

TESTS FOR PLANAR EVENTS IN e^+e^- ANNIHILATION [☆]Geoffrey C. FOX and Stephen WOLFRAM ¹*California Institute of Technology, Pasadena, CA 91125, USA*

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We present a new class of observables which distinguish events containing two or three hadron jets from those containing a larger number. These observables, which essentially measure the coplanarity of events, are calculable in QCD perturbation theory. Their use should allow the mechanism of Υ decay to be determined.

According to QCD, e^+e^- annihilation into hadrons at high center of mass energies (\sqrt{s}) proceeds dominantly through the process $e^+e^- \rightarrow q\bar{q}$, with some contribution from higher-order mechanisms such as $e^+e^- \rightarrow q\bar{q}G$. On vector meson resonances composed of heavy quark pairs (such as ψ and Υ , denoted generically ζ), QCD suggests that hadrons should be produced primarily through $e^+e^- \rightarrow \zeta \rightarrow GGG$, and should therefore form three jets. In this paper, we discuss tests for this mechanism, which distinguish it, for example, from those in which the hadrons are distributed isotropically rather than forming jets. In a previous paper [1], we considered the class of observables defined by (the P_l are the Legendre polynomials)

$$H_l = \sum_{i,j} \frac{|p_i||p_j|}{s} P_l(\hat{p}_i \cdot \hat{p}_j), \quad (1)$$

where the sums run over all particles in an event, and the \hat{p}_i are unit vectors along the momenta p_i . These observables provide a measure of the "shapes" of events in e^+e^- annihilation and allow some discrimination between isotropic and three-jet hadron production on resonance. For idealized two-jet events, $H_{2l} = 1$ and $H_{2l+1} = 0$, while for isotropic events $H_l = 0$ for $l \neq 0$. Three-jet events lead to intermediate values of the H_l . To make this more quantitative and include the effects of the fragmentation of quarks and gluons

to hadrons, one must perform a detailed theoretical calculation [1]. Perhaps the most distinctive feature of three-jet events is the approximate coplanarity of the final state particles. Unfortunately, this property has no simple consequences for the H_l . However, if instead one considers observables of the form

$$\begin{aligned} \Pi_S &= \sum_{i,j,k} \frac{|p_i||p_j||p_k|}{(\sqrt{s})^3} (\hat{p}_i \times \hat{p}_j \cdot \hat{p}_k)^2 S(\hat{p}_i, \hat{p}_j, \hat{p}_k) \\ \Psi_A &= \sum_{i,j,k} \frac{|p_i||p_j||p_k|}{(\sqrt{s})^3} (\hat{p}_i \times \hat{p}_j \cdot \hat{p}_k) A(\hat{p}_i, \hat{p}_j, \hat{p}_k), \quad (2) \end{aligned}$$

where the functions S and A are respectively symmetric and antisymmetric polynomials in the scalar products of the unit vectors, then for coplanar events, the Π and Ψ vanish. These observables, therefore, provide a definitive test for coplanarity and hence should allow clean discrimination of two- and, particularly, three-jet final states from more complicated structures. The simplest example of the Π class of observables has $S = 1$ and will be denoted Π_1 , while the simplest non-trivial member of the Ψ class (denoted by Ψ_1) has

$$\begin{aligned} A &= [(\hat{p}_i \cdot \hat{p}_k)^2 (\hat{p}_k \cdot \hat{p}_j) + (\hat{p}_j \cdot \hat{p}_i)^2 (\hat{p}_i \cdot \hat{p}_k) \\ &\quad + (\hat{p}_k \cdot \hat{p}_j)^2 (\hat{p}_j \cdot \hat{p}_i) - (\hat{p}_i \cdot \hat{p}_j)^2 (\hat{p}_j \cdot \hat{p}_k) \\ &\quad - (\hat{p}_k \cdot \hat{p}_i)^2 (\hat{p}_i \cdot \hat{p}_j) - (\hat{p}_j \cdot \hat{p}_k)^2 (\hat{p}_k \cdot \hat{p}_i)]. \end{aligned}$$

Note that while the Π are scalars, the Ψ are pseudoscalars, so that when averaged over events, $\langle \Psi \rangle = 0$. Of course, $\langle \Psi^2 \rangle$, for example, need not vanish.

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In [1] we argued that the moments of the H_l should be infrared stable when computed in QCD perturbation theory. This result should also hold for the Π and Ψ . In general, divergences in the mean values of observables are canceled if the observables take on the same value for all physically indistinguishable processes. One requirement is, therefore, that the addition of very soft particles should not affect the value of the observable. This is guaranteed for the Π and Ψ by the presence of a term proportional to the total momenta of the particles. The other condition for infrared stability is that the observables should be linear in the momenta of collinear particles. This is clearly satisfied by the Π and Ψ .

We showed in [1] that the H_l correspond to moments of two-detector energy correlation functions which are formed from the product of the energies incident on each of two detectors [2]. The Π and Ψ may be related to momenta of the analogous three-detector energy correlations ⁺¹. We sketch this relation below.

Let us define the multipole moments of an event by (the Y_l^m are the usual spherical harmonics)

$$A_l^m = \sum_i \frac{|p_i|}{\sqrt{s}} Y_l^m(\Omega_i), \quad (3)$$

where the angles Ω_i are measured with respect to a set of axes chosen in the event. The H_l defined in eq. (1) may then be written as

$$H_l = \left(\frac{4\pi}{2l+1} \right) \sum_{m=-l}^{+l} |A_l^m|^2, \quad (4)$$

which is clearly a rotational invariant and hence independent of the choice of axes used to measure the angles Ω_i . The three-detector energy correlation function may be decomposed in terms of natural generalizations of the H_l , given by

$$T_{l_1 l_2 l_3} = (4\pi)^{3/2} \sum_{m_1 m_2 m_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \quad (5)$$

$$\propto A_{l_1}^{m_1} A_{l_2}^{m_2} A_{l_3}^{m_3}$$

⁺¹ Observables involving products of four or more momenta arising from energy correlations between four or more detectors do not appear to have any immediate application [5].

where the 3- j symbol serves to combine the three spherical tensors into a rotational invariant ⁺². The H_l represent a special case of these observables:

$$T_{l_1 l_2 0} = (-1)^{l_1} \sqrt{2l_1 + 1} \sigma_{l_1 l_2} H_{l_1}. \quad (6)$$

For planar events, the three-detector energy correlation function clearly vanishes unless the three detectors lie in a plane. As we describe in detail elsewhere [2], this property of the three-detector energy correlation may be translated into the vanishing of certain linear combinations of the $T_{l_1 l_2 l_3}$ for planar events. These combinations fall into two classes corresponding to the Π and Ψ observables. Those involving only $T_{l_1 l_2 l_3}$ with $l_1 + l_2 + l_3$ even correspond to the Π and, for example

$$\Pi_1 = \frac{-2}{45} \{ \sqrt{14} T_{222} + 3\sqrt{5} T_{220} - 5 T_{000} \}. \quad (7)$$

If $l_1 + l_2 + l_3$ is even, then $T_{l_1 l_2 l_3}$ is real, but if it is odd, then the $T_{l_1 l_2 l_3}$ are purely imaginary. However, for planar events, all the $T_{l_1 l_2 l_3}$ must be real ⁺³ so that all $T_{l_1 l_2 l_3}$ with odd $l_1 + l_2 + l_3$ must vanish in that case. The Ψ may be written in terms of these $T_{l_1 l_2 l_3}$ and, for example,

$$\Psi_1 = \frac{8}{35} \text{Im}[T_{234}]. \quad (8)$$

The formulae for the simpler Π and Ψ are given in table 1. Note that momentum conservation implies that $T_{l_1 l_2 l_3}$ vanishes if any of its indices $l_i = 1$. We have nevertheless retained such $T_{l_1 l_2 l_3}$ in table 1 so that our results may be applied to incomplete final stages where momentum is not conserved among the particles used to calculate the Π and Ψ .

In the approximation of free final quarks and gluons, events of the types $e^+e^- \rightarrow q\bar{q}(G)$ and $e^+e^- \rightarrow \zeta \rightarrow GGG$ will give zero for all the Π and Ψ . For an exactly isotropic event, however, all the $T_{l_1 l_2 l_3}$ vanish except for $T_{000} = 1$. In this case, therefore, $\Pi_1 = \frac{2}{9}$, $\Pi_2 = 0$, $\Pi_3 = 0$, $\Pi_4 = \frac{2}{15}$ and all $\Psi = 0$.

In order to simulate real hadronic events, we use the phenomenological model for quark and gluon frag-

⁺² Note that the $T_{l_1 l_2 l_3}$ vanish for l_3 outside the range $||l_1 - l_2|$ to $|l_1 + l_2|$ (triangle inequality) or if the sum $l_1 + l_2 + l_3$ is odd and two of the l_i are equal (symmetry property of the 3- j symbols).

⁺³ If the plane formed by the x and z axes is chosen to be in the plane of the event, then from (3) all the A_l^m are real so that the $T_{l_1 l_2 l_3}$ deduced from (5) will also be real.

Table 1

Examples of observables which vanish for coplanar events.

$$\begin{aligned}
\Pi_1 &\equiv \sum_{i,j,k} \frac{|p_i| |p_j| |p_k|}{(\sqrt{s})^3} (\hat{p}_i \times \hat{p}_j \cdot \hat{p}_k)^2 = -\frac{2}{45} [\sqrt{14} T_{222} + 3\sqrt{5} T_{220} - 5T_{000}] \\
\Pi_2 &\equiv \sum_{i,j,k} \frac{|p_i| |p_j| |p_k|}{(\sqrt{s})^3} (\hat{p}_i \times \hat{p}_j \cdot \hat{p}_k)^2 [(\hat{p}_i \cdot \hat{p}_j) + (\hat{p}_j \cdot \hat{p}_k) + (\hat{p}_k \cdot \hat{p}_i)] \\
&= \frac{2}{525} [12\sqrt{21} T_{332} + 15\sqrt{7} T_{330} + 42 T_{321} - 7\sqrt{6} T_{211} - 35\sqrt{3} T_{110}] \\
\Pi_3 &\equiv \sum_{i,j,k} \frac{|p_i| |p_j| |p_k|}{(\sqrt{s})^3} (\hat{p}_i \times \hat{p}_j \cdot \hat{p}_k)^2 [(\hat{p}_i \cdot \hat{p}_j)(\hat{p}_j \cdot \hat{p}_k) + (\hat{p}_k \cdot \hat{p}_i)(\hat{p}_i \cdot \hat{p}_j) + (\hat{p}_j \cdot \hat{p}_k)(\hat{p}_k \cdot \hat{p}_i)] \\
&= \frac{-2}{3675} [12\sqrt{154} T_{433} + 84\sqrt{3} T_{431} - 6\sqrt{21} T_{332} - 21\sqrt{7} T_{330} - 126 T_{321} - 49\sqrt{6} T_{211} + 49\sqrt{3} T_{110}] \\
\Pi_4 &\equiv \sum_{i,j,k} \frac{|p_i| |p_j| |p_k|}{(\sqrt{s})^3} (\hat{p}_i \times \hat{p}_j \cdot \hat{p}_k)^2 [(\hat{p}_i \cdot \hat{p}_j)^2 + (\hat{p}_j \cdot \hat{p}_k)^2 + (\hat{p}_k \cdot \hat{p}_i)^2] \\
&= \frac{-2}{3675} [20\sqrt{77} T_{442} + 140 T_{440} + 12\sqrt{70} T_{422} + 25\sqrt{14} T_{222} + 63\sqrt{5} T_{220} - 245 T_{000}] \\
\Psi_1 &\equiv \sum_{i,j,k} \frac{|p_i| |p_j| |p_k|}{(\sqrt{s})^3} (\hat{p}_i \times \hat{p}_j \cdot \hat{p}_k) [(\hat{p}_i \cdot \hat{p}_k)^2 (\hat{p}_k \cdot \hat{p}_j) + (\hat{p}_j \cdot \hat{p}_i)^2 (\hat{p}_i \cdot \hat{p}_k) + (\hat{p}_k \cdot \hat{p}_j)^2 (\hat{p}_j \cdot \hat{p}_i)] \\
&= -(\hat{p}_i \cdot \hat{p}_j)^2 (\hat{p}_j \cdot \hat{p}_k) - (\hat{p}_k \cdot \hat{p}_i)^2 (\hat{p}_i \cdot \hat{p}_j) - (\hat{p}_j \cdot \hat{p}_k)^2 (\hat{p}_k \cdot \hat{p}_i) = \frac{8}{35} \text{Im}[T_{234}]
\end{aligned}$$

mentation into hadrons developed by Field and Feynman [3]. To investigate the discrimination between planar and non-planar events provided by our observables, we shall compare events due to $e^+e^- \rightarrow \zeta \rightarrow \text{GGG}$ with ones which give the same single hadron momentum ($z = 2|p|/\sqrt{s}$) distribution but which arise from non-coplanar configurations of quarks and gluons. We chose two models for non-coplanar events. In the first (referred to as '6-jet'), we consider the production and decay of a pair of heavy quarks into three particles. This model was introduced in [1]. Although it gives rise to events which are non-planar and contain six hadron jets, it happens that with our quark and gluon fragmentation functions, they have roughly the same z distributions as $e^+e^- \rightarrow \zeta \rightarrow \text{GGG}$ events. For our second model (referred to as 'isotropic'), we generated $e^+e^- \rightarrow \zeta \rightarrow \text{GGG}$ events and then rotated the momentum of each of the particles randomly. This procedure

gives roughly isotropic events but at the cost of some violation of momentum conservation.

In fig. 1, we show the distributions of simulated hadronic events in Π_1 at three center of mass energies while fig. 2 gives their distributions in H_2^{+4} . In both cases, the free quark and gluon predictions are considerably modified by fragmentation to hadrons. This effect is particularly marked for the Π_1 distributions. Nevertheless, even at $\sqrt{s} = 10$ GeV (corresponding to the Υ region), the distributions allow clear discrimination between different mechanisms. Of course, at higher \sqrt{s} , the effects of fragmentation become less important, and the various processes are yet more

⁺⁴ $e^+e^- \rightarrow q\bar{q}(\text{G})$ denotes the sum of the processes $e^+e^- \rightarrow q\bar{q}G$ and $e^+e^- \rightarrow q\bar{q}$, calculated through $O(g^2)$. According to QCD, $e^+e^- \rightarrow q\bar{q}(\text{G})$ should be the dominant process away from resonances. Details are given in [1].

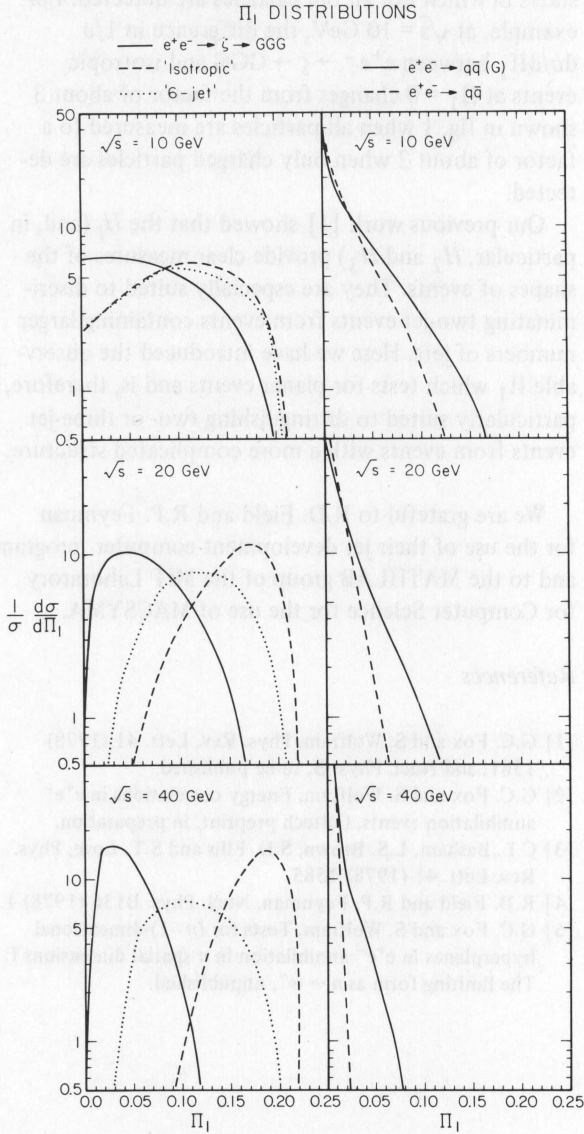


Fig. 1. The distributions $1/\sigma \, d\sigma/d\Pi_1$ of simulated hadronic events in the coplanarity parameter Π_1 for various center of mass energies (\sqrt{s}). $e^+e^- \rightarrow \zeta \rightarrow \text{GGG}$, "isotropic" and "6-jet" are three illustrative mechanisms for heavy resonance (ζ) decay. According to QCD, $e^+e^- \rightarrow q\bar{q}(\text{G})$ should be the dominant process of resonance [1]. In the free quark and gluon approximation, the processes $e^+e^- \rightarrow \zeta \rightarrow \text{GGG}$, $e^+e^- \rightarrow q\bar{q}$ and $e^+e^- \rightarrow q\bar{q}(\text{G})$ should lead to $\Pi_1 = 0$. In the same approximation, the '6-jet' process leads to a roughly flat distribution in Π_1 over its kinematically allowed range ($0 \leq \Pi_1 \leq 2/9$). Completely isotropic events have $\Pi_1 = 2/9$. Note that in this and fig. 2, all curves are calculated by considering only hadrons with momenta above 0.5 GeV.

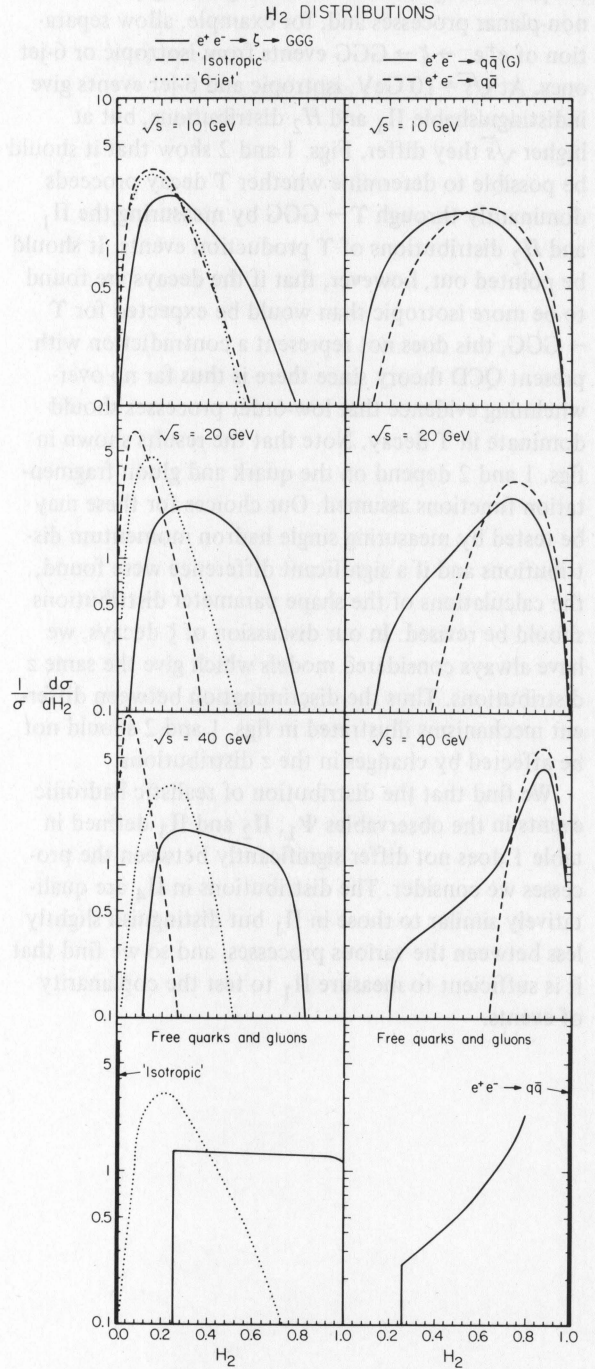


Fig. 2. The distributions $1/\sigma \, d\sigma/dH_2$ of simulated hadronic events in the shape parameter H_2 , for the various center of mass energies (\sqrt{s}). The corresponding distributions in the free quark and gluon approximation are also given.

clearly separated. Note that the distributions in Π_1 are particularly suitable for distinguishing planar from non-planar processes and, for example, allow separation of $e^+e^- \rightarrow \zeta \rightarrow \text{GGG}$ events from isotropic or 6-jet ones. At $\sqrt{s} = 10$ GeV, isotropic and 6-jet events give indistinguishable Π_1 and H_2 distributions, but at higher \sqrt{s} they differ. Figs. 1 and 2 show that it should be possible to determine whether Υ decay proceeds dominantly through $\Upsilon \rightarrow \text{GGG}$ by measuring the Π_1 and H_2 distributions of Υ production events. It should be pointed out, however, that if the decays are found to be more isotropic than would be expected for $\Upsilon \rightarrow \text{GGG}$, this does not represent a contradiction with present QCD theory since there is thus far no overwhelming evidence that low-order processes should dominate in Υ decay. Note that the results shown in figs. 1 and 2 depend on the quark and gluon fragmentation functions assumed. Our choices for these may be tested by measuring single hadron momentum distributions and if a significant difference were found, the calculations of the shape parameter distributions should be revised. In our discussion of ζ decays, we have always considered models which give the same z distributions. Thus the discrimination between different mechanisms illustrated in figs. 1 and 2 should not be affected by changes in the z distributions.

We find that the distribution of realistic hadronic events in the observables Ψ_1 , Π_2 and Π_3 defined in table 1 does not differ significantly between the processes we consider. The distributions in Π_4 are qualitatively similar to those in Π_1 but distinguish slightly less between the various processes, and so we find that it is sufficient to measure Π_1 to test the coplanarity of events.

Our observables can also be used to analyse final states in which not all the particles are detected. For example, at $\sqrt{s} = 10$ GeV, the difference in $1/\sigma d\sigma/d\Pi_1$ between $e^+e^- \rightarrow \zeta \rightarrow \text{GGG}$ and isotropic events at $\Pi_1 = 0$ changes from the factor of about 3 shown in fig. 1 when all particles are measured to a factor of about 2 when only charged particles are detected.

Our previous work [1] showed that the H_i (and, in particular, H_2 and H_3) provide clear measures of the shapes of events. They are especially suited to discriminating two-jet events from events containing larger numbers of jets. Here we have introduced the observable Π_1 which tests for planar events and is, therefore, particularly suited to distinguishing two- or three-jet events from events with a more complicated structure.

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