^{\prime}\right)\right) /\left(\left|C_{A}\right|^{2}+\left|C_{A}^{\prime}\right|^{2}\right) \sim-1$,
implying

$$
\begin{equation*}
\mathrm{c}_{\mathrm{A}} \sim \mathrm{C}_{\mathrm{A}}^{\prime} . \tag{3.7.6}
\end{equation*}
$$

We note that, since the ratio $C_{A} / C_{A}^{\prime}$ is evidently not pure imaginary, the experiment of whet al. established not only parity nonconservation, but also charge conjugation noninvariance.

Having considered parity noninvariance in a pure Gamow-Teller transition, we now examine its effect on the neutron decay, which is a mixed transition. Since this is an $\frac{1}{2} \rightarrow \frac{1}{2}$ process, we write $\left|M_{G T}\right|=\sqrt{3}$, $V_{F}=1$. We now define a number of parameters, and simplify them on the assumption that the interaction occuring in neutron decay has the form $\mathrm{V}, \mathrm{A}$ : (53)
$\xi=\left|G_{V}\right|^{2}+3\left|G_{A}\right|^{2}$,
$a \xi=\left|G_{V}\right|^{2}-\left|G_{A}\right|^{2}$,
$A F=-2\left(\left|G_{A}\right|^{2}+\operatorname{Re}\left(G_{V} G_{A}^{*}\right)\right)$,
$B F=2\left(\left|G_{A}\right|^{2}+\operatorname{Re}\left(G_{V} G_{A}{ }^{*}\right)\right)$,
$D \xi=2 \operatorname{Im}\left(G_{V} G_{A}^{*}\right)$,

1. Formula: $2 \mathrm{Ce}\left(\mathrm{NO}_{3}\right)_{3} \cdot 3 \mathrm{Kg}\left(\mathrm{NO}_{3}\right)_{2} \cdot 24 \mathrm{H}_{2} \mathrm{O}$.
where
$C_{i}=C_{i}^{\prime}=G_{i} / \sqrt{2}$.
We find that a is the angular correlation factor in polarized neutron decay, A and B respectively the electron and antineutrino asymmetry parameters, and $D$ the electron-antineutrino correlation coefficient. Presence of the latter term would indicate both T and C noninvariance. Experimentally (54), a beam of about $10^{8} 8 \%$-polarized thermal neutrons was obtained by reflecting a collimated neutron beam at a glancing angle (8') from a cobalt mirror magnetized at a normal to the beam's direction of propagation. The neutrons were then admitted into a vacuum chamber in which a number of them decayed. Disintegration electrons were recorded by a scintillation counter on the right hand side of the chamber, in time coincidence with the recoil protons which were detected by an electron multiplier. A grid was placed at the opposite side of the beam from the proton detector, and a potential difference of 12000 V was applied between the grid and the first dynode of the electron multiplier. Since it was important to have as high an intensity as possible, the recording units were made as large as possible: the scintillation counter head had a diameter of 15 cm and the first dynode of the electron multiplier had an area of about $225 \mathrm{~cm}^{2}$. Measurements of the anisotropy of electrons in neutron decay were made by polarizing the beam towards and away from the electron detector. These demonstrated that about $20 \%$ more electrons were emitted opposite to the spin direction than along it. Comparing this with the theoretical distribution
$I(\theta)=1+\Delta(v / c) \cos$,
we obtain
$\mathrm{A} \quad=\quad-0.114 \pm 0.019$.
The angular distribution of decay antineutrinos was deduced by observing both electron and proton anisotropy, yielding
$\mathrm{B} \quad=0.88 \pm 0.15$.
The parameter $D$ was calculated by measuring simultaneously the directions in which the electron and antineutrino were emitted with respect to the neutron spin. No indication of time-reversal noninvariance was revealed,
implying that the constants $G_{V}$ and $G_{A}$ were relatively real. Defining $R=G_{V} / G_{A}$,
we may write
$A=-2(1+R) /\left(1+3 R^{2}\right)$,
$B=-2(1-R) /\left(1+3 R^{2}\right)$.
Our experimental results for A and B are consistent with $\mathrm{A}=0, \mathrm{~B}=1$, and substituting these values in the relations (3.7.11) we obtain
$G_{A} \sim-1.25 G_{V}$,
and thus we have demonstrated that the interaction occuring in beta decay has the form $V-A$. This is attractive, since the combination $V-A$ is invariant under Fierz reordering (3.3.17a).

We now investigate our second $P$ pseudoscalar (3.7.1b), which corresponds to the longitudinal polarization of electrons emitted from nonoriented nuclei. Since we now do not sum over electron polarization directions, the relations (3.5.7) become (51), assuming a $\mathrm{V}-\mathrm{A}$ interaction, $I(q)=1 /(2 \pi)^{5} q^{2}\left(E_{\max }-E\right)^{2} d \Omega_{e} \int X d \Omega_{v}, \quad(3.7 .13 a)$ $x=-1 /\left(2 E_{v}\right) \sum_{i, j}\left(n_{i} N_{j}^{*}\right)_{\substack{\text { av } \\ \text { pol } \\ \text { pol }}} \bar{u}_{e}^{(+)(r)}\left(q_{e}\right) p_{i} j \gamma_{n} q_{v(n)} x$

$$
\begin{equation*}
X \quad Y_{4} p_{j}^{*} Y_{4} u_{e}^{(+)(r)}\left(q_{e}\right) \quad, \quad(i, j=V, A) \tag{3.7.13b}
\end{equation*}
$$

where $r$ denotes the decay electron polarization index. Since integration over all possible neutrino momenta must evidently yield zero, only terms involving $\gamma_{4}$ and $E_{v}$ survive, and, using properties of the ganma matrices and of the spinors $u$, we find that (3.7.10b) may be written explicitly:

$$
\begin{align*}
\int X d \Omega_{v} & =2 \pi\left(\left|M_{F}\right|^{2} G_{F}^{2}+\left|M_{G T}\right|^{2} G_{G T}^{2}+u_{e}^{*(+)(r)}\left(q_{e}\right) \gamma_{5} u_{e}^{(+)(r)}\left(q_{e}\right) X\right. \\
X & \left.\left(\left|M_{F}\right|^{2} \operatorname{Re}\left(C_{V} C_{V}^{\prime}\right)+\left|M_{G T}\right|^{2} \operatorname{Re}\left(C_{A} C_{A}^{\prime}\right)\right)\right) . \quad(3.7 .14) \tag{3.7.14}
\end{align*}
$$

The remaining variable term in this expression may be evaluated using the standard formula

where

$$
\xi^{(1)}=\left[\begin{array}{l}
1  \tag{3.7.15b}\\
0
\end{array}\right], \quad \xi^{(2)}=\left[\begin{array}{l}
0 \\
1
\end{array}\right]
$$

Thus the expression ( 3.7 .15 a ) is given by the expectation value of the spin projected along the direction of motion of the particle. Analysing the spin itself along this direction, we find

$$
\begin{align*}
u^{*(+)(r)}(\underline{q}) Y_{5} u^{(+)(r)}(\underline{q}) & =-v, \quad r=1,  \tag{3.7.16}\\
& =\quad v, \quad r=2 .
\end{align*}
$$

We now introduce a new parameter known as helicity, which can adopt either of the values $\pm 1$. A particle whose spin is parallel to its direction of motion is defined to have positive helicity, and one whose spin is antiparallel to it, negative helicity. We may now rewrite (3.7.16) in the form $u^{*(+)(r)}(\underline{q}) \gamma_{5} u^{(+)(r)}(\underline{q}) \quad=\quad-v \mathrm{~h}$.
Substituting $(3.7 .17)$ in $(3.7 .14)$ we obtain
$\int \mathrm{Xd} \Omega_{\mathrm{V}}=2 \pi\left(\xi-h v\left(\left|M_{P}\right|^{2} \operatorname{Re}\left(C_{V}^{*} C_{V}^{\prime}\right)+\right.\right.$

$$
\begin{equation*}
\left.+\quad\left|M_{G T}\right|^{2} \operatorname{Re}\left(C_{A}^{*} C_{A}^{\prime}\right)\right) \tag{3.7.18}
\end{equation*}
$$

Experimentally, we measure the polarization of the outgoing electron, defined by

$$
\begin{equation*}
P=\frac{\text { number of electrons with } h=1 \quad-\quad \text { number with } h=-1}{\text { all electrons }} . \tag{3.7.19a}
\end{equation*}
$$

Introducing this parameter into $(3.7 .18)$ we have
$P \xi=-V\left(\left|M_{F}\right|^{2} \operatorname{Re}\left(C_{V}^{*} C_{V}^{\prime}\right)+\left|M_{G T}\right|^{2} \operatorname{Re}\left(C_{A}^{*} C_{A}^{\prime}\right)\right)$.
We note that, for processes involving a positron rather than an electron, the right hand side of $(3.7 .19 b)$ changes sign.

We now discuss the experimental determination of the electron polarization in beta decay. The first method $(55,56)$ is to measure the azimuthul asymmetry in a beam of decay electrons after single coulomb scattering through large angles from heavy nuclei (Mott scattering), When a transversely-polarized electron is scattered by a nucleus, the interaction cross-section depends upon the relative orientation of its spin and of that of the scattering nucleus. The magnetic moment of the electron is taken
to be aligned in a direction opposite to that of its spin vector $\underline{6}$. In the rest frame of the approaching electron, the nucleus appears as a positive current, and hence it produces a magnetic field $H$ parallel to its orbital angular momentum $\underline{r}$. Thus, when $\underline{6}$ and $\underline{\underline{x}}$ are parallel, there is a repulsive force between the electron and the nucleus, and when they are antiparallel, an attractive one. Obviously, electrons will be scattered to both sides of nuclei because of ordinary electrical forces, but more tend to be scattered on the side corresponding to $\underline{r}$ and $\underline{6}$ entiparallel. However, in order to utilize Mott scattering, we must first translate the longitudinal polarization of the decay electrons into transverse polarization. There exist a number of possible methods for effecting this transformation. One is to apply a transverse static electric field (56) to the electron beam. This leaves the spins of nonrelativistic particles unchanged, while their direction of motion is varied continuously. However, if the electrons are relativistic, then their spins tend to precess in the same direction as their momentum vectors. This problem may be overcome by bending the electron beam through an angle greater than the necessary $90^{\circ}$. A second method of changing the sense of electron polarization is to produce fields, E and $\underline{H}$, so that they are mutually perpendicular, and
$|\underline{E}| /|\underline{H}|=v_{e}$.
An electron beam is then introduced at a normal to both fields, and electrons of a particular energy, corresponding to $v_{e}$, remain undeflected, since the effects of the electric and magnetic fields cancell each other out. However, the spin axes of the electrons are rotated through the desired angle, since they are affected only by the magnetic field. A further method (57) is to introduce the electron beam into a semicircular arc of, for example, aluminium foil, in which multiple scattering occurs. This does not affect the spin axes of the electrons, but, in a few cases, will result in a $90^{\circ}$ change in their direction of motion, thus translating any longitudinal polarization into transverse polarization. Experimentally, it is necessary to have a very thin foil in which Nott scattering may occur, in order to reduce the probability of
multiple scattering, which can simulate true Kott scattering. The metal nuclei in the foil are polarized by applying a transverse magnetic field. Any anisotropy in the distribution of scattered electrons may be checked by reversing the magnetic field and observing whether the preferential scattering direction is correspondingly reversed. Usually, the parameter $D=(N(\phi)-N(\phi+\pi)) /(N(\phi)+N(\phi+\pi))=$

$$
\begin{equation*}
=h_{e} a(\theta) \sin \tag{3.7.21}
\end{equation*}
$$

where $N(\phi)$ is the counting rate at an angle $\phi$ to the beam and $a(\theta)$ is the right-left asymmetry parameter, is measured in experiments. a( $\theta$ ) must be calculated theoretically, the most important considerations tending to be nonuniformities in the beam striking the scatterer, the effects of multiple scattering within the scattering foil, and depolarization resulting from the finite extension of the source. Many experiments, using either the orossed-fields or double-scattering method, have been performed, demonstrating that (58)
$\mathrm{h}_{\mathrm{e}} \sim \quad-\mathrm{v}$.
Substituting this result in (3.7.19b), we have
$\left(2 \operatorname{Re}\left(c_{i}^{*} c_{i}^{\prime}\right)\right) /\left(\left|c_{i}\right|^{2}+\left|c_{i}^{\prime}\right|^{2}\right)=1$

$$
\begin{equation*}
(i=V, A) . \tag{3.7.23a}
\end{equation*}
$$

This is satisfied if and only if we set
$C_{i}^{\prime}=C_{i}$,
and since $c_{i} \neq 0$, our result is incompatible with parity invariance (3.3.20), and thus we are forced to conclude that beta decay is parity nonivariant.

An alternative method for measuring the longitudinal polarization of beta decay electrons is N户ller or electron-electron scattering. It has been found (59) that the cross-section for this process is dependent upon the relative spin orientations of the interacting electrons, Using the Born approximation, we find that
$(6 \uparrow \uparrow) /(6 \uparrow \downarrow)=\left(E^{2}\left(1+6 x+x^{2}\right)-2 E(1-x)+1-x^{2}\right) /$

$$
\begin{equation*}
/\left(8 E^{2}-2 E\left(4-5 x+x^{2}\right)+4-6 x+2 x^{2}\right) \tag{3.7.24}
\end{equation*}
$$

1. This assumes that only one virtual photon is ever exchanged.
where $B$ is the total electron energy and $X=(1-2 q)^{2}$. From (3.7.24) we may deduce that the spin dependence of the scattering cross-section is most pronounced when both electrons have the same energy. N $\phi$ ller scattering may be differentiated from the background of ordinary coulomb scattering by a fast-coincidence technique and by energy and angular selections. In the experiment of Frauenfelder et al. (60), electrons scattered from a magnetized Deita-Kax foil ${ }^{1}$ were detected by two anthracene scintillator crystals connected in fast coincidence. By altering the angle of the detectors, it was calculated that
```
he = - (1.0 \pm0.11) v.

The longitudinal polarization of decay electrons and positrons gives rise to circular polarization both of their bremsstrahlung \({ }^{2}\), and of \(e^{+}-e^{-}\)annihilation radiation. The former reaches a maximum for high energy-transfers in the forward direction, the degree of polarization being \(P_{\max }=\left(1-\left(1 / 2 E_{e}\right) P_{e}\right)\).
The earliest experiment on this principle utilized a \({ }^{90} \mathrm{Y}\) source mounted above a 13 -cm-long magnetized iron cylinder, below which was placed an \(8 \times 8\)-cm NaI (TI) scintillation counter (61). The measured antisymmetric effect was found to be given by \(2\left(N_{-}-N_{+}\right) /\left(N_{-}+N_{+}\right)=-0.07 \pm 0.005\),
where \(N_{+}\)denotes the counting rate when the magnetic field was in the same direction as the bremstrahlung, and \(N_{\text {_ }}\) when it was opposite to it. The negative value of the bremstrahlung circular polarization (3.7.27) definitely indicates that the electron has negative helicity. The small magnitude of (3.7.27) is due to the low momentum-transfer involved in the production of \({ }^{90} \mathrm{Y}\) decay electron bremsstrahlung. Correcting for electron energy and for distortions in the iron magnetic field, we predict

\section*{\(h_{e} \sim-v\).}
1. A type of steel foil (effective atomic number 26).
2. Litterally, 'braking radiation' (Geman). It is emitted by all charged particles when their state of rest or uniform motion is altered, in the form of photons whose frequency is proportional to the energy change involved.

The annihilation of free helical positrons by unpolarized electrons will obviously give rise to circularly-polarized annihilation radiation (usually photons). Even for comparatively low positron energies ( \(\sim 1 \mathrm{MeV}\) ), we find that the forward annihilation gamma rays are almost \(100 \%\) polarized. Experimentally, (62) positrons from a pure Fermi decay were momentumanalysed by a thin-lens spectrometer, and were then brought to rest and hence annihilated. The gamma rays produced were analysed by a standard polarimeter, yielding
\(h_{\gamma}=+(95 \pm 14) \%\).
- As in electron-electron (Nøller) scattering, the cross-section for electron-positron annihilation is dependent upon the relative spin orientations of the two particles. This fact has been used (63) to determine the helicity of positrons in \({ }^{64} \mathrm{Ga}\) decay, but depolarization effects in the annihilation medium tend to render the result inaccurate. An ingenious method for measuring the helicity of beta-decay positrons was suggested and used by Page and Heinberg (64). It involves the formation of positronium, which undergoes the decay
\(\mathrm{e}^{+} \mathrm{e}^{-} \longrightarrow 2 \gamma, \quad 3 \gamma\)
in about \(10^{-7} \mathrm{~s}\). In the absence of a magnetic field, the \(\left.\mathrm{m}=0\right)^{1}\) substates of the singlet \((J=0)\) and triplet \((J=1)\) states of positronium are described by the wave functions
\(1 / \sqrt{2}\left(A_{-} B_{+}-B_{-} A_{+}\right)\),
\(1 / \sqrt{2}\left(A_{-} B_{+}+B_{-} A_{+}\right)\),
respectively, where A denotes the 'spin up' and B the 'spin down' state. The suffixes correspond to electrons and positrons. In the presence of an 'upward' magnetic field \(\underset{H}{ }\), the wave functions (3.7.31) become \(1 / \sqrt{2}\left((1-a) A_{-} B_{+}-(1+a) B_{-} A_{+}\right)\), \(1 / \sqrt{2}\left((1+a) A_{-} B_{+}+(1-a) B_{-} A_{+}\right)\),
where \(\left.a=2 y /\left(1+y^{2}\right), \quad y=\left(\left(1+w^{2}\right)^{\frac{1}{2}}-1\right) / w\right)\),
\(W=4 \mu(H) / s, \mu\) being the electron magnetic moment \({ }^{2}\) and \(s\) the mass-
1. \(m\) is here the magnetic quantum number, which describes the magnetic state of an electron orbit in an atom. It may adopt any integral value between \(+\ell\) and -1 .
2. \(\mu_{e}=1.0012596567 \mathrm{eh} / 2_{m_{e}} c\) (Cohen, op. cit.)
splitting between the triplet ( \(J=1\) ) and singlet ( \(J=0\) ) states for \(\underline{H}=0^{) 1}\). Thus we may deduce that the ratio of the number of positrons with their spin 'up' to that with their spin 'dow' is
\[
\begin{equation*}
R=(1 \mp a) /(1 \pm a) \tag{3.7.33}
\end{equation*}
\]
for the triplet and singlet respectively. This implies that a positron which is completely polarized in the direction opposite to that of the field H is preferentially captured into the triplet state of positronium. The singlet state decays into two oppositely-directed photons in about \(10^{-10} \mathrm{~s}\), and the triplet into three coplanar gamma rays after about \(10^{-7} \mathrm{~s}\), and on the basis of this difference in lifetime and decay mode, it should be possible to differentiate singlet and triplet states in a population of positronium. However, under some circumstances, one of the particles in a \(J=1\) state may 'spin-flip', and the state may decay via the singlet mode. Theoretical considerations (65) demonstrate that, for \(\underline{H} \sim 10^{4} \mathrm{G}\), the singlet accounts for about \(85 \%\) of all \(2 \gamma\) decays, and, using this fact, Lundby et al. have obtained (65) a tentative value for the helicity of positrons in beta decay. Nevertheless, Page and Heinberg (64) devised an alternative method for discriminating between triplets and singlets decaying into the \(2 \gamma\) mode. Since the singlet is only about \(10^{-10} \mathrm{~s}\) old when it undergoes decay, it retains some of its formation kinetic energy, and hence the two photons may emerge at any angle around \(180^{\circ}\). However, the triplet lives longer, and thus it has usually lost most of its original energy by the time it decays, so that, if two photons are produced, they will tend to emerge strictly at \(180^{\circ}\). By analysing angular correlation spreads, Page and Heinberg (64) have show that the positrons in \({ }^{22_{N a}}\) decay have positive helicity.

Finally, we discuss the last of our three P pseudoscalars: (3.7.1c). This corresponds to the polarization and angular distribution of electrons emitted from oriented nuclei. If a polarized nucleus decays by beta emission into an exited state of its daughter nucleus, then the spin of the resultant nucleus will be correlated with the direction of electron emission if the electron has a definite helicity. If the excited nuclide decays before its spin axis has been altered, then the circular polarization 1. When a magnetic field is applied, the Zeeman or splitting effect occurs.
of the decay ganma ray will be proportional to \(\cos \theta\), where \(\theta\) is the angle between the electron and gamma ray emission directions. Experimentally (66), we must determine the sense of photon polarization. Initially, we may measure either the effect of the interaction of the gamma rays with a polarized sample or the polarization of secondary particles whose emission was caused by incident gamma rays. The types of interaction of gamma rays with matter are, within the required energy range: the photoelectric effect; formation of electron-positron pairs; nuclear photo-effect, and Compton scattering by electrons. Since there is, at present, no means of polarizing electrons in inner atomic shells, no direct estimate of gammaray polarization may be made using the photoelectric effect. Nevertheless, observation of the longitudinal polarization of ejected photoelectrons does provide some infomation concerning the helicity of the incident gamma photon, although numerous correction factors render it inaccurate. Both \(e^{+}-e^{-}\)pair formation and the nuclear photo-effect methods rely on good knowledge of the fine structure of atomic nuclei, which is not, as yet, available. The cross-section for Compton scattering is dependent upon the angle between the spin axes of the interacting electron and photon. A Klein-Nishina formula adapted for polarized initial particles may be used (66) to demonstrate that the cross-section for forward Compton scattering is greatest when the spins of the electron and photon are antiparallel, and for backward scattering, when they are parallel. If we polarize electrons by magnetization, then we may measure the helicity of incident gamma rays by observing the differential cross-section ( \(\mathrm{d} \sigma / \mathrm{d} \Omega\) ) for scattering into a particular solid angle element. However, background radiation caused by scattering from surrounding objects tends to complicate this measurement. An alternative method is to calculate the probability of absorption of the photon by iron magnetized in a particular direction. The major sources of inaccuracy in this method are scattering from the magnet coils and the difficulty in determining the absorption length of magnetized iron. The most accurate measurement of e \(-\gamma\) angular correlation was made by Schopper (67). In his experiment, gamma rays from \({ }^{60}\) co decay were absorbed in a sample
of iron whose direction of magnetization was reversed every 200 s . The scattered photons were detected by an NaI (T2) scintillator, and the results obtained (38) were consistent with \(h_{e}=-v\).

\subsection*{3.8 The Two-Component Neutrino Hypothesis.}

With our knowledge of the type and strength of interactions occuring in neutron decay, we may now rewrite our original Hamiltonian (3.3.6): (68)
\[
\begin{align*}
& H_{I}=\left(c_{V} / \sqrt{2}\right) \int \tilde{\psi}_{p} \gamma_{T} \psi_{n} \tilde{\psi}_{e} \gamma_{I}\left(1+\gamma_{5}\right) \psi_{v} \\
&-\left(c_{A} / 2\right) \int \tilde{\psi}_{P} \gamma_{r} \gamma_{5} \psi_{n} \tilde{\psi}_{e} \gamma_{I}\left(1+\gamma_{5}\right) \psi_{v}+ \\
&+ \text { Herm. conj. } \tag{3.8.1}
\end{align*}
\]

We observe that, in this expression, a factor \(\left(1+Y_{5}\right)\) always preceeds the neutrino field operator. Since \(\left(1+\gamma_{5}\right)=\left(1+\gamma_{5}\right) \gamma_{5}\),
the neutrino wave function must always be invariant under the 'chirality' transformation
\[
\begin{equation*}
\psi_{\mathrm{v}} \longrightarrow \gamma_{5} \psi_{\mathrm{v}} \tag{3.8.3}
\end{equation*}
\]

As we saw in 3.6, this is only the case for massless particles, and hence the neutrino mass must be precisely zero. On this assumption, we analyse the neutrino spin along its direction of motion, and thus the Dirac equation yields \({ }^{1}\) :
\[
\begin{aligned}
& \left(1+Y_{5}\right) u_{v}{ }^{(+)(r)}\left(\underline{q}_{v}\right)= \\
& \\
& =\left[\begin{array}{rr}
1 & -1 \\
-1 & 1
\end{array}\right]\left[\begin{array}{l}
\xi^{(r)} \\
\underline{\sigma}_{z} \xi^{(r)}
\end{array}\right] \frac{1}{\sqrt{2}}=\left[\begin{array}{c}
\left(1-\underline{\sigma}_{z}\right) \xi^{(r)} \\
-\left(1-\underline{\sigma}_{z}\right) \xi^{(r)}
\end{array}\right] \frac{1}{\sqrt{2}}
\end{aligned}
\]

We see that all components of the neutrino spinor vanish for \(r=1\), and hence
\[
\begin{gather*}
(1 / \sqrt{2})\left(1+\gamma_{5}\right) \psi_{v}=\left(1+\gamma_{5}\right) \sum_{q}\left(u^{(+)(2)}(\underline{q}) e^{j q x} a^{(2)}(\underline{q})\right. \\
u^{(-)(1)}(-q) e^{-j q x} b_{b}+(1)  \tag{3.8.5}\\
(q) .
\end{gather*}
\]
1. Using the gamma-matrix representation in which \(\gamma_{5}\) is not diagonal.

We are thus forced to conclude that only antineutrinos with positive helicity ( \(r=2\) ), and only neutrinos with negative helicity ( \(r=1\) ) may ever be produced. We may therefore reduce our usual four-component neutrino spinor to a two-component one. This is known as the 'two-component neutrino hypothesis'.

We now discuss the experimental determination of the neutrino helicity (69). In a decay of the form \(\mathrm{A}\left(\mathrm{O}^{-}\right)+\mathrm{e}^{-} \longrightarrow \mathrm{B}\left(\mathrm{I}^{-}\right)+\mathrm{v} \longrightarrow \mathrm{C}\left(\mathrm{O}^{+}\right)+\gamma\),
we see that, by angular momentum conservation \({ }^{1}\), the nucleus \(B\) must have a spin opposite to that of the emitted neutrino. Furthermore, when the excited nuclide B returns to the ground-state \(C\), the circular polarization of the decay photon will be \(h_{v} \cos \theta\), where \(\theta\) is the angle between the neutrino and photon momentum vectors. In order to select only those gamma rays which are emitted in a direction opposite to their associated neutrinos, we may employ the phenomenon of nuclear resonance scattering. When a photon is emitted or absorbed by a nucleus, the latter recoils with energy \(\mathrm{E}_{0}^{2} / \mathrm{M}\). Thus, when a photon is absorbed, the energy available for excitation is only \(E_{0}\left(1-E_{0} / M\right)\). In order to make resonance possible, extra energy, amounting to that lost in the nuclear recoil, must be supplied to the photon. This occurs if the decaying nucleus has already recoiled in a direction opposite to that induced by gamma-ray emission. Such an extra recoil could be provided by the emission of a neutrino whose direction of motion was precisely opposite to that of the photon. The Doppler shift in the gama-ray energy caused by the nuclear recoil is given by
\(\Delta \mathrm{E}=\left(\mathrm{E}_{0} \mathrm{E}_{\mathrm{V}} \cos \theta\right) / \mathrm{M}\),
and hence the total energy for excitation trensferred by the gamma ray will be
\(\mathrm{E}=\mathrm{E}_{0}+\left(E_{0} E_{v} \cos \theta\right) / M-E_{0}^{2} / M\).
We now deduce that the condition for resonant absorption is
\(E_{V} \cos \theta=E_{0}\).
1. Obviously we assume that any electrons captured from the atomic \(K\) shell will be s-wave.

For cases in which the neutrino has almost the same energy as the photon, the resonance condition is fulfilled only for those photons emitted opposite to the neutrino. Thus, by measuring the circular polarization of the gamma rays inducing resonance, we may deternine the helicity of the neutrino.

In 1958, Goldhaber et al. (70) attempted to measure the neutrino helicity using the process
\[
\begin{align*}
{ }^{152} 2_{\mathrm{Eu}}\left(0^{-}\right) & +e^{-} \longrightarrow{ }^{152} \mathrm{Sin}^{*}\left(1^{-}\right)+\mathrm{v}(900 \mathrm{keV}) \\
& { }^{152} \operatorname{Sin}\left(0^{+}\right)+Y(961 \mathrm{keV}) . \tag{3.8.8}
\end{align*}
\]

The lifetime of the \(I^{-}\)state is \((7 \pm 2) \times 10^{-14} \mathrm{~s}\), which is short enough to assume that the nucleus will not lose any recoil energy between neutrino and photon emission. Ganma rays from the \({ }^{152}\) ku source impinged on a ring scatterer consisting of 1700 g of \(\mathrm{Sm}_{2} \mathrm{O}_{3}\). Any photons arising from resonant scattering were detected by means of an NaI (II) scintillation counter, which was shielded from the primary gamma rays by a lead block. In order to determine the sense of gamma-ray polarization, the photons were made to traverse 3 mean free paths of magnetized iron before hitting the ring scatterer. An electron whose spin is antiparallel to that of a photon may absorb the latter's unit of angular momentum by spin-flip; when it is parallel, it may not. By measuring the change in counting-rate when the direction of the magnetic field was reversed, it was deduced that the gamma-rays were \((67 \pm 15) \%\) left-handed \((h=-1)\), in agreement with the theoretical prediction of \(84 \%\) for \(h_{v}=-1\).

There exists one further method of deriving the two-component theory of the neutrino. This requires a new representation for the gamma matrices:
\[
\begin{gather*}
\gamma_{5}=\left[\begin{array}{rr}
1 & 0 \\
0 & -1
\end{array}\right] \quad Y_{4}=\left[\begin{array}{ll}
0 & 1 \\
1 & 0
\end{array}\right] \\
\gamma_{r}=\left[\begin{array}{cc}
0 & -j \underline{\sigma}_{r} \\
j \underline{\sigma}_{r} & 0
\end{array}\right] \tag{3.8.9}
\end{gather*}
\]

Defining two two-component wave functions \(v\) and \(w\), the Dirac equation for zero mass now reads
\(j Y_{T} q_{r} u=\left[\begin{array}{l}(\underline{\underline{\sigma} q}-E) w \\ -(\underline{\underline{q}}-E) v\end{array}\right]=0\),
which may be resolved into two uncoupled relations:
\((\mathrm{B}+\underline{\underline{q}}) \mathrm{v}=0\),
\((\mathrm{B}-\underline{\underline{q}}) \mathrm{v}=0\).

Thus the projection of the spin along the direction of motion is \(-E /|q|\) for \(v\), and \(E /|q|\) for \(w\). Hence, massless particles with positive energy must have negative helicity if they correspond to the solution v , and positive helicity if to w . Since w is the antiparticle solution in ( 3.8 .10 ), we now inferr that all neutrinos have negative helicity while all antineutrinos have positive helicity. The equations (3.8.11) were first proposed by Weyl (71), but they were condeaned by Pauli because they implied parity noninvariance (72). However, after the experiment of wh et al., they were revived by Salan (73), Lee and Yang (74), and Landau (75).

Finally, we examine a number of useful predictionsmade by the two-component neutrino theory. We assume that in a Fermi decay ( \(\Delta \mathrm{J}=0\) ), the recoiling nucleus takes up no momentum. We consider the case in which electron and antineutrino emerge in opposite directions with equal momenta. Since the antineutrino has positive helicity, so also must the electron. However, if we assume that the electron has enough energy to be satisfactorily described by a two-component wave function, then we predict that it should have negative helicity, and hence that its spin should be parallel to that of the antineutrino. This would imply nonconservation of angular momentum, and consequently we are forced to conclude that the situation described is impossible. From this we may inferr that the e-v angular correlation factor should vanish for \(\theta=\pi\), which is consistent with experiment and with (3.5.13).

The two-component neutrino theory may be used (76) to derive the correct V - A structure of beta decay. Since the antineutrino has positive helicity, it follows that, in the \(m_{e}=0\) approximation, the electron must have negative helicity. By angular momentum conservation, this implies that the nucleon spins must both be parallel to the antineutrino spin.

We find (76) that the matrix element for beta decay assuming two-component leptons is given by
\[
\begin{align*}
M_{\text {if }}= & (1 / \sqrt{2})\left(\bar{u}_{p} u_{n}\right)\left(\bar{u}_{e}\left(G_{v}+G_{A} \sigma_{3}\right) u_{v}\right)+ \\
+ & \left(2 G_{A} / \sqrt{2}\right)\left(\bar{u}_{p} \sigma_{-} u_{n}\right)\left(\bar{u}_{e} \sigma_{+} u_{v}\right), \tag{3.8.12}
\end{align*}
\]
where \(\sigma_{+}\)and \(\sigma_{-}\)are the spin-raising and spin-lowering operators respectively. Thus the second term of (3.8.12) represents the nucleon spin-flip amplitude. In our case, this is zero. By allowing the 'helicity operator' \(\sigma_{3}\) to act either on \(\bar{u}_{e}\) or on \(u_{v}\), we obviously obtain \(g \sim G_{V}-G_{A}\),
in agreement with data from polarized neutron decay (3.7.12). Among the other predictions of the two-component neutrino theory is that the magnetic moment of the neutrino should vanish (73), so long as it only takes part in weak interactions.

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\subsection*{4.1 Phenomenology of Muon Decsy.}

In 1936, Anderson and Neddermeyer (1) obtained a number of cosmic ray cloud chamber tracks at mountain altitudes which were attributed to a particle of approximate mass \(100 \mathrm{KeV} / \mathrm{c}^{2}\). In 1940, Willians and Roberts (2) obtained cloud chamber photographs showing a negative particle of mass \(120 \mathrm{HeV} / \mathrm{c}^{2}\), which decayed into an electron. By comparing Geiger-counter counting rates at different altitudes, the lifetime of the new particle, which was named the muon, was established as about \(2 \mu s\). Rasetti made a more direct deternination of the muon lifetime. He placed two Geiger counters above a 10 cm thick iron absorber, and two others below. When a muon was stopped in the iron, as indicated by the anticoincidence of the second pair of counters, the time before the emergence of its charged decay product was measured. However, only about half of all the muons stopped in the iron appeared to decay. This was explained by assuming that, in cosmic rays, there exist equal numbers of mons, which are negatively-charged, and positively-charged antimuons. The antimuons are repelled by the Coulomb fields of the iron nuclei, and are thus free to decay, but the muons are captured by atomic nuclei. As captured muons cascade towards the nucleus, they emit \(x\)-rays as they jump from one electron orbit to the next. The energies of these x-rays may be measured to within ten or twenty electronvolts, and, using the Bohr formula \({ }^{1}\), we may calculate the muon mass as
\(105.66 \pm 0.015 \mathrm{NeV} / \mathrm{c}^{2}\).
The currently acknowledged value for the muon mass is (3)
\(105.65948 \pm 0.00035 \mathrm{KeV} / \mathrm{c}^{2}\).
The muon lifetime was initially measured by track length in nuclear emulsions, and then by accelerator time-of-flight measurements. The current value is (4) (2.1994 \(\pm 0.0006\) ) \(\times 10^{-6} \mathrm{~s} . \quad\) (4.1.3)

Since the electron in muon decay is not monoenergetic, we may deduce that there are two unobserved neutral particles present in the decay, and thus
the simplest possible muon decay scheme, taking into account conservation laws, is
\(\mu^{-} \longrightarrow e^{-}+v+\bar{v}\).
However, we must be careful not to make the assumption that the neutrino and antineutrino on the right-hand side of the decay (4.1.4) are similar to those which we studied in chapter 3. As we shall see, they are, in fact, not similar. A number of other muon decay modes have been suggested to date (5):

Decay
Branching ratio
ey
\[
\begin{aligned}
& <1.6 \times 10^{-5} \\
& <6 \times 10^{-9} \\
& <2.2 \times 10^{-8}
\end{aligned}
\]
zerr
er
A few other modes are also consistent with conservation laws, but they have not been observed. These include
\[
\begin{align*}
& \mu \longrightarrow e v \bar{v}_{\gamma} .  \tag{4.1.5}\\
& \mu \longrightarrow e v \bar{v} e \bar{e} . \tag{4.1.6}
\end{align*}
\]

\subsection*{4.2 The Theory of Nuon Decay.}

Since we are once asain concemed with four femion fields, which we assume to interact at a point, we imitate our neutron decay Hamiltonian (3.3.5), and, writing the neutrino and antineutrino in (4.1.4) as \(v_{1}\) and \(v_{2}\) reepectively, we have
\(H_{I}=\sum i \tilde{\psi}_{e}(x) 0_{i} \psi_{v_{1}}(x) \widetilde{\psi}_{v_{2}}(x) F_{i} \psi_{\mu}(x)\)
where
\(F_{i}=(1 / \sqrt{2}) o_{i}\left(c_{i}+c_{i} Y_{5}\right)\).
Using the Fierz reordering relations (3.3.39) and (3.3.40), we may prove that this Hamiltonian is unique. From (4.2.1), we way write immediately the matrix element for muon decay as
\[
\begin{gather*}
M_{i f}=\sum \bar{u}_{\theta}^{(+)}\left(p_{e}\right) o_{i} u_{\mu}^{(+)}\left(p_{\mu}\right) \bar{u}_{v_{1}}^{(+)}\left(p_{v_{1}}\right) p_{i} u_{v_{2}}^{(-)}\left(-p_{v_{2}}\right) x \\
\int d^{3} x e^{j x\left(p_{\mu}-p_{e}-p_{v_{2}}-p_{v_{1}}\right)} . \tag{4.2.3}
\end{gather*}
\]

From (4.2.3) we may deduce the relation for the transition probability per unit time for the emission of an electron with a momentum in the region \(d^{3} p_{\theta}\), independent of the momenta of the two neutrinos:
\[
\begin{align*}
(\delta w / \delta t)= & \left(d^{3} p_{e}\right) /(2 \pi)^{5} \iint d^{3} p_{v_{1}} d^{3} p_{v_{2}} \delta\left(p_{\mu}-p_{e}-p_{v_{1}}-p_{v_{2}}\right) x . \\
& x\left|\sum_{i} \bar{u}_{e}^{(+)}\left(p_{e}\right) o_{i} u_{\mu}^{(+)} \bar{u}_{v_{1}}^{(+)}\left(p_{v_{1}}\right) p_{i} u_{v_{2}}^{(-)}\left(-p_{v_{2}}\right)\right|^{2} \tag{4.2.4}
\end{align*}
\]

Since the two neutrinos in the decay (4.1.4) are not observed, we sum over their polarization directions, obtaining
\[
\begin{align*}
& \sum_{r_{v_{1}}, r_{v_{2}}}\left|\sum_{i} \bar{u}_{\theta}^{(t)}\left(p_{\theta}\right) o_{i} u_{\mu}^{(t)}\left(p_{\mu}\right) \bar{u}_{v_{1}}^{(t)}\left(p_{v_{1}}\right) p_{i} u_{v_{2}}^{(-)}\left(-p_{v_{2}}\right)\right|^{2} \\
& =\left(1 /\left(4 E_{v_{1}} E_{v_{2}}\right) \sum_{i, j} \bar{u}_{e}^{(+)}\left(p_{e}\right) o_{i} u_{\mu}^{(+)}\left(\underline{p}_{\mu}\right) \bar{u}_{\mu}^{(+)}\left(p_{\mu}\right) o_{j}^{+} u_{e}^{(+)}\left(p_{e}\right) \quad z\right. \\
& \mathrm{x} \operatorname{Tr}\left(j \gamma p_{v_{1}} P_{i} j Y p_{v_{2}} F_{j}^{+}\right) \text {, } \tag{4.2.5}
\end{align*}
\]
where
\(o_{j}=Y_{4} 0_{j}^{*} \gamma_{4}=\gamma_{4} o_{j} Y_{4}\).
Using the fact that \(O_{i}\) and \({O_{i}}^{+}\)differ only by a sign, and substituting with \((4.2 .5)\) in (4.2.4) we have
\[
\begin{align*}
& (\delta w / \delta t)=\left(d^{3} \underline{p}_{\theta}\right) /(2 \pi)^{5} \sum_{i, j} \vec{u}_{e}^{(t)}\left(\underline{p}_{e}\right) o_{i} u_{\mu}^{(t)}\left(\underline{p}_{\mu}\right) \vec{u}^{(+)}\left(\underline{p}_{\mu}\right) o_{j} x \\
& x u_{e}^{(+)}\left(p_{e}\right) \cdot \frac{1}{2} \iint d p_{v_{2}} d p_{v_{1}} \delta\left(\underline{p}_{v_{1}}{ }^{2}\right) \delta\left(p_{v_{2}}{ }^{2}\right) \theta\left(p_{v_{1}}\right) \theta\left(p_{v_{2}}\right) \quad \chi \\
& x \quad \delta\left(p_{\mu}-p_{e}-p_{v_{1}}-p_{v_{2}}\right) \operatorname{Tr}\left(j \gamma p_{v_{1}} o_{i}\left(c_{i}+c_{i}^{\prime} \gamma_{5}\right) j \gamma \quad x\right. \\
& \left.x p_{v_{2}}\left(c_{j}-c_{j}^{\prime} \gamma_{5}\right) o_{j}\right) \tag{4.2.7}
\end{align*}
\]

We now take the special case of (4.2.7) for unpolarized decaying muons, and, using the techniques outlined in Kallen: Elementary Particle Physics, AddisonWesley 1964, pp. 380-386, we have \((\delta W / \delta t)=\left(d^{3} p_{e}\right) /\left(384 \pi^{4} E_{e} E_{\mu}\right) \quad\left(3\left(\left|c_{S}\right|^{2}+\left|c_{S}^{\prime}\right|^{2}+\left|c_{p}\right|^{2}+\left|c_{p}^{\prime}\right|^{2}\right) \quad X\right.\)
\[
\begin{align*}
& X Q^{2}\left(p_{e} p_{\mu}\right)+2\left(\left|c_{V}\right|^{2}+\left|c_{V}^{\prime}\right|^{2}+\left|c_{A}\right|^{2}+\left|c_{A}^{\prime}\right|^{2}\right)\left(Q^{2}\left(p_{e} p_{\mu}\right)+\right. \\
& \left.\left.+2\left(p_{\theta} Q\right)\left(p_{\mu} Q\right)\right)+2\left(\left|c_{T}\right|^{2}+\left|c_{T}\right|^{2}\right)\left(4\left(p_{e} Q\right)\left(p_{\mu} Q\right)-Q^{2}\left(p_{e} p_{\mu}\right)\right)\right) \tag{4.2.8}
\end{align*}
\]
where \(Q\) is the difference between the muon and the electron momenta. We define the intensity of outgoing electrons, \(I(x)\), by \((\delta W / \delta t)=I(x) d x\),
where
\(x=\left(2 B_{e}\right) / / \pi_{\mu}\).
Thus we find that
\(I(x)=\left(m \mu^{5}\right) /\left(16 \pi^{3}\right) c^{2} x^{2}(1-x+(2 / 3) \rho((4 / 3) x-1))(4 \cdot 2 \cdot 11)\) where \(c^{2}\) is the average coupling constant, defined
\(c^{2}=(1 / 16)\left(\left|c_{S}\right|^{2}+\left|c_{S}^{\prime}\right|^{2}+\left|c_{P}\right|^{2}+\left|c_{P}^{\prime}\right|^{2}+4\left(\left|c_{V}\right|^{2}+\left|c_{V}^{\prime}\right|^{2}+\left|c_{A}\right|^{2}+\left|c_{A}^{\prime}\right|^{2}\right)+\right.\) \(\left.+6\left(\left|c_{T}\right|^{2}+\left|c_{T}\right|^{2}\right)\right)\)
and where the Wichel parameter (6) is defined by
\[
\rho c^{2}=(3 / 16)\left(\left|c_{V}\right|^{2}+\left|c_{V}^{\prime}\right|^{2}+\left|c_{A}\right|^{2}+\left|c_{A}^{\prime}\right|^{2}+2\left(\left|c_{T}\right|^{2}+\left|c_{T}^{\prime}\right|^{2}\right)\right) \quad \text { (4.2.13) }
\]

However, \(\rho\) and \(C\) are the orly quantities which are experimentally measurable, and we need nineteen equations to bolve for our ten complex coupling constants. Evidently, these cannot be produced with only two parameters.

We may obtain a value for \(C\) using the equation
\(1 / \Gamma_{\mu}=\int_{0}^{1} I(x) d x=\left(m_{\mu}^{5} c^{2}\right) /\left(192 \pi^{3}\right)\),
where \(T_{\mu} \mu\) is the muon lifetime. Substituting the values (4.1.2) and (4.1.3) for the muon mass and lifetime repectively, we obtain, from (4.2.14)
\(c=1.431 \times 10^{-65} \mathrm{Jm}^{3}\),
in near agreement with our previous value in neutron decay (3.4.47). The difference is probably caused by the need for radiative corrections to both values (7). We now define (8) the following combinations of coupling constants:
\(a=\left|c_{s}\right|^{2}+\left|c_{s}^{\prime}\right|^{2}+\left|c_{p}\right|^{2}+\left|c_{p}^{\prime}\right|^{2}\)
\(b=\left|c_{V}\right|^{2}+\left|c_{V}^{\prime}\right|^{2}+\left|c_{A}\right|^{2}+\left|c_{A}^{\prime}\right|^{2}\)
\(c=\left|c_{T}\right|^{2}+\left|c_{T}^{c}\right|^{2}\)
\(\alpha=\left|c_{S}^{2}\right|^{2}+\left|c_{s}^{\prime}\right|^{2}-\left|c_{p}^{\prime}\right|^{2}-\left|c_{p}^{p}\right|^{2}\)
\(\beta=\left|c_{V}\right|^{2}+\left|c_{V}^{\prime}\right|^{2}-\left|c_{A}\right|^{2}-\left|c_{A}^{\prime}\right|^{2}\)
\(a^{\prime}=C_{S} C_{P}^{\prime *}+C_{S}^{\prime} C_{p}^{*}+c_{S}^{*} c_{p}^{\prime}+C_{S}^{\prime *} c_{p}\)
\(b^{\prime}=c_{V} c_{A}^{\prime *}+c_{V}^{\prime} c_{A}^{*}+c_{V}^{*} c_{A}^{\prime}+c_{V}^{* *} c_{A}\)
\(c^{\prime}=C_{T} \mathrm{C}_{\mathrm{T}}^{\prime *}+\mathrm{C}_{\mathrm{T}}{ }^{*} \mathrm{C}_{\mathrm{T}}\)
\(\alpha^{\prime}=\mathrm{C}_{S}^{\prime} \mathrm{C}_{\mathrm{P}}^{\prime *}+\mathrm{C}_{S}^{\prime} \mathrm{C}_{\mathrm{P}}^{*}-\mathrm{C}_{S}^{*} \mathrm{C}_{\mathrm{P}}^{\prime}-\mathrm{C}_{\mathrm{S}}^{* * \mathrm{C}_{\mathrm{P}}}\)
\(\xi^{\prime}=C_{V} c_{A}^{*}+C_{V}^{\prime} C_{A}^{*}-C_{V}^{*} C_{A}^{\prime}-C_{V}^{1 *} C_{A}\)
and in terms of these, we redofine the Kichel parameter
\[
p=(3 b+6 c) /(a+4 b+6 c)
\]
and also define four other asymetry parameters
\[
\eta=(\alpha-2 \beta) /(a+4 b+6 c)
\]
\[
\begin{align*}
& \xi=\left(-3 a^{\prime}-4 b^{\prime}+14 c^{\prime}\right) /(a+4 b+6 c)  \tag{4.2.28}\\
& \delta=\left(3 b^{\prime}-6 c^{\prime}\right) /\left(3 a^{\prime}+4 b^{\prime}-14 c^{\prime}\right) \\
& A=(a+4 b+6 c)
\end{align*}
\]

We have used these parameters because they are the commonest in experimental work. For antimuon decay, we change the sign of \(\}\). The equality between the average coupling constants for neutron and muon decay (4.2.15), (3.4.47) justify the assumption of a \(V-A\) model of muon decay. However, there is little or no justification for the assumption that no pseudoscalar interaction occurs in muon decay, since this was simply rendered undetectable in noutron decay, and was never shown to be zero. In muon decay, velocities are sometimes great enough for the P interaction to become important, but as we shall see, it tums out that it is, in fact, absent. Using the V - A theory, we immediately have
\(\xi=-\left(c_{V} C_{A}^{*}+c_{V}^{*} c_{A}\right) /\left(\left|c_{V}\right|^{2}+\left|c_{A}\right|^{2}\right)\)
\(A=8\left(\left|c_{V}\right|^{2}+\left|c_{A}\right|^{2}\right)\)
\(\rho\) and \(\delta\) have been measured by analysing muon decay electron spectra, and the best results are (9)
\[
\begin{align*}
& \rho=0.752 \pm 0.003,  \tag{4.2.35}\\
& \delta=0.755 \pm 0.009 . \tag{4.2.36}
\end{align*}
\]
(4.2.35) and (4.2.36) agree excellently with our theoretical prediction (4.2.31), indicating that our assumption of zero P interaction was indeed correct. If we now make the assumption
\[
\begin{equation*}
c_{V}=-c_{A}, \tag{4.2.37}
\end{equation*}
\]
which is effectively correct, then we predict
\(\xi=1\).
\(\eta=0\).
\(\eta=0\).
The experimental results for \(\xi\) contain relatively little error, and give
\(\xi=0.972 \pm 0.013\),
in agreement with (4.2.38). The results for \(\eta\) are less accurate, since they rely on the use of very low-energy muons. They give
\(\eta=-0.12 \pm 0.21\),
and thus our predictions still lie within the limits of experimental error.

\subsection*{4.3 The Two Neutrinos.}

If the two neutrinos in the muon decay (4.1.4) are each other's antiparticles, then we might expect them to annihilate each other, making the branching ratio for the decay mode
\(\mu \longrightarrow e \gamma\)
much higher than its experimental value of under
\[
\begin{equation*}
2.2 \times 10^{-8} \tag{4.3.2}
\end{equation*}
\]

Initially, there are a number of possible reasons for this behaviour. First, the two particles could be identical. However, Gell-Nann (10) and Feinberg (21) showed that this implied zero vector interaction in muon decay, which is obviously unreal. Second, the two neutrinos could be distinct particles. The feasibility of an experiment to demonstrate this hypothesis was increased by the suggestion of Lee and Yang (12) in 1960 that one of the neutrinos in muon decay was associated with the electron, wille the other was associated with the muon. The first problem encountered in setting up an experiment to test this theory was the production of neutrinos of the desired energy. This was overcome by Pontecorvo (13) who sugsested the use of neutrinos in pi decay. The pions would be produced by the interaction of a high-energy proton beanl with a suitable target. In 1960, Schwartz, Steiberger and Lederman showed that the alternating gradient proton synchrotron \({ }^{2}\) at Brookhaven could provide a sufficiently high-energy beam to produce the necessary energy and intensity of neutrinos. Thus, in 1962, Danby, Gaillani, Goulianos, Lederman, Nistry, Schwartz and Steiberger (14) began to search for evidence of two distinct types of neutrino. The pxinciple of their experiment was to detemine whether neutrinos produced in the decay
\(\pi^{+} \longrightarrow \mu^{+}+\mathrm{v}\)
would, when absorbed by protons and neutrons, produce only muons, or both muons and electrons according to the reactions
\(\overline{\mathrm{v}}+\mathrm{p} \longrightarrow \mathrm{n}+\mathrm{e}^{+}\),
\(\overline{\mathrm{v}}+\mathrm{p} \longrightarrow \mathrm{n}_{3}+\mu^{+}\).
The cross-section \({ }^{3}\) for the reactions (4.3.4) and (4.3.5) was estimated to be about \(10^{-36} \mathrm{~mm}^{2}\) for neutrinos with an energy of 2000 MeV . Hence, it was
necessary to have a high flux of high-energy neutrinos. Dandy et al. decided to use half their available beam energy: 15 GeV . The 15 GeV proton beam, which had an intensity of \(2 \times 10^{11}\) particles per second, was made to strike a beryllium target. The pions and other particles produced wich were within a \(14^{\circ}\) cone were allowed to decay in a 21 min straight decay tube. At the end of this, when about \(10 \%\) of the pions had decayed, the beam was filtered through 13 mif of iron, thus stopping all particles except neutrinos. The latter were unaffected by the iron, and continued in their original straigit trajectories. They then encountered a number of spark chambers with dimensions \(1(1 / 3)\) m \(Z 1(1 / 3)\) m \(X\) \((1 / 3) \mathrm{m}\), each containing nine 25 mu-thick aluminiwn plates. The total mass of material in the spark chambers was about 10 NF . Since the number of true neutrino events was expected to be very small, it was essential that all possible spurious events should be avoided. This was done in two ways. First, the spars chambers were surrounded by scintillation counters which were in anticoincidence with those placed betweon the spark chambers. The neutrinos entering the spark chambers would not interact with the extermal scintillation counters, but when they materialized into muons or electrons, they would trigger the internal ones. The requirenent for triggering the spark chamber was no count in the external scintillation counters, and counts in two of the internal ones. This eliminated a considerable number of cosmic ray events. The second method for avoiding spurious events was to activate the spark chambers only for a very short time while the neutrinos were actually in their vicinity. This was achieved by pulsing the main synchrotron beam every \(25 \mu \mathrm{~s}\). Within each puise, there were twelve smaller 20 ns pulses, each separated by 220 ns from the next. The spark chambers were switched on by a signal from a Cerenkov counter pointing at the proton bean target. Over 1700000 beam pulses had been accepted from the synchrotron, but only a few hundred photographs had been taken by the end of the experiment, demonstrating that the methods for selecting desired events were very efficient.

The next problem was to distinguish which photographs showed the effects of neutrino absorption, and which were simply spurious cosmic ray and main bean events. By running the spark chamber apparatus on its own, it was establishod that the muons in cosmic rays had momenta of about \(300 \mathrm{NeV} / \mathrm{c}\), and hence produced
much longer tracks than any of the particles from the accelerator. Thirty-four photographs were eliminated because they contained tracks of this type. It was possible that a number of neutrons might have triggered the anticoincidence circuit, and a number of short-track events were discounted because of this possibility. Finally, twenty-two photographs showing a vertex as expected, remained. The evidence against the fact that these might have been caused by neutrons was firstly that there was no attenuation in the number of events along the length of the spark chamber array, and secondly, that the removal of 1.2 m of iron shielding, which would have been expected to cause a significant increase in the number of neutron events, caused no difference in the number of one-vertex events. By placing a lead block in front of the bean target, it was established that the spark chamber one-vertex photographs were the result of pion or kaon decay products, since this reduced the event rate from 1.46 to 0.3 events per \(10^{16}\) protons, since the pions and kaons were then not allowed to decay.

Pinally, it was necessary to identify the secondary tracks produced by neutrino absorption. It was found that single tracks traversed about 8.2 mp of aluminium, no case of nuclear interaction baing observed. Had the particles been electrons or pions, a number of nuclear interactions would have been expected, and these did not, in fact, occur. Furthermore, electron tracks tend to be erratic, but the tracks photographed were straight. In all, 51 photographs contained muon tracks, whereas only 6 photographs showed electron showers. The latter were probably due to neutrinos produced in kaon decay. Thus it was concluded that there exist two distinct types of neutrinos: one associated with the electron and one with the muon. On the basis of the 'two neutrino experiment' it was suggested that an electron and a muon number should be attributed to every particle. The electron and e-neutrino are assigned electron numbers of +1 , while the pooitron and anti-e - neutrino are assigned electron numbers of -1 . All other particles are thought to have zero electron number. Kuon number is allotted analogously. Electron and muon number appear to be conserved in all reactions, explaining why the decay
\(\mu \longrightarrow e+\gamma\)
has never been observed.

Recalling our Hamiltonian for muon decay (4.2.1), it seems possible that this might initially be constructed from simpler units such as
\(\bar{\psi}_{e} Y_{r}\left(1+\gamma_{5}\right) \psi_{v_{e}}\)
and
\(\psi_{\mu} \gamma_{r}\left(1+Y_{5}\right) \psi_{V_{\mu}}\).
(4.4.1) and (4.4.2) are strongly reminiscent of the electromegnetic current \(S_{r}(x)=e \psi_{e}(x) \gamma_{r} \psi_{e}(x)\).
This is known as a local operator, since it is dependent on a single point \(x\) in space-time. The current (4.4.3) obeys the causality commutation relation
\(\left[S_{Y}(x), S_{B}\left(x^{\prime}\right)\right]=0\)
where \(x\) and \(x^{\prime}\) are two spacelike soparated points (see 2.6). Bohr and Rosenfeld (15) showed that when a system of elementary charges in a state \(A\) may be treated macrocosmically by the classical approximation, the matrix element \(\langle A| S_{r}(x)|A\rangle\)
is simply the classical current density. The electromagnetic interaction Hamiltonian has the form
\(H_{I}=S_{r}(x) A_{r}(x)\).
This is identical to the Ramiltomian in classical electrodynamics. From (4.4.3), we should be able to obtain the electromagnetic charge density by integrating over all space:
\(P_{A}(t)=\int\langle A| S_{0}(\underline{x}, t)|A\rangle d^{3} x\).
(4.4.7) is turned into a conserved quantity by the classical equation
\[
\begin{equation*}
\left(\partial / \partial x_{r}\right) s_{r}(x)=0 \tag{4.4.8}
\end{equation*}
\]

Returning to muon decay, we see that this could be considered as the interaction of the two currents (4.4.1) and (4.4.2), represented as ( \(\vec{v}_{e} e\) )
and
\(\left(\vec{v}_{\mu} \mu\right)\).
We may now ask whether the complete weak interaction Hamiltonian for leptonic reactions written in current fonm contains any other leptonic current-current
interactions. The currents (4.4.1) and (4.4.2) are both charged, and we now consider the possibility that neutral currents also exist. First, there are terms of the type
\[
\begin{align*}
& (\overline{\mathrm{e}} \mathrm{e})(\overline{\mathrm{e}} \mathrm{e}),  \tag{4.4.11}\\
& (\bar{\mu} \mu)(\bar{\mu} \mu), \\
& (\overline{\mathrm{e}} \mathrm{e})(\bar{\mu} \mu) .
\end{align*}
\]

If these have coupling constants of the same order as that for muon decay, then it is obvious that their effects will be almost unobservable, because of the fact that all the reactions (4.4.21), (4.4.12) and (4.4.13) can occur via second-order electromagnetic processes with a much higher transition probability. For example, (4.4.11) would occur electromagnetically as \(\overline{\mathrm{e}} \mathrm{e} \longrightarrow \gamma \longrightarrow \overline{\mathrm{e}} \mathrm{e}\).
Thus, at present, there is no experimental evidence conceming the neutral current terms \((4 \cdot 4 \cdot 11),(4 \cdot 4.12)\) and (4.4.13). However, if the same currentcurrent interaction occurs in semileptonic as well as pure leptonic processes, then the failure to observe decays such as
\(\mathrm{K}^{\circ} \longrightarrow \mu^{+} \mu^{-}\)
might indicate that these neutral current tems were not in fact present. The term
\((\bar{\mu} e)(\bar{e} e)\)
appears to be absent, if it is of the same strength as the muon decay term, since decays such as
\(\mu^{ \pm} \longrightarrow e^{ \pm} \quad e^{+} \quad e^{-}\)
have a branching ratio of less than (16)
\(1.5 \times 10^{-7}\).
The coupling
\((\bar{e} \mu)(\bar{\mu} \mu)\),
which would cause such decays as
\(\mu \longrightarrow e \gamma \quad\),
if it is present, must have a very small amplitude, since the branching ratio for \((4.4 .20)\) is less than (4.3.2). The similar term
\(\left(\begin{array}{ll}\bar{\mu} & e\end{array}\right)(\bar{\mu} e)\),
because of the failure to observe such processes as
\(e^{-}+e^{-} \longrightarrow \mu^{-}+\mu^{-}\),
which cannot occur electromagnetically, is assumed to be non-existent. In fact, all the terms \((4 \cdot 4 \cdot 16),(4 \cdot 4 \cdot 19),(4 \cdot 4 \cdot 21)\) may be discounted since they all violate the conservation of muon and electron number mentioned in 4.3 . We now consider the so-called charged 'self-current' terms, which may be present in the weak interaction leptonic Hamiltonian:
\(\left(\bar{e} v_{e}\right)\left(\bar{v}_{e} e\right)\),
\(\left(\bar{\mu} v_{\mu}\right)\left(\bar{v}_{\mu} \mu\right)\).
These consist of the self-coupling of the electron current and of the muon current respectively. The tern \((4.4 .23)\) might be detectable by the reaction \(\mathrm{v}_{\mathrm{e}}+\mathrm{Z} \longrightarrow \mathrm{z}+\mathrm{e}+\mathrm{e}^{-}+\mathrm{v}_{\mathrm{e}}, \quad\) (4.4.25) but, to date, this has not been observed. If the elastic scattering process \(\overline{\mathrm{v}}_{\mathrm{e}}+\mathrm{e} \longrightarrow \overline{\mathrm{v}}_{\mathrm{e}}+e\)
could be shown to occur, then this would demonstrate the existence of (4.4.23). However, at an energy of 10 KeV , the cross-section for (4.4.26) is only \(1.7 \times 10^{-44} \mathrm{~cm}^{2}\).

At higher energies, it is greater, but the neutrino flux tends to be smaller. The matrix element for the scattering process (4.4.26) is given by
\(K_{i f}=(g / \sqrt{2})\left(\widetilde{\psi}_{e} F_{r} \psi_{v}\right)\left(\check{\psi}_{v} F_{r} \psi_{e}\right)\)
where
\(F_{r}=Y_{r}\left(1+Y_{5}\right)\).
Using the Fierz reordering relation ( \(3 \cdot 3 \cdot 40\) ), we may interchange \(\psi_{e}\) and \(\psi_{v}\), obtaining
\(k_{i f}=-(g / \sqrt{2})\left(\bar{\psi}_{e} \bar{F}_{r} \psi_{\theta}\right)\left(\bar{\psi}_{v} F_{r} \psi_{V}\right)\)
Similarly, we may obtain the matrix element for electron-antineutrino
scattering by replacing the neutrino bracket by its complex conjugate:
\(u_{\text {if }}=-(\mathrm{g} / \sqrt{2})\left(\widetilde{\psi}_{e} P_{r} \psi_{e}\right)\left(\widetilde{\psi}_{\bar{v}} \gamma_{r}\left(1-\gamma_{5}\right) \psi_{\vec{v}}\right)\).
Since, by definition,
\(\sigma=W / \phi\),
where \(\phi\) is the particle flux per \(\mathrm{m}^{2}\) per second, and \(W\) is the transition
rate, we obtain, using (3.4.5):
\(\sigma_{v e}=\sigma_{0}\left(2 E^{2}\right) /(1+2 E)\),
\(\sigma_{\text {ie }}=\left(\sigma_{0} E\right) / 3 \cdot\left(1-1 /(1+2 E)^{3}\right)\)
where
\(\sigma_{0}=2 \mathrm{~g}^{2} \mathrm{~m}^{2} / \pi \sim 8.3 \times 10^{-45} \mathrm{~cm}^{2}\)
and where E is the incident neutrino onergy in eV . The results (4.4.33) and (4.4.34) could also have been obtained by an elementary consideration of units, and this method is outlined in Okun': Weak Interactions of Elementary Particles, Pergamon 1965, p. 66.

The best evidence for self-current terms in the leptonic weak interaction Hamiltonian comes from astrophysics. Since neutrinos are extremely unreactive, their mean interaction-free path in stellar material is much greater than that for photons, and thus, once produced, most neutrinos will carry off energy into space without interaction. There appear to be three major possible reactions leading to neutrino pair production in stars:
\(\begin{array}{lll}\gamma+e^{-} \longrightarrow e^{-}+v_{e}+\bar{v}_{e}, & \\ e^{+}+\mathrm{e}^{-} \longrightarrow \mathrm{v}_{\mathrm{e}}+\overline{\mathrm{v}}_{\mathrm{e}} & \text { (4.4.36) } \\ \Gamma \longrightarrow \mathrm{v}_{\mathrm{e}}+\overline{\mathrm{v}}_{\mathrm{e}} & , & \text { (4.4.37) } \\ \mathrm{r} & \text { (4.4.38) }\end{array}\)
where \(\Gamma\) is a so-called 'plasmon'. We find that the photoproduction process (4.4.36), which is a form of Compton scattering (17), dominates the neutrino pair production for otellar objects with densities of between 10 and \(10^{4} \mathrm{~g} \mathrm{~cm}^{-3}\) for temperatures ranging from \(5 \times 10^{7}\) to \(5 \times 10^{8} \mathrm{~K}\). Beyond about \(5 \times 10^{8} \mathrm{~K}\), an increasing density of electron-positron pairs exist in equilibrium with the thermal electromagnetic radiation, and hence the pair annihilation process (4.4.37) takes over the neutrino pair production at this point. The total cross-section for the reaction (4.4.37) is given by (18)
\(\sigma=\frac{G^{2} m^{2}}{3_{\pi}} \frac{(E / m)^{2}-1}{v}\),
where \(E\) is the c.m.s. energy \({ }^{4}\) of the electron-positron pair, and \(v\) is the relative velocity of its component particles. \(m\) is the electron mass. We assume that all the processes \((4.4 .36),(4.4 .37)\), and (4.4.38) have the same strength as muon decay, and hence we introauce a factor of \(G^{2}\), where \(G\) is the muon decay coupling constant (4.2.15). It may be found (19) that the total neutrino emissivity resulting from (4.4.37) is given by
\(\mathrm{K}_{\mathrm{V}}^{(37)}=\frac{4.3 \times 10^{24}}{P}\left(\frac{T}{10} 10\right)^{9}\),
whereas that for \((4.4 .36)\) is given by
where \(Y\) is the reciprocal of the mean number of electrons per nucleon in the star. We note that (4.4.37) is even more temperature-dependent than (4.4.36). The last method of neutrino pair production which we shall discuss is that of 'plasmon' decay. A 'plasmon' is a quantum of excitation of stellar plasma which resonance accelerates \({ }^{5}\) electrons, which radiate neutrino pairs. A plasmon with wave number \(k\) has an energy \(E\) given by \(\mathrm{E}=\sqrt{\mathrm{E}_{\mathrm{p}}{ }^{2}+k^{2}}\).
The expression (4.4.42) implies that the plasmon is a particle of finite rest mass \(\mathrm{E}_{\mathrm{p}}\) decaying into a neutrino pair. In fact, plasmons are bosons \({ }^{6}\), and decay via the electromagnetic current (4.4.3), which, in turn, interacts with the weak current (4.4.23). At temperatures of higher than \(5 \times 10^{8} \mathrm{~K}\), the plamon decay process is thought to dominate neutrino pair production.

We have discussed the possible cause and nature of stellar neutrino pair emission above, and we now consider its results and hence the evidence in favour of it. For stars of about 1000 solar luminosities, and with raciii of about \(1 / 10\) of the solar radius, it is thought that their central density is about \(10^{5} \mathrm{~g} \mathrm{~cm}^{-3}\) and their temperature is about \(3.5 \times 10^{8} \mathrm{~K}\). At this temperature and pressure, the plasmon decay process ( 4.4 .38 ) should have a significant effect on the evolution of a star, provided that the self-current term (4.4.23) is present in the weak Hamiltomian. The star will, at this point, tend to contract, finally becoming a white dwarf. At the stage when its luninosity is about 100 solar luminosities, the star should remain at the same radius for \(10^{4}\) yr if neutrino pair emission does not occur, and \(4 \times 10^{3} \mathrm{yr}\) if it does happen. When the star is at 10 solar luminosities, its lifetime would be \(3 \times 10^{5} \mathrm{yr}\) wi thout neutrino emission, and \(4 \times 10^{3}\) with neutrino emission. Similariy, when the star has reduced its magnitude to 1 solar luminosity, it should remain in this stage for \(2 \times 10^{6} \mathrm{yr}\) if neutrino emission does not occur, and \(2 \times 10^{5} \mathrm{yr}\) if it does occur. When the star evolves finally into a white dwarf, ite internal temperature will be too low for neutrino enission to occur. Thus, neutrino emission would reduce the number of stars with a luminosity of between 1.00 and 1 solar luminosity by a factor of about 10. A search for these so-called 'gap' stars has set an upper limit on their lifetime of \(6 \times 10^{5} \mathrm{yr}\), tending to favour the existence of neutrino emission from stars and hence the self-current terms.

In 1955, Davis (20) suggested the assignment of a quantum number known as leptonic charge or lepton number to all particles. The electron, muon, and neutrinos would have a lepton number of +1 , winile their antiparticles would have \(L=-1\). All other particles would have \(L=0\). Thus, for any particle \(L=\theta+\mu\),
where e and \(\mu\) are its electron and muon number respectively (see 4.3). The analogue of lepton number in the strong interaction \({ }^{7}\) is baryon number. The latter is carried by all particles subject to the strong interaction. If baryon number is conserved, then the proton must be stable, since it is the lightest baryon. Stuckleberg and Wigner have studied the atability of the proton, and have thus deduced that baryon number is conserved to better than one part in \(10^{43}\). From studying leptonic and semileptonic reactions, it is easy to see that lepton number is also conserved. Thus the electron must be stable, since it is the lightest charged lepton. The neutrinos cannot decay, since they are the ligintest leptons.

We now consider some of the evidence for lepton conservation. If this conservation law holds good, then the nuclear decay \((Z, A) \longrightarrow(Z+2, A)+2 e^{-}\),
known as double beta decay, should never occur, since it violates the law of the conservation of lepton number. The decay \((Z, A) \longrightarrow(Z+2, A)+2 e^{-}+2 \bar{v}_{e}\)
is, however, permitted. The decay \((4.5 .3)\) may be distinguished from ( \(4.5 \cdot 2\) ) in experiments by observing the electron energy distribution. In (4.5.2), the electrons share the full decay energy, whereas in (4.5.3), some of this is removed by the antineutrinos. If (4.5.2) were allowed, then it should have a rate about \(10^{5}\) times greater than that of the permitted decay (4.5.3). Recently, it has been suggested that double beta decay occurs in nature (21.): \(\mathrm{Te}^{130} \xrightarrow{\beta \beta} \mathrm{Xe}^{130}\)
and there is convincing geological evidence to support this view. The measured half-life for the process \((4.5 .4)\) is \(10^{21.34} \pm 0.12 \mathrm{yr}\). The theoretical prediction for a neutrinoless decay is \(10^{16.3 \pm 2} \mathrm{yr}\) and that for a decay with
neutrinos is \(10^{22.5} \pm 2.5\) (22). The latter agrees better with experimental evidence than the former, but since it is impossible to observe the two electrons in (4.5.4) experimentally, this experiment does not rule out lepton number violation. However, the figures quoted above set an upper limit on the lepton number violating amplitude in double beta decay of \(3 \times 10^{-3}\).

Wore direct evidence for lepton conservation comes from a study of antineutrino capture. The antineutrino in beta decay has a lepton number of -1 , and hence the process
\(\vec{v}_{e}+(Z, A) \longrightarrow(Z+1, A)+e^{-}\)
camnot occur. However,
\(\overline{\mathrm{v}}_{\mathrm{e}}+(\mathrm{Z}, \mathrm{A}) \longrightarrow(Z-1, A)+e^{+}\)
does not violate lepton number conservation. Antineutrinos from atomic piles have been observed to produce positrons according to (4.5.6) (23), and no electrons have been found due to the interaction of antineutrinos and c \({ }^{37}\).

We now consider a possible alternative assignment of lepton number than that discussed above. The electron, positron, e-neutrino and anti-e-neutrino retain the same lepton number as before, but the lepton number of the muon and associated particles change sign, according to the suggestion of Konopinsiki and Mahmoud (24). Thus the \(\mu^{+}\)is now a lepton, while the \(\mu^{-}\) becomes an antilepton. The four types of neutrinos still remain, but, since both the \(v_{e}\) and the \(v_{\mu}\) are left-handed (i.e. they have a helicity of -1 ), the \(\mathrm{v}_{e}\) is now a left-handed lepton state, while the \(\mathrm{v}_{\mu}\) is a left-handed antilepton. Thus we have dispensed with the need for an electron and muon number. This means that we may use a single 4-component field to describe the neutrinos instead of the usual two 2-component ones. The latter are separated purely because of the existence of muon and electron number. Let \(\phi_{v_{\mu}}(x)\) and \(\phi_{r e}(x)\) be the field operators for the muon and electron neutrinos respectively. Since we know the helicity or chirality of the states \(v_{\mu}\) and \(v_{e}\), we may write \(\phi_{v_{\mu}}(x)=\frac{1}{2}\left(1-\gamma_{5}\right) \psi_{v_{\mu}}(x)\)
and
\(\phi_{v_{e}}(x)=\frac{1}{2}\left(1+\gamma_{5}\right) \psi_{v_{e}}(x)\).
Since the only distinction between \(\psi_{v_{e}}(x)\) and \(\psi_{v_{\mu}}(x)\) now lies in their
helicities, it is unnecessary to distinguish between them. Setting
\(\psi_{v_{\mu}}(x)=\psi_{v e}(x)=\psi_{v}(x)\), we obtain
\[
\begin{align*}
& \phi_{v_{\mu}}(x)=\frac{1}{2}\left(1-\gamma_{5}\right) \psi_{v}(x),  \tag{4.5.10}\\
& \phi_{r_{e}}(x)=\frac{1}{2}\left(1+\gamma_{5}\right) \psi_{v}(x) . \tag{4.5.11}
\end{align*}
\]

Thus the muon decay Hamiltonian becomes
\[
\begin{gather*}
H_{I}=(G / \sqrt{2})\left(\bar{\psi}_{\mu}^{C}(x) \gamma_{T}\left(1-Y_{5}\right) \psi_{V}(x)\right)\left(\tilde{\psi}_{e}(x) \gamma_{r}\left(1+Y_{5}\right) \psi_{V}(x)\right) \\
+ \text { Herm. conj., }
\end{gather*}
\]
where \(\psi_{\mu}^{C}\) denotes the complex conjugate of the muon field. This Hamiltonian is invariant under two gauge transformations: the lepton gauge
\(e(x) \longrightarrow e^{j \AA} e(x)\),
\(\mu(x) \longrightarrow e^{-j A} \mu(x)\),
\(\mathrm{v}(\mathrm{x}) \longrightarrow \mathrm{e}^{j A} \mathrm{v}(\mathrm{x})\),
the symbol for the particles standing for their wave functions; and the so-called 'second gauge group':
\(e(x) \longrightarrow e^{j B} e(x)\),
\(\mu(x) \longrightarrow e^{j B} \mu(x)\),
\(\mathrm{v}(\mathrm{x}) \longrightarrow \mathrm{e}^{\mathrm{jB} Y_{5}} \mathrm{v}(\mathrm{x})\).
Thus we have two conserved quantum numbers in our new formulation, and these, in fact, are equivalent to our original quantum numbers e and \(\mu\).

At this point, we briefly consider the status of the muon with respect to the electron. These two particles appear to be identical, except for their large mass difference. Ross suggests (25) the possibility that the muon consists of an electron with a zero mass particle, which he calls the 'wavon', in orbit around it. He points out that if the 'wavon' has neutrino-like properties, then it would be unaffected by strong or electromagnetic forces from the central electron, and the weak interaction between the two particles would be negligable, since they would be spatially separated. Thus the only significant force within the Ross model of the muon would be gravitation. Using special relativity, Ross claims to be able to justify the comparative stability of the muon, and to predict a muon-electron mass ratio of 206.55 , which is close to the measured value of 206.77 . Purther, he suggests that the wavon, which must have a spin of one or zero to justify the muon spin,
might be composed of a bound state \(\bar{v}_{e}-v_{\mu}\), which is attractive, since it would explain the muon decay. However, Ross' model fails to explain the existence of the mu-neutrino, and could only be verified by very high-energy electron-muon scattering experiments.

CHAPTER FIVE: THE HADRONIC STRUCIURE OF THE WEAK INTBRACIION.

\subsection*{5.1 The Guantum Numbers of the Strong Interaction.}

The strongly-interacting particlea or hadrons are grouped together in isotopic multiplets, accoriing to their mass. Nultiplets consist of between one and four particles, each carrying a different electric charge. Thus, if the electromagnetic interaction could be damped, then particles in the same multiplet would appear identical, since they would cease to have any charge, and the slight mass-splitting between them would dissappear. As regaris the strong interaction, however, the proton and neutron are identical, and the equality of \(p-p, n-n\), and \(p-n\) couplings has been verified to great accuracy (1). This property of the strong interaction is known as charge independence. In 1936, Cassen and Condon (2), following an earlier suggestion by Heisenberg, postulated that the proton and neutron were simply different facets of the same particle, their difference in charge being caused by the fact that their 'isospin' \({ }^{1}\) was differently projected in charge space. We may write the wave function of the nucleon as
\(\left|\psi_{\text {nucleon }}\right\rangle=\psi(x) \xi_{\text {spin }} ?_{\text {charge }} \quad\),
where the first factor on the right-hand side is the ordinary nonrelativiatic wave function for the nucleon, the second factor is a state vector describing its spin, and the third is a similar state vector describing its charge. There are two basic states for the spin vector: \(-\frac{1}{2}\) and \(\frac{1}{2}\), the two possible projections for a spin of \(\frac{1}{2}\). Similarly, there are two basic states for the charge vector: p and n , which we denote as \(\eta_{+}\)and \(\eta_{\text {- respectively. We may specify whether }}\) a given nucleon is a proton or a neutron by stating its eigenvalue under the matrix operator \(T_{3}\). The latter is defined, for a multiplet containing two particles:


In analogy with the real spin component \(\sigma_{z}\), we choose as our representation
of \(\mathrm{T}_{3}\) the Pauli spin matrix (1.7.18):
\(T_{3}=\left[\begin{array}{rr}1 & 0 \\ 0 & -1\end{array}\right]\)
Often it is useful to have an operator which transforms the proton into the neutron and vice-versa. Thus we define
\(T_{1} \eta_{+}=\eta_{-}\)
\(T_{1} \eta_{-}=\eta_{+}\)
Once again, we use the Pauli matrix representation, so that
\(T_{1}=\left[\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right]\)
We also introduce, in complete analogy to real spin,
\(\mathrm{T}_{2}=-j \mathrm{~T}_{3} \mathrm{~T}_{1}=\left[\begin{array}{rr}0 & -j \\ j & 0\end{array}\right]\)
A further two useful operators are defined
\(T^{I}=\frac{1}{2}\left(T_{1} \pm j T_{2}\right)\),
so that
\(\mathrm{T}^{+}=\left[\begin{array}{ll}0 & 1 \\ 0 & 0\end{array}\right]\)
\(\mathrm{T}^{-}=\left[\begin{array}{ll}0 & 0 \\ 1 & 0\end{array}\right]\)
Thus
\(\mathrm{T}^{+} \mathrm{n}=\mathrm{p}\)
\(\mathrm{T}^{+} \mathrm{p}=0\)
\(T{ }^{-} \mathrm{n}=0\)
\(T^{-} p=n\).
Since our representation and formalism for isospin is identical to that for real spin, we may now define a vector I such that
\(I=\frac{1}{2}\left(T_{1}, T_{2}, T_{3}\right)\),
so that (see Appendix B)
\(\left[\mathrm{T}_{\mathrm{k}}, \mathrm{T}_{1}\right]=j \mathrm{~T}_{\mathrm{m}} . \quad(\mathrm{k}, 1, \mathrm{~m}\) cyclic \()\)
The total isotopic spin I defined by ( 5.1 .16 ) must be a conserved quantity. However, since it was based only upon the characteristics of the strong interaction, there is no reason to suppose that it is conserved in anything except the atrong
interaction, and, in fact, we find that it is not. The third component of isospin, \(I_{3}\), is nevertheless also conserved in the electromagnetic interaction, since it is charge-dependent. From (5.1.16) we see that isospin is given by \(I=(K-1) / 2\),
where M is the multiplicity of a partioular particle, i.e. the number of particles in its multiplet. Thus, for the nucleons, \(I=\frac{1}{2}\).

We now consider field quantization using isospin. We may introduce a aingle eight-component field for the nucleon, to replace our original two spinor ones. We distinguish between different components of this field by the two indices \(z\) and \(T\), which denote the spin and isospin, respectively, of each component. \(z\) may assume values between 1 and 4 inclusive, and \(T\) either \(\frac{1}{2}\) or \(-\frac{1}{2}\). Hence our field becomes \(\psi_{z, T}(x)=\sum_{p} \sum_{r=1}^{2}\left(e^{j p x_{u}}{ }_{z}^{(+)(r)}(\underline{p}) a_{\mathrm{r}}^{(r)}(\underline{p})\right.\) \(+e^{-j p x_{u}}{ }_{z}^{(-)(r)}(-\underline{p}) b_{T}^{t(r)}(\underline{2})\), (5.1.20) where \(u_{z}^{( \pm)(r)}(\underline{p})\) are, as usual, plane wave solutions to the Dirac equation, of the form (1.8.5), with positive and nesative energies respectively, polarization state \(r\), and momentum \(\mathfrak{p}\). The operator a \((r)\) is the destruction operator for both protons and neutrons with polarization \(r\), and similarly, \(b_{I}+(r)\) is the creation operator for antiprotons and antineutrons with polarization \(r\). Por the pions, we must introduce three separate fields: a Hermitian one for the \(\pi^{0}\), and complex ones for the \(\pi^{ \pm}\). Since the pions have zero spin, they may be described by scalar fields of the form (2.2.3):
\[
\begin{align*}
& \phi_{0}(x)=\sum \underline{k}(1 / \sqrt{2 \omega})\left(e^{j k x} a_{0}(\underline{k})+e^{-j k x} a_{0}^{+}(\underline{k})\right),  \tag{5.1.21}\\
& \phi(x)=\sum \underline{k}(1 / \sqrt{2 \omega})\left(e^{j k x} a_{+}(\underline{k})-e^{-j k x} a_{-}^{+}(\underline{k})\right), \\
& \phi^{\dagger}(x)=\sum_{k}(1 / \sqrt{2 \omega})\left(-e^{j k x} a_{-}(\underline{k})+e^{-j k x} a_{+}^{+}(\underline{k})\right),
\end{align*}
\]
in obvious notation. The minus aign in front of the creation and destruction operators of the \(\pi^{I}\) in (5.1.22) and (5.1.23) originates from the phase convention \({ }^{2}\). The charge of the pions is given by
\[
\begin{equation*}
q=e \sum \sum_{\underline{k}}\left(a_{+}^{+}(\underline{k}) a_{+}(\underline{k})-a_{-}^{+}(\underline{k}) a_{-}(\underline{k})\right), \tag{5.1.24}
\end{equation*}
\]
which is equivalent to \((2.2 .23)\). The operator \((5.1 .24)\) is, in fact, identical to \(T_{3}\) except for the factor of \(e\). We might continue to consider interacting
isospin-formulated fields, but this study has little or no application in weak interaction theory. A simple treatment of the subject may be found in Kallen: Elementary Particle Physics, Addison-Wesley 1964, pp. 67-144.

We now mention two other quantun numbers which are commoniy used in both strong and weak interaction theory. The first of these is hypercharge, Y , which is defined by
\(Y=2 \bar{Q}\),
where \(\bar{Q}\) is the average charge of the particles in a particular multiplet. We know that
\(Q / e=T_{3}-\frac{1}{2}\).
Pollowing the suggestion of Pais (3), Cell-Mann (4) and Nishijima (5) postulatep, in 1953 and 1955 respectively, that (5.1.26) could be expressed more elegently by introducing a new quantum number called strangeness \({ }^{3}\), s , according to the formula
\(Q / e=(B+S) / 2+I_{3}\).
Alternatively, we may say that strangeness is twice the distance by which a given multiplet is displaced from the standard multiplet. For baryons, this is taken as the nucleons, and for mesons, the pions. The displacement of a particular multiplet is found by taking the difference between its centre of charge or average charge, \(\bar{Q}\), and the centre of charge of the standard multiplet. Thus, empirically, we find that
\(S=Y-B\).

\subsection*{5.2 The Conserved Vector Current Hypothecis.}

We recall that the vector coupling constants for neutron beta decay and for mpon decay are almost equal, wereas the axial vector coupling constants for these two processes differ by a factor of about \(20 \%\). A poseible explanation for this fact is that, initially, both the vector and the axial vector coupling constants for the two processes are equal, but that then, strong interaction effects occur in the neutron decay, and much weaker electromagnetic offects occur in the muon decay. The process affecting the coupling constant is 'renormalization' \({ }^{4}\). We know that the vector current in neutron decay may be written:
\(J_{r}^{\prime \prime \prime}(x)=g \bar{\psi}_{p}(x) Y_{r} \psi_{n}(x)\).
Using isospin, (5.2.1) becomes
\(-(\sigma / \sqrt{2})\left(\bar{\psi}(x) Y_{r} T^{+} \psi(x)\right)\)
or
\(-(g / 2 \sqrt{2})\left[\bar{\psi}(x), \gamma_{2^{2}} T^{+} \psi(x)\right]\).
The electromagnetic current (4.4.3) of the nucleon system is
\(J_{r}^{e l}(x)=j e\left(\bar{\psi}_{p}(x) Y_{T} \Psi_{p}(x)\right)\),
or, in isospin formalism:
\((j e / 4)\left[\bar{\psi}(x), Y_{r}\left(1+T_{3}\right) \psi(x)\right]\).
From (5.2.5) we see that (5.2.4) may be decomposed into an isospin soalar and an isospin vector (an isoscalar and isovector):
\(J_{r}^{\mathrm{el}}(x)=J_{r}^{S}(x)+J_{r}^{V}(x)\),
where
\(J_{r}^{S}(x)=(j e / 4)\left[\bar{\psi}(x), Y_{r} \psi(x)\right]\)
and
\(\frac{J}{r}_{V}^{J^{V}}(x)=(j e / 4)\left[\bar{\psi}(x), Y_{r} T_{3} \psi(x)\right]\).
Obviously the current (5.2.1) may be considered as another component of (5.2.8), so that
\(J_{r}^{V}(x)=j 2 \mathrm{~g} / \mathrm{e} \mathrm{J}_{\mathrm{r}}^{\mathrm{V}}(+)(\mathrm{x})\).
Unfortunately the electromasnetic nuclion current (5.2.4) does not obey a continuity equation of the form (1.4.12). However
\(\frac{\partial J^{e l}}{\partial x_{r}}+\frac{\partial J^{\pi}}{\partial x_{r}^{r}}=0\)
\(J\) being defined
\(J_{r}^{\pi}(x)=j e\left(\frac{\partial \phi^{\top}(x)}{\partial x_{r}} \phi(x)-\phi^{\dagger}(x) \frac{\partial \phi(x)}{\partial x_{r}}\right)\)
where \(\phi\) is the complex pion field in (5.1.22) and (5.1.23). By decomposing the latter field into real and imaginary components, we may write
\[
\begin{equation*}
\left(\partial J_{r}^{S}(x) / \partial x_{r}\right)+\left(\partial / \partial x_{r}\right)\left(\tilde{J}_{r}^{V}(x)+J_{r}^{\pi}(x)\right)=0 \tag{5.2.12}
\end{equation*}
\]

Thus each term in (5.2.12) is conserved. We assume that the interaction which we are considering is invariant under rotations in isospin space, and hence other components of the second tern in (5.2.12) will also be conserved;
\(\left(\partial / \partial x_{r}\right)\left(\underline{J}_{r}^{V}(+)(x)+{J_{r}}^{\pi(+)}(x)\right)=0\).

We see that the first term in \((5 \cdot 2.13)\) is simply the weak vector nucleon. current (5.2.9). However, we have not yet proved that the latter is conserved on its own. Setting
\(J_{r}^{W^{\prime}}(x)=(j g / e) \sqrt{2}\left(J_{r}^{V(+)}(x)+J_{r}^{\pi(+)}(x)\right)\),
we find that
\(J_{r}^{W^{\prime}}(x)=E \bar{\psi}_{p}(x) \gamma_{r} \psi_{n}(x)+E \sqrt{2}\left(\phi_{0}(x)\left(\partial \phi^{+}(x) / \partial x_{r}\right)-\right.\)
\[
\left.-\phi^{+}(x)\left(\partial \phi_{0}(x) / \partial x_{r}\right)\right)
\]
where \(\phi_{0}, \phi\) and \(\phi^{+}\)are defined in (5.1.21), (5.1.22) and (5.1.23) respectively. From ( 5.2 .15 ), it is easy to deduce that
\(\left(\partial J_{r}^{W \prime}(x) / \partial x_{r}\right)=0\)
so that
\(\left(\partial J_{r}^{W}(x) / \partial x_{r}\right)=0\).
Thus the weak vector current in neutron decay is unaffected by the strong pion current, winich explains the near equality of the true vector coupling constants in muon and neutron decay. \((5.2 .17)\) is known as the conserved vector current or CVC hypothesis. It is natural to ask whether a similar relation to ( 5.2 .17 ) might hold for the axial vector current in neutron decay. However, as we saw above, the axial vector coupling constant in muon decay is about \(20 \%\) more than that in neutron decay, and thus it seems likely that axial vector current is not conserved. In fact, by similar reasoning as that employed above for the vector current, we may show that \(\left(\partial J_{r}^{A}(x) / \partial x_{r}\right)=f_{\pi} m_{\pi}^{2} \phi(x)\),
where \(J^{A}\) is the axial vector current, \(\phi(x)\) was defined in (5.1.22) and \(f_{\pi}\) is a constant dependent on the strength of the strong interaction. (5.2.18) is known as the partial conservation of axial vector current or PCAC.

\subsection*{5.3 The Structure of the Weak Hadronic Current.}

There exist in nature three different types of weak reaction. The first type are termed leptonic, and contain only leptons. The second type, the so-called semileptonic reactions, involve both leptons and hadrons, and the third type, which are called hadronic reactions, consist purely of hadrons, which also react via the strong interaction. We denote the weak hadronic current
by \(\mathrm{J}_{\mathrm{r}}^{\mathrm{H}}\), and the weak leptonic current, containing such terms as (4.4.9) and (4.4.10), and also possibly neutral and self-charged current terms, by \(\mathrm{J}_{x}^{\mathrm{L}}\). Since in semileptonic processes, such as the neutron decay (3.1.1), the hadronic current couples with the leptonic one, the former must be a charged current, since at least the leptonic current terms (4.4.9) and (4.4.10) are charged. This is expressed by saying that, for the hadronic current \(J_{T}^{H}\), \(\Delta Q=1\)
so that when \(J_{r}^{H}\) acts on a system, it raises the total charge of the hadrons present by one unit of charge Since the current \(J_{r}^{L}\) also raises the charge of all leptons in a system by le, the Hamiltonian for semileptonic processes must be of the form
\(H_{I}=(G / \sqrt{2})\left(J_{r}^{H}(x) J_{r}^{\mathrm{J}}(x)\right)+\) Herm. conj.
in order to obey charge conservation. As \(\bar{J}_{r}\) is evidently a
\(\Delta Q=-1\)
current, the hadronic current may couple with itself to produce a Hamiltonian
\(H_{I}=(G / \sqrt{2})\left(J_{r}^{H}(x) \bar{J}_{r}^{-H}(x)\right)+\) Herm. conj.
(5.3.4) is thought to be responsible the pure hadron decays of strange particles, such as
\(\wedge^{0} \longrightarrow \mathrm{p} \pi^{-}\).
We note that the total weak Hamiltonian is probably of the current-current form:
\(H_{I}=(G / \sqrt{2})\left(J_{r}^{H}(x)+J_{r}^{\perp}(x)\right) \cdot\left(\bar{J}_{r}^{H}(x)+\bar{J}_{r}^{L}(x)\right)\)
A useful method of summarizing our present knowledge of weak interaction couplings is the Puppi tetrahedron, which has, at its vertices, the currents:
pn
\(e^{+} v_{e}\)
p त
\(\mu^{+} v_{\mu}\).
There exiats also a charge-conjugate Puppi tetrahedron (6), and the question of whether vertices of this are coupled to those of the normal tetrahedron was discussed in 4.4 .

We now consider the assignments of the three important quantum numbers, Y, I and \(G\), to the current \(J_{r}^{H}\). Since both hypercharge-conserving and hypercharge-
changing semileptonic reactions occur in neture, terms of each type must be included in \(J_{Y}{ }^{H 1}\). Let \(\Delta Y\) be the difference between the hypercharges of the initial and final states in a reaction. Theoretically,
\(J_{r}^{H}=a_{0} J_{r}^{0}+a_{1} J_{r}^{1}+a_{2} J_{r}^{2}+a_{-1} J_{r}^{-1}+a_{-2} J_{r}^{-2} \ldots(5.3 .11)\) where the numbers \(0,1,2,-1,-2, \ldots\) associated with each term correspond to the value of \(\Delta Y\) forthose tems, a is a parameter which allows the various terms in the current \({\underset{r}{r}}_{H}\) to be coupled with different strengths to the leptonic current. Thus the semileptonic interaction Hamiltonian becomes
\(H_{I} \quad=\quad \frac{G^{0}}{2} J_{r}^{0} J_{r}^{-L}+\frac{G^{1}}{2} J_{r}^{I} J_{r}^{L} \quad \ldots\).
ilithout is selection rule for \(\Delta Y\), the hadronic current could contain all the terms on the right-hand side of ( \(5 \cdot 3.11\) ), including an infinite number of coupling constants. However, it is found that the partial conservation law \(|\Delta x| \leqslant 1\)
is obeyed in all reactions, so that only the terms corresponding to \(\Delta Y=\) 0,1 and -1 remain on the right-hand side of \((5.3 .11)\). There is little evidence for \(|\triangle Y| \geqslant 2\) reactions, although the decay \(Z^{\circ} \longrightarrow \mathrm{pe}^{-} \mathrm{v}\),
which has \(|\Delta Y|=2\), has only been shown to have a branching ratio of under (7) \(1.3 \times 10^{-3}\), (5.3.15)
which nevertheless sets a small maximum \(|\Delta Y| \geqslant 2\) amplitude for the hadronic current. We usually asoume that no \(|\Delta Y| \geqslant 2\) transitions occur. Another selection rule which appears to apply to all \(|\Delta Y|=1\) semileptonic processes is \(\Delta Y=\Delta Q\),
implying that the change in hypercharge in an interacting system of hadrons is equal in sign to the change in their total charge. Processes of the form \(\Delta Y / \Delta Q=-1\)
almost certainly do not occur, althougin the upper limit on the branching ratio for decays of the type
\(\mathrm{K}^{0} \longrightarrow \pi^{+}+1^{-}+\overline{\mathrm{v}}_{1}\)
where 1 is either an electron or a muon, is (8) only \(\sim 10^{-2}\).
However, on the assumption that \((5.3 .16)\) always holds true, our complete hadronic current becomes:
\(J_{r}^{H 1}=a_{0} J_{r}^{0}+a_{1} J_{r}^{1}\).
There exist two further important selection rules for semileptonic reactions: the \(\Delta I=\frac{1}{2}\) and \(\Delta I=1\) rules. The \(\Delta I=1\) rule states that in any \(\Delta Y=0\) semileptonic process, the chance in total isospin must satisfy \(\Delta I=1\).
In hypercharge-changing reactions, total isospin cannot be conserved because of the Gell-Mann - Nishijima - Nakano (9) (GNN) relation
\(Q=I_{3}+Y / 2\).
In fact, in \(|\Delta y|=1\) processes, it is found that
\(\Delta I=\frac{1}{2}\).
Both \((5.3 .21)\) and \((5.3 .23)\) appear to be obeyed to a high degree of accuracy in the weak interaction.

We now consider the symmetry properties of the \(\Delta Y=0\) hadronic current. Since the latter has \(\Delta Q=1\), (5.3.22) implies
\(\Delta I_{3}=1\).
Similarly, the current \(\underset{\sim}{\mathcal{H}}\) corresponds to \(\Delta Y=0, \Delta Q=-1\), so that
\(\Delta I_{3}=-1\).
In terms of isospin, the \(C\) operator may be considered as an operator which reverses the sign of the third component of isospin of a particle, ie. it reflects it in the plane \(I_{3}=0\). Alternatively, we may say that
\(c=e^{j \pi I_{2}}\)
where \(I_{2}\) is the generator of rotations about the \(I_{2}\) axis in isospace. Using the operator ( \(5 \cdot 3.26\) ) we may decompose the currents \(\mathcal{J}_{r}^{2}\) and \(\mathcal{J}_{r}^{H}\) with \(\Delta Y=0\) : \(J_{I}^{H(0)}=\frac{1}{2}\left(J_{I}^{H(0)}+e^{-j \pi I_{2}} J_{r}^{-H(0)} e^{j \pi I_{2}}\right)+\)
\[
\begin{equation*}
+\frac{1}{2}\left(J_{r}^{H(0)}-e^{j \pi I_{2}} \bar{j}_{r}^{-H(0)} e^{j \pi I_{2}}\right), \tag{5.3.27}
\end{equation*}
\]
\(\bar{J}_{r}^{H(0)}=\frac{1}{2}\left(\tilde{J}_{r}(0)+e^{-j \pi I_{2}} J_{r}^{H(0)} e^{j \pi I_{2}}+\right.\)
\[
+\frac{1}{2}\left(\tilde{j}_{r}^{H(0)}-e^{j \pi I_{2}} J_{r}^{H(0)} e^{j \pi I_{2}}\right),
\]
so that
\(e^{j \pi I_{2}} J_{r}^{H(0)} e^{-j \pi I_{2}}=\frac{1}{2}\left(J_{r}^{H(0)}+e^{-j \pi I_{2}} J_{r}^{H(0)} e^{\left.j \pi I_{2}\right)}-\right.\)
\[
\begin{equation*}
-\frac{1}{2}\left(j_{r}^{-H(0)}-e^{-j \pi I_{2}} J_{i}^{H(0)} e^{j \pi I_{2}}\right) \text {. } \tag{5.3.29}
\end{equation*}
\]

From (5.3.29) we see that, in general, \(\underset{\mathrm{j}}{\mathrm{j}(0)}\) need not necessarily be the charge conjugate of \(J_{r}^{H(0)}\). However, if either term on the right-hand side of (5.3.27)
vanishes, then
\(e^{j \pi I_{2}} J_{r}^{H(0)} e^{-j \pi I_{2}}= \pm \frac{J_{r}(0)}{J_{r}}\).
(5.3.30) is known as the charge symetry condition (10). Those terms in the hadronic current wiich yield a positive right-hand side of (5.3.30) are said to be first-class tems, and those which make it negative are callei necondclass (11). We find that if both T invariance and (5.3.30) hold, then secondclass terms must be absent. However, if both first- and second-class terns are present in the hadronic current, then this implies T violation. If CPT holds, then T violation implies CP violation. If I invariance does hold, then the current must not obey (5.3.30). An operator related to the charge symmetry one (5.3.26) is the G parity operator defined \(G=C e^{j \pi I_{2}}\),
where \(C\) is the stendard charge conjugation operator, which, unlike (5.3.26), reverses the sense of a figure in isospace. \(A\) possible \(G\) parity scheme for the \(J_{r}^{H(0)}\) current is
\(G V_{r} G_{-1}^{r-1}=V\),
\(G A_{r} G^{-1}=-A\),
where \(V\) and \(A\) are the vector and axial vector termas in the current \(J_{r}^{H(0)}\) respectively. We shall not onter into a verification of (5.3.32) here, but this may be found in Karshak, Riazuddin, Rvan: Theory of Weak Interaction in Particle Physics, willey-Interscience, 1969, pp. 108-109. (5.3.32) and (5.3.33) may be shown to imply that only first-class terms appear in the matrix elements of \(J_{r}^{H(0)}\).

We now consider the current \(J_{r}^{H(1)}\) with
\(\Delta Q=Y=1\).
From ( 5.3 .34 ) and \((5.3 .22)\), we see inmediately that
\(\Delta I_{3}=\frac{1}{2}\).
(5.3.35) is not the only possible \(I_{3}\) assignment for \(J_{r}^{H(1)}\), but it has been found to be the only one occuring in nature. We find that, wnlike the \(J_{r}^{H(0)}\) current, the vector component of the \(J_{I}^{H(1)}\) current is not precisely consorved under the effect of the strong interaction, unless two particles can have the same spin, parity and mass while having charges and hypercharges differing by one unit. This situation is not found in nature, and hence we are forced to conclude that the vector term in \(\mathrm{J}_{\mathrm{r}}^{\mathrm{H}}(2)\) is not conserved.

\subsection*{5.4 Form Factors.}

Before we may consider form factors in the weak interaction, we must first discuss them with reference to the electromagnetic interaction, in terms of which they were originally fommlated. For a proton, for example, unaffected by strong interactions, the probability or crosa-section for elastic electron scattering is given by the Dirac formuls, which assumes the proton to be a point with spin \(\frac{1}{2}\) and masmetic moment \(e / \lambda / \partial N_{p} c\) :
\(\frac{d \sigma}{d \Omega}=\frac{e^{4} \cos ^{2}(\theta / 2)}{4 p_{0}^{2} \sin ^{4} \frac{\theta}{2}\left[1+\frac{2 p}{u_{p}} \sin ^{2} \frac{\theta}{2}\right]}\left(1+\frac{q^{2}}{2 p_{p}^{2}} \tan ^{2} \frac{\theta}{2}\right)\)
where \(M_{p}\) is the proton mass, \(p_{0}\) is its initial three-monentum, and \(q\) is the four-momentum transfer between the election and the proton during the scattering process. The second term in (5.4.1) is due to magnetic scattering, and is absent from the Wott scattering cross-section from atomic nuclei. For a real proton, we must take into account the virtual pairs of hadrons surrounding it due to the fact that it takes part in strong interactions, and also the anomalous magnetic moment over and above the Dirac prediction of 2 . Thus we define two form factors, the electric or charge form factor \(F\), and the magnetic form factor \(G\). These two form factors are real functions of the four-momenturn transfer squared, \(q^{2}\). Thus, for the proton
\(P(0)=1\),
\(G(0)=\mu_{p} \sim 2.79\) n.프
where \(\mu_{\mathrm{p}}\) denotes the magnetic moment of the proton, and n.m. stands for the units nuclear magnetons \({ }^{5}\). For the neutron
\(F(0)=0\),
\(\mathrm{G}(0)=\mu_{\mathrm{n}} \sim-1.91 \mathrm{n} \cdot \mathrm{m}\).
We find that the corrected cross-section has the form
\(\frac{d \sigma}{d \Omega}=\left(\frac{d \sigma}{d \Omega}\right)_{\text {Mott }}\left[\left(\frac{p^{2}+\left(q^{2} / 4 M^{2}\right) c^{2}}{1+\left(q^{2} / 4 M^{2}\right)}\right)+\frac{q^{2}}{4 M^{2}} \cdot x^{2} \tan ^{2}(\theta / 2)\right]\)
(5.4.6) is known as the Rosenbluth formula (22). The important feature of it is that
\(\left(\frac{d \sigma}{d \Omega}\right) /\left(\frac{d \sigma}{d \Omega}\right)_{\text {Mott }}=A\left(q^{2}\right)+B\left(q^{2}\right) \tan ^{2}(\theta / 2)\).
Thus, if we were to plot, the cross-section per element solid angle against \(\tan ^{2}(\theta / 2)\) for differing values of the incident momentum \(p_{0}\) and the four-momentum transfer square \(q^{2}\), we should obtain a straight line. This is a direct consequence of the fact that we have made use of the Born approximation (13). This assumes that in any electromagnetic interaction, only one virtual \({ }^{6}\) photon is ever exchanged. The best test to dewonstrate that only one-photon exchange ever takes place in e-p elastic collisions is to make a Rosenbluth plot, as described above, for the processes
\(e^{+} p \longrightarrow e^{+} p\),
\(e^{-} p \longrightarrow e^{-} p\)
and then to compare the curves obtained. If two or more photons are exchanged, then the curves will be different and will be non-linear. However, at least at energies below 10 GeV in the c.r.s., the curves remain linear, so that we are justified in making use of the Borm approximation.

We now briefly discuss the usual interpretation and significance of the nucleon form factors. Following the experiments of Hofstadter et al. (14) in 1961, it was established that the nucleon consisted of a hard spinning pointlike core, surrounded by a virtual meson cloud which spent about \(3 / 10\) of its time outside the central nucleon core. It vas found that the charge-density distribution corresponded very closely to a normal distribution, and that it fell away to zero about 1.4 fill from the centre of the particle. The mean radius of the nucleon was thus calculated to be about 0.74 fm . The form factor is basically a measure of the probability that the nucleon will not diaintegrate and shake off one or more pions during the collision. For large \(q^{2}\), this probability becomes very small indeed. Attempts were made a first (25) to analyse the situation assuming that the pion was the only mediator of the strong interaction, as Yukawa had originally suggested (16). However, these met with little success, and Namou (17) postulated that there existed one or more hadrons in the nucleon cloud which coupled directly with the photon. These mesons must have the same quantum numbers as the photon, i.e. they must have zero charge, spin-parity of \(1^{-}\)and odd \(C\) parity. We now define two new
form factors: an isoscalar form factor:
\(G_{S}=\frac{1}{2}\left(G_{p}+G_{n}\right)\),
and an isovector one:
\(G_{V}=\frac{1}{2}\left(G_{p}-G_{n}\right)\).
From \((5 \cdot 4.10)\) and \((5 \cdot 4.11)\), we may deduce that
\(G_{p}=G_{S}+G_{V}\),
\(G_{n}=G_{S}-G_{V}\),
and similarly for the electric form factors. We see that \(G_{S}\) has the same sign for the proton and the neutron, whereas \(G_{V}\) has opposite signs for the tro particles. Thus the meson responsible for \(G_{S}\) will have zero isospin, and that responaible for \(G\) will have \(I=1\). The mesons which fit these requirements are the \(\omega(783)\) and the \(\rho^{\circ}(770)\) respectively.

In the weak interaction, we may also define form factors. We use two form factors, the vector one: \(G_{V}\left(q^{2}\right)\), and the axial vector one: \(G_{A}\left(q^{2}\right)\). In the matrix elements for hadronic weak processes, wo must include six form factors: \(f_{V}\left(q^{2}\right), \varepsilon_{V}\left(q^{2}\right), h_{V}\left(q^{2}\right), f_{A}\left(q^{2}\right), \varepsilon_{A}\left(q^{2}\right), h_{A}\left(q^{2}\right)\), all of which are scalar functions of \(q^{2}\). These contain all possible information conceming the modifications to the basic weak interaction caused by the existence of virtual strong interactions. In order to produce a complete model for the weak interaction, we mist determine the nature and structure of these form factors. The best method of so doing is to employ dispersion theory. However, this is beyond the scope of this book. A good introduction to dispersion theory is S.Mandelstam: Disporsion Relations in Strons-Couplinc Physion, Rep. Prog. Phys., 25, pp. 99-162 (2962), which is reproduced in ed. Fronsdnl: Lecture Notes on Woak Interactions and Topics in Dispersion Physics from the Second Borgen International School of Physics - 1962, Benjamin 1963, pp. 269 et. seq. Using arguments in dispersion theory, we find that
\(\varepsilon_{V}\left(q^{2}\right)\),
\(f_{V}\left(q^{2}\right)\)
receive contributions from states with \(J^{P G}\) of \(1^{-+}\), of wich the resonant pion states \(2 \pi\) and \(4 \pi\) are the lowest mass examples. The former correaponds to the \(\rho(770)\) meson.
\(\left(\varepsilon_{V}\left(q^{2}\right)\left(m_{A}-m_{B}\right)+q^{2} n_{V}\left(q^{2}\right)\right)\)
where \(A\) and \(B\) are the initial and final hadrons in the reaction respectively, receives contributions from \(\mathrm{J}^{\mathrm{PG}}=0^{+-}\)states, of the form \(5 \pi, 7 \pi\). The \(5 \pi\) resonance with the lowest mass is probably \(\delta(970)\), although this may in fact be a bound state of the \(I=1 K \bar{K}\) syatem and not the pion syotem (18). \(g_{A}\left(q^{2}\right)\)
receives contributions from \(J^{P G}=1^{+-}\)states, such as \(3 \pi\) and \(5 \pi\), or, in terms of particles, \(A_{1}\) (1100).
\(\left(g_{A}\left(q^{2}\right)\left(m_{A}+m_{B}\right)+q^{2} f_{A}\left(q^{2}\right)\right)\)
is dependent upon \(0^{-}\)states such as the pion. \(\left(h_{A}\left(q^{2}\right)\right)\)
receives contributions from \(1^{++}\)states of the form \(4 \pi\), \(6 \pi\), such as \(B\) (1235). For the \(|\Delta Y|=1\), the situation is similar, except that the contributing resonant states are all of the form \(\mathrm{K} n \pi\), and thus have a strangeness of 1 . For \((5 \cdot 4.14)\) and ( \(5 \cdot 4.15\) ) the corresponding strange resonance is \(K^{*}(1420)\); for (5.4.16) probably \(K(725)\); for (5.4.17) \(\mathrm{K}_{\mathrm{A}}^{*}(1320)\); for (5.4.18) the kaon; and for (5.4.19) probably \(\mathrm{K}_{\mathrm{A}}^{*,}(1230)\). Weak form factors may roughly be interpreted as measures of the spatial distribution of the 'weak charge' of particles, or of the characteristics leading them to behave in certain ways in weak interactions.

\subsection*{5.5 Weak Narnetis.}

We know that, when free from strong interaction effects, the vector current term in the beta decay matrix element is
\[
v_{r}=\bar{u}_{p} Y_{r} u_{n}
\]
and the axial vector ourrent term is
\(A_{r}=\bar{u}_{p} \gamma_{r} \gamma_{5} u_{n}\).
If, however, we reintroduce strons effects, we have
\(V_{r}^{\prime}=\bar{u}_{p}\left(f_{V} \gamma_{r}-g_{V} \sigma_{r g} q_{B}-h_{V} q_{r}\right) u_{A}\),
and
\(A_{r}^{\prime}=\bar{u}_{p}\left(f_{A} Y_{r}-g_{A} \sigma_{r s} q_{s}-h_{A} q_{r}\right) Y_{5} u_{n}\),
where
\(\sigma_{r B}=\gamma_{r} \gamma_{\varepsilon}-\gamma_{s} \gamma_{r}\)
and where q is the four-momentum of the particles. By analogy with electrodynamics, we may deduce that in \((5.5 .3)\) and (5.5.4) both \(h_{V}\) and \(h_{A}\) are zero. Tests of this hypothesis are rendered difficult because the terms containing h are momentum-dependent, and hence are very small in most weak processes. In (5.2.9) we considered the weak current of the form (5.5.1) as another component of the vector electromagnetic current. Using this approach, we may evaluate the vector form factors occuring in the matrix element for a transition between the nucleon states (i.e. neutron decay and crossed equivalents \({ }^{7}\) ) in terms of the nucleon isovector electromagnetic form factors defined in (5.4.11): \(E_{V}\left(q^{2}\right) \quad \Rightarrow F_{V}\left(q^{2}\right)\),
\(f_{V}\left(q^{2}\right)=\left(\mu_{p}-\mu_{n}\right) /\left(2 \mathbb{m}_{N}\right) \quad G_{V}\left(q^{2}\right)\).
We may also write down the boundary conditions for the case
\(\mathrm{q}=0 \quad:\)
\(g_{V}(0)=1\),
by the CVC hypothesis, and
\(f_{V}(0)=\left(\mu_{p}-\mu_{n}\right) /\left(2 m_{N}\right) \sim(3.7) /\left(2 m_{N}\right)\).
By analosy with electromagnetism, the tem (5.5.7) is known as the veak magnetic form factor, and its offects are known as weak magnetism (19).

There are basically three types of reactions in which weal magnetisen may be detected: favourable beta decays, muon capture in atoms and high-energy neutrino reactions. Since the term containing \(g_{V}\) in (5.5.3) is momentumdependent, we wish to maximize this momentum in order to detect weak magnetic effects. Thus, in order to detect weak magnetisu in beta decay, we consider, as was suggested by Gell-Kann (20), the \(\Lambda=12\) triad: \(B_{5}^{12}, C_{6}^{12}, N_{7}^{12}\), and assume perfect charge independence. All three members of the triad, which have \(I=1, J^{P}=I^{+}\)in their excited level, are connected to the \(I=0, J^{P}=0^{+}\) \(\mathrm{C}^{12}\) ground-state by allowed transitions:
\(\mathrm{B}^{12} \longrightarrow \mathrm{C}^{12}+\mathrm{e}^{-}+\overline{\mathrm{v}}_{\mathrm{e}} \quad\),
\(\mathrm{N}^{12} \longrightarrow \mathrm{C}^{12}+\mathrm{e}+\mathrm{v}_{\mathrm{e}} \quad\),
\(\mathrm{C}^{12 *} \longrightarrow \mathrm{C}^{12}+\mathrm{Y}\)
(5.5.11) and (5.5.12) are allowed Gamow-Teller transitions with large energy releases, and \((5 \cdot 5.13)\) is a magnetic dipole transition. The presence of weak
magnetic effects in these decays would cause first-order forbidden transition effects, which we mentioned in 3.5. In the absence of all forbiden transition effects, we should obtain pure \(\mathrm{Fempi}_{\text {s }}\) spectra \(F\left(\mathrm{E}, \mathrm{E}_{0}\right)\), where E is the energy of the electron or positron, and \(E_{0}\) is the endpoint energy (see 3.4), for the \(B^{12}\) and \(\mathrm{N}^{12}\) decays. \(\mathrm{B}_{0}\) is slightly different for \(\mathrm{B}^{12}\) and for \(\mathbb{N}^{12}\). Experimentally, the decay spectra are of the form
\(N\left(E, E_{0}\right)=P\left(E, E_{0}\right)(1 \pm(8 / 3) \mathrm{aE})\),
or, dividing through by \(\mathrm{P}\left(\mathrm{E}, \mathrm{E}_{0}\right)\),
\(S\left(E, E_{0}\right)=N\left(E, E_{0}\right) / P\left(E, E_{0}\right)=(1 \pm(8 / 3) a E)\).
The term linear in \(E\) in \((5.5 .15)\) arises from interference between the axial vector interaction, which causes the allowed Gamow-Teller transition, and the vector interaction, which is responsible for first forbidden effects, if these are indeed present. For the decay (5.5.11) the last term in ( \(5 \cdot 5.15\) ) becomes
\((1-(8 / 3) a E)\),
and for (5.5.12):
\((1+(8 / 3)\) a \(E)\).
Weak magnetism should produce a nonzero coefficient a, which is connected to the bandwidth or uncertainty in the energy of the photon emitted in \(\mathrm{c}^{12 *}\) decay (5.5.13). By comparing the spectra in (5.5.11) and (5.5.12) we may obtain a value for \(a\). The ratio of the departures from the allowed transition spectra for these decays is given by
\(S\left(E, B^{12}\right) / S\left(E, N^{12}\right)=k .(1+A(1+\delta A) E) f(E)\),
whore \(f(\mathbb{E})\) is a correction for the inner Bremsstrahlung \({ }^{8}\) which unavoidably accompanies beta decay. \(f(E)\) is dependent upon the endpoint energy \(E_{0}\). Weak magnetism predicts the gradient of the line in ( \(5 \cdot 5.18\) ) to be \(\mathrm{A}=(1.33 \pm 0.15) \% \mathrm{KeV}^{-1}\).
The experimental value of A obtained by Kayer-Kuckuck and Kichel (21) was \(A \quad-\quad(1.13 \pm 0.25) \% \mathrm{KeV}^{-1}\).
Thus the prediction of weak magnetism is within experimental error limits. Similar experiments have also been conducted on the \(\mathrm{A}=8\) triad. Here again, the results favoured weak magnetism, but were not accurate enough to verify it. The second method for detecting weak magnetic effects is to study high-energy
muon absorption:
\(\mu^{-}+p \longrightarrow v_{\rho}+n\).
We write a symetric four-fermion Hamiltonian for the process (5.5.21):
\[
\begin{gather*}
\mathrm{H}_{\mathrm{I}}=(g / \sqrt{2}) \bar{\psi}_{\mathrm{n}}(x) \gamma_{r}\left(1+\gamma_{5}\right) \Psi_{\mathrm{P}}(x) \psi_{\mathrm{V}}(x)\left(1-\gamma_{5}\right) \gamma_{T} \psi_{\mu}(x)+ \\
\\
+ \text { Herm. conj., }
\end{gather*}
\]
and thus the matrix element becomes
\(n_{i f}=(g / \sqrt{2}) \bar{u}_{n}^{(+)}\left(p_{n}\right) Y_{r}\left(I+Y_{5}\right) u_{p}^{(+)}\left(p_{p}\right) \bar{u}_{v}^{(+)}\left(p_{v}\right)\left(I-Y_{5}\right) X\)
\[
x \quad Y_{T}{ }^{(+)}\left(p_{\mu}\right) \quad \int d^{3} x e^{j\left(p_{p}+p_{\mu}-n_{n}-p_{V}\right) x},
\]
assuming that all the particles taking part in (5.5.21) may be described by plane waves. The matrix element ( 5.5 .23 ) must be modified, since the muon in \((5.5 .21)\) is already bound to the proton. We assume that all particles in both the initial and final states are nonrelativistic, and, applying the transition rate formula (3.4.10) to our new matrix element, the lifetime for the capture process, \(T_{c}\), is given by
\(\frac{1}{T_{c}}=\frac{96 \pi \alpha^{3}}{T_{\mu}} \eta\left(1+3 x^{2}\right)\left[1+4 x \frac{1-x}{1+3 x} \sim(J(J+1)-(3 / 2))\right]\)
where \(\eta\) is a kinematical factor defined

and where \(J\) is the total spin of the muon and the capturing atom, \(T_{\mu}\) is the lifetime of the free muon (4.1.3), \(\propto\) is the fine structure constant, the universal coupling constant for the electromagnetic interaction, with value \(\alpha=e^{2} /\) h/c \(=1 / 137.03604(11)\),
and
\(x=\left(g_{A} / g_{V}\right)\).
If we had been able to assume that the proton and neutron masses vere equal, and that the muon mass was negligable compared with the nucleon, then the value of \(\eta\) would have been near to unity. Using experimental masses and substituting in (5.5.25) we find that
\(\eta=0.5786\),
so that we are not justified in making this assumption. Substituting experimental
values for the constants in ( 5.5 .24 ), we obtain
\(1 / \mathrm{T}_{\mathrm{c}}=123(4-2 J(\mathrm{~J}+1)) \quad \mathrm{s}^{-1}\).
Since the possible J assigrments are 0 and 1 , we have, for \(J=0\),
\(1 / T_{c}=492 \mathrm{~s}^{-1}\),
and for \(\mathrm{J}=1\) :
\(1 / T_{c}=0 \quad s^{-1}\),
assuming that
\(x=-1\),
as in ordinary muon decay. With (5.5.32), we predict (5.5.31) that no muon capture will occur from a \(J=1\) state. However, capture of this type does occur, and so we must consider in what way ( 5.5 .24 ) should be modified. Since axial vector current is not conserved under the influence of the strong interaction, we might expect our assumption (5.5.32) to be slightly incorrect. If we now set \(x\) equal to the ratio of axial vector to vector coupling constants observed in neutron decay, \((5.5 .29)\) becomes
\(1 / T_{c}=159(1-0.994(2 J(J+1)-3)) \mathrm{s}^{-1}\),
and thus for \(\mathrm{J}=0\),
\(1 / T_{c}=633 \mathrm{~s}^{-1}\),
and for \(\mathrm{J}=1\),
\(1 / T_{c}=1 \quad s^{-1}\).
However, ( 5.5 .33 ) is still not correct, since other strong interaction effects, which do not occur in neutron decay, affect the lifetime of muon capture. Taking account of these effecte, we obtain (22)
\(1 / T_{c}=169(1-0.945(2 J(J+1)-3)) \mathrm{s}^{-1}\),
so that
\[
\begin{array}{lll}
1 / T_{c}=636 & s^{-1} & (\mathrm{~J}=0)  \tag{5.5.37}\\
1 / \mathrm{T}_{\mathrm{c}}=13 & \mathrm{~s}^{-1} & (\mathrm{~J}=1) .
\end{array}
\]

We have assumed throughout that the muon is initially in a Bohr-type orbit around the proton. However, theoretical (23) and experimental (24) investigations have shown that the ionic bonding \((\mathrm{p} \mu \mathrm{p})^{+}\)is, in fact, more common than the simple muonic atom. Taking this possibility into account, Weinberg calculates (25), expressing the lifetimes for the two spin states together, that \(300 \mathrm{~s}^{-1} \leqslant 1 / \mathrm{T}_{\mathrm{c}} \leqslant 565 \mathrm{~s}^{-1}\).

Experiments give (26)
\(1 / T_{c}=480 \pm 70 \mathrm{~s}^{-1}\),
which is within the limits of Weinberg's prediction, once again favouring weak magnetism, which was used to determine coupling constants, but not producing conclusive evidence for it.

Finally, we consider the evidence for weak magnetism from hign-energy neutrino scattering:
\(\overrightarrow{\mathrm{v}}+\mathrm{p} \longrightarrow\) antilepton +n ,
\(\mathrm{v}+\mathrm{n} \longrightarrow\) lepton +p .
We may predict the differential cross-sections \({ }^{9}\) for the processes ( \(5 \cdot 5.41\) ) and (5.5.42) using the isovector electromagnetic form factors \(P^{V}\left(q^{2}\right)\) and \(G^{V}\left(q^{2}\right)\) and the weak form factors \(f_{A}\left(q^{2}\right), f_{V}\left(q^{2}\right), g_{A}\left(q^{2}\right)\), and \(g_{V}\left(q^{2}\right)\). Hich-energy electron-nucleon scattering experiments (26) have shown that
\(P^{V}\left(q^{2}\right)=\left(G^{V}\left(q^{2}\right)\right) /\left(G^{V}(0)\right)=1 /\left(1+\left(q^{2} / m_{V}^{2}\right)^{2}\right) \simeq\)
\[
\begin{equation*}
\simeq \varepsilon_{V}\left(q^{2}\right)=\left(i_{V}\left(q^{2}\right)\right) /\left(i_{V}(0)\right) \tag{5.5.43}
\end{equation*}
\]
where
\(m_{V} \sim 0.84 \mathrm{GeV}\).
The equation (5.5.43) has been verified up to a momentum transfer, q, of \(5 \mathrm{GeV} / \mathrm{c}\). From similar experiments, and from the so-called 'double-pole' model in dispersion theory, we have
\(\left(g_{A}\left(q^{2}\right)\right) /\left(g_{A}(0)\right)=1 /\left(1+q^{2} / m_{A}^{2}\right)^{2}\),
where \(m_{A}\) is a mass parameter which must be determined by experiment. By graphing inferred neutrino spectra, the best value for \(m_{A}\) is (27) \(m_{A}=0.8 \pm 0.15 \mathrm{GeV}\).
Within the limits of experimental error, \(m_{V}(5.5 .44)\) and \(m_{A}(5.5 .46)\) appear to be equal, demonstrating that, up to about 4 GeV , vector and axial vector form factors have a similar \(q^{2}\) dependence. Thus we see that the weak magnetic form factor is nonzero, in accordance with the hypothesis of weak magnetism. This constitutes the best experimental evidence in favour of weak magnetism obtained to date.

In 4.4 we saw that the charged lepton current may be defined
\[
\begin{align*}
J_{r}^{L}=j \psi_{\bar{v}_{e}} Y_{r}\left(1+\gamma_{5}\right) & \psi_{e}+j \psi_{\bar{v}_{\mu}} \gamma_{r}\left(1+\gamma_{5}\right) \psi_{\mu}+ \\
& + \text { Herm. conj. } \tag{5.6.1}
\end{align*}
\]

By electron-muon universality, we now write the semileptonic weak current as
\(J_{r}^{S}=j\left(V_{r}+A_{r}\right) J_{r}^{L}+\) Herm. conj.,
where \(V_{r}\) and \(A_{r}\) are the vector and axial vector pure hadron currents respectively. From (5.6.1) and (5.6.2), we deduce that
\(H^{L}+H^{S}=-(G / \sqrt{2})\left(J_{r}^{L}+J_{r}^{H}\right) J_{r}^{L}+\) Herm. conj..
where \({ }_{J_{r}}^{H}\) is the total \(\Delta Q=1\) hadron current:
\(J_{r}^{H} \quad a J_{r}^{0}+b J_{r}^{1}\),
a and b being real constants, and \(J_{r}^{0}\) and \(J_{r}^{1}\) the hypercharge-conserving and hypercharge-changing currents used in 5.3. As we shall see in chapter 8 , both \(J_{r}^{0}\) and \(J_{r}^{2}\) are members of the same \(\operatorname{SU}(3)\) octet, and this fact led Cabibbo to postulate (28) that
\(a^{2}+b^{2}=1\),
or
\(a=\cos \theta\),
\(b=\sin \theta\),
where \(\theta\) is the so-called 'Cabibbo angle'. The condition (5.6.5) is equivalent to the assumption that the sum of the squares of both vector and axial vector \(\Delta Y=0\) and \(\Delta Y=1\) coupling constants are equal to the square of the total pure hadron coupling constant. A number of arguments involving SU(3) may be used to justify (5.6.5), and these are discussed in chapter 8 . \((5.6 .6)\) and \((5.6 .7)\) allow us to write for the total weak current:
\(J_{r}=J_{r}^{I}+\cos \theta J_{r}^{0}+\sin \theta J_{r}^{1}\),
so that the total weak Hamiltonian (5.3.6) becomes
\(H_{I} \quad=\quad-(G / 2)\left(J_{r} J_{Y}\right)\).
One consequence of (5.6.9) is that, by choosing a suitable value for \(\theta\) : \(\theta \sim 0.2\),
we predict the \(2 \%\) reduction of \(V_{r}^{0}\) compared to \(J_{r}\) mentioned in 5.2 , and with the same choice \((5.6 .10)\) for \(\theta\), as demanded by the Cabibbo model, we may account for the reduction by a factor of 20 in semileptonic \(|\Delta Y|=1\) decays rates compared to hypercharge-conserving ones.

We now examine the possibility that the Hamiltonian (5.6.9) may explain pure hadronic as well as semileptonic weak interactions. Reactions between beryons and mesons, for example, would be described by the terms \(J_{r}^{0} \bar{J}_{r}^{1}+\bar{J}_{r}^{0} J_{r}^{1}\)
as well as by the self-current hadron interactions
\(\mathrm{J}_{r}^{0} \overline{\mathrm{~J}}_{r}^{0}\)
and
\(\mathrm{J}_{r}^{1} \mathrm{~J}_{r}^{1}\).
An interesting feature of the Cabibbo model is that it automatically predicts parity violation in hadronic processes, since \(J_{r}^{0}\) and \(J_{r}^{1}\) contain both vector and axial vector parts. If (5.6.9) is indeed precisely true, then it is evident that, if semileptonic processes are CP-violating, then so also must pure hadronic ones, and vice-versa. Similarly, CP conservation in one type of interaction implies the same effect in the other type. The attempted confirmation of this prediction is discussed in 7.4. If hadron interactions do occur through the term (5.6.11), then we might expect their coupling constant to be reduced by a factor of \(\sin \theta \cos \theta\), which is almost certainly not observed. Writing
\(J_{r}^{0}\left(I=1, I_{3}=1\right)=V_{r}^{0}(1,1)+A_{r}^{0}(1,1)\),
\(J_{r}^{1}\left(\frac{1}{2}, \frac{1}{2}\right)=V_{r}^{1}\left(\frac{1}{2}, \frac{1}{2}\right)+A_{r}^{1}\left(\frac{1}{2}, \frac{1}{2}\right)\),
we are forced to conclude that hadronic weak interactions involve
\(\Delta I=3 / 2\)
if they may be described by (5.6.11). Thus, unless the component (5.6.15) is aignificantly suppressed by strong interaction dynamics, a possibility considered below, the Hamiltonian (5.6.9) must be generalized so that it
includes also neutral hadron current terms. If this is the case, then the \(\Delta I=\frac{1}{2}\)
rule may be built into \(H_{I}\) by the replacement
\(J_{r}^{0}(1,1) \bar{J}_{r}^{1}\left(\frac{1}{2},-\frac{1}{2}\right) \longrightarrow J_{r}^{0}(1,1) \bar{J}_{r}^{-1}\left(\frac{1}{2},-\frac{1}{2}\right)-J_{r, 0}^{0}(1,0) \mathcal{J}_{r}^{1}, 0\left(\frac{1}{2}, \frac{1}{2}\right)\),
\(\bar{J}_{r}^{0}(1,-1) J_{r}^{1}\left(\frac{1}{2}, \frac{1}{2}\right) \longrightarrow \mathrm{J}_{r}^{0}(1,-1) J_{r}^{1}\left(\frac{1}{2}, \frac{1}{2}\right)-J_{r, 0}^{0}(1,0) J_{r, 0}^{1}\left(\frac{1}{2}, \frac{1}{2}\right)\),
where \(J_{r, 0}^{0}(1,0)\) and \(J_{r, 0}^{1}\left(\frac{1}{2},-\frac{1}{2}\right)\) are \(\Delta Y=0\) and \(\Delta Y=1\) neutral currents. We see that the currents \(J_{r}^{0}(1,1), J_{r, 0}^{0}(1,0)\) and \(J_{r}^{0}(1,-1)\) constitute an isovector (isospin vector), and \(J_{r}^{1}\left(\frac{1}{2}, \frac{1}{2}\right)\) and \(J_{r, 0}^{1}\left(\frac{1}{2},-\frac{1}{2}\right)\) form two components of an isospinor. Another method of ensuring the obeyence of the rule \((5.6 .16)\)
is to postulate that the combinations
\(J_{r, 0}^{\prime 0}(0,0) \bar{J}_{r, 0}^{1}\left(\frac{1}{2}, \frac{1}{2}\right)\),
\(J_{r, 0}^{0}(0,0) J_{r, 0}^{1}\left(\frac{1}{2},-\frac{1}{2}\right)\),
where
\({ }_{\mathrm{J}, 0}^{\prime 0}(0,0)\)
is an \(I=0, \Delta Y=0, \Delta Q=0\) current, are responsible for hadronic reactions. There is no experimental evidence at present in favour of neutral hadron currents, but \(\mathrm{SU}(3)\) symmetry does indicate that they should exist (see chapter 8). It is now possible to write a weak Hamiltonian whose \(|\Delta Y|=1\)
component obeys (5.6.16):
\(\underset{\text { where }}{H_{T}}=-(G / 2)\left(J_{r} \bar{J}_{r}-J_{r, 0} \bar{J}_{r, 0}\right)\),
\(J_{r}=J_{r}^{J}+a J_{r}^{0}(1,1)+b J_{r}^{1}\left(\frac{1}{2}, \frac{1}{2}\right)\),
\(J_{r}=\bar{J}_{r}^{L}+a \bar{J}_{r}^{0}(1,-1)+b \bar{J}_{r}^{1}\left(\frac{1}{2},-\frac{1}{2}\right)\),
\(J_{r, 0}=a J_{r, 0}^{0}(0,0)+a J_{r, 0}^{0}(1,0)+b J_{r, 0}^{1}\left(\frac{1}{2},-\frac{1}{2}\right)\),
\(\bar{J}_{r, 0}=a J_{r, 0}^{\prime 0}(0,0)+a J_{r, 0}^{0}(1,0)+b \bar{J}_{r, 0}^{1}\left(\frac{1}{2}, \frac{1}{2}\right)\).

However, until the neutral current strength has been accurately determined (see 6.8 ), the neutral lepton currents which could be added to \((5.6 .25\) )
and (5.6.26) might still be absent from the total Hamiltonian. Thus there exist two separate possibilities: first, the Hamiltonian contains only charged-current terms, and its \(\Delta I=3 / 2\) is suppressed by the strong interaction; and second, the Hamiltonian does, in fact, involve neutral lepton currents with a strength comparable to that of the charged lepton currents. We now examine the predictions of these two models. Both indicate weak \(\Delta Y=0\) hadron interactions. Assuming the Cabibbo form of universality (5.6.5), (5.6.9) becomes
\[
\begin{align*}
H_{I}(\Delta Y=0)= & -(G / \sqrt{2})\left(\cos ^{2} \theta\left(J_{r}^{0}(1,1) \bar{J}_{r}^{0}(1,-1)\right)+\right. \\
& \left.+\sin ^{2} \theta\left(J_{\Gamma}^{1}\left(\frac{1}{2}, \frac{1}{2}\right) J_{r}^{-1}\left(\frac{1}{2},-\frac{1}{2}\right)\right)\right), \tag{5.6.27}
\end{align*}
\]
whereas (5.6.22) yields
\(H_{I}(\Delta Y=0)=-(G / \sqrt{2})\left(\cos ^{2} \theta\left(J_{r}^{0}(1,1) \bar{J}_{r}^{0}(1,-1)-\left(J_{r, 0}^{0}(1,0) J_{r, 0}^{0}(1,0)+\right.\right.\right.\) \(+J_{r, 0}^{0}(1,0) J_{r, 0}^{0}(0,0)+J_{r, 0}^{0}(0,0) J_{r, 0}^{0}(1,0)+\) \(\left.\left.+J_{r, 0}^{0}(0,0) J_{r, 0}^{0}(0,0)\right)\right)+\sin ^{2} \theta\left(J_{r}^{1}\left(\frac{1}{2}, \frac{1}{2}\right) J_{r}^{1}\left(\frac{1}{2},-\frac{1}{2}\right)-\right.\) \(\left.\left.-J_{r, 0}^{1}\left(\frac{1}{2},-\frac{1}{2}\right) \bar{J}_{r, 0}^{1}\left(\frac{1}{2}, \frac{1}{2}\right)\right)\right)\).
\(\Delta Y=0\) weak interactions are usually difficult to study experimentally because they are masked by strong interactions with a much larger amplitude. However, weak effects do give rise to parity-violating nuclear transitions, so that a nuclear state \(K\) no longer possesses a well-defined parity, but consists of a mixture of parities:
\(|\mathrm{K}\rangle=\left|K^{ \pm}\right\rangle+e\left|K^{7}\right\rangle\),
where \(f\) is the parity-violating amplitude. From (5.6.27) or (5.6.28) we may predict (29)
\(f \sim 10^{-7}\).
Experiments use, for example, the fact that for the \(\gamma\) ray connecting the excited state of \(\mathrm{Ta}{ }^{181}\left(482 \mathrm{keV}, 5 / 2^{+}\right)\)to the \(7 / 2^{+}\)ground state, the normal ML transition matrix is suppressed by nuclear structure effects \({ }^{10}\) (30), so that the parity-violating El transition matrix produces an observable effect. Recent studies of the circular polarization in the gamma rays from \(\mathrm{Ta}^{181}\) and \(\mathrm{Lu}{ }^{175}\) indicate that (31)
\(f_{T a}=(0.4-4) \times 10^{-7}\),
\(f_{L u}=(2-8) \times 10^{-7}\),
in good agreement with the theoretical predictions for \(f\). It has been suggested that experiments on the isospin properties of \(H_{I}(\Delta Y=0)\) might provide some basis for a discrimination between the models (5.6.9) and (5.6.22). In the 'charged-current' theory, we note the \(\cos ^{2} \theta\) term contains only \(\Delta I=0\) and \(\Delta I=2\) components, while the \(\sin ^{2} \theta\) term involves only the \(\Delta I=1\) component. Thus this model predicts the ratio of the \(\Delta I=1\) amplitude to the \(\Delta I-0\) one to be in the order of \(\tan ^{2} \theta\). However, in the 'neutral-current' hypothesis (5.6.28), both the \(\Delta I=0\) and the \(\Delta I=1\) amplitudes are proportional to \(\cos ^{2} \theta\), and hence are of the same order of magnitude. Since
\(\sin \theta \sim 0.21\),
the strength of the \(\Delta I=1\) component is about 20 times greater in the 'neutral-current' model then in the 'charged-current' or ' \(\Delta I=\frac{1}{2}\)-enhancement' model. Thus, if isospin dependence in weak nuclear interactions could be detected, then we could decide between (5.6.27) and (5.6.28). No accurate measurements of the type necessary have yet been made, although a number of candidates, such as \(0^{16}\left(8.8 \mathrm{MeV}, 2^{-}, I=0\right) \longrightarrow C^{12}\left(0^{+}, I=0\right)\),
have been put forward as suitable reactions in which to observe significant isospin dependence.```

