

Parton and Hadron Production in e^+e^- Annihilation*

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Largely based on a talk given at

XV Rencontre de Moriond, March 1980

ABSTRACT

The production of showers of partons in e^+e^- annihilation final states is described according to QCD, and the formation of hadrons is discussed.

* Work supported in part by the U.S. Department of Energy under Contract

No. DE-AC-03-79ER0068. *and by*

† Fleischmann Foundation Grant No. 30175.

Introduction

In these notes, I discuss some attempts to describe the development of hadron final states in e^+e^- annihilation events using QCD. A few features (barely visible at available energies) of this development are amenable to a precise and formal analysis in QCD by means of perturbation theory. For the most part, however, existing theoretical methods are quite inadequate: one must therefore simply try to identify the dominant physical phenomena to be expected from QCD, and make estimates of their effects, with the hope that results so obtained will provide a good approximation to eventual exact calculations. In so far as such estimates are necessary, precise quantitative tests of QCD are precluded. On the other hand, if QCD is assumed correct, then existing experimental data may be used to investigate its behavior in regions not yet explored by theoretical means.

These notes make brief excursions into many topics: some further explanation and details may be found in Refs. [1,2,3].

The Stages of an e^+e^- Annihilation Event

In QCD perturbation theory, an e^+e^- annihilation event is initiated by the decay of the virtual photon (γ^*) into a quark and an antiquark. If the QCD coupling constant were zero, then the q, \bar{q} would propagate freely to infinity, and would therefore be produced on their mass shells. In fact, the q, \bar{q} need propagate only for a finite time before interacting or radiating, and may therefore be produced with a distribution of invariant masses (μ), usually peaked at low values, but with a power-law tail extending up to the kinematic limit $\mu = Q$ imposed by the mass Q ($\equiv \sqrt{s}$) of the γ^* . Large initial quark invariant masses should be dissipated predominantly by radiation of gluons: the outgoing q, \bar{q} should emit gluons at a rate decreasing roughly inversely with (proper) time, thereby converting their invariant masses into transverse momenta of the produced gluons, and spreading their energy and color into a cone of finite aperture. The emitted gluons may also have invariant masses up to those of their parent quarks, and hence may themselves radiate more gluons (and occasionally, another $q\bar{q}$ pair), generating a cascade or shower of partons, as illustrated in Fig. 1. However, even in perturbation theory, such free emissions cannot continue unchecked forever: as the invariant masses of the partons become small, back reactions in which emitted partons reinteract with their parents or other ambient partons should become increasingly important. Eventually, these reactions, together, perhaps, with qualitatively new phenomena not visible

in perturbation theory, should cause the system of partons to condense into color singlet hadrons. The magnitude of the critical invariant mass μ_c , below which free perturbative emissions no longer dominate is presumably determined by the masses of hadrons, and by the renormalization group invariant mass Λ (which gives the position of the infrared Landau divergence in the leading log effective coupling constant $\alpha_s(\mu^2) \simeq 1/\beta_0 \log(\mu^2/\Lambda^2)$). Phenomenological comparisons (mentioned below) suggest that μ_c is probably of order 1-2 GeV, and give some hints on the transformation of a system of partons below this critical mass into hadrons. As the mass Q of the original γ^* (c.m.s. energy in the e^+e^- collision) is increased, the extent of the period during which free perturbative emissions dominate should correspondingly increase: at sufficiently high energies phenomena occurring below μ_c should become irrelevant. However, it will turn out that for most purposes, the residual effects decrease slower than $O(\mu_c/Q)$, and are by no means negligible compared, for example, with $O(\alpha_s(Q^2))$ hard gluon emissions at presently available energies ($Q \lesssim 35$ GeV).

A parton off-shell by an amount $\gtrsim \mu$ will typically survive without radiating ("decaying") for a proper time $\tau \lesssim 1/\mu$ [F.1]. The transverse momentum between its "decay" products is kinematically bounded by $k_T^2 \lesssim \mu^2$. At very early times, gluons may be emitted with large transverse momenta $O(Q)$, leading to clearly separated additional parton "jets". As discussed in the next section, the partons produced in each "decay" tend to have much smaller invariant masses than their parents, and thus tend to survive for much longer times before decaying themselves. Partons emitted at later times must therefore be progressively much more closely collinear with their parent partons, and their existence should thus affect the final angular distribution of energy over smaller and smaller regions.

In so far as the partons emitted in each decay tend to have much smaller masses than the decaying parton, their energies may remain of the same order as the energy (mass) of the decaying parton. Typically, the energies of partons emitted by the decays of partons with invariant masses $\sim \mu$ decrease only logarithmically with μ (c.f. "scaling violations" in $\langle z \rangle$); their average wavelength therefore remains for a long time $O(1/Q)$. On the other hand, the distance traveled by the decaying partons $\sim 1/\mu$. Hence, the distance between successive emissions soon typically becomes much larger than the wavelengths of the decaying or emitted partons: thus the amplitudes for successive emissions should not interfere appreciably. Hence, so long as the energy of an emitted parton is sufficiently large,

but its invariant mass is not too close to the mass of its parent, its decay should be independent of its production, and the spectrum of its decay products well described by an independent classical probability distribution. This point is crucial in simplifying the discussion on the development of parton final states below. Any gluons with $k_T \sim Q$ must be emitted very soon after the γ^* decay and must arise from very short-lived virtual quarks, typically with $\mu \sim Q \sim |\vec{p}|$. The wavelengths of these virtual quarks (and the gluons they radiate) are therefore no longer than their decay paths: interference between amplitudes for successive gluon emissions, or radiations from q and \bar{q} , could thus be important. Explicit calculation of the first gluon emission to $O(\alpha_s)$ in perturbation theory suggests, however, that such effects are numerically insignificant. On the other hand, at very large times, quantum mechanical interferences are undoubtedly crucial: the organization of the final state into color singlet hadrons with definite masses may be viewed as resulting from destructive interference between the amplitudes for producing illegal final states.

The Leading Pole Approximation

In the classical approximation discussed above, the development of a shower (or jet) of partons is described by a sequence of independent "decays" of off-shell partons into partons with smaller invariant masses. To leading order in $\alpha_s(t)dt/t$, it is sufficient to consider only (quasi-)two-body "decays". Then the probability for a parton of type j_0 to have an invariant mass squared in the range $\mu^2 = t \rightarrow t + dt$ and to "decay" into partons of types j_1, j_2 carrying (roughly) fractions z and $(1-z)$ of its longitudinal momentum is given by

$$\mathcal{P}_{j_0 \rightarrow j_1 j_2}(t, z) dt = \frac{\alpha_s(\tilde{t})}{2\pi t} P_{j_0 \rightarrow j_1 j_2}(z) dt \quad (1)$$

if the invariant masses of the final partons are sufficiently small that the "decay" is kinematically allowed, and $\mathcal{P}(t, z) = 0$ otherwise. The (Altarelli-Parisi [4]) distributions $P(z)$ for the various possible $O(\alpha_s)$ decays are [F.2]

$$P_{q \rightarrow qG}(z) = C_q \left(\frac{1+z^2}{1-z} \right) \quad (2)$$

$$P_{q \rightarrow Gq}(z) = C_q \left(\frac{1+(1-z)^2}{z} \right) = P_{q \rightarrow qG}(1-z)$$

$$P_{G \rightarrow q\bar{q}}(z) = \frac{1}{2} (z^2 + (1-z)^2)$$

$$P_{G \rightarrow GG}(z) = 2C_G \left(\frac{(1-z+z^2)^2}{z(1-z)} \right),$$

where the color "charges" are given by $C_q = (N_c^2 - 1)/(2N_c) = 4/3$, $C_G = N_c = 3$. The probability for each decay is uniform in azimuthal angle. The \tilde{t} which appears as the argument of the effective coupling in (1) is $0(t)$: its exact value will be discussed below. As explained below, the z in (1,2) is identified as the $E + |\vec{p}|$ fraction of each daughter parton with respect to its parent (as measured in the γ^* rest frame [F.3]). In the rest frame of the decaying parton, $2z-1$ is roughly the cosine of the angle between the spin direction of the decaying parton and the (oppositely-directed) momenta of its decay products: the parton decays are not isotropic. (The spin of the decaying parton depends on its momentum with respect to its parent, and ultimately with respect to the original γ^* .)

The differential cross-sections for multiparton production obtained by suitable application of the probabilities (1,2) to all possible decay chains could, in principle, also be found by explicit evaluation of all the contributing high order Feynman diagrams. The results of the latter exact (but intractably complicated beyond $O(\alpha_s^2)$) approach must agree with the approximation, at least to leading order in $\alpha_s dt/t$ for each emission. However, the probabilistic interpretation of the resulting differential cross-section as a sequence of independent "decays" will not in general be manifest: individual diagrams apparently involving interference between different decay chains may appear to be important. To establish the probabilistic interpretation directly from individual diagrams, one must use particular gauges, in which the gluon spin is explicitly constrained to be orthogonal to its momentum, at least when the gluon approaches its mass shell, thereby preventing propagation of unphysical gluon polarization states. This is achieved in an axial gauge, for which the gluon polarization tensor is $\Delta_{\mu\nu}(k) = -g_{\mu\nu} + (k_\mu \eta_\nu + k_\nu \eta_\mu)/k \cdot \eta + O(k_\mu k_\nu)$, where η is a fixed four-vector, to which the gluon spin is approximately orthogonal. Then the decay probabilities $\mathcal{B}(t,z)$ may be calculated from explicit diagrams involving "incoming" off-shell partons (but, in the leading pole approximation, with on-shell outgoing partons): interference diagrams are explicitly relegated to nonleading order in $\alpha_s dt/t$. (Corrections to the leading pole approximation $O(\alpha_s dt/t_0)$ probe details of the off-shell extrapolation; in calculating these, one must consider explicitly the "decay" by which the off-shell parton was generated.) Different choices for the gauge vector η share the leading pole contribution differently among the various radiating partons: for example, if η is chosen nearly along the momentum of some parton, then the diagrams involving radiation from that parton will give no leading pole contribution [F.4]. To obtain directly the probabilities

(1,2), one must choose η away from the momenta of radiating partons: the z appearing will then be $E + |\vec{p}|$ fractions evaluated in the rest frame of η . Of course, the sum of all diagrams regardless of gauge reproduces leading pole results obtained by applying the probabilities (1,2) for radiation from all possible partons.

As discussed above, the "leading pole approximation" (1,2) undoubtedly becomes inadequate when $t \lesssim \mu_c^2$: the later development must be treated by other means. This limitation also affects the z distributions in $\mathcal{B}(t, z)$, which become inaccurate when z approaches 0 or 1 too closely. Consider the decay of a parton 0 into two partons, 1 and 2. For the z of the final partons to be maximal, they must have zero invariant mass. In this case, parton 1 has $z_1 \equiv (E_1 + |\vec{p}_1|) / (E_0 + |\vec{p}_0|) = 2E_1 / (E_0 + |\vec{p}_0|)$. By energy conservation, $E_1 = E_0 - E_2 \leq E_0$, and hence $z_1 \leq 2E_0 / (E_0 + |\vec{p}_0|) = 1 - (E_0 - |\vec{p}_0|) / (E_0 + |\vec{p}_0|) = 1 - t_0 / (\bar{z}_0^2 Q^2) \leq 1$, where \bar{z}_0 is the $E + |\vec{p}|$ fraction for parton 0 with respect to the original γ^* momentum. Since we require $t_0 \gtrsim \mu_c^2$, the minimum "z loss" for (1,2) to remain accurate is $\sim \mu_c^2 / Q^2$: the soft divergences in $P(z)$ for gluon emission are always avoided.

The classical and iterative nature of parton evolution for $Q \gtrsim \mu \gg \mu_c$ makes this phase of jet development eminently suited to investigation by Monte Carlo methods. The parton showers shown in Fig. 1 were generated using a Monte Carlo computer program [2], taking $Q = 200$ GeV, $\mu_c = 1$ GeV, $\Lambda = 0.5$ GeV. Virtual partons were drawn to travel for a proper time $\Delta\tau \sim 1/\Delta E \sim 1/(E - |\vec{p}|)$ before radiating. Note that all parton trajectories are curved by the semilogarithmic scales used to display the events.

The parton showers in Fig. 1 resulting from the "decay" of a high invariant mass parton are in many respects analogous to electromagnetic showers, initiated by the entry of a high-energy electron or photon into matter [F.5]. In the latter case, the initial particle is on its mass shell, but is repeatedly "poked" off shell by interactions with nuclei and generates a shower by successive Bremsstrahlung radiation and pair production. Eventually, when the energies of produced e, γ fall below some fixed critical value, interactions with ambient atomic electrons (ionization losses) become important, so that free radiation, as described by probabilities analogous to (1,2), no longer dominates. The radiation from an accelerated, and hence off shell, electron may be found directly by solution of classical electrodynamics equations with suitable boundary conditions. The explicit simulation of photon emissions may be considered as a Monte Carlo solution of these equations. For the QCD case, however, the classical equations are much less tractable, and Monte Carlo methods become almost obligatory. The primary reason for this is that gluon decays cause parton showers to have a much more dendritic structure than their QED counterparts: this extra cascading means, for example, that the gluon potential

at a point samples sources not only on the surface, but also throughout the volume of its past light cone (hence a "pulse" of gluon radiation will become dispersed, even propagating through the vacuum).

Perturbative Corrections to the Leading Pole Approximation

To $O(\alpha_s^0)$, the γ^* decay is described by the two-body decay probability $\mathcal{D}_{\gamma \rightarrow q\bar{q}}(Q^2, z) = O(\alpha/Q^2)$. At $O(\alpha_s)$, terms describing emission of one gluon must be added. In the leading pole approximation, the relevant quark decay is independent of the γ^* decay, and the differential cross-section is given by the product $\mathcal{D}_{\gamma \rightarrow q\bar{q}}(Q^2, z) \mathcal{D}_{q \rightarrow qG}(t, z_1) dt = O(\alpha/Q^2 \alpha_s dt/t)$. However, when the invariant mass \sqrt{t} of the virtual quark approaches its kinematic maximum $\simeq Q$, corrections $O(\alpha/Q^2 \alpha_s dt/Q^2)$ may become important, and invalidate the independent emission approximation. Such corrections may be included by introducing a three-body γ^* decay probability $\mathcal{D}_{\gamma \rightarrow q\bar{q}G}^3(Q^2, t; z_1, z_2)$ defined simply as the piece of the complete $\gamma^* \rightarrow q\bar{q}G$ differential cross-section not accounted for by the successive independent two-body decays $\gamma^* \rightarrow q\bar{q}$, $q \rightarrow qG$. The contribution of $\mathcal{D}_{\gamma \rightarrow q\bar{q}G}^3$ to integrated cross-sections is typically $O(1/\log(t))$ relative to the dominant independent decay term

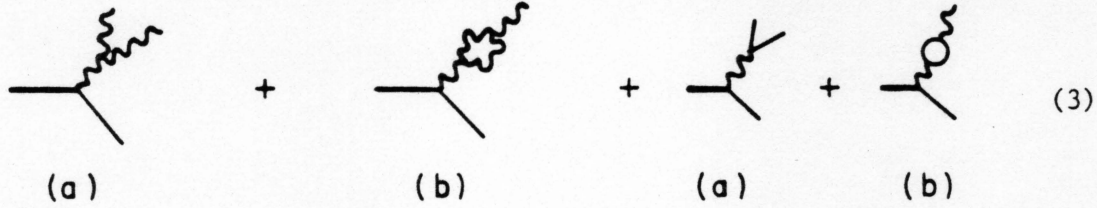
$\mathcal{D}_{\gamma \rightarrow q\bar{q}} \mathcal{D}_{q \rightarrow qG}$. It turns out that with the identification for z made here (see above), $\mathcal{D}_{\gamma \rightarrow q\bar{q}G}^3$ is identically zero away from $z = 1$. However, if one considers the decay of a scalar photon γ_S^* , then the independent two-body decay term remains as for a vector photon, but the three-body decay term becomes $\mathcal{D}_{\gamma_S \rightarrow q\bar{q}G}^3(Q^2, t; z_1, z_2) dt = \alpha/Q^4 4\alpha_s/3\pi dt$ (where z_1, z_2 refer to q, \bar{q} , respectively, $z_{1,2} < 1$, and t is the q, \bar{q} invariant mass). Even at the kinematic boundary $t \rightarrow Q^2$, $\mathcal{D}_{\gamma_S \rightarrow q\bar{q}} \mathcal{D}_{q \rightarrow qG} dt = \alpha/Q^2 8\alpha_s/3\pi dt/Q^2$, so that the three-body decay term provides a rather small correction away from the end point, and the sharp cutoff assumed for the two-body decay probabilities is adequate.

Just as for $\gamma^* \rightarrow q\bar{q}G$, the approximation of successive independent two-body parton decays may receive significant corrections when a parton produced in the first decay does not have an invariant mass much smaller than its parent. To account for these corrections, one may again introduce a three-body decay probability $\mathcal{D}_{j_0 \rightarrow j_1 j_2 j_3}^3(t, t'; z_1, z_2)$ [F.6]. The exact differential cross-section for production of n partons by γ^* decay is then given by a sum of terms: the first results from $(n-1)$ independent two-body decays; the second from one three-body decay and $(n-2)$ two-body decays; the final term represents a single n -body decay. In most cases, the successive terms in this series should be progressively much smaller; the first term, corresponding to the simple leading pole approximation should then provide an adequate estimate for the complete differential cross-section. Observable properties of the parton final state are given as integrals of this differential cross-section. It appears that the results obtained by using the leading pole approximation for the differential cross-section but

keeping the exact kinematic boundaries agree (apart from overall normalization discussed below) well with relevant regions of available complete explicit calculations [F.7], suggesting that \mathcal{B}^3 may indeed be neglected. Note that in many discussions (e.g., [12]) of the "leading log approximation", further kinematic approximations are made on integrating the leading pole approximation differential cross-section (typically, all interdependence in the limits of t and z integrations is ignored, and the μ_c cutoff is implemented only in t , but not z , integrals). The results of this procedure are often inaccurate: a good estimate of the differential cross-section is wasted by maltreatment of the kinematic limits of integration. Inclusive observables in the parton final state (e.g., single parton energy or k_T distributions, or H_2 distributions) often receive divergent contributions close to the kinematic boundaries at each order in α_s (e.g., $O(\alpha_s \log^2(k_T^2/Q^2))^k$) terms arise in the relative transverse momentum distribution of the leading q, \bar{q} , which dominates the $1-H_2$ distribution). When summed to all orders in α_s using the leading pole approximation, the contributions exponentiate to provide "radiation damping" at the kinematic boundary (e.g., the relative transverse momentum distribution between the leading q, \bar{q} becomes $\sim \exp(-\alpha_s \log^2(k_T^2/Q^2))$, as discussed in Ref.[1]). All \mathcal{B}^3 contributions are formally $O(1/\log(t/\Lambda^2))$ relative to the \mathcal{B}^2 leading pole terms (in practice they appear to have small coefficients): since perturbative methods apply only when $t \gtrsim \mu_c^2 \gg \Lambda^2$, this factor probably ensures small \mathcal{B}^3 contributions in the relevant region. In addition, multiple decay probabilities corresponding to "Bose-Einstein correlations" between gluons should be suppressed by powers of $1/N_c$ because a larger number of colors gives a smaller probability for a set of gluons to be indistinguishable. Note that for small t , the two body decay probabilities $\mathcal{B}(t, z)$ receive $O(m^2 dt/t^2)$ corrections from finite light quark current masses, and presumably suffer $O(\mu_c^2 dt/t^2)$ "higher twist" corrections from the onset of hadron formation.

I now discuss the origin and form of the effective coupling $\alpha_s(\tilde{t}) \sim 1/\beta_0 \log(\tilde{t}/\Lambda^2)$ appearing in the decay probabilities $\mathcal{B}(t, z)$ of eqs. (1,2). The $\mathcal{B}(t, z)$ give leading pole approximations to the probabilities for two-body decays of off-shell partons, summed over all subsequent interactions of the decay products. All parton interactions receive virtual corrections which exhibit ultraviolet divergences; such divergences may be renormalized by a subtraction at an invariant mass μ_R . Renormalized quantities, such as the physical coupling constant $\alpha_s(\mu_R^2)$, depend on the value of this renormalization mass: different choices for μ_R leave measurable cross-sections unchanged by altering contributions from explicit virtual corrections so as to cancel the changes in $\alpha_s(\mu_R^2)$. At $O(\alpha_s)$, the probability for the decay

$q^* \rightarrow qG \sim \alpha_s(\mu_R^2)/t P_{q \rightarrow qG}(z)$. At $O(\alpha_s^2)$, the decay probability receives its leading corrections from the diagrams [F.8]



In (3a), the outgoing gluon is not on its mass-shell, but may have an invariant mass t' up to the kinematic limit $t_{\max} \simeq z(1-z)t$ imposed by the mass of the decaying quark. The total correction due to diagrams (3a) is roughly

$$\alpha_s(\mu_R^2) \int_0^{t_{\max}} \frac{dt'}{t'} \int_{\delta}^{1-\delta} [P_{G \rightarrow GG}(z') + P_{G \rightarrow q\bar{q}}(z')] dz' \sim$$

$$\alpha_s(\mu_R^2) \log\left(\frac{t_{\max}}{\delta}\right) [2\log\delta + (2F-33)].$$

In diagram (3b), the final gluon must be on-shell, so that the result does not depend on t_{\max} : it serves simply to cancel the divergences from (3a), yielding total correction to the $q^* \rightarrow qG$ decay probability $\sim (1 + \alpha_s(\mu_R^2)/6\pi (2F-33)\log(t_{\max}/\mu_R^2))$ where μ_R^2 is the renormalization mass introduced in subtracting the divergences of (3b). In higher orders, the dominant diagrams involve several virtual corrections followed by real pair production: the diagrams form a geometric series whose sum is $(1 - \alpha_s(\mu_R^2)/6\pi (2F-33)\log(t_{\max}/\mu_R^2))^{-1} \simeq \alpha_s(t_{\max})/\alpha_s(\mu_R^2)$. Hence the total probability for the decay $q^* \rightarrow qG$, summing over all subsequent fates for the produced $q, G \sim \alpha_s(t_{\max})/t P_{q \rightarrow qG}(z)$. If μ_R^2 is chosen to be t_{\max} , then explicit higher order diagrams will provide no (leading pole) corrections to $\mathcal{P}_{q \rightarrow qG}(t, z)$: all such corrections will have been included implicitly in the effective coupling $\alpha_s(t_{\max})$ [F.9].

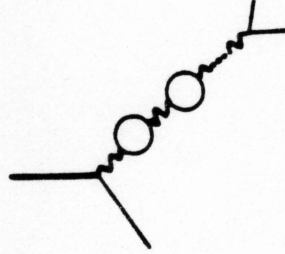
The argument t_{\max} of $\alpha_s(t_{\max})$ appearing in the decay probabilities (1,2) is roughly the relative transverse momentum (squared) between the products of the parton decay. If $\alpha_s(t)$ rather than $\alpha_s(t_{\max})$ were used in $\mathcal{P}(t, z)$ then the three-body decay probabilities $\mathcal{P}^3(t_1, t_2; z_1, z_2)$ would contain $O(\alpha_s^2 \log(z(1-z)))$ terms which become large near the kinematic boundary: such terms are summed and accounted for by use of $\alpha_s(t_{\max})$ in the two-body decay probabilities. In obtaining the usual form $\alpha_s(t_{\max}) \sim 1/\beta_0 \log(t_{\max}/\Lambda^2)$ from the diagrams (3), one assumes that intermediate gluons may have arbitrarily small invariant masses. As discussed above, the $\mathcal{P}(t, z)$ used in the calculation become inaccurate for $t \lesssim \mu_c^2$: to be consistent with the treatment of real emissions, one should assume that no parton may have an

invariant mass $\leq \mu_c$. In this case, the higher order corrections imply a form [F.10] $\alpha_s(\tilde{t}) \sim 1/\beta_0 \log((\tilde{u} + \mu_c)^2/\Lambda^2)$: this freezes at a large fixed value for small \tilde{t} , and does not diverge at the Landau point $\tilde{t} \simeq \Lambda^2$. For some purposes, this behavior is analogous to providing an effective mass $\simeq \mu_c$ for gluons. (Note that the exact form of the corrected decay probabilities could in principle be obtained by Monte Carlo integration over all possible final states accessible from each decay considered: the procedure is, however, quite unwieldy, and probably unnecessary in view of the small changes expected (see below).)

In addition to the logarithmic $O(\alpha_s^2)$ terms from (3) accounted for by use of $\alpha_s(\tilde{t})$, there are also constant, nonlogarithmic $O(\alpha_s^2)$ terms which provide contributions to both two- and three-body decay probabilities. The values of these terms depend on the renormalization prescription used: arbitrary constant terms may be removed with the divergences in (3b): whenever divergences, such as those in (3b), are subtracted (renormalized) away, a nondivergent remainder is in general left. This remainder may be removed by redefinitions of "bare" parameters, such as the coupling constant α_s . The value of α_s to be inserted into the Lagrangian is, of course, not known a priori (and, in fact, must be divergent to cancel the divergences in the perturbation series), but may be determined by fitting theoretical calculations to experimental data. The fitted numerical value for α_s will depend on the remainder removed (in such a way that the output experimental prediction used for the fit remains unchanged). The value of the remainder is thus determined by the "renormalization prescription" used to remove the divergences, and to define physical parameters such as α_s . Theories such as QCD possess the properties of being renormalizable and infrared factorizable, whose meaning is that the number of distinct divergences (due to large and to small momentum configurations) at a given order in perturbation theory is limited: once prescriptions have been devised and applied to subtract these divergences in a limited number of processes, definite parameter-free physical predictions may be obtained for any other processes to the same order in perturbation theory [F.11]. Hence the "value of α_s " to be used in $\langle H_2 \rangle$ (or $\langle \text{thrust} \rangle$) in e^+e^- annihilation at a given order could be deduced (in principle), for example, from the experimental e^+e^- total cross-section. If, say, both processes are calculated only to $O(\alpha_s)$, then higher order terms in each perturbation series will lead to $O(\alpha_s^2)$ errors in the predictions. Such errors can be corrected for by modifying the α_s to be used by a calculated $O(\alpha_s^2)$ correction, thereby absorbing the higher order terms into the numerical value of α_s . Of course, the correction will, in general, be different for different processes. Note that all renormalization prescriptions introduce some renormalization mass μ_R : the logarithmic dependence of α_s on μ_R was discussed

in the preceding paragraph. To $O(\alpha_s^2)$, all nonlogarithmic terms can be accounted for by using effective μ_R differing by numerical factors determined from explicit $O(\alpha_s^2)$ calculations in different processes: this procedure fails, however, beyond $O(\alpha_s^2)$.

As a simple, but revealing, example of higher-order nonlogarithmic corrections to parton production, I consider the QED-like processes



which modify $\mathcal{P}_{q \rightarrow qG}(t, z)$. Diagrams with only this structure may be selected by taking the formal limit that the number of fermion flavors goes to infinity. Then a simple calculation [5] reveals multiplicative corrections to the lowest-order $\mathcal{P}_{q \rightarrow qG}(z)$ of the form $\sum_k k! (-1)^k (4F\alpha_s/9\pi)^k \zeta(k) + (4iF\alpha_s/9\pi)^k \pi^k \theta(-\mu_R^2)$, k even, where k is the number of loops. The basic origin of the embarrassing $O(k! \alpha_s^k)$ terms is with the Landau singularity. The fermion vacuum polarization corrections to a gluon propagator with invariant mass $\sqrt{p^2}$ may be summarized (in the limit $F \rightarrow \infty$) by the leading log effective coupling $\bar{\alpha}_s(p^2) = \alpha_s(\mu^2) / [1 - (4F\alpha_s(\mu^2)/9\pi) \log(p^2/\mu^2)]$. Then the finite part of the diagrams (4) involves roughly $\int_0^t d(p^2) \bar{\alpha}_s(p^2)$, which introduces $\sum_k k! (-\alpha_s)^k$. The alternating sign in these corrections makes them formally amenable to Borel summation (with result $-\xi e^\xi \text{Ei}(\xi)$, $\xi = (9\pi/4F\alpha_s)$). However, if the integral over p^2 had run over the Landau singularity (at $\Lambda^2 \approx \mu^2 \exp(-\beta_0/g^2(\mu^2))$, where the denominator in the leading $\log \bar{\alpha}_s(p^2)$ vanishes), then all terms would have had the same sign, and no reordering would allow the divergence as $k \rightarrow \infty$ to be removed. This behavior occurs in, for example, corrections to $g_\mu - 2$ due to multiple electron loops in QED [6]. In QCD (forsaking the limit $F \rightarrow \infty$), the Landau singularity Λ^2 is at small, rather than large, invariant masses: in a purely perturbative calculation (with no μ_c imposed), the p^2 integration in evaluating, for example, corrections to parton decay probabilities runs across the singularity, and $O(k! \alpha_s^k)$ terms are expected. As discussed in the preceding paragraph, any corrections would be irrelevant if they could be absorbed universally by a change of renormalization prescription. Unfortunately, diagrams requiring the same renormalization may or may not involve integration over the Landau singularity, so that the corrections cannot be absorbed universally. (Perhaps by defining separate "Landau divergent" and "Landau convergent" α_s , this particular class of corrections could be avoided.)

At low orders, corrections, e.g., $O(\pi^k/k! \alpha_s^k)$ or $O(\pi^k \alpha_s^k)$ may be important [7]. Such terms arise in comparing processes with incoming and outgoing partons or spacelike and timelike Q^2 (some are visible in (4) if $\mu_R^2 < 0$), and result from unitarization corrections: for some external kinematic configurations, intermediate lines may reach their mass shells, thereby sampling the second term in the propagator $1/(t-i\epsilon) = 1/t + i\pi\delta(t)\theta(t)$. Clearly such terms are usually associated with logarithms, and may then mathematically be obtained by changing signs in arguments of logarithms, (unitarity specifies the relevant Riemann sheet) according to the signs of external kinematic invariants. Hence they may often be summed in parallel with the logarithms, usually forming exponential or geometric series [F.12].

Despite these indications, one might hope that on summing all diagrams to a given order in α_s , tolerable corrections would result. Even if each diagram gave $O((\alpha_s/\pi)^k)$ [F.13] (as would be obtained if its internal loop integration was uniform up to kinematic and renormalization cutoffs) with random sign, the total would be $O(\sqrt{k!} N_c^k \alpha_s^k)$ since the number of diagrams is $O(k!)$. (Indications from g_e^{-2} in QED that diagrams tend to cancel [8] are probably accidental, since individual gauge invariant diagram sets may grow $\sim \alpha_s^k k!$ (as in the example discussed above), and available QCD calculations (e.g., [9]) reveal mainly constructive, rather than destructive arrangements of signs, and large numerical coefficients.) The large observed value of α_s means that higher order terms in the perturbation series will not become small for many orders before eventual divergence (as they presumably do in QED) and reliable truncation of the (perhaps asymptotic) perturbation series becomes impossible. In the face of these apparently insuperable difficulties, I shall use only the lowest order $\mathcal{B}(t,z)$, with the hope that, as appears phenomenologically for the case of very high orders in QED perturbation theory, the correction terms will eventually conspire to be small. To make some allowance for higher order corrections, I allow an arbitrary normalization correction to α_s , but assume the lowest-order kinematic structure [F.14]. (For $\mu_c \gg \Lambda$, this corresponds simply to treating Λ as a free parameter, unconstrained by other determinations of α_s .) Note that existing higher order calculations (e.g., for $q\bar{q} \rightarrow \gamma^* X$) have found large corrections only in the lowest-order kinematic configuration: the next order calculations will, however, undoubtedly exhibit large corrections to any kinematic configuration accessible at the lower order.

The Structure of Parton Final States

By using the leading pole approximation discussed above, and assuming that higher order corrections may alter the overall normalization but not

the kinematic structure of the "decay" probabilities $\mathcal{B}(t,z)$, one may trace the development of parton final states in e^+e^- annihilation until the invariant masses of the partons are degraded below the critical mass μ_c . As mentioned above, the use of Monte Carlo methods [2] allows exact account of kinematic constraints to be taken, yielding in many cases important corrections to results found by "asymptotic" analytical techniques (e.g., [12,1]).

Figure 2 shows the mean total multiplicity of partons produced before the cutoff μ_c as a function of Q , with Λ taken as 0.5 GeV. For smaller μ_c , more partons are radiated before the evolution is curtailed. Note that the detailed quantitative behavior of the parton multiplicity is somewhat sensitive to the details of the imposition of the μ_c cut [F.15]: qualitative features are, however, entirely insensitive. Nearly all the partons are gluons: the curve for mean $q + \bar{q}$ multiplicity with $\mu_c = 1$ GeV given in Fig. 2 indicates that in this case, an average of one secondary quark per event is achieved only at $Q \approx 100$ GeV. (Note that only three quark flavors are considered, since only these may be excited by the momentum transfers $\sim \mu_c$ which dominate the development of the parton final state.) The results in Fig. 2 approach slowly the asymptotic form [11, 12, 1] $\langle n \rangle \sim \exp[2.3(\sqrt{\log(Q^2/\Lambda^2)} - \sqrt{\log(\mu_c^2/\Lambda^2)})]$ when $Q \gtrsim 100$ GeV. $\langle n_{q+\bar{q}} \rangle$ at asymptotic Q should lag $\langle n_g \rangle$ only by a power of $\log(Q^2/\Lambda^2)$: at accessible energies, however, the suppression is numerically large. In QED, the asymptotic multiplicity distributions are of the Poisson form (so that $f_k \equiv \langle n!/(n-k)! \rangle - \langle n \rangle^k = 0$): a sequence of photons is emitted independently from an off-shell electron line, typically with energies much lower than the electron energy so that kinematic correlations are absent. In QCD, however, each emission changes the color of the radiating parton, destroying the independence. Moreover, the much larger total multiplicity is dominated by radiation of gluons from low energy gluons: kinematic correlations are therefore significant, and the multiplicity distributions deviate from the Poisson form (asymptotically [12, 14], $P(n) \sim \Gamma(\xi+n)/\Gamma(\xi)A^n/n!$ with $A \rightarrow 1$, and the constant ξ depending on the cutoff prescription. The dispersion $\sqrt{\langle n^2 \rangle - \langle n \rangle^2} / \langle n \rangle$ of the distribution remains roughly constant over the Q range shown in Fig. 2 (with a value ≈ 0.31 for $\mu_c = 1$ GeV), rather than decreasing $\sim 1/\sqrt{\langle n \rangle}$ as for a Poisson distribution.

For $Q \leq \mu_c$, no gluons are emitted, so that the quark energy distribution is simply $D_q(z) = \delta(1-z)$, where $z = 2E/Q$. As Q increases, progressively more gluons are emitted, and the quark energy spectra soften. Figure 3 shows the mean fractional energy carried by gluons $\langle E_g/Q \rangle$ as a function of Q . The percentage of events in which no gluons are emitted above the critical invariant mass μ_c is also marked, and decreases rapidly with increasing Q . The standard leading log approximation often used to

estimate the evolution of moments of the z distributions is obtained from the leading pole approximation (1, 2) by making the kinematic approximation that all emitted partons are collinear with their parents, but nevertheless formally allowing the invariant mass of each emitted parton to run up to the mass of its parent. The leading log approximation for the evolution of a quark energy spectrum $\delta(1-z)$ at $t = \mu_c^2$ up to $t = Q^2$ gives

$$\langle E_G/Q \rangle \simeq 0.64 \left(1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu_c^2)} \right)^{0.62} \right); \quad (5)$$

this form is compared with the complete result (for $\mu_c = 1$ GeV) in Fig. 3. The effects of the kinematic approximations fall off only rather slowly with Q , mainly because of the importance of multigluon emissions, in which each gluon has only a small fraction of the total available energy Q . At asymptotically large Q , $\langle E_G/Q \rangle$ tends logarithmically from below to the "equilibrium" value $16/25$; the final limit is independent of μ_c or Λ , but is approached more rapidly for smaller μ_c/Λ . For most of the curves in Fig. 3, the $\alpha_s(\tilde{t})$ in the decay probability (1) was approximated by $\alpha_s(t) = 1/\beta \log(t/\Lambda^2)$, with $\Lambda = 0.5$ GeV. However, as discussed above, the choice $\alpha_s(\tilde{t}) \simeq \alpha_s([\sqrt{z(1-z)t} + \mu_c]^2)$ should account for subleading log higher order corrections (with a momentum-independent subtraction scheme used for renormalization): the result for $\mu_c = 1$ GeV with this form shown in Fig. 3 suggests that these corrections are quite insignificant. Changes in the overall normalization of α_s due to higher order corrections alter the $\langle E_G \rangle$ in roughly the same manner as changes in μ_c .

Each off-shell parton "decay" imparts a relative transverse momentum $k_T^2 \simeq z(1-z)t$ between its products. If the transverse momentum distributions in the individual "decays" had finite variance, then the central limit theorem implies that the resulting total k_T distribution should be roughly Gaussian. In fact, the power law k_T distributions in each decay implied by the dt/t factor in the decay probability give rise to a power law tail in the average single parton k_T distributions measured with respect to the initial q, \bar{q} directions, as illustrated in Fig. 4. (This is mathematically analogous to the tail of the (Molière) multiple Coulomb scattering transverse momentum distribution for electrons traversing matter.) The results in Fig. 4 are all for $\mu_c = 1$ GeV, $\Lambda = 0.5$ GeV: a larger μ_c removes small k_T partons, but does not affect "hard" partons with $k_T \gtrsim \mu_c$, and thus increases the $\langle k_T \rangle$.

As discussed above, partons emitted at early times typically have large transverse momenta with respect to their parents, but because of the form of the decay probabilities (1, 2), partons emitted later are progressively more collinear with their parents. This ordering leads at sufficiently high Q to considerable clustering in the angular distribution of

energy for the parton final state [F.16]. Figure 5 shows the distribution of partons in the northern hemisphere of a reasonably typical simulated event at $Q = 100$ GeV, with $\mu_c = 1$ GeV, $\Lambda = 0.5$ GeV. (The original quark was directed towards the north pole. The event displayed in Fig. 5 is somewhat more isotropic than the average.) The angular clumping of the partons is evident. Observables which are sensitive to the angular distribution of energy in the final state only at large angular scales should probe only the total momenta of the clumps, and be insensitive to the precise distribution of the momenta between their individual constituent partons [F.17]. Since typically the angle between a pair of partons produced by the decay of a parent with invariant mass μ is $\sim \mu/(Q\bar{z})$ (where \bar{z} is the fraction of the original γ^* energy carried by the parent), observables which probe the final state energy distribution in angular bins of width $\geq \theta$ should be sensitive only to decays with $\mu \geq \theta Q\bar{z}$. In as far as the final formation of hadrons affects only invariant masses $\mu \leq \mu_c$ ($= 0(1 \text{ GeV})$), so the distribution of hadronic energy over angular scales $\theta \gg \mu_c/(Q\bar{z})$ should reflect the structure of the parton system. A convenient set of observables for measuring the final state angular energy distribution is given by [F.18] [15]

$$H_\ell \equiv \sum_{i,j} \frac{|\vec{p}_i| |\vec{p}_j|}{Q^2} P_\ell(\hat{p}_i \cdot \hat{p}_j) \quad (6)$$

where the sum runs over all final particles, including the case $i = j$. For a final state consisting of just two massless particles (e.g., $\gamma^* \rightarrow q\bar{q}$), $H_{2\ell} = 1$, $H_{2\ell+1} = 0$, while for an isotropic final state, $H_\ell = 0$ ($\ell > 0$). The H_ℓ are the coefficients in a Legendre expansion of the energy correlation [15, 17] between two point detectors as a function of their relative angle. For high ℓ , the Legendre polynomials may roughly be approximated by $P_\ell(\cos\phi) = \theta(|\phi| - 1/\ell) + (-1)^\ell \theta(|\pi - \phi| - 1/\ell)$, so that the H_ℓ lump together systems of partons subtending angles $\leq 1/\ell$, and probe the evolution of the final state only at $\mu \geq Q\bar{z}/\ell$. Figure 6 shows $\langle H_2 \rangle$ as a function of Q for various μ_c with $\Lambda = 0.5$ GeV. At low Q , no emissions are possible above μ_c , and $H_2 \sim 1$. As Q increases, the effects of the cutoff μ_c decrease $\sim \mu_c/Q$ (this linear behavior reflects the linearity of H_2 in final particle momenta necessary to ensure infrared finiteness). At high Q , $\langle H_2 \rangle$ approaches the result $\langle H_2 \rangle \simeq 1 - 1.4 \alpha_s(Q^2)$ [F.19] obtained by treating only the first emission in perturbation theory. (As discussed above, however, higher order terms may modify the normalization of $\alpha_s(Q^2)$.) A small $O(\alpha_s^2)$ deviation remains even at high Q . Note that $\langle H_2 \rangle$ reaches its asymptotic form at much lower Q than did $\langle E_G \rangle$, mainly because multigluon effects are suppressed by $O(\alpha_s)$ rather than $O(1/\log(1/\alpha_s))$. The distributions $1/\sigma d\sigma/dH_2$ for $\mu_c = 1$ GeV at various Q are given in Ref. 2. The $\log(1-H_2)/(1-H_2)$ divergence at

$O(\alpha_s)$ exponentiates to provide "radiation damping" at $H_2 \rightarrow 1$ when higher order emissions are summed. Figure 6 also shows the behavior of $\langle H_8 \rangle$ when $\mu_c = 1$ GeV, and the form of $\langle H_8 \rangle$ at $O(\alpha_s)$ in perturbation theory. H_8 is more sensitive to later emissions than H_2 . Typically, at large ℓ , the effects of successive emissions on $\langle H_\ell \rangle$ decrease as $(\alpha_s \log^2 \ell)^k$, while the cutoff provides $O(\ell \mu_c / Q)$ corrections. As $\ell \rightarrow \infty$, $\langle H_\ell \rangle \rightarrow \langle \sum_{i=1}^{\ell} E_i^2 / Q^2 \rangle \sim \mu^2 / Q^2 \langle n \rangle$.

The emission of a single gluon deflects energy in an event from the q, \bar{q} line. Emission of two or more gluons spreads energy outside a plane. The deviation of a final state from coplanarity may be measured, for example, by [15] $\Pi_1 \equiv \sum_{i,j,k} |\vec{p}_i| |\vec{p}_j| |\vec{p}_k| / Q^3 (\hat{p}_i \cdot \hat{p}_j \cdot x \hat{p}_k)^2$ [F.20].

If the quarks produced in the γ^* decay were exactly on-shell and massless, then their angular distribution with respect to the original e^+, e^- direction would be $(1 + \cos^2 \theta)$. When the quarks are off-shell, their original angular distribution becomes instead roughly $(1 + (1 - 2(t_1 + t_2)/Q^2) \cos^2 \theta)$; the gluons radiated remain roughly uniform in azimuth with respect to the e^+, e^- direction [15].

Beyond Free Emission

The approximation of free (and independent) parton emissions used in the previous section becomes inaccurate when the invariant masses of the partons in an event have fallen so low that the rate of emissions from them no longer dominates over the rate of interactions between them. Above, I have simply truncated all radiation at this critical point, parametrized by a critical parton invariant mass μ_c . However, for an accurate description of even the most inclusive experimental measurements at available Q , it is essential to venture beyond the critical point, and model the final formation of hadrons.

The first simplifying assumption which I shall make is that the cross-section for production of a given final hadron state involves only an incoherent sum over parton states at the critical point. This is in keeping with the free emission approximation for parton production, which provides cross-sections for parton configurations at the critical point: it suggests that interferences between different parton states which are transformed into the same hadron state [F.21] should vanish. The processes of hadron formation are taken to be sensitive only to the probabilities, and not the amplitudes, for the possible configurations of the parton system. As a simple, but somewhat inappropriate analogue, consider a reaction in which many e^+, e^-, γ are produced: the assumption requires that interferences between processes in which a given positronium atom arises from an e^+e^- system and an $e^+e^-\gamma$ system should cancel. This occurs in so far as the amplitudes for population of the various states of the interfering systems have random phases (as would follow from classical free emission).

A further (but to some extent related) assumption is that the evolution beyond the critical point depends only on the local structure of the parton system over small spacetime volumes at the critical point. Hence, suitable sets of partons at the critical point will evolve to form hadrons independently, and irrespective of the processes by which they were produced. For example, I shall below often assume that low-mass color singlet clusters of partons present at the critical point condense directly into hadrons, independently from each other. If the formation of hadrons does not occur in some such local and universal manner, there seems little hope of obtaining useful predictions from QCD without very detailed knowledge of the structure of hadrons. If, even at high Q , the whole parton system at the critical point acted cooperatively to generate the final hadrons, then the disposition of partons could be largely irrelevant, and perturbative parton production above the critical point would be rendered invisible. I shall entirely neglect this possibility, and will assume that the processes of hadron formation act universally and locally: the precise constitution of the independent parton systems is, however, unknown; several possibilities will be discussed below.

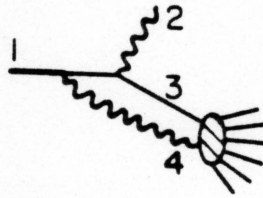
For some purposes, it may, at sufficiently large Q , be adequate to make the approximation that each individual parton in the final state (rather than, say, each color singlet cluster of partons) "decays" independently into hadrons. (This approximation fails to account for color conservation, and therefore must be violated eventually.) It is conventional, for example, to define "fragmentation functions" which describe inclusive hadron spectra in parton decays. In this approximation, the function $F_{p \rightarrow h}(z, \mu_0^2)$ gives the probability for a hadron of type h to carry a fraction z of the (roughly) longitudinal momentum of the parton p , whose invariant mass is less than μ_0 . If the perturbative evolution of the final state is truncated when all parton invariant masses fall below μ_0^2 , then some inclusive properties of the final hadron state may be found by taking each parton to decay according to a suitable $F(z, \mu_0^2)$. In this case, changes in Q affect only the perturbative evolution: the decay of the parton system below μ_0^2 may be described by the same $F(z, \mu_0^2)$, regardless of Q^2 . Thus the approximation allows the change ("scaling violations") in single hadron inclusive energy spectra as a function of Q to be estimated without explicit knowledge of the processes of hadron formation. The approximation fails, however, when μ_0 and Q are small enough that many pairs of partons at μ_0^2 have invariant masses $\leq \mu_c$, and therefore may act cooperatively in forming hadrons, thereby necessitating introduction of a further joint two-parton fragmentation functions $F_{p_1 p_2 \rightarrow h}(\mu_0^2, z_1, z_2)$. Since only the Q variation of the single hadron distributions is required, such cooperative hadron

formation processes are irrelevant unless they depend on Q . Hence, the effects of processes such as



(7)

in which a parton acts cooperatively with the last gluons which it radiated before reaching μ_0 are accounted for by use of the physical fragmentation functions for the radiating parton. On the other hand, for example, the process



(8)

leads to Q^2 -dependent $O(\mu_c^2/Q^2)$ ("higher twist") corrections. The probability for parton 2 in (8) to be emitted in a kinematic configuration which allows partons 3 and 4 to have an invariant mass $\leq \mu_c$ decreases $\sim \mu_c^2/(\mu_1^2(z_3+z_4)^2)$, or formally $\sim \mu_c^2/Q^2$. The higher twist corrections (8) proportional to the joint fragmentation function $F_{qG \rightarrow h}(\mu_0^2; z_1, z_2)$ are therefore roughly of order μ_c^2/k_T^2 for hadrons of transverse momentum k_T with respect to the direction of parton 1. The most important higher twist corrections plausibly result when parton 1 is the original q or \bar{q} . In this case, the single hadron transverse momentum spectra receive corrections $O(\alpha_s \mu_c^2/k_T^2)$, yielding $O(\alpha_s \mu_c^2/Q^2)$ higher twist corrections to the Q development of single hadron energy spectra. (Numerically, the $O(\alpha_s)$ suppression may be more than compensated for by the smallness of $\langle k_T^2 \rangle$ relative to Q^2 . Note that higher twist corrections to energy spectra should become more important for lower energy hadrons at a given Q^2 .) The dominant, leading pole piece of the cross-section for the decay of parton 1 in (8) into 2, 3, 4 corresponds to independent emissions of 1, 2, 3, 4. As discussed in the previous section, there are "subleading log" correction terms, some of which depend on the process by which parton 1 was created (e.g., spin 1 or spin 0 " γ^* " decay): hence higher twist corrections will be no more universal to different processes than are subleading log corrections [18]. Note that in models (such as that discussed below) which describe the complete formation of hadrons from independent parton systems, higher twist terms are automatically present, and their nonuniversal nature accounted for by the form of corrections to leading pole parton decay probabilities. As discussed at the beginning of this section, the apparently plausible

assumption is nevertheless made in such a model that processes such as (8) cannot interfere with processes in which the gluon 4 is absent.

In addition to single hadron inclusive spectra, fragmentation functions may also be used to describe multihadron spectra [12, 13]. However, the different hadrons may arise either from the decays of separate partons, or as some of the decay products of a single parton. The possibility of the latter contribution requires introduction of new multiple fragmentation functions $F_{p \rightarrow h_1 h_2 \dots}(\mu_0^2, z_1, z_2, \dots)$ which may be determined only from experiment. This contribution may be removed by requiring that no hadron pairs considered have invariant masses $\leq \mu_0^2$, and therefore all must have originated from distinct partons. Nevertheless, the fragmentation function approach to multihadron spectra is increasingly affected by higher twist corrections, and is rendered largely impractical by the prohibitive number of parameters to be determined from experimental data. A more complete dynamical model for hadron formation is therefore required (which implicitly provides estimates for fragmentation functions).

The mechanisms for parton production discussed above were based on perturbation theory. However, it is possible, especially when $\alpha_s(\mu^2) \gtrsim 1$, that partons may be produced by effects not visible in perturbation theory (e.g., $O(\exp(-1/\alpha_s))$). (Note that such effects cannot be classified by a twist expansion: their coefficient functions as well as operator matrix elements would not be amenable to perturbative treatment.) In a uniform QED electric field (corresponding to a potential $V(r) = |e|r$), e^+e^- pairs with transverse momenta \vec{p}_T with respect to the field direction are generated spontaneously at a rate [19] $\simeq \exp(-2\pi(m_e^2 + |\vec{p}_T|^2)/|e\epsilon|)$. An e^+e^- pair in the uniform electric field separated by a distance d has a potential energy $2e\epsilon d$; for $d \geq (m_e^2 + |\vec{p}_T|^2)^{1/2}/2|e\epsilon|$, this potential energy exceeds the energy necessary to generate a real pair. The energetically favored state containing a real pair is reached by quantum mechanical tunneling, but at an exponentially small rate. The $O(\exp(-1/e))$ result for this rate exhibits an essential singularity at $e = 0$, and therefore cannot be obtained by any perturbation expansion about $e = 0$. The presence of real pair production eventually results in shielding, so that a uniform electric field cannot be maintained for an infinite time. In QCD, uniform color magnetic as well as electric fields are unstable [F.22], because the gluon magnetic moment deviates from the Dirac value. The dominant higher-order perturbative corrections to the decay rate may be accounted for by the replacement of the coupling constant in the exponential by the effective coupling $\sim g(|\vec{p}_T|)$ (appropriate for scattering of the pair from the potential). With this form, the rise of the pair production rate at small p_T would be damped for $p_T \lesssim \Lambda$.

The results for spontaneous nonperturbative pair production in uniform fields also hold for separating point charges in $1 + 1$ - dimensional QED or QCD (where $V_{\text{Coulomb}}(r) \propto r$) [20], or in $3 + 1$ -dimensional QCD if as yet unknown effects concentrate color flux into a tube between the separating q, \bar{q} [19]. If instead, the color electric field is taken to have the perturbative dipole form resulting from a static $q\bar{q}$ pair separated by a distance a , then the potential difference between two points $\sim 2gr/a^2$ for $r \ll a$; again spontaneous pair production should occur, at a rate $\sim \exp(-\pi(p_T^2 + m^2)a^2/(g(1/a)g(p_T)))$ for $\sqrt{(p_T^2 + m^2)} \leq 1/(g^2 a)$ (and vanishing with a higher power in the exponent for larger p_T). If the q, \bar{q} sources (and hence the field generated by them) were indeed static, then the only source of pairs would be such nonperturbative spontaneous production. However, in practice, the q, \bar{q} are accelerated at the γ^* decay point, and then move rapidly apart, generating a time-dependent field usefully parametrized by elementary gluon excitations and resulting in the perturbative pair production discussed extensively above. Note that pairs produced in the latter manner exhibit power-law, rather than exponential, damping in p_T . Perturbative parton production will modify the field in which nonperturbative tunneling may occur. Typically, at late times, newly-emitted gluon pairs will provide sites for spontaneous production with small a and thus high fields. Any spontaneously-produced gluons will be very closely collinear with the separating gluons; they would form a polarization cloud, which would bleach the color of the high momentum gluons, and reduce their reinteraction cross-section [F.23]. Their effects will probably be important, however, only long after the free emission approximation has failed.

The spontaneous nonperturbative production of partons discussed above occurs by tunneling from a state containing just the field to a state containing, in addition, a parton pair, but having the same energy as the original state. The exponent in the tunneling probability is (minus) the action associated with the classical propagation of the partons through the field in imaginary time. As well as those which lead to additional parton production, there may also occur tunneling processes between identical states, which serve to alter the amplitude for the persistence (propagation) of the state. For example, the parton propagators receive $O(\exp(-2\pi/\alpha_s))$ corrections from processes in which tunneling occurs by way of an instanton solution to the (sourceless) classical field equations. Such corrections are presumably $O(\exp(-2\pi/\alpha_s(\mu^2))) \sim O((\Lambda^2/\mu^2)^4)$ and therefore almost certainly irrelevant.

It is very difficult to make realistic estimates on the failure of the free parton emission approximation. Certainly the simple cutoff μ_c on the invariant mass of individual partons used above is an oversimplification: presumably invariant masses of pairs of partons are also involved. (Since

small invariant mass parton pairs often arise from decay of a single small invariant mass parton, these prescriptions are at least roughly equivalent). In addition, the cutoff will not be sharp: the approximation will progressively become more inaccurate. (This behavior could perhaps be parametrized by choosing the value of μ_c for each parton from a distribution, rather than taking a fixed value.) Nevertheless, the mere and undoubtedly correct assumption that the parton invariant masses determine the validity of the approximation already has the important consequence that hadrons form after a time $\sim 1/\mu_c$ in the rest frame of the original q, \bar{q} and therefore at a distance $\sim Q/\mu_c^2$ in the γ^ rest frame. Hence, as suggested by Fig. 1, the longitudinal extension of the parton system before hadron formation increases with Q . The locus of points in the γ^* rest frame at which hadrons form is roughly a cylinder of length $\sim Q/\mu_c^2$ and breadth $\sim 1/\mu_c$. If the high invariant mass q, \bar{q} were produced by a high p_T interaction in a nucleus, then hadrons should form only far outside the nucleus, and therefore no additional secondary hadron production should occur in the nucleus. The similarity of high p_T jet production from nucleons and nuclei observed experimentally [21] supports this picture. If instead, hadrons formed at a fixed time in the γ^* c.m.s., then different structure would be expected. In this case, hadron formation would occur roughly on the surface of a sphere (with $r \sim 1/\mu_f$) in the γ^* rest frame. Moreover, this alternative (which violates the locality assumption necessary to justify consideration of partons) implies a cutoff $\mu_c \sim \sqrt{Q\mu_f}$ on parton invariant masses which asymptotically yields no scaling violations in single hadron spectra.

The first corrections to the free emission approximation presumably arise within perturbation theory from the increasing importance of reinteractions. After many emissions, the density of partons in some regions of phase space will become so high that invariant masses of pairs of partons are often smaller than the invariant masses of other individual partons. In this case, the rate of exchanges between the partons will exceed the rate of radiation from a single parton, and the free emission approximation will fail. This effect occurs to some extent for a final state containing many e^+e^- or e^-p pairs in QED. At first, reinteractions result in energy loss through Bremsstrahlung; finally, when the invariant masses of the pairs fall below the masses of the Coulomb bound states, many of the charged particles combine into neutral atoms (c.f. recombination in a cooling plasma, e.g., in the early universe). (Note that the structure of a normal positronium or hydrogen atom depends crucially on the nonzero mass of the electron: when $m_e \rightarrow 0$ the "atom" becomes either infinitely extended [F.24] ($Z\alpha \leq 1$) or generates e^+e^- pairs until its charge neutralization radius $\simeq 0$ ($Z\alpha \geq 1$). It is not clear whether the nonzero current u, d masses are crucial in the dynamics of hadron formation; their kinematic effects

will be mentioned below: their importance may be gauged by differences between hadron systems produced in W^* decay (from $\tau \rightarrow W^* \nu_\tau$) and γ^* decay at a given Q .) In QCD, the increase of the effective coupling at large distances presumably leads to reinteractions which ultimately collect all partons into color singlet bound states. The invariant mass μ_c below which such effects dominate is perhaps determined by the point at which $\alpha_s(\mu^2) \gtrsim 1$, (c.f. the critical charge for zero radius atoms in massless QED mentioned above) and is therefore plausibly a few times Λ .

Hadron Formation

According to the assumptions discussed at the beginning of the previous section, reinteractions below the critical point affect only local sets of partons. The minimal groups of partons which may form hadrons independently are color singlets. A color singlet system is defined to transform according to the trivial representation of $SU(3)_c$. Even if a system has zero eigenvalues of the two commuting generators of $SU(3)_c$, it will not in general be a color singlet (c.f. a state with $j_z = 0$ need not have total angular momentum $j = 0$). To determine, for example, whether a $q\bar{q}$ system is a color singlet, one must know not only its total "color magnetic quantum numbers" (τ_3, τ_8), but also the amplitudes for the possible arrangements of the quark "color magnetic quantum numbers" (c.f. the state $(u\bar{u}+d\bar{d})/\sqrt{2}$ has $I = 0$ while $(u\bar{u}-d\bar{d})/\sqrt{2}$ has $I = 1$; both have $I_3 = 0$). In the free emission approximation for parton production, the "color magnetic quantum numbers" are conserved at each vertex; the phase of the amplitude for each emission is random, so that the final partons are statistically distributed among the possible $SU(3)_c$ representations. Hence, for example, a color neutral (i.e., with zero color magnetic quantum numbers) $q\bar{q}$ pair produced has probability $1/2$ to be in an $SU(3)_c$ singlet or an $SU(3)_c$ octet, respectively. Similarly, a color neutral GG pair has probability $1/6$ to be a color singlet (in the limit $N_c \rightarrow \infty$, $g^2 N_c$ fixed, a vanishingly small fraction of GG pairs are color singlets). One might perhaps imagine that final state interactions would mould the amplitudes for different parton states so as to produce particular color representation configurations: such effects would violate the locality assumed, and will therefore be ignored here.

There are several distinct classes of color singlet parton systems which may be considered. First, one might collect all color singlet systems at μ_c delimited by a quark and an antiquark [22]. Unfortunately, the very low multiplicity of secondary $q\bar{q}$ pairs at realistic Q (evident in Fig. 2) causes such $q\bar{q}GG\dots$ systems to have masses $\approx Q$ (although at truly asymptotic Q/μ_c , their masses should become $O(\mu_c)$ [22]); in this case, the final hadron production would not be local. A second scheme for color singlet identification consists in forming the minimum invariant mass color

singlet $q\bar{q}GG..$ or $GGG..$ clusters at μ_c . The gluon systems in this case often have rather large masses, because of the low probabilities for color neutral gluon systems to be color singlets. In a third scheme, on which I concentrate here, each gluon at μ_c is forcibly split into a $q\bar{q}$ pair. Each quark carries one of the spinor color indices of the gluon; every quark is connected by a group theoretical string to its color conjugate antiquark, so as to form a color neutral pair. The formation of color singlet systems from these color neutral $q\bar{q}$ may be estimated by combining with probability $1/2$ pairs of color neutral $q\bar{q}$ systems which arise from splitting of a common gluon ancestor (i.e., the pairs $\bar{q}_1 q_1$, $\bar{q}_1 q$ where $q_1 \bar{q}_1$ originate from forcible splitting $G \rightarrow \bar{q}_1 q_1$ of a "final" gluon are amalgamated into a single color singlet system with probability $1/2$). The prescription of splitting each gluon to a $q\bar{q}$ pair may perhaps be regarded as the assumption that any gluonium mesons formed decay infinitely quickly into $q\bar{q}$ mesons. (This assumption contradicts the $N_c \rightarrow \infty$ indication of narrow gluonium states, but is supported by the experimental absence of narrow gluonium states.) With this prescription, the mass spectrum of color singlet parton systems is very strongly damped ($\sim \mu^{-1}$ or μ^{-2}), and very nearly independent of Q [2] (except at $\mu \sim Q$ where the spectrum is usually infinitesimal), yielding a mean color singlet mass $\langle \mu_{cl} \rangle \simeq 2\mu_c$. (Recall that in the parton production model of Ref. [2], the "final" partons produced from parents with $\mu \simeq \mu_c$ were taken to be on their mass shells; if this assumption were relaxed, then $\langle \mu_{cl} \rangle \simeq 5\mu_c$, yielding rather massive clusters. If clusters were required to be color neutral, but not color singlet, then $\langle \mu_{cl} \rangle \simeq 1.2\mu_c$.) Of course, while these clusters represent essentially the minimal parton systems which can form hadrons independently, it is certainly possible that several such clusters may often act cooperatively, for example, if their joint invariant mass is below some fixed μ_{cl}^{\max} . In splitting gluons into $q\bar{q}$ pairs, I arbitrarily choose the momenta of the quarks to be uniformly distributed over the allowed range (no results are sensitive to this choice) and to have flavors u , d , s with equal probabilities. Just as the color representations of the parton clusters can be determined only statistically, so also their total angular momenta are not determined (the σ_z for each parton could be traced, but the orbital angular momentum is entirely undetermined). Nevertheless, I shall below approximate the clusters to decay isotropically in their rest frames, thereby implicitly assuming zero total angular momenta.

If indeed the local color singlet parton clusters described above are formed by reinteractions below μ_c , one must then determine how each cluster should decay into hadrons. The discussion above assumes that the clusters may have arbitrary masses. Perhaps, however, each cluster may instead represent a definite meson resonance, with discrete mass. The energy levels

of "atoms" bound by nonconfining potentials always become dense close to ionization. A "cooling" e^+e^- pair may thus be treated classically until it lies in the energy band just below the ionization limit; then the atom cascades by quantum mechanical radiation into the ground state of definite discrete energy. For a confining potential, all the energy levels are discrete, suggesting that "atoms" must be directed immediately into discrete levels. This phenomenon must be described by quantum mechanics and would contravene the locality assumption made above. Nevertheless, in a second quantized treatment, the higher levels may decay to lower ones: then the widths of the excited levels may increase faster than their spacings, so that the higher levels merge, effectively yielding a continuous energy band. (This phenomenon occurs for a hydrogen atom in 2-dimensional QED.) This possibility may well be realized for meson resonances: in a constituent model, their level density rises $\sim (m/m_0)^P$, while phenomenologically their widths $\Gamma \sim 0.1 m$. In this case, the available meson (or cluster) masses essentially represent a continuous band: clusters formed in the band may then decay to light mesons with definite masses [F.25]. (The smoothness of the e^+e^- cross-section above $Q \approx 1$ GeV suggests that the band of allowed cluster masses extends down to $Q \approx 1$ GeV.) The decay properties of the clusters may to some extent be estimated from measured meson resonances, together with low-energy e^+e^- annihilation final states [F.26]. All evidence suggests that quasi-two-body decays are universally dominant. For clusters below ≈ 1.5 GeV, an adequate model is to allow decay into pairs of the lowest-lying 0^+ , 1^- , 1^+ , 2^+ mesons, with equal matrix elements for each final spin state (so that the decay branching ratios are determined by the available phase space). This scheme yields a roughly linear increase of multiplicity with mass (for $\mu_{cl} \leq 1.5$ GeV), apparently as an accidental consequence of the properties of the low-lying mesons. Strange meson production is suppressed simply by the larger K mass and by the larger number of π than K produced in decays of low-lying meson resonances. The approximate constancy of the total multiplicity in e^+e^- annihilation from ≈ 1.5 GeV up to ≈ 4 GeV suggests that clusters with masses in this range decay directly to pairs of light mesons, without cascading through clusters of intermediate mass. (Quantum numbers usually leave only one quasi-two-body decay channel open to the known meson resonances, preventing determination of the mass distributions for their decay products [F.27].) The decay products of low mass clusters are thus taken to have masses comparable to their parents. As the cluster masses increase, the product masses remain unchanged, so that the decay momenta increase. This behavior provides a rather smooth transition to the parton decays at larger invariant masses, where daughter partons have much smaller masses than their parents (see eqs. (1, 2)). In as far as the free emission approximation

describes the decay of clusters with sufficiently large mass to lighter clusters, the value of the parameter μ_c should be irrelevant: changes in μ_c over a certain range would simply assign a different fraction of the hadron production process to the free emission stage and to the phenomenological cluster decay stage, leaving the results unchanged. In practice, however, only rather small changes in μ_c exhibit this behavior.

The model defined above purports to describe all features of e^+e^- annihilation final states. A comprehensive investigation will be reported elsewhere [3]; here I make only a few very brief comments. As discussed above, a crucial feature of the model is that it exhibits the locality of hadron formation necessary to justify use of QCD perturbation theory at early times. Previous models (e.g., [15, 25]) have failed dismally in this respect: they typically take each produced parton with energy E in the γ^* c.m.s. to decay into hadrons like one jet of an e^+e^- event with $Q = 2E$ in the original γ^* c.m.s. (usually as parametrized by the Field-Feynman model [26]). In this way, the invariant mass of the hadron jet resulting from the parton decay is $\approx \sqrt{\Lambda E}$: the energy of the parton in the c.m.s. of the complete event is crucial in determining its decay to hadrons, and the locality postulate is totally violated. For this reason, any detailed agreement between such models and experimental data should in no way be construed as support for QCD.

The measured mean charged multiplicity in e^+e^- annihilation is roughly constant at small Q , increasing from ≈ 3 at $Q = 1.5$ GeV to ≈ 4 at $Q = 5$ GeV. At higher Q , $\langle n_{ch} \rangle$ increases rapidly, becoming ≈ 6 at $Q = 10$ GeV and ≈ 12 by $Q = 30$ GeV [F.28]. This increase presumably reflects the rapid rise in parton multiplicity at high Q evident in Fig. 2. Given that the quasi-two-body cluster decay model described above reproduces the observed $\langle n_{ch} \rangle$ for $Q \leq 2$ GeV, the $\langle n_{ch} \rangle$ obtained at higher Q agrees with data to within about 30% for any μ_c in the range 1-2 GeV (with $\Lambda = 0.5-0.8$ GeV). Almost any plausible cluster decay model suggests $\langle n_{ch} \rangle / \langle n \rangle \approx 0.6$. The hadron multiplicity distributions should roughly follow the parton ones, and be broader than Poisson at high Q . Single hadron energy spectra at $Q \gtrsim 4$ GeV are also in adequate agreement with data so long as $\mu_c \leq 2$ GeV (a 10% softening in $\langle z \rangle$ between $Q = 10$ GeV and $Q = 30$ GeV is expected). Whereas in the original Field-Feynman model, the charge-weighted z distributions for each jet were essentially monotonic, they exhibit considerable oscillations in the present model, especially at small z , although the charges of very high z hadrons still reflect those of the original quark (c.f. [27]). The transverse momentum spectra of single hadrons obtained are roughly in accord with the experimental data so long as $\mu_c \leq 2$ GeV (the $\langle p_T \rangle$ increase slowly with μ_c as for partons). (Note that, for example, at

$Q = 10 \text{ GeV}$, $\langle p_T \rangle$ measured with respect to the sphericity axis is about 10% larger than that with respect to the original $q\bar{q}$ direction; in the former case, consideration of charged hadrons alone effects an $\approx 10\%$ reduction in $\langle p_T \rangle$.) Note that the $\langle p_T \rangle$ rises slowly with increasing Q , in contrast with the roughly constant behavior implicit in the Field-Feynman model. Shape parameters, which measure the large-scale angular distribution of energy in the final state, provide an important probe of the processes of hadron production. Recall that successive partons emitted tend to be progressively more collinear, so that only the first few emissions can have a significant effect on the "shapes" of the events. If μ_c is small, then the hadron clusters formed typically have small masses, and release little transverse momentum in their decays; hence the final hadrons are concentrated along the directions of the first few emitted partons, and the shape parameters for the final states are close to those obtained in low-order perturbation theory. Experimental results for shape parameters indicate that actual final states are much less lumpy, strongly suggesting $\mu_c \gtrsim 1 \text{ GeV}$ (note that an increased effective $\alpha_s(Q^2)$ resulting from large higher-order corrections could account for the observed $\langle H_\ell \rangle$ but not the H_ℓ distributions). With such values for μ_c , $\langle \mu_{c\ell} \rangle \gtrsim 2 \text{ GeV}$: the hadrons from decays of different clusters thus overlap considerably in phase space (hence analysis methods used to extract properties of clusters in multiparticle production in low p_T hadron collisions [28] could not discern these "superclusters", but only the lighter clusters resulting from their decays).

Extensions

For simplicity, I have considered above only quarks with vanishing rest masses (all partons nevertheless receive effective masses $O(\mu_c)$ from their finite propagation). Small quark masses introduce $O(m^2 dt/t^2)$ (or possibly $O((m^2/t)^{(1-O(\alpha_s))} dt/t)$) mass corrections to the quark decay probabilities in eq. (1, 2)). (These are a species of higher twist corrections; the mass insertions responsible for them are analogous to the insertions of extra collinear gluons responsible for $O(\mu_c^2 dt/t^2)$ higher twist terms.) When $t \leq m_Q^2$, the quark decay probabilities are kinematically constrained to vanish. For the $O(\alpha_s)$ process $\gamma^* \rightarrow Q\bar{Q}G$ (Q denotes a heavy quark), a very good approximation to the exact differential cross-section is provided by using the usual $m_Q = 0$ $\mathcal{P}_{Q\rightarrow QG}(t, z)$ for $t > m_Q^2$ but setting $\mathcal{P}_{Q\rightarrow QG}(t, z) = 0$ for $t \leq m_Q^2$ (z is, as always, interpreted as the $E + |\vec{p}|$ fraction, thus accounting for Nachtmann scaling corrections). It seems likely that this prescription will also be satisfactory for multiple gluon emissions. Note that in the effective coupling $\alpha_s(\tilde{t})$, the available \tilde{t} is now $\sim (t - m_Q^2)$, rather than $\sim t$. For t close to m_Q^2 , the product Q in a $Q^* \rightarrow QG$ "decay" will have small velocity relative to its parent, so that reinteractions

will be important. Thus the free emission approximation in this case should fail when $t \lesssim (m_Q + \mu_c)^2$: the effective μ_c cutoff for emissions from heavy quarks should therefore be $\mu_c^Q \approx (m_Q + \mu_c)$. For $m^2 \ll t$, the production of a $Q\bar{Q}$ pair from γ^* or G^* should be suppressed relative to $q\bar{q}$ production by a factor $v/2(3-v^2) = 1 - O(m^4/t^2)$ where $v = \sqrt{1 - 4m^2/t}$ is the relative velocity of the outgoing Q, \bar{Q} . $\mathcal{P}_{\gamma \rightarrow Q\bar{Q}}(t, z)$ and $\mathcal{P}_{G \rightarrow Q\bar{Q}}(t, z)$ should thus be reduced by this factor for $t > 4m_Q^2$, and set to zero below threshold $t < 4m_Q^2$ (as t decreases, the angular, or equivalently, z distribution for the decay flattens slowly, becoming isotropic (i.e., flat) at threshold). Just above threshold, the produced Q, \bar{Q} have small relative velocity, and thus undergo extensive final state interactions. At $O(\alpha_s)$, gluon exchange in $\gamma^* \rightarrow Q\bar{Q}$ gives $\Delta\sigma/\sigma \lesssim 4\pi\alpha_s/3v$. In higher orders, the cross-section exhibits isolated peaks at the positions of $Q\bar{Q}$ resonances below threshold for $(Q\bar{q})(Q\bar{q})$ meson production, and further resonant peaks just above threshold. Note that such effects are probably less important for $G^* \rightarrow Q\bar{Q}$ than for $\gamma^* \rightarrow Q\bar{Q}$ because in the former case, the $Q\bar{Q}$ cannot bind into a hadron because of their color. Nevertheless, in all cases, the peaks and valleys in the cross-sections should average out when smeared over a range $\gtrsim \mu_c^2$ around $t = 4m_Q^2$, and the lowest order result should suffice. In the prescription for forming color singlet parton systems described above, each gluon at μ_c is forcibly split into a light $q\bar{q}$ pair: this process, if it occurs at all, is undoubtedly not of perturbative origin, and one may guess that heavy $Q\bar{Q}$ production by it would be suppressed by $\sim \exp(-m_Q^2/(\alpha_s \mu_c^2))$. Hence secondary $Q\bar{Q}$ pairs should be generated only at the perturbative stage, and thus be rare.

Heavy quarks should eventually combine with light quarks and gluons to form color singlet clusters with masses $\approx m_Q + 3\mu_c$. These clusters should then decay into $(Q\bar{q})$ mesons and light hadrons, with branching ratios determined approximately by available phase space (just as for light mesons, " D^* " as well as " D " production should probably be included explicitly, thereby accounting correctly for $D^* \rightarrow D\gamma$ decays: for very heavy Q , $(Q\bar{q})$ meson masses and branching ratios may presumably be estimated from potential models). The ground state (presumably pseudoscalar) $(Q\bar{q})$ mesons will then decay weakly (the lifetimes for these weak decays are much longer than the time necessary for the decaying meson to form, and they should therefore be treated separately). The decay may proceed either through separate $Q \rightarrow q'W^{(*)}$ decay of the heavy quark, or for purely hadronic modes, by W exchange with the spectator \bar{q} . With the first mechanism, the $W^{(*)}$ is produced roughly uniformly (isotropic) in z ; its subsequent decay to a fermion pair may be approximated as independent, and described by the distribution (for $t \ll m_W^2$) $\mathcal{A}_{W \rightarrow f\bar{f}}(t, z) dt \approx g^2/16\pi^2 t dt/m_W^4 (z^2 + (1-z)^2)$, with the various flavors of quarks and leptons weighted with mixing angles according to

their appearance in the weak current. When $m_Q \geq m_W$, real W production is permitted; the relevant decay probabilities are analogous to those for G (or γ) production. The second decay mechanism may be effective for purely hadronic modes; its relative importance may depend on the charge of the decaying $(Q\bar{Q})$ meson. The final $q'\bar{q}''$ pair generated by this mechanism is isotropic in the $(Q\bar{Q})$ rest frame: the various possible quark flavors are weighted by the requisite mixing angles. Other decay modes, such as $(Q\bar{Q}) \rightarrow q'G\bar{q}$ or $(Q\bar{Q})^0 \rightarrow GG$ probably have very small branching ratios. The partons emitted in $(Q\bar{Q})$ decays may be off-shell, and thus radiate, producing hadrons as in γ^* decay. Note that heavy lepton decays may be treated analogously to the first mechanism for $(Q\bar{Q})$ decays.

Below threshold for $(Q\bar{Q})(\bar{Q}\bar{Q})$ production, $(Q\bar{Q})$ resonances (e.g., ψ , T; denoted generically ζ) should be produced in γ^* decay. The lightest such resonance presumably decays mainly to GGG , $GG\gamma$, or $\gamma^* (\rightarrow q\bar{q})$; radiation from these partons may be treated as described above. (Note that the gluons in the decay $\zeta \rightarrow GGG$ are distributed almost uniformly in the available phase space.) Excited $(Q\bar{Q})$ mesons may decay either to lower-lying $(Q\bar{Q})$ states, or directly to lighter partons; the branching ratios may be estimated from potential models. For ψ , a cutoff $\mu_c \approx 1.5$ GeV permits almost no radiation from the GGG produced, and usually combines them into just one hadron cluster; this then decays identically to the single hadron cluster produced by γ^* decay with $Q = m_\psi$. Of course, in the model described here, both quark and gluon "fragmentation functions" are completely determined.

The hadron clusters discussed in the previous section are taken to decay into light meson pairs according to available phase space. For clusters with masses above ≈ 2 GeV, baryon pair production is also possible. To be consistent with other assumptions, no suppression of baryon pair production beyond phase space restrictions should be introduced. Then, at asymptotic Q , $\langle n_p/n_\pi \rangle$ should tend slowly to a constant determined by $m_p/\langle \mu_{cl} \rangle$.

In addition to gluons, virtual quarks produced in γ^* decay may also radiate real or virtual photons, and, if they have sufficiently large masses, real or virtual W^\pm , Z^0 , and perhaps Higgs (H) particles. The probabilities for photon emission are just as for gluon emission (after the replacement $4/3 \alpha_s(\tilde{t}) \rightarrow \alpha_{em} \equiv \alpha$). Direct photon production in the decays of the hadron clusters may probably be ignored. Just as outgoing or incoming quarks may emit gluons, so also incoming e^+, e^- may emit photons. Most of the resulting electromagnetic radiative corrections may be treated by the direct Monte Carlo methods discussed above and in Ref. [2].

In the discussion above, the polarizations of quarks and gluons have not been traced explicitly. It is simple to include spin-dependent decay

probabilities [4], but the likely presence of orbital angular momentum in cluster formation makes deduction of final hadron polarizations difficult.

In this paper, I have concentrated on hadron production in e^+e^- annihilation. Processes which involve partons in the initial state may be treated by largely analogous methods. The valence quarks in incoming hadrons may plausibly be taken to be distributed in the hadron rest frames, for example, Gaussian (c.f. nonrelativistic harmonic oscillator wavefunctions) in all four momentum components so as to have a mean invariant mass $\approx \mu_1$. (There is no difficulty in boosting this momentum probability distribution.) Prior to interactions involving high invariant masses Q (e.g., absorption of a highly spacelike virtual photon (deep inelastic scattering), or production of a highly timelike γ^* (Drell-Yan process)), these nearly on-shell partons may radiate (small timelike invariant mass) partons, and themselves attain progressively more spacelike invariant masses (up to $\sim iQ$). The probabilities for these decays are, in the free independent emission approximation, essentially the same as those for the decays of timelike invariant mass partons considered above, with suitable reinterpretation of z . (The optimal argument \tilde{t} of $\alpha_s(\tilde{t})$ becomes $\sim -(1-z)|t|/z$ rather than $\sim z(1-z)t$, as in the timelike case; hence $O(\alpha_s^2)$ contributions to the decay probabilities in the two cases differ by $O(\log z)$ terms.) The emitted gluons and "sea quarks" may be considered to provide extra constituents of the incoming hadrons: their momentum distributions will as usual "evolve" as Q^2/μ_1^2 increases. Note that with this model, the "constituent" partons will exhibit a distribution of transverse momenta with respect to the incoming hadron direction. After the hard scattering, remaining partons may be off-shell with timelike invariant masses up to $\sim Q$, and may radiate just as in γ^* decay. Partons from the initial hadrons which do not participate in the hard scattering should preserve their original momenta (roughly collinear with the incoming hadrons) until they are combined with other partons in the formation of color singlet hadron clusters. The model outlined here should allow discussion of all high-transverse momentum hadron processes [29]. Comparison with results from triggered experiments will, however, require nontrivial importance sampling in theoretical Monte Carlo calculations. One may speculate that the methods used to describe high transverse momentum scatterings between incoming partons could also be applied to low transverse momentum multiparticle production processes. The incoming valence partons would exchange a small momentum, with a cross-section given by one-gluon exchange, but with its Coulomb singularity at small Q^2 regularized for $Q^2 \leq \mu_1^2$, by the presence of additional partons from the incoming hadrons, which shield the color of the valence quarks at distances $\gtrsim 1/\mu_1$. This small momentum transfer, say $O(\Lambda)$, causes the incoming high-energy (but nearly on-shell) partons to be "poked" off shell to

invariant masses $\sim \sqrt{\Lambda s}$, where \sqrt{s} is the c.m.s. energy in the parton collision. These off-shell partons would then radiate just like partons with the same invariant masses produced in γ^* decay. At sufficiently high s , the transverse momentum distribution of the final hadrons should therefore broaden.

Acknowledgments

I am grateful to R. D. Field and G. C. Fox for their contributions to various aspects of this work (especially Monte Carlo computer programs). I am also grateful to several experimenters at PETRA for discussions regarding their data and its analysis, and to A. E. Terrano for comments on the manuscript of these notes.

Footnotes

- [F.1] The configuration space propagator for a zero rest mass particle whose invariant mass is required to exceed μ^2 is given by

$$\begin{aligned}\Delta(r^2) &\sim \int_{p^2 > \mu^2} e^{ip \cdot r} \frac{d^4 p}{p^2} \sim \frac{1}{|r|} \int_{\mu}^{\infty} \frac{\sin(|p||r|)}{|p|} d|p| \\ &= \frac{-1}{|r|} \text{si}(\mu|r|),\end{aligned}$$


where $\text{si}(x)$ is the complementary sine integral, which goes through its first zero at $x \approx 1.9$.

- [F.2] For comparison, with scalar quarks, but vector gluons, $P_{q_s \rightarrow q_s G}(z) = C_q 2/(1-z)$, while for spinor quarks, but scalar gluons, the soft divergence disappears, and $P_{q \rightarrow q G_s}(z) = C_q (1-z)/2$. (For these cases, $\alpha_s(t)$ should also be modified.)
- [F.3] In the limit $E \rightarrow \infty$, this becomes the light cone momentum fraction p^+ , and is Lorentz invariant.
- [F.4] With this choice, z becomes $E + \vec{p} \cdot \vec{\eta} / |\vec{\eta}|$ fraction: some radiated partons will then travel backwards with respect to their parent partons, thereby populating the region of phase space usually associated with emissions from the impotent parton (with $p \parallel \eta$). A table of the explicit contributions from individual diagrams in different gauges is given in the second paper of Ref. [1]. Note that in the "planar" gauge $\eta = q$ with no $k_\mu k_\nu$ or $\eta_\mu \eta_\nu$ term, the lowest-order interference diagram vanishes exactly, rather than being simply suppressed: perhaps this behavior continues in higher orders.
- [F.5] As mentioned in the final section of these notes, the analogy with electromagnetic shower development is even closer for hadron production by low- p_T scatterings of high-energy incident partons.

- [F.6] In the operator product expansion approach, integrals over $\mathcal{D}_{\gamma \rightarrow q\bar{q}G}^3$ provide $O(\alpha_s)$ corrections to the coefficient function. Integrals over the $\mathcal{D}_{j_0 \rightarrow j_1 j_2}^2$ provide the leading log anomalous dimensions, and much of the subleading log anomalous dimensions. $\mathcal{D}_{j_0 \rightarrow j_1 j_2 j_3}^3$ completes the subleading log anomalous dimensions.
- [F.7] The differential cross-sections for $\gamma^* \rightarrow q\bar{q}GG$, $\gamma^* \rightarrow q\bar{q}q'\bar{q}'$ are given in Ref. [30] ; the $O(\alpha_s)$ loop corrections to $\gamma^* \rightarrow q\bar{q}G$ necessary for a complete treatment are under investigation [31] .
- [F.8] Log(t) terms from vertex corrections cancel those from quark self-energy insertions by the axial gauge Ward identities.
- [F.9] In leading log approximation estimates, it is often convenient to account for virtual corrections to parton propagation as possible off-shell parton decay modes, thereby introducing a negative (usually divergent) $\delta(1-z)$ term in the $P(z)$. If $\mu_R^2 = t_{\max}$, then the resulting $P(z)$ satisfies $\int_0^1 P(z)dz = 0$, so that the sum of probabilities for all possible fates of the off-shell parton explicitly sums to one.
- [F.10] The form is very similar to that obtained for massive electrons in QED, or for massless electrons in a magnetic field, or confined within a finite volume.
- [F.11] These subtleties are not usually visible in the treatment of QED with massive electrons. The reason is that a particular renormalization prescription (momentum space subtraction at $q^2 = 0$) is overwhelmingly convenient, because it causes all higher order terms in α to vanish exactly in the low-energy limit for various processes (e.g., Compton scattering), so that measurements of these processes may determine the precise value of α to be used in this renormalization scheme, without the need to calculate higher order terms in the perturbation series. To deduce α in this scheme from other processes (e.g., $g_2 - 2$) requires explicit calculation of higher order terms before comparison with experiment. (The Thomson limit of $\gamma e \rightarrow \gamma e$ is exactly $8/3(\alpha/m)^2$, but $g_e - 2$ contains higher order terms $\alpha/2\pi(1-0.66\alpha/\pi-\dots)$.) Note that in massless QED, the removal of infrared divergences from incoming $e\gamma\dots$ composites again spoils the simple scheme. In QCD, all such low energy limits of perturbation theory are entirely irrelevant, since the theory has a strong coupling in that domain.
- [F.12] As discussed in the final section of these notes, in processes involving initial hadrons (e.g., $\gamma^* N \rightarrow X$ or $NN \rightarrow \gamma^* X$), the incoming

partons initially have small invariant masses $O(\mu_c)$. As they approach the collision, they may radiate timelike invariant mass partons, and themselves acquire progressively more spacelike invariant masses, up to $O(Q^2)$ (where Q is the momentum transferred in the hard scattering; typically γ^* momentum). In most cases, the t_{\max} appearing in $\alpha_s(t_{\max})$ will be positive, as in radiation from timelike mass final partons, and so no $O(\pi)$ terms should be introduced between the two cases. Consider, however, the emissions from incoming partons just before the hard scattering. Integrating these over available phase space typically gives $\sim \int_{\mu_c^2}^{-|Q^2|} dt/t \int_{-|Q^2|/t}^1 dx/x \sim \log^2(-|Q^2|/\mu_c^2)$. On the other hand, the virtual exchanges which cancel the infrared divergence at $\mu_c \rightarrow 0$ give roughly $-\int_{\mu_c^2}^{Q^2} dt/t \int_{Q^2/t}^1 dx/x \sim -\log^2(Q^2/\mu_c^2)$, yielding a total $\sim \log^2(Q^2/-|Q|^2)$. Clearly, comparisons between rates for hard scatterings of incoming partons involving positive Q^2 (e.g., $NN \rightarrow \gamma^* X$) will differ from those with $Q^2 < 0$ (e.g., $\gamma^* N \rightarrow X$) by $O(\alpha_s \pi)$ terms. (The exponentiation of the corresponding double log series demonstrates that such terms sum to a correction $\sim \exp(c\alpha_s \pi)$ where c is the color charge of the incoming partons.) For outgoing partons, the real emission term becomes $\sim \log^2(|Q^2|/\mu_c^2)$, again allowing some $O(\pi)$ differences with incoming parton processes. However, away from the hard scattering, the sign of Q^2 has no $O(\pi)$ effect on corrections to decay probabilities. Nevertheless, the decay probabilities for incoming and outgoing partons may differ by $O(\pi\alpha_s)$ terms. For outgoing partons, imposition of the cutoff μ_c prevents any intermediate partons from reaching their mass shells. Incoming partons presumably have a spread of invariant masses with variance $O(\mu_c)$, extending both to timelike and spacelike values. If an incoming parton begins with timelike invariant mass, it must pass $t = 0$ before reaching spacelike mass: between radiations it may propagate on shell, thereby introducing $O(\alpha_s \pi)$ terms, not present for outgoing partons. It seems likely that this effect is a consequence of the infinite life of incoming hadrons, and is not an artifact of the initial parton mass spectrum considered.

[F.13] One possible method for estimating the contributions of multiloop diagrams would be to consider the diagrams in the limit that the spacetime dimensionality $n = 4 - \epsilon \rightarrow 0$, so that no loop integrals remain, and the diagrams at a given order must merely be counted:

unfortunately, the numerical results of this "approximation" are not even close to those obtained when $n = 4$. Another, more hopeful, but more complicated, method of approximation consists of performing a hyperspherical (Gegenbauer) expansion on each propagator (see [10]), but retaining only, say, the zeroth (spherically symmetric) term (e.g., in $n = 4$, $1/(p \cdot q)^2 \rightarrow 1/|p||q| \text{Min}[|p|/|q|, |q|/|p|]$). Then loop integrals reduce to scalar integrals over $|p|$, etc., but with quite complicated integrands. For the ϕ^3 diagram  the method gives $3/2$ compared to the exact result $3/2 \zeta(3)$. Note that the contributions from large Gegenbauer index terms fall off like $1/l^N$, where $N \sim$ (number of loops). Another possible approach would be direct numerical evaluation of Euclidean momentum integrals: the necessary Monte Carlo integration would, however, be very time consuming because of the canceling divergent integrands.

- [F.14] This approach would, of course, fail in the (seemingly unlikely) event that the constant factors systematically changed from one emission to the next.
- [F.15] In particular, the multiplicity depends on the assumption (discussed below) that "final partons" which can radiate no further because of the μ_c cut have zero invariant mass (rather than masses $\simeq \mu_c$).
- [F.16] Thus the fractal dimensions of the sets of points representing the directions of parton momenta are much smaller than 2. (The angular structure of the events is not, in fact, not exactly self-similar - the effective fractal dimension changes logarithmically with the angular scale considered.) These results should be contrasted with those for electromagnetic showers in matter initiated by very high energy electrons or photons. In that case, transverse momenta are imparted predominantly by Coulomb scattering from nuclei: the maximum k_T from each collision is $\sim \sqrt{E\Lambda}$ (where Λ is a fixed mass determined by the inverse nuclear size); since the energies of shower particles decrease only very slowly, there is no clumping in the final state, and the momenta of the e^\pm, γ are spread roughly uniformly. (This behavior is evident in extended air showers initiated by high energy cosmic rays.)
- [F.17] Clearly observables with this property must be linear in the energies of collinear sets of partons (so that sphericity does not

qualify). Such observables are formally classified as "infrared" finite", since divergences appearing in their mean value from decays with $k_T \rightarrow 0$ cancel just as in the total cross-section σ . (The final phase space is weighted uniformly in the calculation of σ ; it is slightly, but continuously, corrugated in the calculation of the average values of infrared finite observables.)

- [F.18] H_2 is related to the eigenvalues $\lambda_1, \lambda_2, \lambda_3$ of the matrix [16]

$$T_{ab} = \sum_i p_i^a p_i^b / |\vec{p}_i| / \sum_i |\vec{p}_i|$$
 by $(\lambda_1^2 + \lambda_2^2 + \lambda_3^2) = 1/3(2H_2 + 1)$. (The existence of this relation was suggested to me by R. K. Ellis.) Thrust $T = \text{Max}_{\{\hat{n}\}} [\sum_i (\vec{p}_i \cdot \hat{n}) \theta(\hat{p}_i \cdot \hat{n}) / \sum_i |\vec{p}_i|]$ is given roughly by $T \approx (1 \pm 0.1) (1 + \sqrt{H_2})/2$. The $H_{2\ell+1}$ are zero for events with inversion symmetry. For a final state consisting of three massless particles $H_3 = 10/9(1-H_2)^2$. In practical experiments where only incomplete hadronic final states are measured, missing energy may be corrected for by dividing each measured H_ℓ by the observed H_0 : missing momentum significantly affects $H_{2\ell+1}$, but may perhaps be corrected for by boosting to the measured rest frame. Note that with the definition (6), $H_0 = 1 - \sum_i m_i^2/E_i^2 \equiv 1 - \sum_i 1/\gamma_i^2$ for a complete final state containing massive particles.
- [F.19] The complete form, retaining a small mass μ_G for the gluon is [1]

$$\langle H_2 \rangle = 1 - 2\alpha_s / 3\pi (4\pi^2 - 33 - 12\pi(\mu_G/Q) + \dots).$$
- [F.20] For a planar event $\Pi_1 = 0$, while for an isotropic event, $\Pi_1 = 2/9$. In general, $2/9(1-4H_2) \leq \Pi_1 \leq 2/9(1-H_2)^2$. In terms of the eigenvalues λ_i , $\Pi_1 = 6\lambda_1\lambda_2\lambda_3$. $H_2 \geq 1/4$ for planar events.
- [F.21] As would lead, for example, to terms proportional to the sum $\sum_{i,j} |e_i e_j|$ of quark changes rather than $\sum_i e_i^2$ in the total γ^* decay width (the former terms appear only at $O(\alpha_s^3)$ in perturbation theory).
- [F.22] The decay widths may be obtained as the imaginary part of the "vacuum" energy density ρ . For massless quarks and gluons, first-order calculations suggest that $\rho \sim \alpha_s (E^2 + B^2)[E^2 + B^2]$; $\text{Re}[\rho]$ therefore has an absolute minimum at $E = B = 0$, indicating that the usual vacuum with $E = B = 0$ should be stable. I am grateful to J. Sapiirstein for discussions regarding these points.
- [F.23] The separating gluons presumably interact by virtual gluon exchange. Self-energy corrections to the virtual gluon propagator yield an effective coupling which results in antiscreening of the charges. Spontaneous nonperturbative gluon production can roughly be considered as real production of the pairs responsible, through

fluctuations in the exchanged gluon field. It is therefore possible that produced real pairs may also antiscreen the separating gluons, thereby increasing, rather than decreasing the reinteraction cross-section.

- [F.24] The effective QED coupling $\alpha(\mu^2) \sim 1/\log(\Lambda^2/\mu^2 + m_e^2)$ ($\Lambda \sim m_e \exp(-1/\alpha)$) strengthens the usual $1/r$ Coulomb potential at $r \lesssim 1/\Lambda$, and if $m_e = 0$ screens the potential for $r \gtrsim 1/\Lambda$. A deeply bound state with $a_0 \sim 1/\Lambda$ may therefore exist (but be totally unobservable in practical systems).
- [F.25] The probabilities for formation of clusters with different masses may perhaps be estimated from the total e^+e^- annihilation cross-section at that mass, thereby providing a weighting for each event, which may possibly be used to infer the behavior of the high energy total cross-section.
- [F.26] Note that below s, c threshold, the photon is dominantly $I = 1$, $G = +$; final states consisting of an odd number of pions are therefore suppressed. This effect is the result of interference between $\gamma^* \rightarrow u\bar{u}$ and $\gamma^* \rightarrow d\bar{d}$ amplitudes, and cannot be obtained by classical considerations. It results in a considerable enhancement in the fraction of charged hadrons produced at small Q , and must be corrected for in comparisons of models with experimental charged multiplicities.
- [F.27] In any confining potential, the spontaneous nonperturbative parton production discussed above should occur (as in $Z\alpha \gtrsim 1$ atoms (e.g., [23]), typically leading to low-mass decay products. On the other hand, the statistical bootstrap model [24] favors unsymmetrical decays, with constant energy release, and one light, one heavy, product. It thus implies $\langle n_{\text{hadrons}} \rangle \propto Q$, in gross disagreement with data.
- [F.28] For this and other experimental e^+e^- annihilation results, see the contributions from PETRA groups to this conference.

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Spacetime Development of Typical Parton Showers
($Q = 200 \text{ GeV}$, $\mu_c = 1 \text{ GeV}$, $\Lambda = 0.5 \text{ GeV}$)

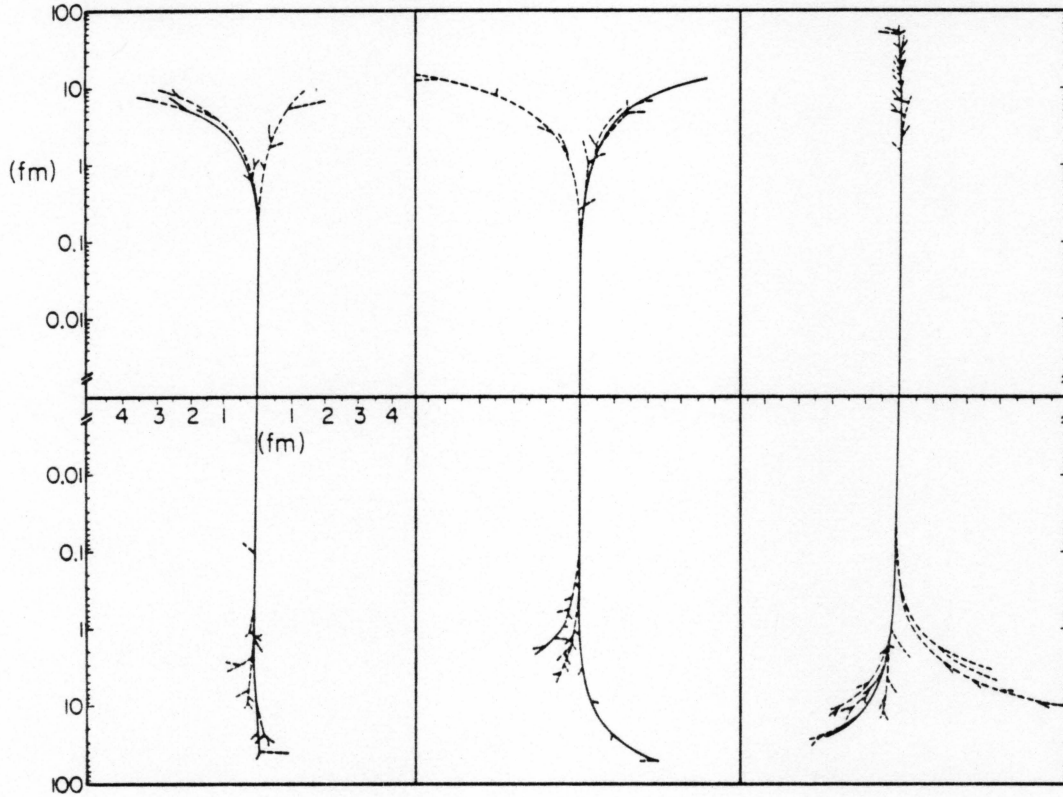


Fig. 1: Spacetime development of typical parton showers initiated by decay of a virtual photon with invariant mass $Q = 200 \text{ GeV}$, traced until each parton has invariant mass below the critical $\mu_c = 1 \text{ GeV}$, and with $\Lambda = 0.5 \text{ GeV}$, generated using the Monte Carlo computer program of Ref. [2].

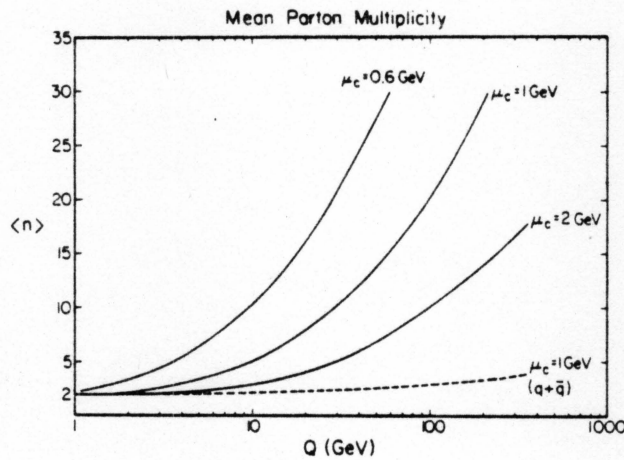


Fig. 2: Mean multiplicity of partons produced in the decay of a virtual photon with invariant mass Q by radiation from virtual partons with invariant masses $\geq \mu_c$. The dashed line gives the multiplicity of quarks and antiquarks. The parton production cross-sections were estimated by a leading pole approximation, with $\Lambda = 0.5 \text{ GeV}$.

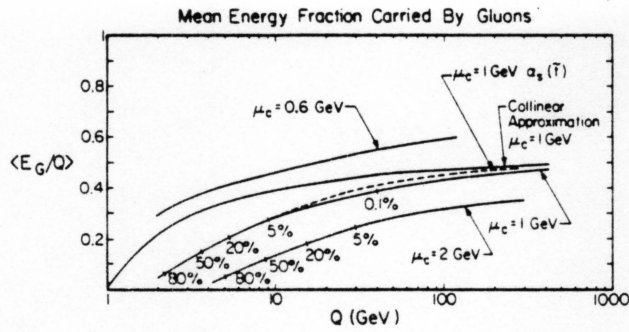


Fig. 3: Mean fraction of total energy in decay of a virtual photon with mass Q carried by gluons. The production of partons by leading pole approximation cross-sections has been truncated when their invariant masses fall below the cutoff μ_c . The percentage of events in which no gluons were emitted above this cutoff is marked. Results obtained by using approximate collinear kinematics (the usual "leading log approximation") are also shown. The consequences of a modification of the effective coupling constant $\alpha_s(\tilde{t})$ for parton "decays" discussed in the text are shown.

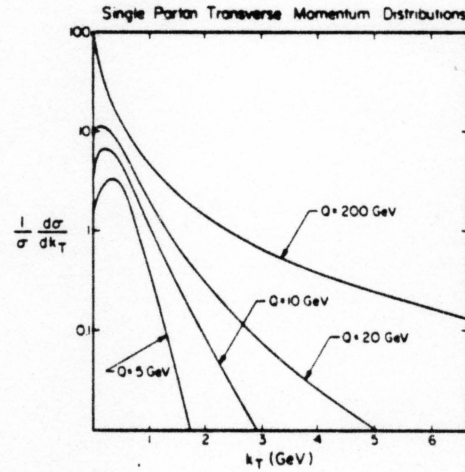


Fig. 4: Transverse momentum distributions for single partons produced in the decay of a virtual photon with mass Q with respect to the primary $q\bar{q}$ pair direction. Parton emissions were truncated by the cut $\mu_c = 1$ GeV.

Northern Hemisphere of a Parton Final State
($Q=100$ GeV, $\mu_c=1$ GeV, $\Delta=0.5$ GeV)

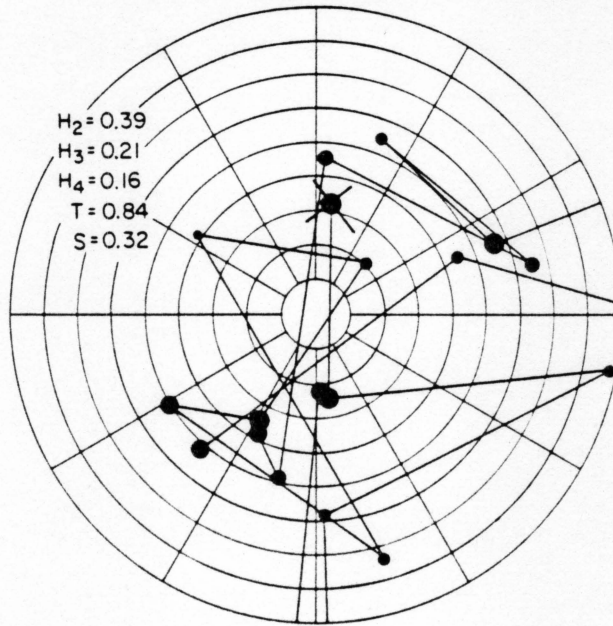


Fig. 5: Distribution of partons in the northern hemisphere of a reasonably typical γ^* decay final state with $Q = 100$ GeV, $\mu_c = 1$ GeV. The sizes of the dots are roughly proportional to the energies of the partons they represent. The original quark produced in the γ^* decay was directed towards the north pole; its final position is marked with a cross. All other partons are gluons. The lines drawn between the parton directions describe the parton color indices. The event shown is somewhat more isotropic than the average; the values of some shape parameters for it are given.

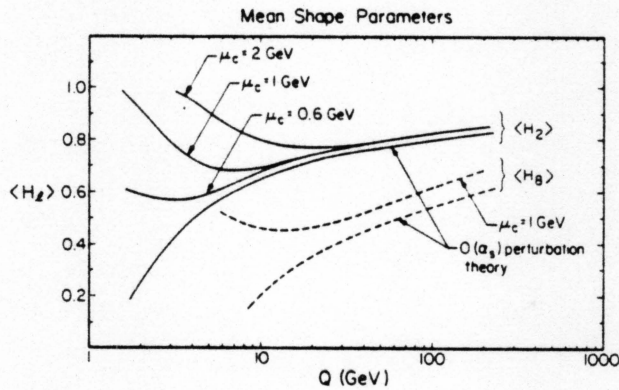


Fig. 6: Mean values of the shape parameters H_2 and H_8 (defined by eq. 6) for parton final states from the decay of a virtual photon with mass Q , and with cutoff μ_c . Results obtained by considering only a single gluon emission at $O(\alpha_s)$ with $\mu_c = 0$ are also shown.