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A Model for Parton Showers in QCD*

by

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ABSTRACT

A Monte Carlo model for the development of parton jets in QCD is described. Explicit low-order calculations are supplemented by leading logarithmic approximations for higher orders.

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Quarks and gluons generated by QCD processes involving large momentum transfers initially have large invariant masses ($\sqrt{t}$); as they travel outwards from their production point, they radiate other partons, which in turn radiate, until each has a small invariant mass ($\sqrt{t_c}$) and the original t has been converted into transverse momenta between partons in the jet. During most of this cascading process, one may neglect interference between the amplitudes for successive emissions: the spectra of radiated partons are adequately described by independent probability distributions, and the evolution of the jet may conveniently be simulated by an iterative Monte Carlo method. The model described below uses this technique to generate parton jets which exhibit essentially all known features of QCD final states, and yields direct and complete QCD predictions for suitably inclusive measurements at sufficiently high energies. To obtain further details, one must venture beyond perturbative QCD, and adopt a model for the formation of hadrons from parton jets. A simple (but phenomenologically accurate) model will be presented in a forthcoming paper [1], and provides complete predictions for hadronic final states (giving multijet events from large angle emissions, and scaling violations in parton energy spectra from multiple small angle emissions). We consider here primarily e^+e^- annihilation, although our methods may be applied to any other processes, including those with partons in the initial state (e.g., deep inelastic ℓN scattering or $\ell\bar{\ell}$ hadroproduction; high energy collisions on hadron or nuclear targets involving only small momentum transfers may perhaps also be treated).

Figure 1 illustrates the spacetime development of typical e^+e^- annihilation events at c.m. energies $\sqrt{s}$ = 20 GeV and $\sqrt{s}$ = 200 GeV generated by our Monte Carlo method. Any emissions at distances $O(\frac{1}{\sqrt{s}})$ (e.g., the first gluon radiated in the upper jet of Fig. 1a) are typically at large angles to the off-shell $q,\bar{q}$, so that they generate separated parton jets (Fig. 1a would probably be classified as a 'three jet event'): the probability for such emissions is $O(\alpha_s(s))$ and thus small. (The wavelengths of any large angle radiations in one jet typically encompass the other jet, so that interferences may occur; to account for this (in practice small) effect, we use the complete explicit $O(\alpha_s)$ differential cross-section for $\gamma^* \to q\bar{q}G$ to describe the first gluon emission in an e^+e^- annihilation event.) As discussed in Refs. [2,3,4], the rate of radiation from an off-shell parton at distances $\gtrsim 1/\sqrt{s}$ decreases roughly inversely with time [F.1] (so that emissions are very roughly equally spaced on the logarithmic longitudinal distance scale of Fig. 1), apart from a logarithmic rise from the effective coupling $\alpha_s(t) \simeq 1/(\beta_o \log(t/\Lambda^2))$ [F.2]. Here $\sqrt{t}$ is the invariant mass of the radiating parton and $\beta_o \simeq (33-2F)/12\pi \simeq 0.7$, with $F \simeq 3$ the number of light quark flavors. The partons are typically emitted at progressively smaller angles: their energies remain $O(\sqrt{s})$ but the typical c.m. distance between emissions approaches $O(\sqrt{s}/t_c)$, so that interferences are small, and the radiations may be treated independently and iteratively.

Perturbation theory suggests that an off-shell parton should always continue to emit progressively softer and more collinear radiation. However, an actual parton propagates only for a finite time before confinement acts and its jet condenses into hadrons: the perturbative evolution treated by our model is curtailed by hadronization when the parton invariant masses fall below a critical value $\sqrt{t_c}$, probably a few times Λ ($= O(0.5\ \mathrm{GeV})$) [F.3]. (The development in Fig. 1 has been cut off with $t_c \simeq 4\Lambda^2 \simeq 1\ \mathrm{GeV}^2$: the parton jets therefore extend a longitudinal c.m.s. distance $\sim (\sqrt{s}/t_c)\langle z\rangle$, where $\langle z\rangle\sqrt{s}$ is the mean 'final' parton momentum.) With some models for hadronization, earlier emissions (with $t > t_c$) may nevertheless be too soft or collinear to affect the structure of the hadron final state. Only emissions which will lead to distinguishable hadron states need be included in the Monte Carlo simulation

(c.f. the evolution of coarse-grained entropy in statistical mechanics). The cuts on elementary parton processes which censor irrelevant emissions depend in detail on the model for hadronization: we discuss several procedures below. Suitably coarse measurements on the final state (which, for example, consider only its energy distribution lumped into large angular bins) are, however, unaffected by the cuts (this insensitivity is the essential characteristic of an 'infrared stable' observable).

The development of a QCD jet from a large invariant mass quark or gluon by a cascade of independent emissions is analogous to the formation of an electromagnetic shower in matter from a high energy electron or photon [F.4]. An electromagnetic shower develops by successive independent Bremsstrahlung and pair production until the $e^{\pm}$ and γ produced fall below some critical energy (governed by inverse atomic sizes) when ionization losses dominate, and the cascade is absorbed (e.g., producing scintillation light). The longitudinal development of QED showers is conventionally treated by moments or master equations [5] just as for their QCD counterparts [F.5]; the transverse profile of a QED shower is, however, dominated by multiple Coulomb scattering, rather than transverse momenta imparted by emissions.

The leading logarithm approximation (LLA) of independent small angle emissions is best investigated using axial gauges $\eta.A = 0$ for the gluon propagator (so that gluon polarization vectors are orthogonal to the fixed vector η). In these gauges, the dominant Feynman diagrams have a simple iterative ladder structure (the squared amplitudes correspond to planar diagrams) as illustrated in Fig. 2a: the total probability is to LLA a product (with suitable kinematic convolutions) of the probabilities for each individual emission. In axial gauges, the LLA probability for a parton of type j_o with invariant mass $\sqrt{t}$ to propagate from its production and 'decay' into approximately massless partons of types j_1 and j_2 carrying (roughly) fractions z and (1-z) of its longitudinal momentum may be calculated directly (e.g., from the diagram of Fig. 2b), yielding ($\Lambda^2 << t << s$):

$$\frac{\alpha_s(t)}{2\pi t} P_{j_o \to j_1 j_2}(z) \qquad (z < 1) \tag{1}$$

where (f labels the F flavors of quarks, which are for now taken effectively massless) [F.6]

$$P_{q_f \to q_f G}(z) = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right) \tag{2a}$$

$$P_{G \to q_f \bar{q}_f}(z) = \frac{1}{2}\left(z^2+(1-z)^2\right) \tag{2b}$$

$$P_{G \to GG}(z) = \frac{6(1-z+z^2)^2}{z(1-z)}. \tag{2c}$$

The choice of η determines the precise interpretation of z: different choices extract probabilities (1) which typically differ by subleading log terms ($\sim 1/t_{max}$ rather than $\sim 1/t$, where $\sqrt{t_{max}}$ is the invariant mass of the preceding decay ($t_{max} \sim s$ for the first emission)). Taking η along the direction of one quark in e^+e^- annihilation is convenient in analytical calculations since it causes all radiation to originate from the other quark. This choice identifies z as the relative longitudinal Sudakov variable, or $E + \vec{p}_3$ fraction of each emitted parton with respect to its parent (which travels along the 3-axis). For the Monte Carlo model, we choose η symmetrically (e.g., along the γ^* direction) yielding equal radiation from each quark, and identifying z as the $E + |\vec{p}|$ fraction [F.7]. In the 'decay' shown in Fig. 2b, therefore, $1 \geq 1 - z \geq (E_o-|\vec{p}_o|)/(E_o+|\vec{p}_o|) = t_o/(E_o+|\vec{p}_o|)^2$ [F.8]. As mentioned above,

avoiding ultimately irrelevant emissions may give tighter constraints on z (e.g., if 'final' partons are to have $\sqrt{t} \gtrsim \sqrt{t_f}$, then $1 - z \gtrsim t_f/s$ always).

In addition to collinear 1/t divergences, the probabilities (1) exhibit further divergences when emitted gluons become soft. As usual, such soft divergences are canceled by virtual processes (for which the discontinuity in, e.g., Fig. 2b is taken through j_o rather than a real $j_1 j_2$ final state), which add to P(z) a divergent $\delta(1-z)$ term such as to give finite results for smoothly-weighted integrals over P(z). The running coupling constant $\alpha_s(t)$ in (1) arises in axial gauges from summing the LLA effects of higher order corrections on each external parton leg. For example, the $O(\alpha_s \log(t/\mu^2))$ corrections to the process $q \to qG$ in Fig. 2b arise from the sum of vacuum polarization insertions on the outgoing gluon leg (quark self energy and vertex correction diagrams cancel by the axial gauge Ward identity) and diagrams in which the produced gluon subsequently decays into two real gluons (or a $q\bar{q}$ pair). When integrated over the available phase space, the infrared divergences cancel between the real and virtual diagrams, but the upper limit t on the invariant mass of the real pair prevents complete cancellation, and introduces a term $\sim \beta_o \alpha_s \log(t/\mu^2)$ multiplied by the lowest order cross-section. In higher orders, the gluon undergoes self-energy corrections before decaying. The sum of the $[\alpha_s \log(t/\mu^2)]^k$ parts of these higher order corrections may be accounted for by an effective running $\alpha_s(t)$ in the lowest order form.

We now describe the iterative procedure for generating LLA parton showers, which has been implemented in a FORTRAN computer program (available from us). For simplicity, we here first assume that the only resolvable emissions are those which pass a cut $z_c \leq z \leq 1 - z_c$ much more stringent than any kinematical constraint; softer emissions serve only to degrade the parent invariant mass, and not to initiate independent jets. We consider a section of shower development illustrated in Fig. 2c. The probability that the parton j_o (which has maximum invariant mass $\sqrt{t_p}$) should evolve until it has $\sqrt{t} \leq \sqrt{t_c}$ (at which mass we terminate its radiation) emitting only unresolvably soft ($z \leq z_c$ or $z \geq 1 - z_c$) partons is given to LLA by a formula directly analogous to the structure function moment evolution equation for deep inelastic scattering:

$$\Pi_{j_o}(t_p,t_c) \simeq \sum_{k=0}^{\infty} \left(\frac{-1}{2\pi}\right)^k \int_{t_c}^{t_p} \frac{\alpha_s(t_1)dt_1}{t_1} \cdots \int_{t_c}^{t_{k-1}} \frac{\alpha_s(t_k)dt_k}{t_k}$$

$$\times \int_{z_c}^{1-z_c} P_{j_o \to all}(z_1)dz_1 \cdots \int_{z_c}^{1-z_c} P_{j_o \to all}(z_k)dz_k$$

$$= \sum_{k=0}^{\infty} \left(\frac{-1}{2\pi\beta_o}\right)^k \frac{1}{k!} \left[\log\left[\frac{\log(t_p/\Lambda^2)}{\log(t_c/\Lambda^2)}\right]\right]^k \left[\int_{z_c}^{1-z_c} P_{j_o \to all}(z)dz\right]^k$$

$$= \left[\frac{\alpha_s(t_c)}{\alpha_s(t_p)}\right]^{-\gamma_{j_o}(z_c)/\beta_o} \qquad (3a)$$

$$\gamma_i(z_c) \equiv \frac{1}{2\pi} \int_{z_c}^{1-z_c} P_{i \to all}(z)dz, \qquad (3b)$$

$$\gamma_i(0) = 0, \qquad (3c)$$

where we have used $\int \frac{[\log \log(t)]^k}{t \log(t)} dt = \frac{[\log \log(t)]^{k+1}}{k+1}$; the crucial $1/k!$ arises from the nesting of the t_i integrations. The effects of virtual diagrams have been included in (3a) by using (3c), which simply states that the total probability for all possible decays of i is one (to LLA). The first step in our Monte Carlo procedure is to determine whether j_o will emit resolvable radiation: if so, its t_o is chosen according to the distribution (1) modified by possible subsequent emissions (the probability for these is given by (3a) with t_c replaced by t_o): the resultant t_o distribution $\Xi_{j_o}(t_o)$ is given by

$$\Xi_{j_o}(t_o)dt_o \simeq \frac{\alpha_s(t_o)}{t_o} \gamma_{j_o}(z_c) \Pi_{j_o}(t_p,t_o)dt_o, \qquad (4)$$

which satisfies

$$\int_{t_c}^{t_p} \Xi_{j_o}(t)dt = 1 - \Pi_{j_o}(t_p,t_c)$$

This step is achieved by generating a random number ξ uniformly between 0 and 1, and then solving for t_o in

$$\xi = \left[\frac{\alpha_s(t_o)}{\alpha_s(t_p)}\right]^{-\gamma_{j_o}(z_c)/\beta_o} \quad ; \qquad (5)$$

if the resulting $t_o \leq t_c$, j_o emits no resolvable radiation. Having determined that j_o is to radiate, and selected its t_o, the type and momenta of its 'decay' products j_1 and j_2 are chosen according to (2); their $E + |\vec{p}|$ fractions satisfy $z_c \leq z \leq 1 - z_c$, and their momenta are distributed uniformly in azimuth. j_1 and j_2 are then evolved as was j_o, and the cascade continues until all partons have chosen to generate no further resolvable radiation with $t \geq t_c$.

The practical implementation of this procedure involves several complications. First, the actual range of z for visible emissions depends on t_o, and is not fixed, as assumed above. This is accounted for by initially choosing z_c in the procedure above to be the minimum over t_o. Then if the generated t_o,z lie outside the kinematic region, the procedure is repeated, but with t_p replaced by the first t_o generated. By this standard Monte Carlo method, the probability (3) is modified to account for the correct (but analytically complicated) kinematic region. In the LLA, the products of each parton 'decay' are formally approximated as massless (since strictly the t_j in LLA are strongly ordered $t_j \gg t_{j+1} \ldots$). Although the t_1,t_2 implied by the LLA are usually small, one must, in practice, account for the effects of t_1/t_o, $t_2/t_o \neq 0$ on the kinematic regions for emissions; the LLA provides no guidance as to the correct procedure (the problem does not yet exist for $\gamma^* \to q\bar{q}G$ at $O(\alpha_s)$: to investigate it, one must evaluate $\gamma^* \to q\bar{q}GG$ in an axial gauge, and it is quite possible that the optimal prescription may differ in higher orders and other processes). Here we use an ad hoc but physically reasonable prescription: results which are sensitive to the details of the procedure cannot in any case be trusted. We first assign z_1, z_2, and then allow t_1, t_2 to run only up to the corresponding kinematic limits ($t_c \leq t_{1,2} \leq z_{1,2} t_o$). (Another, but less satisfactory, procedure would be to take $t_{1,2} \leq t_o$, but to reassign $z_{1,2}$ if the generated $t_{1,2}$ are kinematically forbidden.)

The parton showers in Fig. 1 were generated according to the scheme described above (with $t_c = 1$ GeV^2); virtual partons were taken to survive for a time $\Delta t \sim 1/\Delta E \sim 1/(E-|\vec{p}|)$, while 'final' partons were drawn until the point at which hadrons form in the model of Ref. [1].

Figure 3 shows the distribution of parton final states from e^+e^- annihilation generated according to our procedure in the shape parameter H_2, where [8] (the sum runs over all pairs of final partons, including i = j)

$$H_\ell \equiv \sum_{i,j} \frac{E_i E_j}{s} P_\ell(\cos\phi_{ij}). \qquad (6)$$

For two-parton events, $H_{2\ell} = 1$, $H_{2\ell+1} = 0$, while for isotropic events, $H_\ell = 0$ ($\ell > 0$). The curves of Fig. 3 are insensitive to the value of t_c chosen (as expected from the infrared stability of H_2), except very close to $H_2 = 1$. The main effect of including multiple gluon emissions on the H_2 distribution is to soften the unphysical $\alpha_s \log(1-H_2)/(1-H_2)$ singularity obtained at $O(\alpha_s)$. Replacement of the exact $O(\alpha_s)$ cross-section for the first gluon emission by the LLA makes an indiscernible change to the H_2 distribution (and thus inspires hope as to the accuracy of higher order LLA). The $\langle H_2\rangle$ obtained from the

distributions of Fig. 3 are all very close to the $O(\alpha_s)$ result $\langle H_2\rangle \simeq 1 - 1.4\,\alpha_s(s)$. Results for the thrust observable [9] will be very similar to those obtained for H_2.

Three-parton final states must have $H_2 > 1/4$: to obtain smaller H_2 requires emission of at least two gluons. A direct measure of such four-parton final states is afforded by deviations from coplanarity, which give [10]

$$\Pi_1 = \sum_{i,j,k} (\vec{p}_i \times \vec{p}_j \cdot \vec{p}_k)/(\sqrt{s})^3 (\hat{p}_i \times \hat{p}_j \cdot \hat{p}_k) \neq 0 \quad (\Pi_1 = 2/9 \text{ for an isotropic event}).$$

For continuum e^+e^- annihilation $\langle\Pi_1\rangle \simeq \frac{8}{3}\left(\frac{\alpha_s}{\pi}\right)^2$, while on a heavy resonance (ζ) decaying to three gluon jets, $\langle\Pi_1\rangle \simeq 3(\alpha_s/\pi)$ [F.9].

A standard application of the LLA gives the evolution of parton energy spectra (z distributions) in jets as a function of $\sqrt{s}$. The usual results are recovered in our model by constraining all partons to be collinear. The energy spectra depend slightly on the cuts applied; typically the use of realistic kinematics is most important for the first emission, and changes $\langle z\rangle$ by $\sim \alpha_s(s)$.

To make complete predictions for observed jets, one must treat the formation of hadrons from parton showers at large times. The details of this process are yet unknown, but there are some theoretical and many phenomenological indications that it should be universal: sets of partons with particular quantum numbers and masses should fragment into hadrons in a manner independent of the process by which they were made. If the perturbative evolution of a parton shower is curtailed at comparatively short distances by a large t_c cut (say, $t_c \simeq 10$ GeV^2), then each 'final' quark should presumably form a jet of hadrons roughly like those in e^+e^- annihilation events at $\sqrt{s} \simeq \sqrt{t_c}$ (and analogously for gluons). For our model to be consistent, the final results for hadronic events must be independent of the value of t_c chosen (so long as it is neither too close to the original s nor to Λ^2). (This behavior is familiar from LLA investigations of single hadron distributions, where $\sqrt{t_c}$

is the 'reference mass' Q_o.) In attempting a less phenomenological approach to hadron production, it is interesting to reduce the value of t_c. Although perturbation theory cannot precisely describe the final stages of hadron formation, some properties of the parton system which it prepares may be relevant for hadronization. In particular, one may guess that sufficiently localized color singlet systems of partons should evolve and form hadrons independently [11] from the rest of the final state. A convenient method for identifying such systems is to associate a 'string' with each spinor color index (hence two strings per gluon). Then the final state may be specified as a collection of two-ended strings representing color singlet systems of partons. Figure 4 shows the distributions of invariant masses of such 'strings' in parton final states from e^+e^- annihilation, together with the invariant mass distributions for the nearest (but not necessarily color singlet) pairs of partons in the final state. The mass distributions are essentially independent of the original $\sqrt{s}$, and are power law damped above $m^2 \sim t_c$, so that most strings have $m \simeq \sqrt{t_c}$. (The distributions are, of course, sensitive to infrared cutoffs: other t_c simply scale them, but other forms of cut lead to different damping powers.) If $t_c \simeq 1\ \mathrm{GeV}^2$, then an attractively simple model for hadron formation would be to take each string to condense into hadrons isotropically in its c.m.s., with the final hadrons distributed according to the available phase space volumes. In this model, the mean final hadron multiplicity is proportional to the string multiplicity: using for the string decay parameters those determined from low-energy multiparticle production (c.f. Ref. [12]) gives total multiplicities agreeing with e^+e^- annihilation measurements over the range $5 \leq \sqrt{s} \leq 30$ GeV to within about 20%. Figure 5 shows the transverse momentum distributions for strings produced at various $\sqrt{s}$. The $\langle k_T \rangle$ grows only slowly with $\sqrt{s}$, but the tails on the distributions increase rapidly. The distributions

are sensitive to details of cuts only for $4k_T^2 \leq t_c$. The results for partons track those for strings at large k_T with a constant difference in overall normalization. Detailed predictions for hadron formation will be given in Ref. [1].

Finally, we discuss some extensions of our model. We have considered only emissions from outgoing partons: radiation from incoming partons approaching a collision may also be treated by our Monte Carlo methods. In that case, a radiating parton begins nearly on shell, but its invariant mass becomes progressively more spacelike because of emissions. To leading order, the energy of the incoming parton is shared by emissions just as for outgoing partons in eqs. (1,2).

We have considered only light quarks, with effective masses $m_{eff}^2 \ll t_c$. Heavy quarks (with masses $m_Q \gg \Lambda$) may be included by using massless kernels (1,2) but allowing the quarks to evolve only until $t \simeq m_Q^2$ (it is the mass m_Q of the heavy quarks, rather than finite propagation time, which regularizes small t infrared divergences). The adequacy of this prescription may be seen by comparing with explicit results for the lowest-order process $e^+e^- \to Q\bar{Q}G$.

Emission of hard γ, γ^*, W, Z may be included by adding suitable $O(\alpha_{em})$ kernels to (2); collinear divergences in γ emissions from quarks are cut off by hadron masses, so that $t_c \simeq \Lambda^2$ (a larger t_c would require inclusion of collinear γ in fragmentation functions), whereas for emission from leptons, $t_c \simeq m_\ell^2$. Many QED radiative corrections to cross-sections may be obtained by using a LLA (equivalent γ approximation) for soft photon emissions from leptons and hadrons: our Monte Carlo method may also be used directly to simulate both QED and QCD corrections in, for example, deep inelastic scattering. In this way, one may clarify the effects of QED radiative corrections on scaling violations, previously treated in a less satisfactory manner (e.g., [13]).

To complete the extension of our Monte Carlo model to the first order beyond the LLA, we must include not only the exact (process-dependent) cross-section for the first gluon emission, but also process-independent corrections to each subsequent gluon emission probability, together with corrections accounting for correlations between successive emissions. These effects may be accounted for directly in our Monte Carlo model by calculating and introducing the probabilities (1,2a) for the 'decay' of an off-shell quark into an on-shell quark and one or two gluons (or a $q\bar{q}$ pair) obtained at order α_s^2/t (the leading terms at $O(\alpha_s^2)$ are $O(\alpha_s^2 \log(t/\mu^2)/t)$, but simply represent the iteration of two LLA kernels). These subleading log corrections are essential in determining the optimal choices of kinematic variables and of Λ. The corrections to the total e^+e^- annihilation cross-section from use of an exact, rather than LLA, cross-section for the first gluon emission are rather small. In several other processes, however, the corrections are large. Nevertheless, it is found (e.g., [14]) that they arise primarily from kinematic configurations close to the lowest order one, and therefore do not significantly affect the structures of the final states, but only the total cross-section for the process.

We are grateful to R. D. Field for discussions.

Footnotes

[F.1] c.f. the frequency spectrum of Bremsstrahlung from an instantaneously accelerated classical electric charge.

[F.2] The optimal value of Λ must, as always, be determined by calculation of subleading log terms. It is quite possible that successive emissions involve slightly different effective Λ.

[F.3] In QED, radiation emitted sufficiently late could not be resolved by any practical detector.

[F.4] Low transverse momentum multihadron production processes in which the incident high energy partons are essentially on-shell may perhaps be more directly analogous to electromagnetic showers.

[F.5] After many progressively more contorted analytical attacks, investigations of QED showers finally turned to Monte Carlo techniques [6] which continue to prove fruitful [7]. Perhaps similar development may be traced in QCD showers.

[F.6] Explicit calculation of the leading log ($O(1/t)$) part of Fig. 2(b) gives eq. (2). We assume that all partons are unpolarized; polarizations of quarks and gluons may be treated using suitable spin-dependent kernels.

[F.7] The possibility of this dissection to $O(\alpha_s)$ is revealed by the identity $\frac{1}{(1-x_1)x_3} + \frac{1}{(1-x_2)x_3} = \frac{1}{(1-x_1)(1-x_2)}$, where $x_3 = 2 - x_1 - x_2$ is the gluon fractional energy and $x_{1,2}$ the quark fractions.

[F.8] The equality of the integrated cross-sections to $O(\alpha_s)$ in the two gauges with their corresponding z identifications (quark $E + \vec{p}_3$ fractions z_i satisfying $(1-z_i) \propto (1-x_i)$; gluon $E + |\vec{p}|$ fraction $\propto x_3$) is manifest from

$$\int_0^1 dx_1 \int_{1-x_1}^1 dx_2 \frac{1}{(1-x_1)(1-x_2)} = \int_0^1 dx_1 \int_{1-x_1}^1 dx_3 \frac{1}{(1-x_1)x_3}$$

$$+ \int_0^1 dx_2 \int_{1-x_2}^1 dx_3 \frac{1}{(1-x_2)x_3}$$

[F.9] The possibility of the decay $\zeta \rightarrow GGGG$ giving $\langle \Pi_1 \rangle = O(\alpha_s)$ is a direct test of the non-Abelian nature of the gluon couplings.

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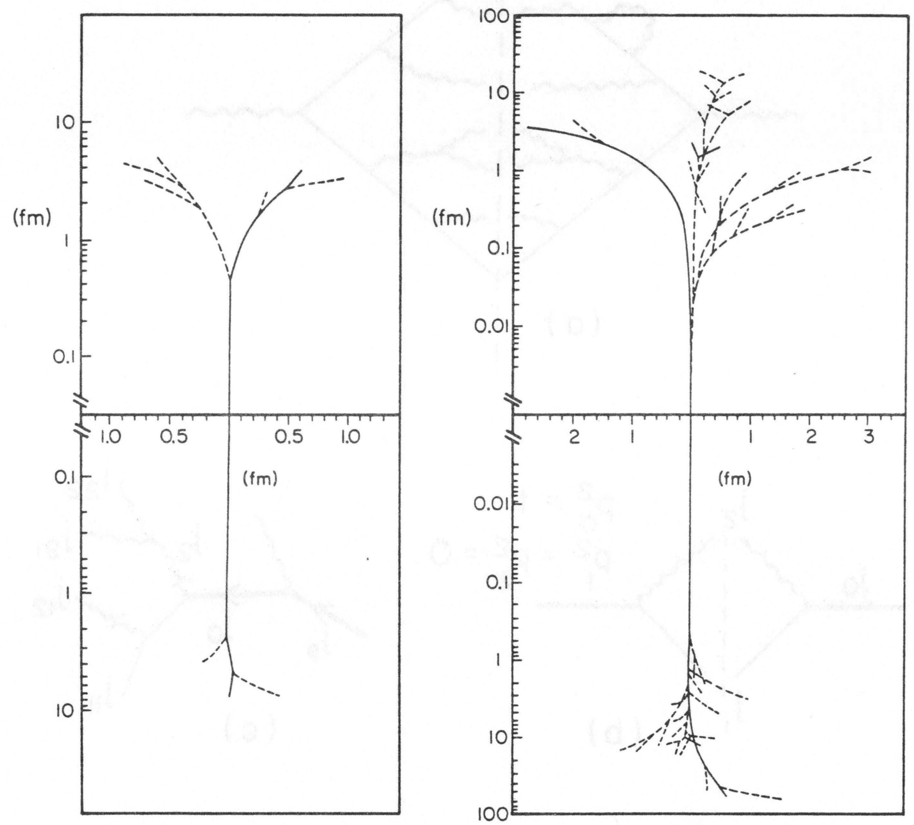
Figure Captions

1. A projection of two typical e^+e^- events evolved to a cutoff mass $\sqrt{t_c}$ = 1 GeV/c. Note that the semi-logarithmic scale causes both a suppressed origin and the curved paths for the evolution of both free and virtual partons. The "coherence" region over which the partons initially produced by the virtual photon are spread would appear as a narrow ellipse of width $O(1/\sqrt{s})$ which is not shown.
2. (a) A typical diagram contributing at the LLA in an axial gauge.

   (b) The diagram described by eq. (2a).

   (c) The section of a parton shower discussed in the text.
3. H_2 distributions calculated by our Monte Carlo method for various e^+e^- c.m.s. energies $\sqrt{s}$ (with a cutoff mass $\sqrt{t_c}$ chosen as 1 GeV). The $O(\alpha_s)$ result from $e^+e^- \to q\bar{q}G$ alone is also shown.
4. The mass distributions for color singlet "strings" and pairs of partons nearest in angle in simulated e^+e^- annihilation events with $\sqrt{s}$ = 10, 20 and 200 GeV.
5. Transverse momentum distributions for color singlet "strings" with respect to the initial $q,\bar{q}$ direction calculated by our Monte Carlo method at various c.m. energies $\sqrt{s}$.

SPACETIME DEVELOPMENT OF TYPICAL PARTON SHOWERS $\sqrt{t_c}$ = 1 GeV



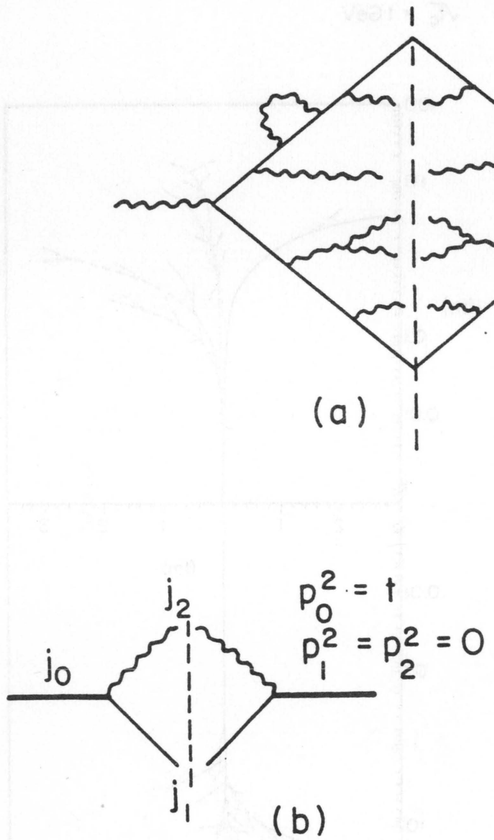
$\sqrt{s}$ = 20 GeV

(a)

$\sqrt{s}$ = 200 GeV

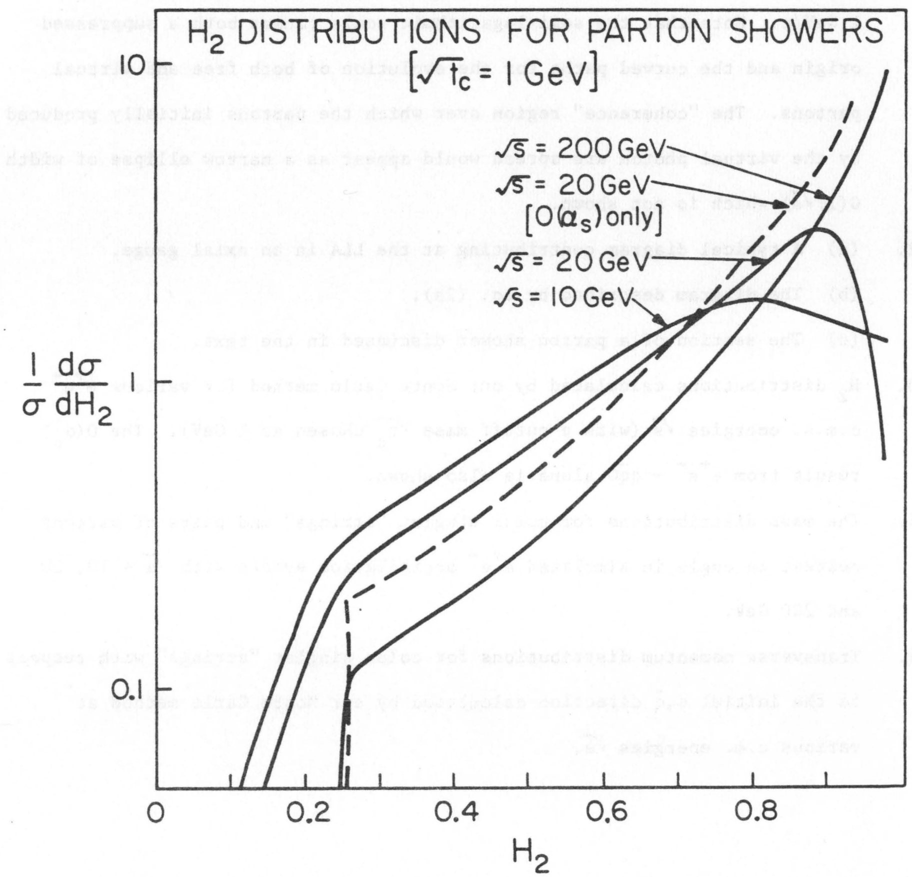
(b)

Fig. 1



(a)
j_0
j_2
j_1
$p_0^2 = t$
$p_1^2 = p_2^2 = 0$
(b)
j_{22}
j_2
j_{21}
j_{12}
j_p
j_0
j_{11}
(c)

Fig. 2



H_2 DISTRIBUTIONS FOR PARTON SHOWERS
$[\sqrt{t_c} = 1\text{ GeV}]$
$\sqrt{s} = 200\text{ GeV}$
$\sqrt{s} = 20\text{ GeV}$
$[O(\alpha_s)\text{ only}]$
$\sqrt{s} = 20\text{ GeV}$
$\sqrt{s} = 10\text{ GeV}$
$\frac{1}{\sigma}\frac{d\sigma}{dH_2}$
10
1
0.1
0
0.2
0.4
0.6
0.8
H_2

Fig. 3

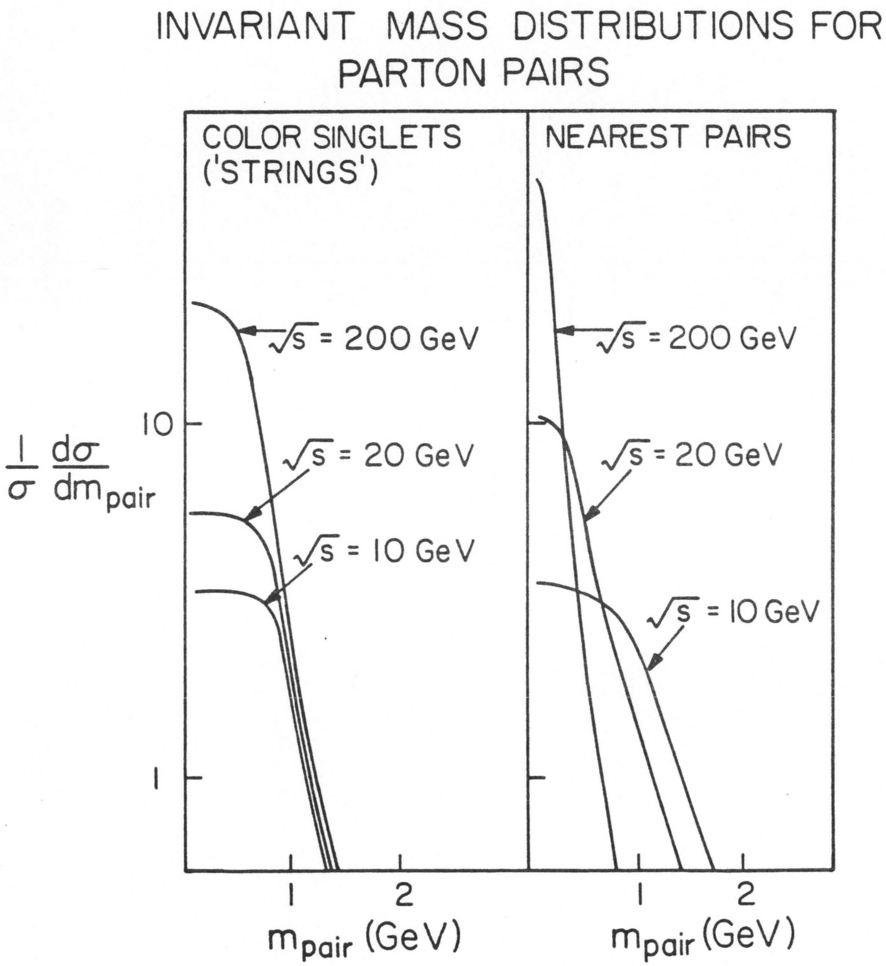

INVARIANT MASS DISTRIBUTIONS FOR PARTON PAIRS
COLOR SINGLETS ('STRINGS')
NEAREST PAIRS
$\sqrt{s}$ = 200 GeV
$\sqrt{s}$ = 20 GeV
$\sqrt{s}$ = 10 GeV
$\sqrt{s}$ = 200 GeV
$\sqrt{s}$ = 20 GeV
$\sqrt{s}$ = 10 GeV
10
1
$\frac{1}{\sigma}\frac{d\sigma}{dm_{pair}}$
1
2
1
2
m_{pair} (GeV)
m_{pair} (GeV)

Fig. 4

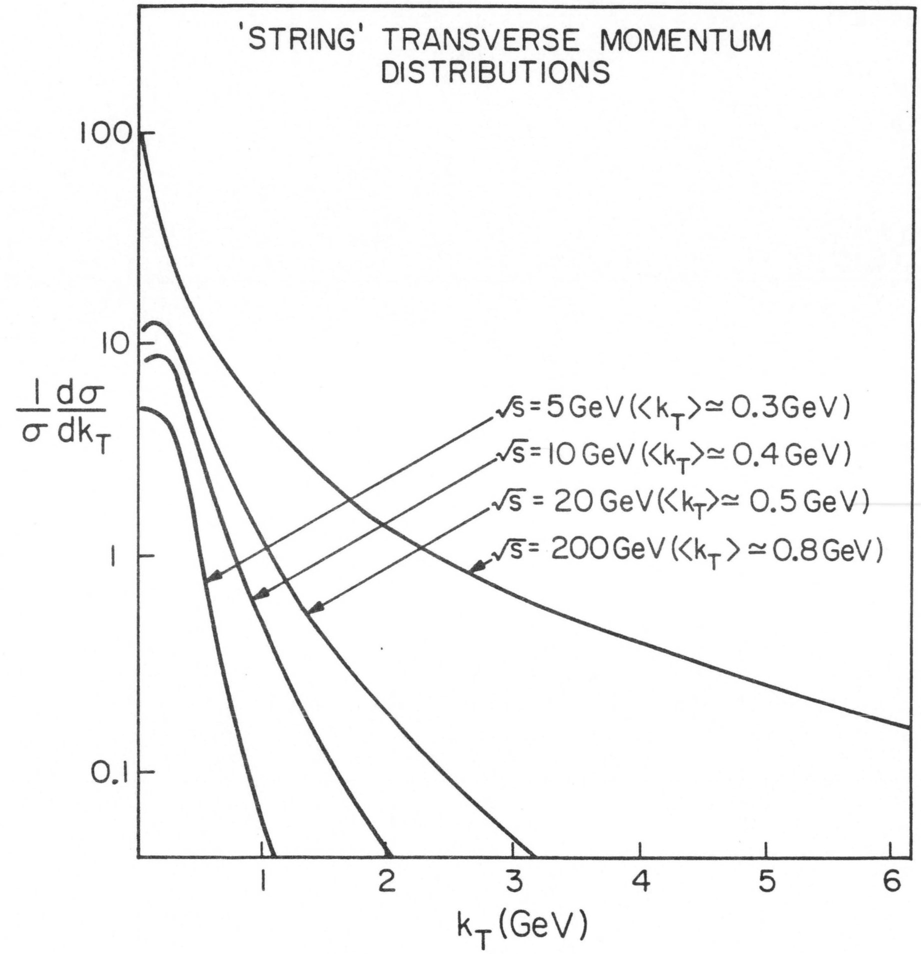

'STRING' TRANSVERSE MOMENTUM DISTRIBUTIONS
100
10
1
0.1
$\frac{1}{\sigma}\frac{d\sigma}{dk_T}$
$\sqrt{s}$ = 5 GeV ($\langle k_T \rangle \simeq$ 0.3 GeV)
$\sqrt{s}$ = 10 GeV ($\langle k_T \rangle \simeq$ 0.4 GeV)
$\sqrt{s}$ = 20 GeV ($\langle k_T \rangle \simeq$ 0.5 GeV)
$\sqrt{s}$ = 200 GeV ($\langle k_T \rangle \simeq$ 0.8 GeV)
1
2
3
4
5
6
k_T (GeV)

Fig. 5