A Relative Complexity Metric for Decision-theoretic Applications in Complex Systems

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Proposed in this paper is a relative complexity metric deduced from the principles of cross-entropy associated with a complex system. Further indicated is the use of such a metric in decision-theoretic applications relevant to a complex system. As an example, the proposed method is applied to modern cellular phone systems in facilitating the so-called hard handoff effort by which, a mobile unit switches to a new base station when the signal from the serving base station becomes too weak (as a result of inevitably prevailing fading conditions). This wireless communication based decision-making scenario is justifiably portrayed as a spatiotemporal exercise in a complex system. The efficacy of the proposed relative complexity metric in facilitating the handoff effort is illustrated *via* simulations and discussed.

1. Introduction

In a large system constituted by a number of interacting subsystems, it is quite often required that a crucial decision be made to accomplish a specific task. This decision is made in spite of existing complexity, prevailing stochasticity, and persisting extensiveness of the system. A typical example is a modern telecommunications system, which can be aptly characterized as a complex system; and, there are a number of engineering tasks accomplished in such systems on the basis of robust decisions taken appropriately in each case, as necessary.

Recently in [1], such decision considerations are considered in asynchronous transfer mode (ATM) telecommunications, pertinent to call admission control procedures. Relevant algorithms and implementation procedures are based on characterizing the global complexity of the ATM service *via* stochastical and fuzzy attributes inherently present in the system. The global complexity addressed thereof refers to the maximum entropy posed by the stochasticity of the entire system. However, the efforts studied in [1] are not directed at comparing the relative complexity between any two constituent parts within the extensive system.

Hence, the present study is devoted to elucidate the relative extent of complexity between two subsets of a complex system. A set of cross-entropy measures are described and indicated thereof as metrics of relative complexity.

Section 2 of this paper describes the relative complexity *vis-à-vis* cross-entropy considerations. Section 3 addresses the use of such relative complexity metrics for comparison against decision-theoretic thresholds. In section 4, the relative complexity based decision-theoretics are applied to handoff strategies in mobile communication systems. Lastly, simulated results are presented and discussed.

2. Entropy-based assay of system complexity

A complex system representing a large ensemble of interacting units invariably has several attributes that are probabilistic in nature. Mostly, such stochastical attributes refer to the spatiotemporal characteristics inherent to the system.

Given the stochasticity profile of a complex system, the associated complexity can be specified in terms of a cohesive parameter that depicts the maximum entropy of the system. This concept is applied in [2] to explain the error behavior in large systems. The underlying principle has also been adopted to describe the neural complexity by one of the authors elsewhere [3]. Further, as mentioned earlier, the relevant considerations have been used in [1] to model the gross complexity of telecommunication systems.

While the considerations presented in [1–3] portray the complexity of the entire system on an extensive basis, studies that focus on elucidating the relative complexity between the systems (or subsystems) are rather sparse. Therefore, indicated here is a method for evaluating a relative complexity parameter. The underlying concept is again based on entropy considerations. However, instead of maximum entropy (which depicts the extensive complexity of the whole system) the present approach is directed at measuring the cross-entropy between two systems (or subsystems). This cross-entropy measure depicts the "distance" between or "divergence" of statistical profiles of the systems being compared. Hence, it is shown (using the approach due to [2]) that the cross-entropy measure can serve as a metric of relative complexity.

Consider a complex system specified by a domain $\mathbf{X}$ as illustrated in Figure 1. Suppose two constituent (interacting) subsystems (i, j) are respectively characterized by two sets of attributes $\{x : i = 1, 2, \ldots, M\}$ and $\{x : j = 1, 2, \ldots, N\}$, where $x \in \mathbf{X}$ as shown in Figure 1.

Further, the system shown in Figure 1 is also assumed to exhibit random variations in time-scale so that the domain representing the complex system is specified by $\{x : i, j; t\} \in \mathbf{X}$, where t denotes the time.

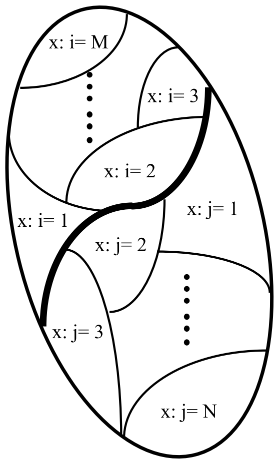

Figure 1. A complex system with constituent spatiotemporal subsystems.

Suppose the randomness associated with the subsets of Figure 1 is expressed in terms of probability density functions (PDFs) $p_x(x = i; t)$ and $q_x(x = j; t)$ corresponding to the sets $\{i\}$ and $\{j\}$ respectively. Now, the maximum entropy concept [2] applied to each group in $\mathbf{X}$ leads to the following functionals:

$$H(s_i) = \ln(M + 1) \approx \ln(M) \qquad \text{since } M \gg 1 \tag{1a}$$

$$H(s_j) = \ln(N + 1) \approx \ln(N) \qquad \text{since } N \gg 1 \tag{1b}$$

where s_i and s_j refer to the metrics of gross complexity corresponding to the extensiveness of the populations M and N of the sets $\{i\}$ and $\{j\}$, respectively. Equation (1) is consistent with the Jaynes principle of maximum entropy or maximum uncertainty [6]. Correspondingly, a class of distribution exists and corresponds to the maximum entropy formalism. It leads to the well-known Shannon entropy, namely,

$$I(x = i) = -\sum_{x \in X} p_x \log[p_x(x)] \qquad \text{for } \{i\} \tag{1c}$$

and

$$I(x = j) = -\sum_{x \in X} q_x \log[q_x(x)] \qquad \text{for } \{j\}. \tag{1d}$$

Eqnuations (1c) and (1d) can be regarded as implicit representations of gross complexity pertinent to the sets $\{i\}$ and $\{j\}$ respectively, *in lieu* of the relations specified by equations (1a) and (1b).

While equation (1) depicts the maximum entropy (H) measuring the gross complexity (s) of the set $\{i\}$ or $\{j\}$, another metric can be specified analogously to measure the relative complexity between these sets. This metric is the cross-entropy functional, which can be written in the following two forms:

$$H(s_i \| s_j) = D(s_i \| s_j) = p_x \log(p_x / q_x) \tag{2a}$$

$$H(s_j \| s_i) = D(s_j \| s_i) = q_x \log(p_x / q_x). \tag{2b}$$

Equation (2) denotes the *statistical distance* (D) between the random attributes of $\{i\}$ *versus* $\{j\}$ or *vice versa*. This cross-entropy measure is also a relative information entity in Shannon's sense. This measure specified *via* equation (2) follows Kullback's minimum (directed) divergence or minimum cross-entropy principle [7].

Writing in the form akin to Ferdinand's [2] representation of gross complexity, the relative complexity can be specified (in terms of the populations M and N) as follows:

$$H(s_i \| s_j) = \ln\left[\frac{M}{(M+N)} \times \frac{(M+N)}{N}\right] \tag{3a}$$

$$H(s_{ij} \| s_i) = \ln\left[\frac{N}{(M+N)} \times \frac{(M+N)}{M}\right] \tag{3b}$$

which, when written in Shannon's entropy format, would lead to equation (2). Hence, in the present study, equation (2) is considered as a metric of relative complexity.

The cross-entropy depicting the relative complexity, in fact, is an expected logarithm of the likelihood ratio (L), namely,

$$L = \frac{[(p_x)_{x=i}]_{\mathrm{I}}}{[(q_x)_{x=j}]_{\mathrm{II}}} \tag{4}$$

where $[p_x(i)di]_{\mathrm{I}}$ and $[q_x(j)dj]_{\mathrm{II}}$ are respective probabilities of observations $\{i\}$ and $\{j\}$ (at any given instant t) when a certain hypothesis $(h_{\mathrm{I}}, h_{\mathrm{II}})$ is true. Corresponding to L, the log-likelihood ratio function (LLR) given by $\log(L)$, is a *discrimination measure*, which provides a choice, whether to choose $\{i\}$ in preference to $\{j\}$ or *vice versa*. That is, LLR is well known [4] as a useful metric in decision-making efforts and is presently considered to depict the relative complexity concurrent to the cross-entropy measure.

The distance-measure or cross-entropy entity indicated earlier is a convex functional of the likelihood ratio. It refers to the difference in the mean values of the LLR under the hypothesis testings h_{I} and h_{II} (in the Neyman–Pearson sense) [4]; that is, whether the decision be made in favor of (or against) the set $\{i\}$, or in favor of (or against) the set $\{j\}$. Hence equation (2) can be specified alternatively (in terms of LLR) as follows:

$$H(s_i \| s_j) = (E[\mathrm{LLR}])_{\mathrm{I}} - (E[\mathrm{LLR}])_{\mathrm{II}} \tag{5a}$$

where $E[.]$ is the expectation operator. Explicitly,

$$(E[\text{LLR}])_{\text{I}} = \int_{(i=x)\in X} (\text{LLR})p(i)di \tag{5b}$$

and

$$(E[\text{LLR}])_{\text{II}} = \int_{(j=x)\in X} (\text{LLR})p(j)dj. \tag{5c}$$

In general, the expectation numbers $(E[\text{LLR}])_{\text{I,II}}$ are known as Kullback–Leibler measures [5] and depict the cross-entropy metrics of equation (2). The cross-entropy is also the minimum entropy functional specifying the relative complexity between $\{i\}$ and $\{j\}$, as indicated *via* equation (3).

In addition to equation (2), there are a number of other cross-entropy functionals developed and elaborated in the literature [5]. In the present study, it is generalized that all such cross-entropy measures can be adopted to denote the relative complexity between two sets; and, it is further indicated that they can be used as appropriate metrics in decision-making algorithms considered in hypothesis testing [4].

The family of cross-entropy measures recast in the present study to depict the relative complexity metric (RCM) follows.

- Kulback–Leibler measure (KL):

$$\text{KL} = \sum_{(i=x)\in X} p(x)\log\left[\frac{p(x)}{q(x)}\right] \tag{6a}$$

or,

$$\text{KL} = \sum_{(j=x)\in X} q(x)\log\left[\frac{q(x)}{p(x)}\right]. \tag{6b}$$

- Bhattacharyya measure (B):

$$\text{B} = -\log(\rho) \tag{6c}$$

where ρ is the Bhattacharyya coefficient, given by

$$\rho = \int_0^\infty \sqrt{p(x)q(x)}\,dx. \tag{6d}$$

- Sharma–Mittal measure (SM):

$$\text{SM} = \frac{1}{(\alpha-\beta)}\left[\sum_x p^\alpha(x)q^{1-\alpha}(x) - \sum_x p^\beta(x)q^{1-\beta}(x)\right] \tag{6e}$$

where $(\alpha > 1, \beta \leq 1)$ or $(\alpha < 1, \beta \geq 1)$.

- Rènyi measure (R):

$$R = \frac{1}{(\alpha - 1)}\left[\sum_x p^{\alpha}(x) q^{1-\alpha}(x)\right] \tag{6f}$$

where $\alpha \neq 1$ and $\alpha > 0$.

- Kapur's measure (K):

$$K = \frac{1}{(\alpha - \beta)}\left[\frac{\sum_x p^{\alpha}(x) q^{1-\alpha}(x)}{\sum_x p^{\beta}(x) q^{1-\beta}(x)}\right] \tag{6g}$$

where $\alpha > 0, \beta > 0$, and $\alpha \neq \beta$.

The above class of metrics fall under the category of so-called Ali–Silvey distance [8], which generates many of the common distance measures that have been used in various applications. The concept of minimum directed divergence measures as proposed in [9] can further be generalized to represent a family of measures known as Csiszàr's f-divergence metrics, indicated in [10] as measures of informativity.

In reference to the various cross-entropy measures indicated above, each one of them specifies a RCM that can be adopted in decision-theoretic efforts (such as the Neyman–Pearson observer). Suppose T is a decision-threshold, the RCM-based decision refers to the condition that,

$$\text{RCM} \underset{h_{\text{II}}}{\overset{h_{\text{I}}}{\lesseqgtr}} T. \tag{7}$$

Equation (7) denotes the decision-theoretic criterion on the hypotheses h_{I} and h_{II}. For example, the RCM (such as the KL measure) implicitly assays the relative complexity of underlying attributes of the two sets $\{x : i, j\}$ being subjected to the Neyman–Pearson test. Thus, the conventional detection problem can now be translated and specified as a choice between two hypotheses h_{I} and h_{II} of equation (7) in respect of an observation, $x = \{i, j\} \in \mathbf{X}$. The basis for this approach is that the metrics of equation (6) contain all the information necessary for a choice to be made between two hypotheses under consideration.

Lemma 1. Given the statistics $y = F(x)$, so that $y_i = F(x = i \in X)$ and $y_j = F(x = j \in X)$, suppose the two hypotheses h_{I} and h_{II} correspond to the PDFs $p(x = i)$ and $q(x = j)$, respectively. These two hypotheses can be discerned, if the statistics indicated above (conditionally to y) are specified by the following inequality:

$$p(x/y) \neq q(x/y). \tag{8}$$

The RCMs of equation (6) do, in essence, satisfy the inequality of equation (8) and therefore depict a sufficient statistic.

Proof. Considering the Kullback–Leibler metric, namely, $H(s_j \| s_i) = p_i \log(p_i/q_i)$ and $H(s_j \| s_i) = q_i \log(p_i/q_i)$, as indicated earlier, they refer to the expected values of the corresponding log-likelihood measures, namely $\text{LLR}(x = i) = \log(p_i/q_i)$ and $\text{LLR}(x = j) = \log(q_j/p_j)$ respectively.

Rewriting equation (8),

$$\frac{p(x)\delta[x - \text{LLR}(x)]}{p(y)} \neq \frac{q(x)\delta[x - \text{LLR}(x)]}{q(y)} \tag{9}$$

where $\delta[.]$ represents the Knonecker delta, which takes the value 1 or 0. This implies that when the event x is realized, the value of y is specified; otherwise, not. (Note: $\gamma(x, y) = \gamma(x)\delta[y - g(x)]$ and $\gamma(y/x) = \delta[y - g(x)]$ where $\gamma = p$ or q and $g(x)$ is an invertible function.) ■

Equation (9) is valid as long as $p(x)/q(x) \neq p(y)/q(y)$. That is, as long as the likelihood ratio is not a constant, the two hypotheses can be discerned.

Lemma 2. Given the two hypotheses h_{I} and h_{II}, a decision-threshold set by an observation level where the RCM becomes invariant can be considered as a metric for a transition of decisions from (h_{I} against h_{II}) to (h_{II} against h_{I}) or *vice versa*.

Proof. It follows from equation (3), that the cross-entropy measure is an implicit metric of relative complexity; and, the cross-entropy as mentioned earlier is an expected value of the LLR. Therefore, the threshold set at the RCM corresponds to a level specified implicitly in terms of the likelihood ratio.

Hence, by Lemma 1, a threshold decision level specified at a constant value of RCM (corresponding to a constant likelihood ratio) can facilitate a transition of decision from (h_{I} against h_{II}) to (h_{II} against h_{I}) or *vice versa*. ■

3. Application of relative complexity metrics in decision-theoretic algorithms

The concept of RCM in decision-theoretic efforts as indicated in section 2 can be demonstrated by considering the so-called handoff (or handover) strategy adopted in mobile communication systems.

Handoff in cellular communication systems refers to a crucial effort by a particular mobile unit changing the serving base station as warranted by dynamic variations in link quality resulting from mobility and interference/fading considerations. Conventionally, the handover initiation is done by a decision algorithm based on received signal strengths (RSSs) from the serving and prospective base stations. That is, the power-difference of the received signals is used as a metric for handoff initiation.

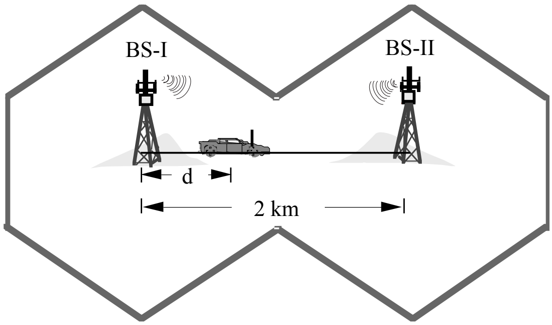

Figure 2. Collinear dispositions of the mobile unit and the base stations BS-I and BS-II.

Demonstrated in the present study is a new algorithmic approach wherein the handoff decision is performed by computing the RCM between the two fluctuating (received) signals. That is, the set of RCMs indicated earlier are computed from the statistics of the RSSs and proposed as alternative metrics for handoff decisions.

To illustrate the efficacy of the metrics adopted towards handoff decisions, the cellular mobile system considered refers to the model due to Vijayan and Holtzman [11,12]. Here, a mobile unit traverses a collinear path between two base stations BS-I and BS-II (Figure 2). A log-normal fading, which affects the link quality between the mobile unit and the base stations is assumed consistent with an urban environment.

As the mobile unit moves through the system coverage area, it would eventually cross the cell boundaries where the RSS from the serving base station may become too weak. As such, a decision should be made in regards to specifying whether the service be shifted (handed over) to the other base station. Relevant considerations on this handoff procedure follows.

As the mobile unit travels away from its serving base station, the linked signal strength (S_1) grows weaker due to path-loss and fluctuates in strength as a result of fast and slow fadings. While the signal S_1 drops and tends to fall below a minimum acceptable level, the signal (S_2) from the approaching base station would grow stronger. At this stage, when S_2 becomes above a certain level (called *add level*), a procedure is initiated to decide whether to transfer (handoff) the link to the stronger station. However, this handover procedure should be done carefully. If handover occurs at every instant when one base station becomes stronger than another (due to the fluctuations involved), then "chatter" (also known as "ping-pongs") or very rapid switching between the two BSs will take place. This would happen especially when the mobile unit moves along a cell boundary. This situation is described by the following handoff decision rules made at specified intervals of time.

- If ($\Delta_f < 0$, the mobile unit is connected to BS-I, and $S_{1f} < 0$), then handoff to BS-II.
- If ($\Delta_f > 0$, the mobile unit is connected to BS-II, and $S_{2f} < 0$), then handoff to BS-I.
- Otherwise do not handoff. (10)

Here, S_{1f} and S_{2f} are averaged (filtered) values of S_1 and S_2 respectively and $(S_{1f} - S_{2f})$ is equal to Δ_f. In the aforesaid handoff scheme, as mentioned before, there is a possibility of extensive ping-pongs/chatters as a result of prevailing fluctuations in RSS. As such, each time a handoff is executed, it amounts to an overhead on the system. That is, with each unnecessary handoff performed, the network bears the responsibility to do signaling, vary the extents of authentication, update the data bases, and perform circuit-switching and bridging as necessary. Such tasks would call for extensive use of network resources and lead to a lack of robustness and reliability of the handoff procedure.

To overcome the burden imposed on the network by the ping-pongs, an element of hysteresis is introduced into the handover algorithm. That is, the handover is facilitated only when the new BS is stronger than the previous one by at least some handover margin. In handoff algorithms, which incorporate this hysteresis, the basic algorithm of equation (10) is modified as follows.

- If ($\Delta_f < -a_1$, the mobile unit is connected to BS-I, and $S_{1f} < \delta$), then handoff to BS-II.
- If ($\Delta_f > +a_2$, the mobile unit is connected to BS-II, and $S_{2f} < \delta$), then handoff to BS-I.
- Otherwise do not handoff. (11)

In equation (11), a_1 and a_2 are known as *hysteresis margins*; and, in general, $a_1 = a_2 = a$. Further, δ is a power level stipulated as the minimum value, so as to avoid dropout. The hysteresis algorithm of equation (11) allows the system to wait until it is more certain that a handoff is necessary and should be performed before it actually does so. This consideration obviously would reduce the number of undesirable back-and-forth handoffs.

Equation (11) in essence, specifies that, if the averaged signal level from the new base station exceeds that from the current base station by a decibles, the handoff is executed. A handoff to BS-I occurs at an "upcrossing of $+a$" provided the previous connection was to BS-II and $S_{2f} < 0$, and a handoff to BS-II occurs at a "downcrossing of $-a$" provided the previous connection was to BS-I and $S_{1f} < 0$. Thus, the hysteresis-specified handoff conditions summarize to: (i) An upcrossing causes a handoff if the previous event was a downcrossing; and, (ii) a downcrossing causes a handoff, if the previous event was an upcrossing. An additional requirement is that the signal from the connected BS should be below a certain reference level in reference to both conditions.

The extent of this handover margin $|a|$ has to be judiciously chosen. If this margin is too large, then the mobile unit may move far into the coverage area of a new cell, causing interference to other users and itself suffering from poor signal quality and eventual call dropout. If $|a|$ is too small, frequent ping-pongs would take place causing undue demand on the resources of the systems. Therefore, a recommended practice is that the optimum handover margin should be set crucially by considering the level of shadowing in the system, since the shadowing effect essentially determines the random variation of signal level along the cell boundaries.

There are two versions of handoff in practice. Suppose during the handoff process that the mobile unit is served by only one base station. Then, the process is known as *hard handoff*. In this case, the communication link at all times is between the mobile and the only base station in service. If, during the handoff process, there are two or more base stations serving the mobile, then the process is called *soft handoff* [7]. Due to hardware limitations, the so-called frequency division multiple access and the time division multiple access systems invariably use hard handoff, while code division multiple access (spread-spectrum) based systems usually use the soft-handoff procedure. The present study refers to hard handoff conditions.

Considering the conventional handoff procedures, the average (filtered) values of S_1 and S_2, namely S_{1f} and S_{2f}, leads to a power-difference metric $\Delta_f = (S_{1f} - S_{2f})$ indicated before. And, as specified *via* equation (11), a handover is initiated from BS-I to BS-II when the serving base station is BS-I and $\Delta_f < -a$; and from BS-II to BS-I when the serving base station is BS-II and $\Delta_f > +a$. This handover strategy is qualitatively illustrated in Figure 3 with a hypothetical set of curves depicting the variations of S_1 and S_2 along the path from BS-I to BS-II. The BS-I signal power at the mobile unit is shown in Figure 3 by a solid line, S_1, while that from BS-II at the mobile unit is shown as a dashed line, S_2. Because the mobile unit is traveling away from BS-I and towards BS-II, S_1 grows weaker due to path-loss. It also varies randomly in strength as a result of shadow fading; on the other hand, S_2 grows stronger because the mobile unit is getting closer to BS-II. S_2 also would vary randomly in strength since it is also subjected to shadow fading. The difference $\Delta_f = (S_{1f} - S_{2f})$ is plotted in the lower half of Figure 3.

Referring to Figure 3, it can be noted that, without the use of hysteresis, a handoff to BS-II at T_1, a handoff back to BS-I at T_2, and again a handoff to BS-II at T_3 take place. Should only one handoff (to BS-II) occur say, at T_4 (in Figure 3), the necessary conditions are that the mea-

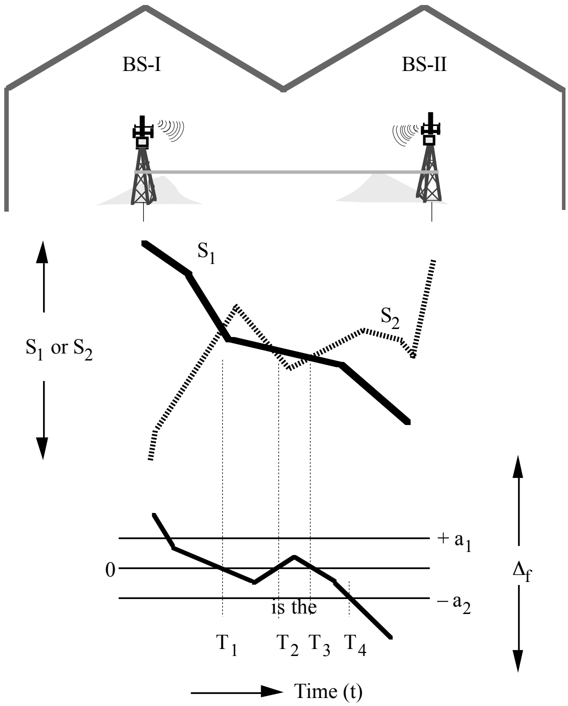

Figure 3. Variation of base station signal strengths at the mobile unit during hard handoff. S_1 is the signal strength due to BS-I, S_2 is the signal strength due to BS-II, $\Delta_f = (S_{1f} \uparrow S_{2f})$, and $a_1 = a_2 = a$ is the hysteresis level. T_1, T_2, and so on are the time instants as illustrated.

sured Δ_f at T_4 be less than $-a$ for a handoff to BS-II; and, if Δ_f is greater than $+a$, then the handoff shifts to BS-I.

In the existing handoff schemes [11–14], the Δ_f metric indicated above is pursued. In the present study, an alternative metric is proposed *in lieu* of Δ_f for the handoff procedure. That is, the question of exceeding the hysteresis level is addressed on the basis of relative entropy between the signals received, rather than on the associated (average) power levels. Senadji and Boashash [15] have considered an alternative approach to the power-difference method for estimating the hysteresis value for handover decision algorithms. However, their method is based on Bayes criterion. Their alternative metric approach amounts to replacing the power-difference value Δ_f with a cross-entropy metric deduced from the statistics of the signals received from the BSs. Hence, the corresponding relative complexity is estimated and observed as a metric against the hysteresis levels. The logistic behind the use of the RCM (*in lieu* of the power-difference metric) is as follows.

It is important and imperative that the handoff decisions be based on as "good information as possible." That is, by exploiting the information content inherent in signal-strength measurements, the quality of handoff decisions can be optimized regardless of the size of the cells and the statistics of signal fluctuations. The signal fluctuations, in general, are "nuisances" impairing the handoff decision procedures implemented. That is, they signify the entropy (or uncertainty) profile of the measurements; and, their presence is rather inevitable. Correspondingly, they pose a complexity of the processing involved.

Notwithstanding the inevitable presence of fluctuations, the handoff algorithms should capture the relative ability of base stations in serving the mobile unit in question. This relative ability depends on the extent of information that can be extracted from the measured RSSs. A cohesive method of comparing the relative information-loss/entropy and extracting useful information contents of the statistics pertinent to the signals received from the base stations participating in the handover efforts, can be done on the basis of complexity measure considerations. It is logical to pursue such an effort, inasmuch as the associated paradigm would account for the entire posentropy and negentropy profile of the stochastical processes involved. Further, to exploit the information learned from each statistic, the handoff algorithm must effect an exchange of soft rather than hard decisions. For a system consisting of two subsystems, the concept behind statistical distance based hypothesis testing is to pass soft decisions from the realm of one set to that of the other, and to iterate this process (on a moving window basis) to produce better decisions. Hence, the research addressed in this paper was motivated to look into feasible aspects of such an approach.

4. Simulated results

To ascertain the efficacy of the RCM based hard handoff algorithm *vis-à-vis* the conventional (power-difference) strategy, simulations were performed on the Vijayan–Holtzman model using MATLAB™ (version 5.3). The parameters used in the simulations are as follows. The mobile unit is presumed to be in a metropolitan area, where the base stations are typically separated by about 2 km. The mobile unit is assumed to be traversing a collinear path between BS-I and BS-II. Further, the speed of the mobile unit is assumed to be $v = 12$ m/s (or 26.8 miles/hour); again, typical in an urban environment. As the mobile unit recedes from BS-I, the RSS (S_1) from BS-I suffers a path-loss of 30 dB per decade of distance [11,12] and fluctuates due to fading conditions. Likewise, the RSS from BS-II, namely, S_2 is also path-loss specified and would fluctuate as a result of fading involved. For smoothing the RSS values, window-averaging is used with a filter constant of (30 m/v). The shadow fading component of signal power (in dB) is assumed as a gaussian process with a standard deviation of 6 dB and a correlation time of (20 m/v). Further, assuming a possible cross-correlation between S_1 and S_2 (due to a common fading environment in the vicinity of the mobile unit), a coefficient of cross-correlation equal to 0.25 is assumed. An appropriate hysteresis level is set for each metric so that minimal ping-pongs and dropouts were observed in the simulations performed with each metric.

Delay for all the metrics is assessed in a relative scale. It refers to the delay measured from the time at the midpoint (50 seconds) to the time at which handoff occurs. The computed results on the delay are listed in Table 1 for all the metrics under consideration. Further indicated in Tables 2 and 3 are percentages of single and/or multiple handoffs and dropouts respectively obtained from the simulations. The values indicated correspond to an ensemble average of 50 trips.

Metric	Delay in seconds (Mean value)	Instant at which handoff occurs (in seconds, counted from the start time at BS-I)	
		Mean value	Standard deviation
Δ_f	13.99	63.99	10.62
KL	14.02	64.02	9.98
B	16.42	66.42	7.80
SM	5.95	55.95	14.24
R	6.76	56.76	14.67
K	18.71	68.71	15.56
LLR	24.65	74.65	13.60

Table 1. Handoff delay characteristics. Delay is measured with respect to the time-instant corresponding to the midpoint between BS-I and BS-II.

Metric type	% Handoffs		
	Single	Multiple	
	0 ping-pongs	3 ping-pongs	4 ping-pongs
Δ_f	98	2	0
KL	96	4	0
B	100	0	0
SM	84	12	4
R	86	10	4
K	96	4	0
LLR	98	2	0

Table 2. Percentage of single and multiple handoffs.

Metric type	Ante-handoff dropouts %	Post-handoff dropouts %
Δ_f	14	0
KL	10	4
B	28	0
SM	12	4
R	12	4
K	22	4
LLR	36	0

Table 3. Percentage dropouts.

5. Concluding remarks

In summary, the study presented here offers a method of comparing two complex systems (or subsystems) in terms of the associated relative entropy between them. The underlying consideration is that the "complexity is a conceptual precursor to entropy" or *vice versa*. A relative complexity concept is proposed thereof, and the cross-entropy functional is indicated as the metric of relative complexity. A family of such metrics is explicitly identified.

The use of a relative complexity metric (RCM) in decision-theoretic efforts is elucidated in terms of likelihood ratio considerations and hypothesis testing heuristics.

A practical application of the RCMs in wireless communication systems is illustrated in reference to handoff algorithms adopted to switch the base stations serving a mobile unit on *ad hoc* basis as dictated by received signal conditions. The vastness of the mobile communication system and the interaction between its constituent subsystems aptly permit that such a system be described as a complex system. The present study offers an assessment of the performance of such a system. In reference to the optimal hysteresis pertinent to the handoff procedure (under the constraints of performance requirements), it can be flexibly ascertained using the multiple options on RCMs proposed. Practical implementation of the proposed strategy can follow the underlying signal selection techniques indicated in [16,17].

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