Nuclear Physics B117 (1976) 109–133


# NEUTRAL WEAK INTERACTIONS AND PARTICLE DECAYS

Stephen WOLFRAM ★
*Theory Division, Rutherford Laboratory, Chilton, Didcot, UK*

Received 2 August 1976
(Revised 27 September 1976)

We discuss decays in which neutral weak interaction effects may be observed. We concentrate on pseudoscalar meson decays, including $P \to \ell^+\ell^-$, decays involving $\bar{\nu}\nu$ and $\eta \to \pi^0\ell^+\ell^-$. We also discuss vector and true scalar meson decays and baryon decays. Our conclusion is that it is difficult, but not impossible, to deduce the form of the neutral weak interactions from measurements on particle decays.

## 1. General introduction

Neutral weak interactions have now been observed experimentally in $\nu$N and $\nu$e scattering. It is important to study their form to see, for example, whether they can be described by current-current Lagrangians, and combined with charged weak currents and electromagnetism in a gauge theory. Data on $\nu$N and $\nu$e interactions cannot, however, distinguish between a wide variety of possible combinations of Dirac interactions [1]. Other sources of information are therefore desirable.

Stellar [2] and atomic [3] effects have been considered, but any results are marred by the difficulties of the ancillary physics. The weak correction to the muon $g$-factor is expected to be only about 1% of the magnitude of the hadronic corrections [4]. Atomic effects are quite promising, and have shown [5] that *CP* violation is absent to $10^{-3}\, G_F$, but any results which are obtained in this manner may hold only at very low energies.

All present data is consistent with the Weinberg-Salam model and $\sin^2\theta_W = 0.35$ [6]. We shall often give the predictions of this theory below.

In the present paper, we consider what can be learned about neutral weak interactions (NWI) from a study of particle decays.

★ Present address: St. John's College, Oxford.

## 2. Pseudoscalar meson decays. General

In any SU($N$) scheme, if the only internal quantum number which NWI can change is isospin, then there will be no decays involving single charged mesons which proceed purely by NWI. It is, however, by no means proven that $\Delta C = 0$ ($C$ denotes charm) in NWI. We discuss some implications of $\Delta C \neq 0$ NWI in subsect. 6.4. There can, of course, be NWI parts in various charged-current decays: these are considered in subsect. 6.3.

Probably the most promising pseudoscalar meson decays in which to detect NWI are those of $\eta$ and $\pi^0$. We list the experimentally more accessible of these, together with experimental limits on their branching ratios, and the sections in which we shall discuss them: (P denotes $\pi^0$ or $\eta$ and $\ell^\pm$, $e^\pm$ or $\mu^\pm$).

| Decay | Experimental limit | Ref. | Section |
|---|---|---|---|
| $P \to \bar{\nu}\nu$ | | | 4.2 |
| $\pi^0 \to e^+e^-$ | $< 8 \times 10^{-6}$ | [7] | 3 |
| $\eta \to e^+e^-$ | $< 3 \times 10^{-4}$ | [7] | 3 |
| $\eta \to \mu^+\mu^-$ | $(2.2 \pm 0.8) \times 10^{-5}$ | [8] | 3 |
| $\pi^0 \to \gamma e^+e^-$ | $(1.17 \pm 0.05) \times 10^{-2}$ | [8] | 6.1 |
| $\pi^0 \to \gamma\gamma\gamma$ | $< 5 \times 10^{-6}$ | [8] | 6.1 |
| $\eta \to \gamma e^+e^-$ | $(2 \pm 0.5) \times 10^{-3}$ | [9] | 6.1 |
| $\eta \to \gamma\mu^+\mu^-$ | | | 6.1 |
| $P \to \gamma\bar{\nu}\nu$ | | | 4.3 |
| $\eta \to \pi^0 e^+e^-$ | $< 4.5 \times 10^{-5}$ | [9] | 5 |
| $\eta \to \pi^0\mu^+\mu^-$ | $< 5 \times 10^{-4}$ | [8] | 5 |
| $\eta \to \pi^0\bar{\nu}\nu$ | | | 4.4 |
| $P \to \ell^+\ell^-\bar{\nu}\nu$ | | | 4.5 |

The decays of $\eta_c^{(\prime)}$ and $\eta'(958)$ could also furnish information on NWI, but we do not expect production of sufficient numbers of these in the foreseeable future to allow measurements of rare decay modes, and so we do not consider them in detail.

NWI effects might also be detectable in $P \to \gamma\gamma$ (see subsect. 6.1) and in $P \to$ hadrons (see subsect. 6.4). The production reactions $e^+e^- \to P$ could also be helpful. These we discuss in subsect. 3.5.

## 3. Pseudoscalar meson decays to charged lepton pairs

### 3.1. Introduction

Gauge invariance forbids one-photon intermediate states in $P \to \ell^+\ell^-$; the largest electromagnetic contributions are of order $\alpha^2$. Thus NWI may be detectable here. In the past $K^0 \to \mu^+\mu^-$ has been studied extensively [10], and stringent bounds set on the strength of $\Delta S \neq 0$ NWI. The analysis of $K^0 \to \mu^+\mu^-$ differs from that for $P \to \ell^+\ell^-$ since it has no electromagnetic component (except through higher-order weak interactions).

Experimentally, the only decay $P \to \ell^+\ell^-$ to have been observed is $\eta \to \mu^+\mu^-$, and this in only one experiment [11] with few events. Measurements of $\pi^0 \to e^+e^-$ are currently being attempted [12] which should be sensitive to $B \sim 5 \times 10^{-8}$, close to the theoretical lower bound. We find that pseudoscalar NWI could probably be detected if they occur in $\pi^0 \to e^+e^-$.

### 3.2. The matrix element

In order to treat the NWI component of $P \to \ell^+\ell^-$, we ignore the neutral weak intermediate boson (Z) propagator and assume a general local first-order interaction. Thus

$$M_\mathrm{w} = \bar{u}(P)[S + V\gamma_\lambda \Sigma^\lambda + T\sigma_{\kappa\lambda}\Sigma^\kappa\Sigma^\lambda$$

$$+ A\gamma_\lambda\gamma_5\Sigma^\lambda + \Pi\gamma_5 + \Lambda\gamma_5\sigma_{\kappa\lambda}\Sigma^\kappa\Sigma^\lambda]\, v(\bar{P}) ,$$

$$P = P_{\ell^-} ,\ \bar{P} = P_{\ell^+} ,\ \Sigma = P + \bar{P} , \tag{3.1}$$

reducing to

$$M_\mathrm{w} = \bar{u}(P)[S - 2Am\gamma_5 + \Pi\gamma_5]\, v(\bar{P}) ,$$

$$m = m_\ell . \tag{3.2}$$

The dimensions of the coupling constants are such as to be directly comparable with charged current ones: $[S] = [1]$, $[V] = [m^{-1}] = [G_\mathrm{F} f_\pi]$, and so on. There is no loop momentum integration in $M_\mathrm{w}$ so we need know the $\pi$ weak form factor only on its mass shell. $CP$ invariance implies $S = 0$.

An interesting test of $CP$ invariance could be achieved by observing the electron asymmetries from the decays of $\mu^\pm$ in $\eta \to \mu^+\mu^-$. Lee's model [13], however, predicts $CP$ (and $P$) violation in Higgs' particle interactions, and thus $CP$ violation here. Invariance under $CP$ demands a $^1S_0$ final state, and, assuming no final-state interactions, violation of this symmetry would be manifest [46] in $\langle \sigma_{\mu^+} \times \sigma_{\mu^-} \cdot P\rangle \neq 0$ or in $\langle \sigma_\mu \cdot P_\mu \rangle \neq 0$. There can be no $P$ violation (a signal for NWI) without $CP$ violation. Nevertheless, net lepton helicities could be an important effect if scalar interactions take part in the decay. $CPT$ invariance provides no useful constraints in these processes.

The $\gamma\gamma$ intermediate state in $P \to \ell^+\ell^-$ has been discussed by a number of authors; a clear survey is given by Quigg and Jackson [14]. They take ($P\rho\omega$ model)

$$M(P \to \gamma\gamma) = \frac{f\epsilon^{\lambda\mu\nu\sigma} k_{1\lambda} k_{2\mu} e_{1\nu} e_{2\sigma}}{i[k_1^2 + m_\mathrm{v}^2]\,[k_2^2 + m_\mathrm{v}^2]} , \tag{3.3}$$

where $k_i$ and $e_i$ are the photon momenta and polarizations and $m_\mathrm{v}$ is the mass of the vector meson which is assumed to saturate them. This yields ($M$ is the pseudoscalar meson mass, $m$ the lepton mass)

$$\Gamma(\mathrm{P} \to \gamma\gamma) = \Gamma_{\gamma\gamma} = \frac{f^2 M^3}{16\pi m_\mathrm{v}^8} . \tag{3.4}$$

For $\mathrm{P} \to \ell^+ \ell^-$ we have

$$M = \overline{u}(P)[\theta\gamma_5 + C\gamma_5]\, v(\overline{P}) , \tag{3.5}$$

where $\theta\gamma_5$ is the $\gamma\gamma$ intermediate state component [14]

$$\theta\gamma_5 = \frac{f\alpha}{(2\pi)^4} \int \frac{\mathrm{d}k(4mk^2 - 2mk\cdot\Sigma + i\gamma\cdot k(M^2 + 2k\cdot\Sigma))\gamma_5}{k^2(k^2 + m_\mathrm{v}^2)[(\Sigma - k)^2 + m_\mathrm{v}^2](\Sigma - k)^2[(k - \overline{P})^2 + m_\mathrm{v}^2]} , \tag{3.6}$$

and $k$ the photon loop momentum, $\Sigma$ the pseudoscalar meson momentum, and $P$ and $\overline{P}$ the lepton and antilepton momenta. We work in the frame $\Sigma = 0$. $k$ is the photon loop momentum. The weak component is

$$C\gamma_5 = (\Pi - 2mA)\gamma_5 . \tag{3.7}$$

In helicity representation,

$$M_{\lambda\overline{\lambda}} = \left(4\pi f\alpha \frac{m}{M} I + MC\right)(-1)^{\overline{\lambda} - 1/2}\delta_{\lambda\overline{\lambda}} ,$$

$$I = \frac{(Y - iX)M^2}{8\pi m_\mathrm{v}^4} , \tag{3.8}$$

where $Y$ is the absorptive part, $X$ the dispersive part of the pure electromagnetic contribution. Hence

$$\Gamma(\mathrm{P} \to \ell^+ \ell^-) = \Gamma = \left\{ 2\alpha^2 \Gamma_{\gamma\gamma} \frac{m^2}{M^2} [X^2 + Y^2] \right.$$

$$\left. + \frac{\alpha m \sqrt{\Gamma_{\gamma\gamma}}}{2\sqrt{\pi M}} \mathrm{Re}[(Y + iX)C] + \frac{M|C|^2}{8\pi} \right\} \left[1 - \frac{4m^2}{M^2}\right]^{1/2} . \tag{3.9}$$

The first term is pure electromagnetic, the third pure weak, and the second an interference term. We discuss numerical estimates in subsect. 3.4. We find that for many values of $C$, the interference term tends to be the smallest, since it involves a lepton mass factor relative to the pure weak term.

Since we have assumed $m_\mathrm{Z}^2 \gg M^2$, the Z in $\mathrm{P} \to \ell^+ \ell^-$ must be off-shell, and thus contributes only to the dispersive part of the amplitude. The rate (3.9) is given by

$$\Gamma = \left\{ 2\alpha^2 \Gamma_{\gamma\gamma} \frac{m^2}{M^2} [X^2 + Y^2] \pm \frac{\alpha m \sqrt{\Gamma_{\gamma\gamma}}}{2\sqrt{\pi M}} X|C| + \frac{M|C|^2}{8\pi} \right\} \left[ 1 - \frac{4m^2}{M^2} \right]^{1/2} . \tag{3.10}$$

The form of this expression requires an accurate knowledge of $X$, since any NWI can always be confused with errors in this quantity. The value of $Y$ is nearly model-independent, since $A(\mathrm{P} \to \gamma\gamma)$ is known, but the calculation of $X$ necessitates a knowledge of the off-shell behaviour $A(\mathrm{P} \to \gamma_\mathrm{v}\gamma_\mathrm{v})$. We discuss these in the appendix.

### 3.3. Modifications to the matrix element

We discuss in appendix A.1 the effect of the $\pi\pi\gamma$ intermediate state on $Y$ in the case $\mathrm{P} = \eta$. Other decay products need not be considered, since they contribute to $Y$ only at higher orders in $\alpha$. The $\gamma\gamma\gamma$ intermediate state is forbidden by $C$ invariance. We shall discuss $\mathrm{P} \to \gamma\gamma\gamma$ in subsect. 6.1. The $\gamma\gamma\gamma\gamma$ and $\mathrm{W}^+\mathrm{W}^-$ intermediate states may safely be ignored.

The Higgs scalar boson ($\phi$) could lead to an interesting term. It appears in any theory in which the Higgs mechanism is used to break the gauge symmetry. This may also be achieved with quantum corrections to the classical potential (requiring no physical Higgs particle). The coupling of a single $\phi$ to other particles is given generally by

$$H^{\phi}_{\lambda\mu} = H\overline{\Psi}_\lambda(\alpha + \beta\gamma_5)\Psi_\mu\phi , \tag{3.11}$$

where $\lambda$, $\mu$ denote particle types.

For $\lambda \neq \mu$, a non-ghost $\phi$ can, in principle, be charged. It should then, however, contribute significantly to weak semileptonic decays, allowing a stringent bound to be put on its mass. We shall not do this here, since we know of no convincing models which involve a charged $\phi$.

For $\lambda = \mu$, *CP* invariance constrains $\alpha = 0$ or $\beta = 0$. Choosing $\alpha \neq 0$ allows a pseudoscalar $\phi$ to contribute to pseudoscalar meson decays at first order. It is possible to construct models in which this occurs. A scalar $\phi$, however, cannot affect $J^{PC} = 0^{-+}$ meson decays, simply because of *CP* invariance. This is the case in the Weinberg-Salam model. We shall discuss the role of $\phi$ further in subsect. 6.5.

### 3.4. Numerical estimates

Since the weak component of $\Gamma(\mathrm{P} \to \ell^+\ell^-)$ is purely dispersive, it cannot reduce any of the unitarity limits for these decays (values of (3.10) for $X = 0$):

$$B(\pi^0 \to e^+e^-) > 4.78 \times 10^{-8},$$

$$B(\eta \to e^+e^-) > 1.72 \times 10^{-9},$$

$$B(\eta \to \mu^+\mu^-) > 4.2 \times 10^{-6},$$

$$B(\eta'(958) \to \mu^+\mu^-) \gtrsim 1.2 \times 10^{-7},$$

$$B(\eta_c'(3450) \to \mu^+\mu^-) > 10^{-6} B(\eta_c'(3450) \to \gamma\gamma)\,. \tag{3.12}$$

Experimental violation of any of these bounds does not appear to be comprehensible in the framework of present theory. It would require both that *CP* is violated, and that the decaying meson is not an eigenstate of *CP* (as in the $K^0$ case), in contradiction with, for example, the quark model.

The model which we have assumed above leads to $X(\pi^0 \to e^+e^-) \sim 3.3$, a value rather insensitive to $m_v$, which is satisfactory. We also obtain $X(\eta \to \mu^+\mu^-) \sim 2.3$, $X(\eta \to e^+e^-) \sim 9$. Ignoring NWI, we than have

$$B(\pi^0 \to e^+e^-) = 6.2 \times 10^{-8},$$

$$B(\eta \to e^+e^-) = 4.4 \times 10^{-9},$$

$$B(\eta \to \mu^+\mu^-) = 1.2 \times 10^{-5} \tag{3.13}$$

$$B(\eta'(958) \to \mu^+\mu^-) = 4 \times 10^{-7},$$

$$B(\eta_c'(3450) \to e^+e^-) \simeq 10^{-9}\, B(\eta_c'(3450) \to \gamma\gamma)\,,$$

$$B(\eta_c'(3450) \to \mu^+\mu^-) = 6 \times 10^{-6}\, B(\eta_c'(3450) \to \gamma\gamma)\,. \tag{3.14}$$

Experimental information on these decays (given in sect. 2) is so far rather scanty, and so no good bounds on the strength of the NWI can be obtained: $\eta \to \mu^+\mu^-$ yields* $C \lesssim 3 \times 10^{-5}$; $\pi^0 \to e^+e^-$, $C \lesssim 4 \times 10^{-6}$; $\eta \to e^+e^-$, $C \lesssim 7 \times 10^{-4}$. We should not, however, ignore weak corrections especially in $\pi^0 \to e^+e^-$ and $\eta \to e^+e^-$: *both need only* $C \sim 10^{-7}$ *for weak and electromagnetic components to be comparable.* Such a value ($\sim G_F M^2 \sim G_F f_P^2$) would be expected for pseudoscalar NWI. Fig. 1 contains a plot of $B(\pi^0 \to e^+e^-)$ against $C$.

A measurement of $B(\eta \to \mu^+\mu^-)/B(\eta \to e^+e^-) \equiv R$ could yield much information on both electromagnetic and weak components in these decays. The P $\rho\omega$ model predicts $R \sim 1200$ for $m_v = m_\rho$ increasing by ~20% when $m_v = 1000$ MeV. Setting

* We assume the values given above for the electromagnetic components.

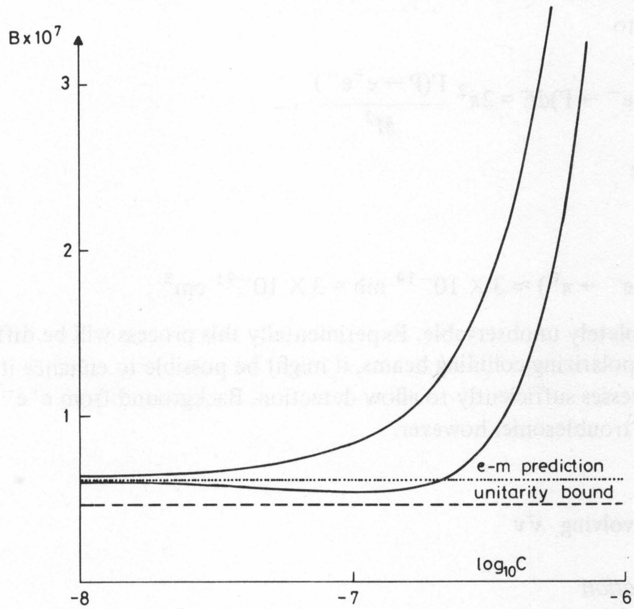


Fig. 1. Graph of $B(\pi^0 \to e^+e^-)$ against the dimensionless weak coupling constant $C$. The upper full curve is for $C > 0$, the lower one for $C < 0$.

$C = 10^{-7}$, however, yields* $R \sim 1050$; $R$ is quite sensitive to NWI.

$\pi^0 \to e^+e^-$ is an isovector interaction, $\eta \to \mu^+\mu^-$ an isoscalar one. Since vector NWI cannot contribute to $P \to \ell^+\ell^-$, the Weinberg-Salam model predicts $C(\eta \to \mu^+\mu^-) = 0$ and $C(\pi^0 \to e^+e^-) = -2G_F f_\pi m \simeq -2G_F Mm$. If this is the correct coupling constant, then we cannot expect to detect its presence in any conceivable experiment.

It is difficult to reconcile the theory (see ref. [16] for an exception) with the experimental value of $B(\eta \to \mu^+\mu^-)$. It could be explained by setting $C \simeq 10^{-5}$, but this would imply $B(\pi^0 \to e^+e^-) \sim 10^{-4}$ (if $C_{\text{isoscalar}} \sim C_{\text{isovector}}$). It could also be explained by setting $m_v \sim 1000$–$2000$ MeV in $\eta \to \mu^+\mu^-$. Such a value of $m_v$ would lead to $B(\pi^0 \to e^+e^-) \sim 5 \times 10^{-7}$. We note, however, that standard form factor measurements indicate $m_v \sim 800$ MeV in the $\pi^0$ case.

### 3.5. The inverse process

$e^+e^- \to P$ may also provide information on NWI. We have simply

$$\sigma(e^+e^- \to P) = 4\pi \frac{\Gamma(P \to e^+e^-)}{(s - M^2)^2 + M^2\Gamma_{\text{tot}}^2(P)}, \tag{3.15}$$

* We assume $\mu e$ universality for NWI, although this is not yet well-established from other data.

which leads to

$$\int \sigma(e^+e^- \to P) dE = 2\pi^2 \frac{\Gamma(P \to e^+e^-)}{M^2} , \tag{3.16}$$

$$s \simeq M^2 .$$

This yields

$$\int \sigma(e^+e^- \to \pi^0) \approx 3 \times 10^{-14} \text{ mb} = 3 \times 10^{-41} \text{ cm}^2 , \tag{3.17}$$

i.e. not completely unobservable. Experimentally this process will be difficult to detect, but by polarizing colliding beams, it might be possible to enhance it over one-photon processes sufficiently to allow detection. Background from $e^+e^- \to e^+e^-P$ could prove troublesome, however.

## 4. Decays involving $\overline{\nu}\nu$

### 4.1. Introduction

The $\overline{\nu}\nu$ system is thought to interact solely by NWI (except by $\gamma W^+W^-$ couplings at the $\lesssim 10^{-7}$ level), and thus it provides a good method for studying NWI. We find also, surprisingly, that the decays are often experimentally accessible.

### 4.2. Decays of the form $P \to \overline{\nu}\nu$

In usual models, these decays cannot occur, since V, A and T couplings cannot contribute. There is no reason to believe, however, that $\nu$ is intrinsically left-handed [15]. The fact that it has appeared so in many experiments may simply be a manifestation of the fact that only charged-current weak interactions have been observed in detail, and it is known that $J_\mu^{\text{leptonic}}$ contains $(1 - \gamma_5)$ which serves to project out a single helicity state. Let us assume that the apparent chirality of neutrinos is an accident arising from the form of the charged weak current. Thus P or S NWI would lead to $B(P \to \overline{\nu}\nu) \neq 0$.

By the very nature of the mass-generating terms associated with the Higgs' mechanism, at least in models similar to that of Weinberg and Salam, $\phi$ cannot contribute to $P \to \overline{\nu}\nu$ since it couples to the trace of energy-momentum tensor of the fundamental fermion fields, i.e. to $m_\nu^2 = 0$. This means that such a contribution would also be absent from $\nu X$ quasielastic scattering.

In the (somewhat unlikely) event that only pseudoscalar NWI exist,

$$\Gamma(P \to \overline{\nu}\nu) = n|\Pi|^2 \frac{M}{8\pi} , \tag{4.1}$$

where $n$ is the number of massless types of neutrino coupling through NWI. Letting $\Pi = f_\pi^2 G_F^2$, $n = 2$ we obtain

$$B(\pi^0 \to \bar{\nu}\nu) \sim 3 \times 10^{-8} ,$$

$$B(\eta \to \bar{\nu}\nu) \sim 10^{-10} . \tag{4.2}$$

There is, however, little reason to take such a small value for $\Pi$.

The process

$$\mathrm{K}^+ \to \pi^+ \pi^0 \atop \quad\quad \llcorner\!\to \bar{\nu}\nu , \tag{4.3}$$

has been suggested [15] as a possible one in which to detect helicity-flipping NWI. Another might be

$$\mathrm{e}^+\mathrm{e}^- \to \psi' \to \gamma\chi \atop \quad\quad\quad \llcorner\!\to \bar{\nu}\nu$$

$$(s \simeq m_{\psi'}^2) . \tag{4.4}$$

The $\gamma$ would be monochromatic and, with high hadron detection efficiency, this decay may be detectable ★. The process

$$\mathrm{P} \to \bar{\nu}\nu\bar{\nu}\nu , \tag{4.5}$$

is experimentally indistinguishable from $\mathrm{P} \to \bar{\nu}\nu$, and it should occur in the Weinberg-Salam model – but with $B \lesssim 10^{-24}$.

### 4.3. Decays of the form $P \to \gamma\bar{\nu}\nu$

$\mathrm{P} \to \gamma\bar{\nu}\nu$ is very similar in structure to $\mathrm{P} \to \gamma\gamma$. In the latter, we take the photons to be dominated by vector mesons; in the former, the $\gamma$ and virtual Z also to be dominated by mesons (fig. 2). The quantum numbers of M (which saturates the Z) depend on the model considered. In the Weinberg-Salam model, for example, M could be a vector meson (such as $\rho$, $\omega$, $\phi$ ...), or an axial vector one ★★ (such as $\mathrm{A}_1$). It could not, however, be an isoscalar $J^P = 1^+$ meson (such as D(1285)). In a pseudo-

★ If $\chi$ is a scalar, rather than pseudoscalar state, then this process measures $S$, rather than $\Pi$, if $CP$ invariance holds.

★★ It cannot be scalar/pseudoscalar, since such particles cannot decay to $\bar{\nu}\nu$ in the Weinberg-Salam model.

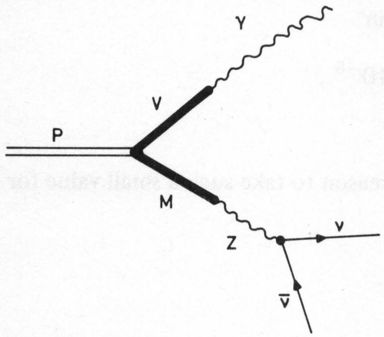


Fig. 2. Diagram for the process $P \to \gamma\bar{\nu}\nu$ in a meson dominance approach.

scalar model *, M could be any $J^{PC} = 0^{-+}$ meson.

In comparing $P \to \gamma\bar{\nu}\nu$ with $P \to \ell^+\nu_\ell\gamma$, which has been calculated by several authors [16] there is some difficulty associated with lepton mass singularities, but we obtain

$$\Gamma(\pi^0 \to \gamma\bar{\nu}\nu) \sim 2 \times 10^2(|V|^2 + |A|^2) + 10^{-2}|\Pi|^2$$
$$+ 10^5(|T|^2 + |\Lambda|^2)\ \mathrm{MeV}\ . \tag{4.6}$$

In the Weinberg-Salam model, this yields ** $B(\pi^0 \to \gamma\bar{\nu}\nu) \sim 3 \times 10^{-13}$. A pseudoscalar model might give $B(\pi^0 \to \gamma\bar{\nu}\nu) \sim 10^{-13}$, a tensor one $B(\pi^0 \to \gamma\bar{\nu}\nu) \sim 10^{-11}$ (taking $T \sim G_F$).

### 4.4. The decay $\eta \to \pi^0\bar{\nu}\nu$

The decay $\eta \to \pi^0\bar{\nu}\nu$ allows a rather clean study of the isovector NWI to be made. Regardless of its branching ratio, it will be difficult to detect experimentally, primarily because it is hard to know whether the initial particles was an $\eta$ without analysing most of its decay products. One method of circumventing this problem might be to use $\pi^+ n \to p\eta$ either in the N(1535) region, or near threshold, and to analyse the proton momentum to check that the recoiling mass was indeed $m_\eta$. Then by increasing the photon detection efficiency, $\pi^0\gamma\gamma$ and $3\pi^0$ decays could be rejected.

* A scalar Z would give no contribution to this decay.

** We take account of the vector mesons by a factor 0.17 in the matrix element deduced from $B(\pi^0 \to \gamma\gamma)$. There is a slight ambiguity in the factor by which $G_F$ must be multiplied to render it dimensionless. We use the vector meson rather than the $\pi^0$ mass here.

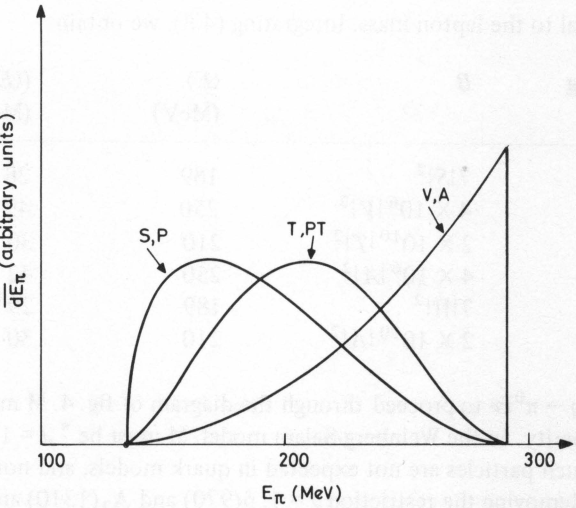


Fig. 3. Graph of $d\Gamma(\eta \to \pi\bar{\nu}\nu)/dE_\pi$ for various coupling types. The curves are normalised to the same $\Gamma_{tot}$.

We now write a general local matrix element for $\eta \to \pi^0\overline{\nu}\nu$ [17]:

$$M = \bar{u}(P)[S + \gamma_\lambda Q^\lambda V + r\cdot(P - \bar{P})T + \gamma_\lambda\gamma_5 Q^\lambda A + \gamma_5 \Pi$$

$$+ r\cdot(P - \bar{P})\gamma_5\Lambda]\,v(\bar{P})\,, \tag{4.7}$$

where $r = P_\pi$; $P = P_\eta$; $Q = P + r$. This yields ($E = E_\pi$, $\mu = m_\pi$, $M = m_\eta$)

$$\frac{d\Gamma}{dE} = \frac{2\sqrt{E^2 - \mu^2}}{16\pi M^3}\,W\,; \tag{4.8}$$

with

$$W = \tfrac{4}{3}(|V|^2 + |A|^2)(E^2 - \mu^2) + \tfrac{1}{2}(|\Pi|^2 + |S|^2)(\rho^2/M^2)$$

$$+ \tfrac{1}{6}(|T|^2 + |\Lambda|^2)(E^2 - \mu^2)\rho^2\,, \tag{4.9}$$

and $\rho^2 = \mu^2 + M^2 - 2ME$.

$d\Gamma/dE$ is given for various NWI in fig. 3. We have assumed that the coupling constants (form factors) are independent of $E$. This appears to be roughly correct for most meson decays *. We note the absence of interference terms in (4.9): these are

* For example, in $\eta \to \gamma e^+e^-$, $f(E) = 1 + (-0.22 \pm 0.45)\rho^2/M^2 \sim 1$ [9].

all proportional to the lepton mass. Integrating (4.8), we obtain

| Coupling type | $B$ | $\langle E\rangle$ (MeV) | $(\langle E^2 - E\rangle^2)^{1/2}$ (MeV) | |
|---|---|---|---|---|
| S | $7\lvert S\rvert^2$ | 189 | 28 | |
| V | $4\times 10^6\lvert V\rvert^2$ | 250 | 42 | |
| T | $2\times 10^{10}\lvert T\rvert^2$ | 210 | 30 | |
| A | $4\times 10^6\lvert A\rvert^2$ | 250 | 42 | |
| PS | $7\lvert\Pi\rvert^2$ | 189 | 29 | |
| PT | $2\times 10^{10}\lvert\Lambda\rvert^2$ | 210 | 30 | (4.10) |

We may take $\eta\to\pi^0\overline{\nu}\nu$ to proceed through the diagram of fig. 4. M must have $I^{GC} = 1^{-+}$, natural parity. In the Weinberg-Salam model, M must be * $J = 1$ and preferentially $P = -$. Such particles are not expected in quark models, and none are known to exist [8]. Removing the restriction $J = 1$, $\delta(970)$ and $A_2(1310)$ are candidates for M. Vector-meson dominance may not be a reasonable approximation in this decay (see subsect. 6.2). The largest alternative diagram contains an electromagnetic tadpole (exchange of a photon between quarks). In either of these models, we expect $B\sim 10^{-11}$. For S, PS, $B\sim 10^{-9}\to 10^{-10}$, for T, PT, $\sim 10^{-12}$.

### *4.5. Decays of the form $P\to \ell^+\ell^-\overline{\nu}\nu$*

$P\to\ell^+\ell^-\overline{\nu}\nu$ is structurally similar to $P\to\gamma\overline{\nu}\nu$; the difference is simply that the $\gamma$ is internally converted. The standard formulae [18] can thus be applied. The largest result is

$$B(\eta\to e^+e^-\overline{\nu}\nu)\simeq 3\times 10^2(\lvert V\rvert^2+\lvert A\rvert^2)+7\times 10^{-4}\lvert\Pi\rvert^2$$

$$+\,2\times 10^6(\lvert T\rvert^2+\lvert\Lambda\rvert^2)\,, \qquad (4.11)$$

yielding $B\sim 10^{-16}$ in the Weinberg-Salam model, and roughly the same for $\Pi$, $T$, $\Lambda$. The process will also be very difficult to detect experimentally: its only difference from $P\to\ell^+\ell^-\gamma$ is that the differential width is even more strongly at small values of the dilepton invariant mass.

## 5. The decay $\eta\to\pi^0\ell^+\ell^-$

There are two possible methods to detect NWI in $\eta\to\pi^0\ell^+\ell^-$. The first is to observe parity violation, and the second to measure the weak corrections to $d\Gamma/dE_\pi$. We discuss these in turn.

* $J = 0$ mesons cannot decay to $\overline{\nu}\nu$ in this model.

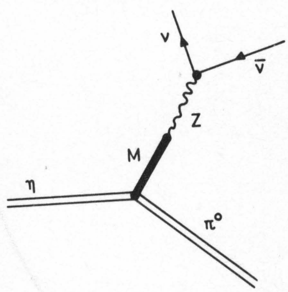


Fig. 4. Diagram for the process $\eta \to \pi^0 \bar{\nu}\nu$.

We shall neglect final-state interactions in $\eta \to \pi^0 \ell^+ \ell^-$. $\ell^+ \ell^-$ energy asymmetries can occur only in the presence of $C$, $T$ violation and final state interactions [19,20] if $CPT$ invariance holds. The possible correlations are then [20] ($\boldsymbol{\sigma}$ is some spin vector, $\boldsymbol{p}_i$ some momentum):

| Correlation | Symmetries violated | |
|---|---|---|
| (1) $\boldsymbol{\sigma} \cdot \boldsymbol{p}$ | $P$ | |
| (2) $\boldsymbol{\sigma} \cdot \boldsymbol{p}_1 \times \boldsymbol{p}_2$ | $T$ | |
| (3) $\boldsymbol{p}_1 \cdot \boldsymbol{p}_2 \times \boldsymbol{p}_3$ | $T$ | (5.1) |

(3) must be zero, since the momenta are coplanar in this case. (1) may be measurable in the case $\ell = \mu$, by analysing the decay electron momentum to find the polarization *. These asymmetries may occur at the $\gtrsim 0.1\%$ level in the Weinberg-Salam model.

$\eta \to \pi \ell^+ \ell^-$ is thought to proceed mainly by a $\gamma\gamma$ intermediate state **. Llewellyn Smith [21] obtains $B(\eta \to \pi^0 e^+ e^-) \sim 7 \times 10^{-10}$ but uses a Lagrangian which forces $e^+e^-$ to be in an S-wave, and thus suppresses the rate by a factor of $m_\ell^2$. Cheng [22] considers various vector meson dominance models, and concludes that $B(\eta \to \pi^0 e^+ e^-) \sim 10^{-8}$. We give his $d\Gamma/dE$ in fig. 5. Fig. 3 gives $d\Gamma(\eta \to \pi^0 \bar{\nu}\nu)/dE_\pi \sim d\Gamma_w(\eta \to \pi^0 e^+ e^-)/dE_\pi$. We note that non-vector NWI would modify the tail of $d\Gamma(\eta \to \pi^0 e^+ e^-)/dE_\pi$.

Our analysis for the NWI mechanism in $\eta \to \pi^0 e^+ e^-$ follows closely that in subsect. 4.4 for $\eta \to \pi^0 \bar{\nu}\nu$. Now $J_M = 0$ is not forbidden but merely suppressed by $m_e$. In the Weinberg-Salam model, we obtain $B$(pure weak) $\sim 10^{-11}$ and $B$(weak-electromagnetic interference) $\sim 5 \times 10^{-10}$, and slightly more for tensor NWI. Probably, no $\eta \to \pi^0 e^+ e^-$ decay has been observed, and the present limit is $B < 4.5 \times 10^{-5}$ [23].

The electromagnetic part of $B(\eta \to \pi^0 \mu^+ \mu^-)$ has never been calculated: from

* We note that this should be the only use made of $p_e$; it cannot be $p$ in (1), for example, since $\mu$ decay is a final-state interaction, and our conclusions only hold in their absence.

** $C$ invariance forbids a $1\gamma$ contribution.

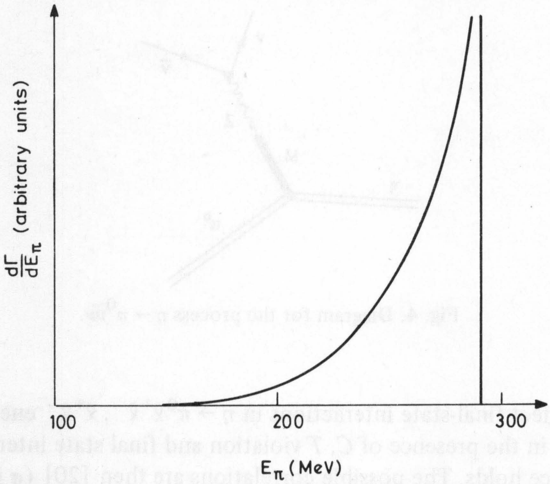


Fig. 5. Graph of $d\Gamma(\eta \to \pi e^+e^-)/dE_\pi$ for a purely two-photon mechanism [22].

Llewellyn Smith's results we obtain $B \sim 10^{-5}$. No such estimate is possible in Cheng's model from existing calculations, since he takes $m_e = 0$. Experimentally $B < 5 \times 10^{-4}$ [24] – not far above the theoretical estimate. NWI contributions will tend to be suppressed in $\eta \to \pi^0\mu^+\mu^-$ compared to $\eta \to \pi^0 e^+e^-$. We obtain $B$(weak-electromagnetic) $\sim 10^{-8}$.

We expect $B(\eta' \to \eta e^+e^-) \sim 10^{-10}$ MeV/$\Gamma_{tot}$, $B(\eta' \to \pi^0 e^+e^-) \sim 5 \times 10^{-10}$ MeV/$\Gamma_{tot}$. The present experimental bounds are [10] $< .1\%$ and $< 1.3\%$ respectively.

## 6. Other meson decays

### *6.1. Photon and Dalitz decays*

In close analogy with $P \to \ell^+\ell^-$, any net circular photon polarization in $P \to \gamma\gamma$ is a signal of *CP* violation or of final-state interactions (thought to be absent in Yang-Mills' theories). In the presence of P, S interactions, this could be as large as $10^{-3}$. In $P \to \gamma\ell^+\ell^-$, $\langle\sigma_\ell \cdot P_\ell\rangle$ may well be $\sim$0.1%, and possibly detectable in the case $\ell = \mu$, $P = \eta$, as [18] $B(\eta \to \mu^+\mu^-\gamma) \simeq 5 \times 10^{-4}$, although this decay has not yet been observed [8].

Perhaps the most interesting meson photon decay is $P \to \gamma\gamma\gamma$. This is forbidden by *C* (but not *CP*) invariance, and its occurrence would be a signal of NWI or electromagnetic *C* violation. The dominant diagram would probably be fig. 6. For V, A,

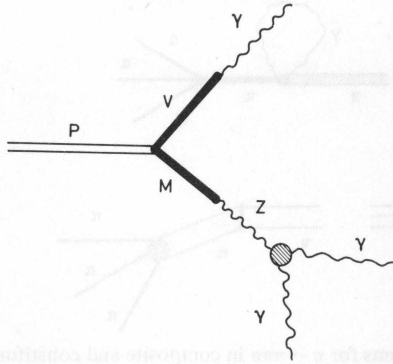


Fig. 6. Diagram for the process $P \to \gamma\gamma\gamma$ by NWI.

NWI, Yang's theorem [26], which we generalize in subsect. 6.6, breaks down * since the virtual Z has a scalar component which can decay to $\gamma\gamma$, so long as NWI violate $C$. In a Yang-Mills' theory, there is no direct $Z\gamma\gamma$ coupling; the lowest-order process involves a virtual fermion loop, yielding $B \sim 10^{-16}$ **. If, however, Z is pseudoscalar then the process is allowed (the $Z\gamma\gamma$ coupling will probably be electromagnetic; c.f. $\gamma W^+W^-$) and we expect $B \sim 10^{-8}$ ***.

*6.2. Decays to hadrons*

The first form of decay which we consider here is pseudoscalar meson → mesons. No decay of this type is fundamentally pure $P$, $C$ violating; $\eta \to \pi^+\pi^-$ [27], for example, violates $CP$. $\eta \to \pi^0\pi^+\pi^-$ has been considered as a possible decay in which to detect electromagnetic $C$-violation [19]; here we discuss its use as a signal for NWI. The conventional [28] diagram for $\eta \to \pi^0\pi^+\pi^-$ is that of fig. 7; i.e. involving an electromagnetic tadpole. The NWI contribution would probably consist of a Z tadpole, so that either a $J^P = 0^-$ or $0^+$ meson could occur in place of the virtual $\pi^0$. However, a $J^P = 0^+$ meson cannot decay to $3\pi$ (c.f. the $\tau$-$\theta$ paradox), and so we must assume that any NWI $C$-violation is a result of an admixture of a $C = -$ meson, which is not allowed in quark models. Ignoring this suppression, we may make the

* Formally, this is because the assumption that a Lorentz transformation may be made to the Z rest frame, fails.

** Alternatively all the photons might come from a virtual quark loop with a $W^\pm$ or Z inside, but this again gives $B \sim 10^{-16}$.

*** It is difficult to estimate the PVM coupling required here; we have used $\rho \to \pi\pi$ for this purpose.

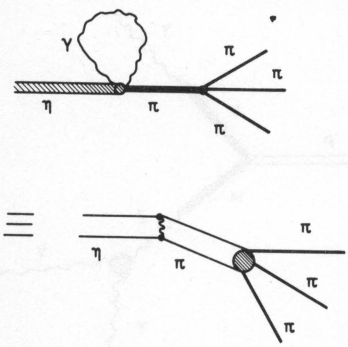


Fig. 7. Diagrams for $\eta \to \pi\pi\pi$ in composite and constituent models.

naive estimate

$$A = \frac{N(E_{\pi^+} > E_{\pi^-}) - N(E_{\pi^+} < E_{\pi^-})}{N(E_{\pi^+} > E_{\pi^-}) + N(E_{\pi^+} < E_{\pi^-})} \sim 10^{-5} . \tag{6.1}$$

Experimentally, $A = (1.2 \pm 1.7) \times 10^{-3}$ [8].

Another possible form of decay is pseudoscalar meson → mesons + lepton pair. Examples of this are

$$\eta \to \pi^+ \pi^- e^+ e^- , \tag{6.2}$$

$$\eta \to \pi^+ \pi^- \bar{\nu}\nu , \tag{6.3}$$

$$\eta \to \pi^+ \pi^- \pi^0 e^+ e^- . \tag{6.4}$$

The decay (6.2) occurs electromagnetically with [29] $B \sim 3.3 \times 10^{-4}$. For (6.3) we estimate $B \sim 10^{-15}$, and for (6.4): $B_{2\gamma} \sim 10^{-9}$ and $B_{\mathrm{NWI}} \ll B_{2\gamma}$.

### *6.3. Charged meson decay*

$P^\pm \to \ell^\pm \nu e^+ e^-$ occurs primarily as an electromagnetic correction to the weak decay $P \to \ell^\pm \nu$, with [30] $B(\pi^\pm \to e^\pm \nu e^+ e^-) \simeq 4 \times 10^{-10}$ and $B(\pi^\pm \to \mu^\pm \nu e^+ e^-) \simeq 3 \times 10^{-6}$. Even weak-electromagnetic interference effects will occur only at the $\sim 10^{-8}$ level, resulting in, for example, $\langle \sigma \cdot p_e \rangle \neq 0$.

$K^+ \to \mu^+ \nu \bar{\nu}\nu$ has been searched for [31] (as a test for strong $\bar{\nu}\nu$ interactions). One may estimate $B \sim 10^{-15}$ for charged-current weak interactions and $\sim 10^{-14}$ for NWI.

### *6.4. Charmed particle decays*

In the Weinberg-Salam-Glashow-Iliopoulos-Maiani model, charm-changing NWI are absent, simply in analogy with the absence of strangeness-changing NWI. Detection or failure to detect $\Delta C \neq 0$ NWI at the level expected would thus put stringent constraints on weak interaction models. The best decay in which to achieve this appears to be

$$\mathrm{D}^{\pm} \to \pi^{\pm} \ell^{+} \ell^{-} . \tag{6.5}$$

There is assumed to be no electromagnetic competition here.

### 6.5. Scalar meson decays

The only $J^P = 0^+$ meson decay which is interesting from the point of view of NWI, and which differs significantly from the analogous $J^P = 0^-$ meson decay is $\mathrm{S} \to \ell^+ \ell^-$. Here only scalar NWI may contribute in the limit of $CP$ invariance*. In the Weinberg-Salam model, therefore, the only non-electromagnetic possible intermediate state is $\phi$. One finds [32]

$$\langle x\overline{x} | \phi \rangle = \frac{ig}{2m_{\mathrm{W}}} \theta^{\mu}_{\mu} , \tag{6.6}$$

where $x$ is a fundamental fermion field whose mass arises from Higgs' mechanism, and $\theta^{\mu}_{\mu}$ is the trace of the stress-energy tensor. The $x$ may be either a quark field or a meson field. We find

$$M_{\phi} \sim \begin{matrix} 2 \times 10^{-7} \\ \\ 10^{-9} \end{matrix} \; \frac{\overline{u}vM^2}{M^2 - m_{\phi}^2} \, m_{\mathrm{q}} \; \mathrm{GeV}^{-1} \begin{matrix} (\ell \equiv \mu) \\ \\ (\ell \equiv \mathrm{e}) \end{matrix} , \tag{6.7}$$

leading to a contribution $\lesssim 10^{-2}$ times the electromagnetic one, i.e. unobservably small. There do exist other models in which either the quark masses are very large or a second $\phi$ is introduced [33], which would lead to a larger weak component in $\mathrm{S} \to \ell^+ \ell^-$, but we deem these unlikely.

Experimentally, the existence of ordinary scalar mesons is still in doubt [34], but the $0^{++}$ state in $\psi' \to \gamma\chi$ may have a measurable $\ell^+ \ell^-$ branching ratio, since it probably contains rather heavy quarks.

### 6.6. Vector meson decays

There exist a number of vector meson decays which could furnish information on NWI. Isgur [35] has analysed $\psi \to \mu^+ \mu^-$, and we have nothing to add. $\psi \to \overline{\nu}\nu$ has been discussed by Rich and Winn [36]. Using a free quark model, they obtain $B(\psi \to \overline{\nu}\nu) \sim 1.5 \times 10^{-5}$. Generally, $B(\psi \to \overline{\nu}\nu) \sim 3 \times 10^3 G^2 n$, where $G$ is a dimensionless NWI coupling constant $\sim\frac{2}{3} \sin^2 \theta_{\mathrm{W}} G_{\mathrm{F}} M_{\psi}^2$ in the Weinberg-Salam model, and

* The electromagnetic ($\gamma\gamma$) contribution is an effective scalar.

$n$ is the number of massless neutrino types. Rich and Winn conclude that

$$\psi' \to \psi\pi\pi \quad \llcorner\!\!\to \nu\nu\,, \tag{6.8}$$

could be observed in $e^+e^-$ colliding beams.

Yang [26] has shown that the decay of a $J = 1$ particle to $\gamma\gamma$ is forbidden by $P$ invariance. It is easy to show ★, however, that this is also forbidden by gauge invariance and Bose statistics, and so we will never observe a decay such as $\psi \to \gamma\gamma$ even in the presence of $P$, $C$ violating NWI.

## 7. Baryon decays

If no $\Delta S \neq 0$ or $\Delta C \neq 0$ NWI exist, then the only $J^P = \frac{1}{2}^+$ baryon decays in which NWI effects might be visible are

$$\Sigma^0 \to \Lambda e^+ e^-\,, \tag{7.1}$$

$$\Sigma^0 \to \Lambda\nu\nu\,, \tag{7.2}$$

and possibly

$$\Sigma^0 \to \Lambda\gamma\,. \tag{7.3}$$

The decay (7.1) has been discussed in ref. [35], where a value of $\sim 10^{-5} \to 10^{-3}$ is obtained for the $\Lambda$ polarization in the Weinberg-Salam model. This could be increased by about an order of magnitude in non-vector theories.

The energy spectrum [37] of $\Sigma^0 \to \Lambda\bar{\nu}\nu$ is given in fig. 8 and

$$\Gamma(\Sigma^0 \to \Lambda\bar{\nu}\nu) \simeq \frac{G^2\Delta^5}{60\pi^3}\,\frac{\Sigma^3}{2M_\Sigma}\,(1 + 3|x|)^2\,,$$

$$x = \frac{A}{V}\,, \qquad \Sigma = M_\Sigma + M_\Lambda\,, \qquad \Delta = M_\Sigma - M_\Lambda\,. \tag{7.4}$$

This yields $B > 10^{-11}$ and perhaps $\sim 10^{-8}$ in the Weinberg-Salam model, more than that expected for $\Sigma^0 \to \Sigma^+ e^-\nu$.

★ We transform to the rest frame of the decaying particle, which has polarization $\epsilon$, and decays to two photons $(k_i, \epsilon_i)$. We satisfy gauge invariance by setting $k_i \cdot \epsilon_i = 0$. Then Bose statistics demands that any possible amplitude constructed be zero (it must also be linear in $\epsilon_i$, $\epsilon$ in correspondence with the annihilation and creation operators).

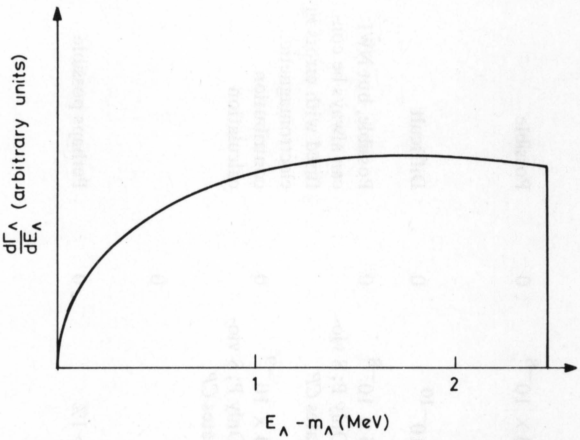


Fig. 8. Graph of $d\Gamma(\Sigma^0 \to \Lambda\bar{\nu}\nu)/dE_\Lambda$ in the Weinberg-Salam model.

Experimentally, the process

$$\begin{array}{l} \qquad\qquad\ \ \ulcorner\!\!\to \pi^+\pi^- \\ \pi^- p \to \Sigma^0 K^0_s \\ \qquad\quad\ \ \llcorner\!\!\longrightarrow \Lambda\nu\nu \\ \qquad\qquad\qquad\ \ \llcorner\!\!\to p\pi^- \,, \end{array} \tag{7.5}$$

could be searched for. $\Sigma^0 \to p\pi^-$ ($B \sim 2 \times 10^{-8}$) and $\Sigma \to \Lambda\gamma$ could be important backgrounds. Thus the $p\pi^-(\Lambda)$ and $\pi^+\pi^-(K^0)$ invariant masses would have to be determined to check that the decaying particle was a $\Sigma^0$.

Since the $\Sigma\Lambda$ relative parity is even, the photon in $\Sigma \to \Lambda\gamma$ must be M1, if the process is purely electromagnetic. However, various Z and W tadpole diagrams serve to admix a small E1 component into this, which could well be $\sim 10^{-4}$. Better theoretical knowledge of the $\Sigma \to \Lambda$ electromagnetic form factor might allow a measurement of such an effect by observing pair production from the photon [38].

## 8. Conclusions

We summarize our conclusions in table 1, from which we have omitted the most hopeless decays.

I am very grateful to Dr D. Sivers for many useful discussions and for reading the manuscript, and to Drs. G.V. Dass, P. Herczeg, R.L. Kingsley, C.H. Llewellyn Smith, R.J.N. Phillips, A.I. Sanda and Professor L. Wolfenstein for useful comments.

Table 1

| Decay | Method of NWI detection (and experiment where applicable) | Quantity measured | Expected magnitude: Weinberg-Salam | Other: V, A | Other: P, S | Other: T | |
|---|---|---|---|---|---|---|---|
| $\pi^0 \to \bar{\nu}\nu$ | $K^+ \to \pi^+\pi^0$, $\pi^0 \to \bar{\nu}\nu$; $e^+e^- \to \gamma\chi$, $\chi \to \bar{\nu}\nu$ | $B$ | 0 | 0 | $3 \times 10^{-8}$ | 0 | Possible |
| $\eta \to \bar{\nu}\nu$ | | $B$ | 0 | 0 | $10^{-10}$ | 0 | Difficult |
| $\pi^0 \to e^+e^-$ | $B(\pi^0 \to e^+e^-) \sim 6.2 \times 10^{-8}$ (2nd order e-m) | $\Delta B$ | $\approx -10^{-10}$ | $\sim -10^{-10}$ | $5 \times 10^{-8}$ Only P; S violates *CP* | 0 | Possible, but NWI can always be confused with errors in electromagnetic contribution calculation |
| $\eta \to e^+e^-$ | $B(\eta \to e^+e^-) \sim 4.4 \times 10^{-9}$ (2nd order e-m) | $\Delta B$ | 0 | $-10^{-10}$ | $4 \times 10^{-9}$ Only P; S violates *CP* | 0 | |
| $\eta \to \mu^+\mu^-$ | $B(\eta \to \mu^+\mu^-) \sim 1.2 \times 10^{-5}$ (2nd order e-m) | $\Delta B$ | 0 | $-10^{-8}$ | | 0 | |
| | $\langle \sigma_\mu \cdot P_\mu \rangle \neq 0 \Rightarrow CP$ violation measure $\mu$ decay e distribution | $\langle \sigma_\mu \cdot P_\mu \rangle$ | 0 | 0 | $\sim 1\%$ | 0 | Perhaps possible |
| $\pi^0 \to \gamma\gamma\gamma$ | $B(\pi^0 \to \gamma\gamma\gamma) \neq 0 \Rightarrow C$ violation | $B$ | $10^{-16}$ | $10^{-16}$ in Yang-Mill's theories | $10^{-8}$ | $\sim 10^{-8}$ | Possible |
| $\eta \to \gamma\mu^+\mu^-$ | $\langle \sigma_Q \cdot P_Q \rangle \neq 0 \Rightarrow P$ violation | $\langle \sigma_Q \cdot P_Q \rangle$ | $\sim 10^{-3}$ | $\sim 10^{-3}$ | $\sim 10^{-3}$ | $\sim 10^{-3}$ | Possible |

Table 1 (continued)

| Decay | Method of NWI detection (and experiment where applicable) | Quantity measured | Expected magnitude | | | | |
|---|---|---|---|---|---|---|---|
| | | | Weinberg-Salam | Other | | | |
| | | | | V, A | P, S | T | |
| $\pi^0 \to \gamma\nu\bar{\nu}$ | $K^+ \to \pi^+\pi^0$ <br> $\hookrightarrow \gamma\nu\bar{\nu}$ | $B$ | $3 \times 10^{-13}$ | $10^{-12}$ | $10^{-13}$ <br> No S | $10^{-11}$ | Difficult |
| $\eta \to \pi^0 e^+ e^-$ | $B(\eta \to \pi^0 e^+ e^-) \sim 10^{-8}$ (2nd order e-m) Modifications to tail of $d\Gamma/dE_\pi$ | $\Delta B$ | $5 \times 10^{-10}$ | $\sim 10^{-9}$ | $\sim 10^{-10}$ | $\sim 10^{-9}$ | Perhaps possible |
| $\eta \to \pi^0 \mu^+ \mu^-$ | $B(\eta \to \pi^0 \mu^+ \mu^-) \sim 10^{-5}$ (2nd order e-m) Modifications to tail of $d\Gamma/dE_\pi$ | $\Delta B$ | $\sim 10^{-8}$ | $\sim 10^{-8}$ | $\sim 10^{-9}$ | $\sim 10^{-8}$ | Difficult |
| | $\langle \sigma_\mu \cdot P_\mu \rangle \neq 0 \Rightarrow P, C$ violation | $\langle \sigma_\mu \cdot P_\mu \rangle$ | $\sim 10^{-3}$ | $\sim 10^{-3}$ | $\sim 10^{-4}$ | $\sim 10^{-3}$ | Possible <br> $\langle \sigma_\mu \cdot P_{\mu^+} \times P_{\mu^-} \rangle \neq 0 \Rightarrow T, C$ violation <br> $\langle P_\pi \cdot P_{\mu^+} \times P_{\mu^-} \rangle \neq 0 \Rightarrow P, T$ violation |
| $\eta \to \pi^0 \nu\bar{\nu}$ | $\pi^+ n \to p\eta$ <br> $\hookrightarrow \pi^0 \nu\bar{\nu}$ | $B$ | ?0 | $\sim 10^{-11}$ | $\sim 10^{-9}$ | $\sim 10^{-12}$ | Perhaps possible |
| $\pi^0 \to e^+ e^- \nu\bar{\nu}$ | | $B$ | $10^{-20}$ | | $10^{-20}$ | | |
| $\eta \to e^+ e^- \nu\bar{\nu}$ | | $B$ | $\sim 10^{-16}$ | | $\sim 10^{-16}$ | | Impossible |
| $\eta \to \mu^+ \mu^- \nu\bar{\nu}$ | | $B$ | $\sim 10^{-16}$ | $10^{-16}$ | $10^{-22}$ | $10^{-14}$ | |

Table 1 (continued)

| Decay | Method of NWI detection (and experiment where applicable) | Quantity measured | Expected magnitude | | | | |
|---|---|---|---|---|---|---|---|
| | | | Weinberg-Salam | Other | | | |
| | | | | V, A | P, S | T | |
| $\pi^0 \to \gamma\gamma$ | $\gamma$ circular polarization $\Rightarrow CP$ violation | $\gamma$ circular polarization | | 0 | $\sim 10^{-3}$ | 0 | Possible |
| $\eta \to \pi^0\pi^+\pi^-$ | $\pi^+\pi^-$ energy asymmetry $\Rightarrow C$ violation | $A$ | $?\sim 10^{-5}$ | | $\sim 10^{-5}$ | | Possible |
| $\psi \to \nu\nu$ | $\psi' \to \psi\pi\pi$<br>$\quad\hookrightarrow \nu\nu$ | $B$ | $10^{-5}$ | | $10^{-5}$ | | Possible |
| $\psi \to \gamma\gamma$ | | $B$ | | | 0 | | Impossible |
| $\Sigma \to \Lambda e^+e^-$ | | $P_\Lambda$ | $10^{-5}$–$10^{-3}$ | | $? \to 10^{-2}$ | ? | Possible |
| $\Sigma \to \Lambda\bar{\nu}\nu$ | $\pi^- p \to \Sigma^0 K^0$<br>$\quad\hookrightarrow \Lambda\nu\nu$ | $B$ | $>10^{-11}$ | | $?\ 10^{-10}$ | | Perhaps possible |
| $\Sigma \to \Lambda\gamma$ | E1 not M1 $\gamma$ | | | $\sim 10^{-4}$ | | | Perhaps possible: signal for charged as well as NWI |

## Appendix

*Calculation of the electromagnetic contribution to $P \to \ell^+\ell^-$*

*A.1. The absorptive part (Y)*

This may be obtained from the unitarity relation of fig. 9 [39]. We assume $n \equiv \gamma\gamma$. For $P = \pi^0$, this is completely justified, but for $P = \eta$, $n \equiv \pi\pi\gamma$ may need to be considered (it has been calculated [4] as $\sim 2 \times 10^{-2}$ $Y(n \equiv \gamma\gamma)$). It may be even more significant in $\eta'$ and $\eta'_c$ decays. For $n \equiv \gamma\gamma$, we obtain

$$Y = \log\left(\frac{M + \sqrt{M^2 - 4m^2}}{2m}\right) \Big/ \left(1 - \frac{4m^2}{M^2}\right)^{1/2} . \tag{A.1}$$

Numerically, this becomes

| meson | leptons | |
|---|---|---|
| | e | $\mu$ |
| $\pi^0$ | 5.6 | |
| $\eta$ | 7.0 | 1.7 |
| $\eta'(958)$ | 7.6 | 2.3 |
| $\eta'_c(3450)$ | 8.9 | 3.5 |

(A.2)

We note that a large direct (weak) decay rate would contribute to the nominally pure electromagnetic $Y$, but we do not expect this to be significant.

*A.2. The dispersive part (X)*

The simplest assumption possible in this calculation is that P is pointlike. This is not satisfactory, however, since $\theta \sim \int dk/k$ for large $k$. We must thus find some way to cut off this integration; we give structure to the meson. Measurements of the pion-form factor [41] indicate that it conforms well to vector-meson dominance predictions. Although this has no direct physical connection with the process under discussion, it suggests that vector-meson dominance should be used there.

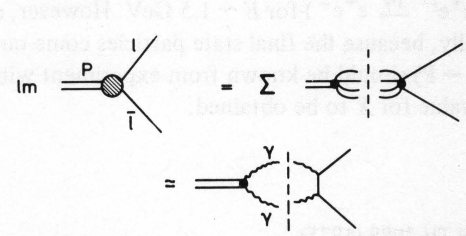


Fig. 9. The unitarity relation for the imaginary part of the amplitude for $P \to \ell^+\ell^-$.

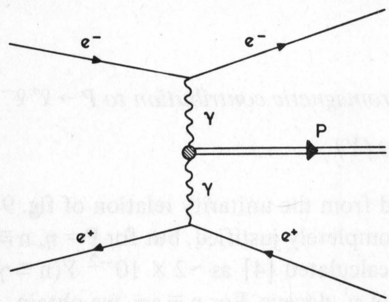


Fig. 10. Diagram for the process $e^+e^- \to e^+e^-P$, which allows the $\gamma\gamma$ form factor of P to be probed.

A number of estimates have been made for $X$. We give some for $\pi^0 \to e^+e^-$:

| Author(s) | Cut-off factor | $X$ for $m_v = m_\rho$ |
|---|---|---|
| 1. Drell [42] | $\theta(k_1^2 + \Lambda^2)$ | 7.11 |
| 2. Berman and Geffen [43] | $\dfrac{m_v^2}{(m_v^2 + k_1^2 + k_2^2)}$ | 3.21 |
| 3. Quigg and Jackson 1 [14] | $\dfrac{m_v^4}{(m_v^2 + k_1^2)(m_v^2 + k_2^2)}$ | 3.33 |
| 4. Quigg and Jackson 2 [14] | $\dfrac{m_v^2}{(m_v^2 + k_1^2)}$ | 3.14 |

(A.3)

We have used model 3, in which one photon is saturated with $\rho$, the other with $\omega$. Model 4 saturates only one photon with a meson. Model 1 takes P to be a uniform sphere of charge in momentum space; it is not at all clear that its radius $\Lambda M$ should be $m_v$. Other models for $X$ include taking $\pi^0$ as a virtual $N\overline{N}$ state [40] (obtaining $X = 7.9$), and a very general one [44] whose results vary by an order of magnitude around those of model 3.

The cut-off factor in $X$ can, in principle, be deduced from measurements on $e^+e^- \to Pe^+e^-$ [45], which proceeds by the diagram of fig. 10. $\sigma(e^+e^- \to Pe^+e^-) \sim \log^3 E$, overtaking $\sigma(e^+e^- \xrightarrow{1\gamma} e^+e^-)$ for $E \sim 1.5$ GeV. However, $e^+e^- \to Pe^+e^-$ is difficult experimentally, because the final state particles come out with small $P_T$. Nevertheless, $A(\gamma_v\gamma_v \to P)$ should be known from experiment within a few years, allowing as accurate value for $X$ to be obtained.

## References

[1] J.J. Sakurai, CERN TH-2099 (1975).
[2] S. Weinberg, Astrophysical implication of the new theories of weak interaction, Talk presented at 7th Texas Symp. on relativistic astrophysics, Dallas, 1974.

[3] F.C. Michel, Phys. Rev. 138 (1965) B408.
[4] B.E. Lautrup, A. Peterman and E. de Rafael, Phys. Reports 3 (1972) 193.
[5] E.A. Hinds, C.E. Loving and P.G.H. Sandars, Phys. Letters 62B (1976) 97.
[6] B.W. Lee, Summary talk, Aachen Neutrino Conf., 1976.
[7] J.D. Davies, J.G. Guy and R.K.P. Zia, Nuovo Cimento 24A (1974) 324.
[8] Particle Data Group, Review of particle properties, Phys. Letters 50B (1974) and references therein.
[9] M.R. Jane et al., Phys. Letters 59B (1975) 103.
[10] M.K. Gaillard, B.W. Lee and R.E. Shrock, Phys. Rev. D13 (1976) 2674.
[11] B.D. Hyams et al., Phys. Letters 29B (1969) 128.
[12] Omicron Collaboration, Rutherford, Laboratory, proposal no. 170, June 1975;
J.S. Frank et al., LAMPF proposel no. 222, Los Alamos, 1975.
[13] T.D. Lee, Phys. Reports 9 (1974) 145.
[14] C. Quigg and J.D. Jackson, UCRL-18487 (1968).
[15] B. Kayser, G.T. Garvey, E. Fischbach and S.P. Rosen, Phys. Letters 52B (1974) 385;
R.L. Kingsley, F. Wilczek and A. Zee, Phys. Rev. D10 (1974) 2216.
[16] S.B. Treiman and H.W. Wyld, Phys. Rev. 101 (1956) 1552;
D.E. Neville, Phys. Rev. 124 (1961) 2037 and references therein.
[17] D.S. Beder and G.V. Dass, Phys. Letters 59B (1975) 444;
G. Kallén, Elementary particle physics (Addison-Wesley, 1964) p. 456.
[18] N.M. Kroll and W. Wada, Phys. Rev. 98 (1955) 1355.
[19] T.D. Lee, Phys. Rev. 139 (1965) B1415;
J. Bernstein, G. Feinberg and T.D. Lee, Phys. Rev. 139 (1965) B1650.
[20] S. Gasiorowicz, Elementary particle physics (Wiley, 1966) p. 508.
[21] C.H. Llewellyn Smith, Nuovo Cimento 48 (1967) 834.
[22] T.P. Cheng, Phys. Rev. 162 (1967) 1734.
[23] M.R. Jane et al., Phys. Letters 59B (1975) 99.
[24] Wehmann et al., Phys. Rev. Letters 20 (1968) 748.
[25] H.S. Mani and H.S. Sharatchandra, Phys. Rev. D10 (1974) 2849;
J. Schecter and M. Singer, Syracuse University preprint COO-3533-42 (SU-4205-42) 1975.
[26] C.N. Yang, Phys. Rev. 77 (1950) 242.
[27] J.J. Thaler et al., Phys. Rev. D7 (1973) 2569.
[28] S. Raby, Phys. Rev. D13 (1976) 2594.
[29] C. Jarlskog and H. Pilkuhn, Nucl. Phys. B1 (1967) 264.
[30] W. Flagg, Phys. Rev. 178 (1969) 2387.
[31] C.Y. Pang et al., Phys. Rev. D8 (1973) 1989.
[32] J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nucl. Phys. B106 (1976) 292.
[33] H. Georgi and S.L. Glashow, Phys. Rev. Letters 28 (1972) 1494.
[34] D. Morgan, Rutherford Laboratory preprint RL-74-063 (1974).
[35] N. Isgur, Phys. Rev. D11 (1975) 3236.
[36] J. Rich and D.R. Winn, Harvard University preprint (1976).
[37] R.E. Marshak, Rinzuddin and C.P. Ryan, Theory of weak Interactions in particle physics (Wiley, 1964) p. 413.
[38] C. Alff et al., Phys. Rev. 137 (1965) B1105.
[39] C.G. Callan and S.B. Treiman, Phys. Rev. Letters 18 (1967) 1083.
[40] M. Pratap and J.T. Smith, Phys. Rev. D5 (1972) 2020.
[41] D. Sivers, S.J. Brodsky and R. Blankenbecler, Phys. Reports 23 (1976) 1.
[42] S.D. Drell, Nuovo Cimento 11 (1960) 693.
[43] S.M. Berman and D.A. Geffen, Nuovo Cimento 18 (1960) 1192.
[44] B-L. Young, Phys. Rev. 161 (1967) 1620.
[45] S.J. Brodsky, Radiative problems and quantum electrodynamics, 1971, Int. Symp. on electron and photon interactions at high energies, Cornell University, 1971.
[46] P. Herczeg, Los Alamos preprints (June, July 1976).