Nuclear Weapon Pits: Burn Them or Bury Them? A Richardsonian Energy Competition Decision Model*

Alvin M. Saperstein
Department of Physics and
Center for Peace and Conflict Studies,
Wayne State University, Detroit, MI 48202, USA

Since decision making in the face of emotionally laden problems and incomplete information is very difficult, it is useful to develop and test alternative decision methodologies and/or criteria. One approach is to enlarge the system in which the decision is to be made, for example, go from a national system to an international one, as well as to alter the decision criteria. This paper is a heuristic test of such an approach: the goal is to see how such a method could be developed, not to produce and validate a definitive policy, national or international.

The test problem is the disposal of nuclear materials from demobilized nuclear weapons—currently a contentious domestic problem in the United States. In the extension of this paper, the criteria are richardsonian stability and the absence of chaos in a model of the world system including several nuclear nations such as the U. S.

A modified Richardson model is created in which two nations compete over finite world energy supplies. Additional civilian energy may also be available from recycling the fissionable nuclear materials from demobilized nuclear weapons. Alternatively, these surplus fissionable materials may be disposed of without civilian use of their nuclear energy. Under fairly general conditions, the demand for Richardson stability in the model and the absence of chaos (or crisis instability) implies that reuse of the nuclear weapon materials for civil energy purposes is more conducive to international stability than is disposal without reuse. Obviously, other decision paradigms, such as economics, also have to be applied before rational choices can actually be made.

1. Enlarging the system of discourse to facilitate dispute resolution

Important national and international policies are frequently driven and constrained by emotional commitments, often hidden, which have no direct connection or relevance to the problem at hand. Arguments for and against these policies stem from contradicting *implicit* axiom sets

*Paper developed from a presentation to the Sixth World Peace Science Congress, Timbergen Institute, Amsterdam, May 24–26, 1999.

and hence pass by each other without rational contact or influence. If all parties to the policy making could appeal to a more fundamental *explicit* set of axioms, acceptable to all, rational discussions might ensue, resulting in better policies.

For example, participants in the ongoing debate about nuclear power, pro and con, often have opposing mind sets, as to the perfectibility of technology and science, which underlie their arguments about economics, energy needs, and relative environmental impacts. Hence there is no mutually acceptable quantitative way to balance one argument against its counter so as to reach rational conclusions. Decisions are made by wearing the opponents down, politically and psychologically. On the other hand, most citizens of the modern world would be in favor of international stability and those policies which diminish the possibility of war, especially large scale nuclear war. Is it possible to make use of this common stability desire to transcend the domestic nuclear power debate? Can we get a better handle on the national energy discussion by enlarging the field of argument to that of international security?

The major interest driving this paper is methodological: can we facilitate the resolution of policy dispute by enlarging the system of discourse and its axioms, assuming that the more general the axiom set, the less contentious it may be. The specific example chosen to explore this methodological question is a small subset of the nuclear power question: What should be done with the nuclear fission components of demobilized nuclear weapons? (There are those who argue this question completely within the realm of nuclear weapons, denying its relevance to civil nuclear power. Thus, by stating the question as a subset of nuclear power, I am already enlarging the field of discussion.) I attempt to couple the question of disposition of surplus nuclear weapons with the concept of enhancing international stability in the hope that agreement on the latter will lead to insights or conclusions about the former, not subject to the contentions, implicit or explicit, that civil nuclear power is, or is not, a good thing.

2. Methodology

Science may be viewed as mankind's rational, collective attempt to cope with its biological, physical, psychological, and social environments. When an appropriate science exists, it can be used to make predictions about the outcomes of different design, engineering, or policy choices. Hence science facilitates the making of technology: policy making is the technology associated with a political science.

The usual scientific procedure for making predictions is to create a model based upon the known science. The model is then "run," analytically or computationally, evolving into the future according to the selected scientific laws. If the model is an accurate representation of reality, and the science is good, the future behavior of the model is a good indicator of the real future: a successful prediction has been made (e.g., [1]).

For example, a standard model of our solar system is a bunch of point masses in a vacuum representing the sun and its planets free to move in space under their mutual influences. The relevant science is dynamics: Newton's laws of motion and the law of universal gravitation. Given an initial specification of its appearance, for example, the present locations and velocities of the solar system's components, the science allows the orbits of the planets to be computed, thus predicting the appearance of the solar system in the future. Experience indicates that the predictions have been very accurate representations of reality, thus strengthening our confidence in the validity of the science and the model. Similarly, the science of statics, plus engineering models of bridge structures, enable us to determine whether the anticipated real bridge will successfully carry the desired loads. The first of these two examples illustrates the use of a science to further science; the second demonstrates how science underpins technology. In both cases, the success of the predictions validates the science.

But what if the relevant science does not exist? To the best of my knowledge, at present there is no science of international relations, nothing from which predictions can be made upon which we can found policies, with confidence, to protect our futures (e.g., [2]). Here we must attempt to simultaneously create both the science and the technology: guess a science and a model, see what technology it implies, test that technology against reality, loop back to the beginning to modify the initial guesses on the basis of that test, and then go around again.

The suggested paradigm is to choose a pressing problem, model it several different ways, making use of the best sciences we have, that is, the best understandings we have of the system in which the problem manifests itself. Then use the models to make predictions of the outcomes of different policy choices. If we get contradictions between the different predictions, if different policies are suggested by the different, but similar, realistic models and sciences, then we know that we are not on the correct path to an appropriate science for the system in question. If the predictions are similar, if the different approaches lead to very similar policy recommendations, then we may surmise that there is some common core of validity in these different approaches, a core which may be part of the hoped for "correct" science.

Quantitative predictions require quantitative models which imply quantitative assumptions and parameters. Ideally, this input should be empirically validated before it is used in the model. Such validation is often an enormous task. Putting off model building until all input is solidly verified may mean that the model is never built. An alternative procedure, adopted here, is to make reasonable heuristic guesses as to the input data for the proposed model. If the model's results prove "interesting," scholars may be motivated to further test its assumptions. If the model, including its associated input ideas, is "uninteresting," for example, its predictions are contrary to fact, there will be no need, nor incentive, for empirical validation studies of its input.

I am attempting to develop a dynamical science of international security, a science which—given the initial state of a system of competitive nation states—will predict the time evolution of that system ([18] and references therein). The initial state includes the set of security policies adopted at the initial time (the present) to deal with a given problem or situation; different policy choices mean different outcomes of the evolution. The preferred policy is that which leads to the preferred outcome, usually a stable or peaceful world. If the policies suggested by "my science" are similar to those suggested by "custom," "common sense," or alternative bodies of understanding, confidence in the validity and applicability of my science, and its recommended policy options, should be enhanced. Thus the envisioned process is the twining of general scientific understanding, the development of models of world system stability, and the resulting constraints placed upon contentious policies in national subsystems. Each thread adds to the strength of the others in the developing rope of useful knowledge and technology.

3. Example problem

The problem I wish to consider is that of the "proper" disposal of the nuclear pits from surplus nuclear weapons. These pits, consisting of fissionable plutonium and/or uranium, are the source of the bang in atomic bombs (nuclear fission weapons); they are also the triggers required to ignite the much bigger bang of hydrogen bombs (nuclear fusion weapons). As the major nuclear weapon powers seem to be embarked on a program of massive reductions in their stocks of nuclear weapons, the question arises as to what to do with these junked bombs and warheads (cf., [3–8]).

The conventional parts of these weapons—the metal, electronics, and chemical explosives—are neither particularly valuable nor dangerous when appropriately trashed, destroyed, or recycled. It is usually easier, cheaper, and safer, to recreate them anew if more weapons are to be constructed. The facilities for trashing and recreating should be available to any moderately sophisticated industrial nation.

Nuclear materials are not so easily available. The required fissionable materials are very difficult to obtain from naturally occurring resources. The chemical and/or physical separations required to obtain bomb-grade fissionable materials, either from natural ores or from civilian nuclear power programs, are beyond the reach of most nations and nonnation groups. On the other hand, the pits are already weapons-grade and are easily recycled into the essential cores of new nuclear weapons. Thus these pits are very valuable to those who would, surreptitiously or openly, wish to manufacture new nuclear weapons. They are also very valuable in that their initial creation represents the expenditure of significant social and natural resources; they represent large amounts of available energy which could be returned to society if used in appropriate civil nuclear power reactors.

The disposal question then is how to make sure that the materials of these pits will not again be available for military purposes. Two suggestions are currently popular: burn them or bury them.

To *burn* them means converting them into fissile fuel for ordinary civilian power reactors. They have to be isotopically diluted down from bomb grade to reactor grade by mixing with natural uranium and, perhaps, be chemically changed from metallic to oxide form. The result is a "swords into plowshares" conversion—destructive power into electric power.

To *bury* them means to mix them with ceramic materials, in the form of "glass logs," the fissile material in each log is diluted so as to be far from critical mass. Then bury the logs in secure and safe locations.

The burn option implies a large commerce and extensive transportation of radioactive fissionable materials, perhaps open to criminal (or national) diversion back to the bomb makers. It also implies an expansion, or continuation, of the nuclear power industry with its popularly perceived problems of public safety, environmental pollution, and disposal of radioactive wastes. However, once the fissionable materials have been burnt—have been fissioned to release their stored energy in a civilian nuclear reactor, they can no longer be used in a weapon, their nuclear energy is gone. Of course, no single pass through a reactor of fissile fuel utilizes all of the energy available in that fuel. In a practical reactor, much fissile material remains in the ashes; also, further fissile materials are created in the reactor from the fertile uranium or thorium in which the fissile uranium or plutonium is necessarily imbedded. Consequently, much reprocessing using many pass-throughs are required before all of the energy is utilized, that is, before all potential military utility is gone.

The bury option also implies the large scale transportation and manipulation of these dangerous materials with their attendant problems. Also, when disposing of the logs, the same problems of long-term safe isolation of radioactive materials from the common environment are operative as in the case of nuclear reactor wastes. Furthermore, the nuclear fission energy is still present in these logs! Careful (because of the intense radioactivity present in the logs due to the reactor ashes also mixed in with the fissile materials), but ordinary, chemical procedures can reclaim the fissile bomb material from the glass. The logs are thus still valuable to potential bomb makers and must be appropriately safeguarded over very long periods of time.

Neither of these approaches really succeeds in returning the contents of "Pandora's Nuclear Box" to their original safe configuration.

Each approach has its strongly vociferous advocates in many of the industrialized nations of the world. Their advocacies are strongly linked to their positions on nuclear power, technological optimism, nuclear weapons, internationalism or "go-it-alone-ism," militarism, environmentalism, energy demands, consumerism, and so on.

4. Enlarging the system in which the example problem is manifested

How do we choose between these two approaches? Any discussion leading to a choice has major economic, environmental, ethical, scientific, and international security components. Much of the discussion is largely emotional and dominated by fear. People do not usually come to discussions of nuclear power with minds open to rational discourse. They are either strongly "for" or "against" it! They either hunger for the cheap energy promised by nuclear power or fear its possible radioactive consequences; they either desire freedom from dependence upon fossil fuels and their associated economic dependency and environmental and health insults or are repelled by the nuclear weapons presumed to be associated with the use of nuclear power; they are either "techies" confident in the problem-solving capabilities of modern science, or are science skeptics, leery that every human problem solution brings a host of new problems.

Hence public discussion of suggested problem solutions, which have a nuclear power component, is very controversial, often dominated by the discussant's nuclear axioms—worship of scientific technology and desire for energy or suspicion of science and technology and fear of radioactivity in war and peace—rather than an overall view of the problem at hand. I seek an alternative (not necessarily better) route to decision making in which the dominant axiom is the need for international security and stability rather than the individual axiomatic views of the nuclear issue. Thus the system of discourse is enlarged from the nuclear weapon state to the world system of many such states. My approach is *via* dynamical modeling of the international system, using models in which means of pit disposal and international security are coupled. The "scientific approach" is an extension of the Richardson approach to international stability (e.g., [9]). The question then becomes: Which of the two suggested options, if either, is more congenial or more detrimental to international stability?

The coupling between means of pit disposal and international security is *via* the additions to national energy resources potentially represented by these pits. The present world inventory of weapons-grade plutonium is about 250 tons [10]. According to [8], plutonium can replace about 1/3 of the fissile uranium in current civilian nuclear reactor cores, which currently use the equivalent of 30 tons of highly enriched uranium per year ("2300 tons of highly enriched uranium would supply about seven years total demand for the world's reactors" (p. xvi)). Thus civil reactors may be able to use about 10 tons of demobilized plutonium per year in the generation of electricity at current rates (ignoring all economic constraints). The 200 tons of Pu said to become surplus during the next 20 years could thus be burned up during the next 20 years. During this period the surplus weapons plutonium could generate 1/3 of the 6% of the world's 1992 primary energy currently supplied by nuclear means ([11], p. 40). Thus the plutonium recycled from nuclear weapons could supply some 2% of the world's total energy needs in the near future, a not-insignificant amount, an amount which may very well influence national resource policy and security considerations. Certainly, the sudden 1% drop in U. S. energy use due to the Middle East "oil crisis" of 1973 ([12], p. 75) led to large fuel price increases and significant changes in national and societal behavior. It is not certain that a gradual decrease of the same magnitude would have similar consequences. Of course, substitution of weapon fissile materials for raw fissiles may have a negative impact upon the uranium mining industry, its labor, and investments. On the other hand, the growing concern with greenhouse warming of the earth's climate [13] might lead to a shift from fossil to nuclear fuels, thus increasing the need for both used and raw fissile materials.

It certainly will do so for the Russians, who forsee great national savings; according to [8], they say that 10 tons of weapons-grade plutonium, "when burnt in an open fuel cycle thermal reactor," would save them 25 billion cubic meters of natural gas, worth $2.1 billion at 1996 export prices. This saving is considerably greater than the estimated [3] billion dollars excess cost to Russia for burning rather than burying its present 50 tons of surplus plutonium.

Since time immemorial, nations have been challenging each other, even going to war, over natural resources; for example, energy, either to protect their own or to gain access to those of others.

> Choucri and North hypothesize that a growing population experiences an increasing demand for basic resources. As technology becomes more advanced, the greater will be the kinds and quantity of resources required by the society. If these demands are not met, the development of new capabilities will be sought, and if these cannot be attained within the nation's boundaries, lateral pressures will be created to attain them beyond the boundaries. Lateral pressures may be expressed through commercial activities, the building of navies and merchant marine fleets, the dispatch of troops into foreign territory, the acquisition of colonial territory or foreign markets, the establishment of military bases abroad, and in other ways. ([2], p. 327.)

These “lateral pressures” have often led to conflict and violence. A good example of this process is the expansionist activities of Japan before World War II, which, pressing upon the commercial, moral, and nationalistic sensibilities of the U. S., eventually led to war. More recent examples of resource-based conflict abound: Iraq’s designs upon Kuwait’s oil resources and attempts to protect its own contributed to its invasion, the resultant war, and its consequences; the U. S. involvement in the Middle East, as well as its “forbearance” on Nigeria’s human rights record, are certainly colored by its desire to control its oil supplies, as is its current contention with Russia and Iran over the routes to move Caspian oil to the west; Israel’s relations with the Palestinians and its Arab neighbors is exacerbated by contentions over water rights, as are the relations between Syria and Turkey; many of the bloody sub-Saharan African conflicts revolve around the control of diamond and metal resources. (And, of course, once war has started, a major military activity is the securing of resources for oneself, denying them to the enemy, e.g., Hitler’s drive to the Caucuses relieving his pressure on Moscow, Allied bombing of Romanian oil fields.)

If a nation had more of its own vital resources (if “these demands” are “met”), that would lessen its needs or desires for those of other’s, hence lessening the lateral pressure, the need for challenge to, or war with, those others. But what if enhancing one’s own resources, thus lessening the *intent* to threaten neighbors, also increased one’s own war-making capability and hence the *capability* of threatening the neighbors? (For a discussion of the relation between “intent” and “capability” in international relations see [14].) Increasing capability may also precipitate a war.

> As the challenger overtakes the erstwhile leader, its more rapid growth rate may breed an excess of self-confidence and tempt it to seek complete victory. The converse danger is that the dominant power, viewing apprehensively the expanding capabilities of its rival, may go to war to defeat the latter while it can.... ([2], p. 330.)

It is certainly true that the additional plutonium energies represent only small parts of the national energy budgets of the nations concerned, perhaps 2% at present rates. But nations have gone to war over very small insults or deprivations.

International stability can be defined, as in [15, 16], as the inability of the mathematical representation of the system to display an exponential increase in armaments: no matter what the initial arms stocks of the hostile competing member states of the system may be, the equations governing the evolution of the system only allow exponential decreases (with, perhaps, superimposed oscillations). If exponentially growing solutions are allowed, the system is unstable; policies defining such a system are to be avoided. Alternatively, in [17, 18], when making a nonlinear extension of Richardson's ideas, I defined stability as the inability of the mathematical model, representing the *nonlinear* world system, to have a chaotic evolution. If some input parameters lead to mathematical chaos, the underlying world system will display crisis instability; the policies represented by such parameters should be avoided. (For earlier recognition of the evidence for chaos in world affairs, see [19, 20].) Europe before World War I was unstable; the assassination of a single political figure led to incommensurate results—the death of many millions. Conversely, Europe after World War II was apparently stable: many "insults," such as the Berlin blockade or the downing of the Korean airliner, did not precipitate a "hot war."

In a previous discussion of the burn *versus* bury issue [18, 21], I used a Richardson model in which the additional energy which becomes available from burning the cores of former nuclear weapons is incorporated into the "suspicion" term of the usual Richardson equations. The result was decreased instability under certain circumstances. The results were then confirmed by examining a nonlinear extension of the Richardson equations for regions of chaotic instability. In nonlinear models, very small changes in parameter values, as would be represented by the civilian energy contribution of the recycled nuclear weapon pits, may have major consequences.

This paper presents an alternate model in which the arms procurement response terms of the linear Richardson equations (the "reaction" term describing one nation's response to the arms acquisitions of it's opponent) are constrained, *more realistically*, by economic prosperity: you cannot buy arms you cannot afford (over the long run). The prosperity, in turn, is governed by energy availability, which brings in the burn *versus* bury dichotomy. Energy availability is also governed by competition for external energy resources, which competition also influences the mutual suspicion term (the "grievance," term which governs the arms responses of the competing nations to each other independently of their actual arms stocks and acquisitions). Richardson's stability criterion is then used to ascertain whether to burn or to bury is the preferable (more international stability inducing) national policy within the model of this paper. Of course, different models and decision paradigms (e.g., game theory or economics), which should also be explored, may lead to different policy suggestions. It then will become necessary to consider "super models" incorporating all of these paradigms in order to practice rational policy making.

5. Basic political assumptions and their possible validation

A number of fundamental assumptions are required to create a model which can address the problem in question. Most of them stem from anecdotal/historical evidence. It is clear that much more research would be required to put the evidence for these assumptions in numerical form. Thus the following assumptions are at best heuristic; if the results of the model they support prove sufficiently interesting to political scientists and practitioners, perhaps these assumptions can be quantitatively tested.

1. In [15, 16] Richardson assumed that it is the international *system*, not its constituent members, that causes wars. Conflict is the result of the natural value-neutral responses of each nation to the natural actions of others, not to the evil intent of any one member of the system. Thus war can be forced upon nations against their natural inclinations. This assumption is supported, for example, by Tuchman's account of the origin of World War I. It is obviously not valid for all wars, for example, Hitler apparently desired and forced World War II (which, however, can be viewed as just an extension of World War I).

Richardson postulated that nations acquired arms in reaction to the arms of their fellow nations; they reduced their arms because of the "fatigue" of supporting large armories (the larger the arms burden, the more the fatigue). All nations had attitudes towards each other of benevolence or grievance which also contributed towards decrease or increase in their military commitments. In the simplest two-nation linear system, which we adopt in this paper for initial model building (it is trivial to extend the model to N nations, not trivial to obtain "reasonable" model parameters), the nations will be referred to as "A" and "B."

Then, the simple, *linear*, Richardson model can be written as

$$\begin{aligned} \dot{x}_A &= a_A x_B - b_A x_A - r_{AB} \\ \dot{x}_B &= a_B x_A - b_B x_B - r_{BA}. \end{aligned} \tag{RE}$$

Here the x represent the commitment of each nation to armaments; they might be measured in the dollar value of the total military burden (e.g., personnel, facilities, and weapons) or; as I prefer, to make the variables dimensionless, as the ratios of these military burdens to the nation's gross national product (GNP). (The theory only makes sense if the x are nonnegative variables; if the model's time evolution leads to negative values, the model is no longer useful.) The dot over the variables $\dot{x}$ on the left-hand side of equation (RE), signifies their time derivatives, that is, the rate (per annum) at which they acquire (positive derivative) or retire (negative) military assets. The a terms are the reaction or response terms. They are usually taken to be positive, meaning—everything else being equal—the more military strength your fellow nation (in the *competitive system*) is perceived to have, the more rapidly you will try to build up your own. The fatigue coefficients b, if positive, imply that the more arms you have, the less is your need to acquire more. (Negative values of the b parameters would lead them to be designated as "military domestic dominance" coefficients; the stronger the military component of the society, the more inclination and power it has to further enhance its strength.) The r terms, if negative, are grievance coefficients: if an r is negative, the nation in question is sufficiently suspicious of its fellow nation that it will endeavor to acquire arms even if it perceives its fellow to have none. Positive values of r represent benevolence and lead to arms reductions in the absence of opposing armaments.

The linear Richardson equation (RE) is easily solved (c.f., [1, 21]); the time variation of the solutions are either exponential growth or decay, superimposed upon oscillations, depending upon the relative values of the reaction and fatigue coefficients. (The grievance parameters do not appear in the expression for the exponents which determine the time evolution!) Richardson assumed a peaceful system required exponential decays for all x; any possibility of exponential growth of armaments signified the possibility of war. There is a large body of literature exploring the validity of the Richardson model; see [9, 22] for some citations.

The parameters a, b, and r represent national characteristics which, we know from common experience, can vary with time (e.g., r for France and Germany once was very negative, now it seems to be quite positive). Richardson assumes that the time rate of change of these parameters occurs over a much longer time scale than is characteristic of the solutions of the equation. (Thus they can be treated as constants while solving the equations.) This assumption is based upon the common perception that national attitudes change much more slowly than do economic or military strength: capability changes faster than intent. The main point of the richardsonian approach is that system dynamics may transcend individual intents.

2. The hostility between member states of the world system, symbolized by the Richardson grievance coefficient, is influenced by their competition over resources such as energy. Thus if an attempt is to be made to model the time dependence of the grievance term, it is not unreasonable to posit a dependence of the term upon competition for energy. Some anecdotal evidence for this assumption has already been given but more numerical evidence, for example, the relation between the fraction of normal resources challenged and the changes in public opinion and/or the probability of war, would obviously be welcome.

Observation of the current world also indicates a jealous suspicion of the non- and lesser-nuclear states towards the more powerful nuclear weapon states. This suggests an assumption that the grievance terms also depend upon the rate of nuclear arms change of the competing nations. Again, more numerical evidence for this might be obtained from public opinion polls of attitude.

3. Arms acquisition cannot depend only upon perception of the opponent's arms stocks. Even if the will to acquire arms is present, the economic means also have to be present. (As partial evidence for this statement, consider the current paucity of arms acquisition by the relatively impoverished Russian government compared with that of the prosperous U. S. Both nations are equally suspicious of each other!) Hence the reaction terms of the Richardson equations should, more realistically, also depend upon the prosperity of the respective competing nations.

4. National prosperity depends upon the availability of resources (among other important factors). Energy is a very important resource; in fact a surplus of energy can make up for the shortages of other important resources (e.g., the energy-costly desalination of ocean or brackish waters when fresh water supplies need augmentation, as in Saudi Arabia). Evidence for the relation between prosperity and energy supplies is implicit in the decline of U. S. GNP during the 1973 Arab oil embargo [12]. Thus energy availability will be included in modeling prosperity which, in turn, is included in modeling national attitudes, that is, the grievance factor.

5. Success in the international competition for resources is usually assumed to depend upon the relative economic, diplomatic, or military strengths of the nation. These strengths, in turn (really not independent of each other), are assumed to stem from national prosperity. Thus a working hypothesis for this paper is that national competitive success increases with increasing national prosperity. Again, this must remain a heuristic assumption until quantitative research supports or refutes it.

6. In the simple model of this paper it is assumed that the nuclear energy which becomes available for civil use as a result of nuclear disarmament is proportional to the rate of nuclear disarmament which, in turn, is proportional to the rate of total disarmament $\dot{x}$. This is the simplest hypothesis which enables us to deal with the burn *versus* bury controversy under the rubric of prosperity and security. Since such civil use is not yet common, there is no evidence to refute or support this assumption. Again, the investigation of its validity probably has to follow, rather than (as we would prefer) precede its heuristic use in model building.

7. In this paper all parameters and variables are taken to be dimensionless. This means they are all scaled relative to some appropriate bases. For example, the x variables represent the ratio of national military dedication to GNP. The results of this paper are purely qualitative and hence further descriptions of the base values are not needed. When, and if, the results of the qualitative model become sufficiently interesting to suggest quantitative use, further—difficult—specification of the bases and the actual parameter values (for given nations and epoch) would be required.

6. The Richardson model, prosperity, and energy dependence

I have modified the original two-competing-nation (designated by A and B) form of the Richardson equation (RE), which relate the rates of arms acquisitions of the two (the time derivatives of the arms stocks, $\dot{x}_A$ and $\dot{x}_B$) to the existing arms stocks of their opponents and themselves, to include some economics (cf., [23]). Nation A cannot respond to the arms stocks of hostile nation B, by acquiring further arms of its own, unless it has some surplus of resources beyond that necessary to minimally sustain its population. The other-nation-reaction-response terms a now incorporate *prosperity factors p*. The fatigue-self-response-terms, now written as β, instead of b, require no modification as they usually are positive, implying arms build-down, which is assumed to be cost free at the margin. Thus equation (RE) can be rewritten as:

$$\begin{aligned} \dot{x}_A &= \alpha_A p_A^2 x_B - \beta_A x_A - r_{AB} \\ \dot{x}_B &= \alpha_B p_B^2 x_A - \beta_B x_B - r_{BA} \end{aligned} \tag{1}$$

where the p appear quadratically in equation (RE) (i.e., $a_A = \alpha_A p_A^2$) to approximate the reality of a cutoff, that is, no armaments may be acquired if p is below some minimum value required to sustain the population. If the p dependence were linear, arms could still be purchased with close to null prosperity (see Figure 1).

The Richardson grievance terms r in equation (RE) represent the relationship between the two nations [24, 25]: the fundamental attitudes of one towards the other, independent of their state of armed watchfulness. If the r are positive, the two parties are basically friendly towards each other and would tend to reduce arms, everything else being okay; negative r imply an inherent hostility and a corresponding arms buildup.

It seems reasonable to assert that the rate at which national prosperity changes depends upon the amount of total productive energy available to the nation, E. For simplicity, a linear relation is assumed for the

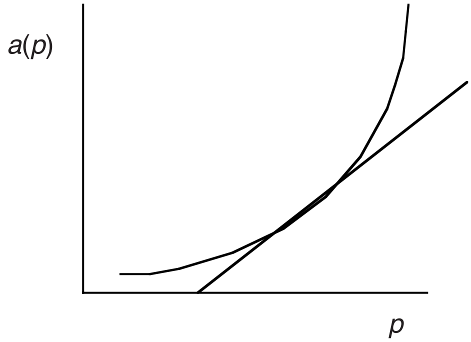

Figure 1.

prosperity time derivatives:

$$\dot{p}_A = q_A^0 + q_A E_A$$
$$\dot{p}_B = q_B^0 + q_B E_B \tag{2}$$

where q_A and q_B are positive rate constants. Each nation may have civilian energy available domestically, denoted by E_A and E_B. They will also import civilian energies E_{AC} and E_{BC}, where C denotes the rest of the world, external to A and B. It is assumed that each nation may also have recourse to additional nuclear energy from surplus nuclear pits. This amount is assumed proportional to the rates of disarmament (this is the burn *versus* bury term):

$$-\rho_A \dot{x}_A \qquad \text{and} \qquad -\rho_B \dot{x}_B \tag{3}$$

where ρ is a positive constant for the burn option. If the time derivatives in equation (3) are negative (decreasing arms stocks means demobilization of nuclear weapons), these represent positive contributions to the national energy supplies in the burn option. In the bury option this term is either not present, or appears with a negative ρ coefficient to represent the nonproductive energy costs of burial. If armaments are increasing (positive derivatives) there is an energy cost to the nation for fabricating new nuclear weapons, energy which is removed from the civilian-prosperity-producing total. (The coefficient ρ may be numerically different in this case but will be positive; there is no burn *versus* bury problem when nuclear weapon stocks are increasing.) Thus the total productive energy available to each nation is written as:

$$E_{AT} = E_A + E_{AC} - \rho_A \dot{x}_A$$
$$E_{BT} = E_B + E_{BC} - \rho_B \dot{x}_B. \tag{4}$$

It is assumed that energy adds to prosperity whether used or traded. Hence, if some of E_A or $\rho_A \dot{x}_A$ is traded to B, or *vice versa*, this would not decrease E_{AT} in equation (2) though it might increase E_{BT}. This possibility of trade is not included in this model, though it should be fairly simple to do so.

The total energy available to A and B in the external world E_C is not infinite. Thus there is a competition between the two for imported energy:

$$E_C = E_{AC} + E_{BC}. \tag{5}$$

Again a simplifying assumption is made: the two competing nations, A and B, are the only significant competitors for the world's energy resources. Eventually, more realistic models will have to include the rest of the world, either collectively (a 3-body model), or individually (an N-body, $N > 3$ model). In this competition, success is assumed to depend both upon relative prosperity p and upon relative military power x; the more of each you have, the more imported energy you may acquire from the limited world stocks. Making the simplest, linear assumption for this energy rivalry results in:

$$E_{AC} = E_C(x_A + p_A)/(x_A + x_B + p_A + p_B)$$
$$E_{BC} = E_C(x_B + p_B)/(x_A + x_B + p_A + p_B). \quad (6)$$

Thus, from equation (2), the prosperity time derivative of each nation is assumed to be given by:

$$\dot{p}_A = q_A^0 + q_A\{E_A - \rho_A \dot{x}_A + E_C(x_A + p_A)/(x_A + x_B + p_A + p_B)\}$$
$$\dot{p}_B = q_B^0 + q_B\{E_B - \rho_B \dot{x}_B + E_C(x_B + p_B)/(x_A + x_B + p_A + p_B)\}. \quad (7)$$

Consequently the prosperity of each nation has a nonlinear dependence on the prosperity of both (since they appear in the denominators as well as the numerators) and on both arms stocks, as well as a linear dependence on their rates of arms acquisition.

Only the direct dependence of prosperity on available energy has been considered so far. But variation in prosperity is also directly related to existing prosperity, to the existing stocks of arms, and to the relationship between the contending nations (cf., [14, 23]). These are taken into account in the following simplistic manner:

$$q_A^0 = \eta_A^0 p_A + \eta_A r_{AB} p_B + \sigma_{AA} x_A + \sigma_{AB} x_B$$
$$q_B^0 = \eta_B^0 p_B + \eta_B r_{BA} p_A + \sigma_{BB} x_B + \sigma_{BA} x_A. \quad (8)$$

The signs of the η coefficients are expected to be positive: usually prosperity produces further prosperity as does trade with a prosperous partner, facilitated by a friendly relationship. Similarly, a negative relation with a prosperous partner, diminishing the possibilities of profitable trade, would tend to decrease prosperity. The sign of the σ coefficients are not obvious to me; large military stocks may enhance or detract from national prosperity. The result of these assumptions, inserting equation (8) into equation (7), is a pair of nonlinear, first-order differential equations for the time variation of prosperity, depending upon existing prosperity, trade propensity, national armories, and competition for energy.

To this point, the relationships (grievance or benevolence) between the two nations has been considered as fixed givens. But it is plausible that the competition over energy between the two and the rate of arms buildup/down should influence the relations between the two. Thus, instead of keeping the r parameters fixed, assume that they change, increasing (becoming more friendly) as the opponent's arms decrease:

$$\dot{r}_{AB} \cong -\dot{x}_B, \qquad \dot{r}_{BA} \cong -\dot{x}_A \quad (9)$$

and decreasing as the opponent acquires a greater fraction of the externally available energy:

$$\dot{r}_{AB} \cong -(E_{BC} - E_{AC}), \qquad \dot{r}_{BA} \cong -(E_{AC} - E_{BC}). \quad (10)$$

Introducing some new *positive* proportionality parameters γ to incorporate the assumptions in equations (9) and (10), and using equation (6) in equation (10), first-order differential equations for the time evolution of the mutual relationships are obtained:

$$\dot{r}_{AB} = r^0_{AB} - \gamma_A \dot{x}_B - \tilde{\gamma}_A E_C (x_B - x_A + p_B - p_A)/(x_A + x_B + p_A + p_B)$$
$$\dot{r}_{BA} = r^0_{BA} - \gamma_B \dot{x}_A - \tilde{\gamma}_B E_C (x_A - x_B + p_A - p_B)/(x_A + x_B + p_A + p_B) \quad (11)$$

where the r^0 are constants. These equations represent the postulated dependence of international attitudes, r (grievance or benevolence), on their arms stocks, armament rates, prosperities, and their competition for external world energy supplies.

7. Some conclusions from the model

The three sets of coupled equations (equations (1), (7), (8), and (11)), are too complicated to be solved analytically and have too many unknown parameters to be solved numerically in general. Some limiting assumptions must be made in order to obtain some useful, general results. Note that if

$$\tilde{\gamma}_A = \tilde{\gamma}_B = \tilde{\gamma}, \qquad \gamma_A = \gamma_B = \gamma \quad (12)$$

that is, if there is symmetry between the way the two nations vary their relationship with respect to energy and arms competition, the energy competition affects both parties the same way. Hence, adding together the two equations of equation (11), the energy competition drops out:

$$\dot{r}_{AB} + \dot{r}_{BA} = r^0_{AB} + r^0_{BA} - \gamma_A \dot{x}_B - \gamma_B \dot{x}_A \quad (13)$$

which can be simply integrated, leading to:

$$r_{AB} + r_{BA} + \gamma(x_A + x_B) = R + (r^0_{AB} + r^0_{BA})t \quad (14)$$

where t is the time variable and R is a constant of integration (equal to the initial net mutual attitude in the absence of arms stocks).

This equation is simply interpreted (remember, the γ are positive): at a fixed time t, as the total arms stocks in the two-nation system goes up, the total relationship between the two goes down (becomes less friendly). This result is in accord with the expectations of common sense, hence increasing the credibility of the assumptions going into this result.

A further result is that, if the system arms stocks are kept constant, and the net initial relationship rate $r^0_{AB} + r^0_{BA} = R^0$ is positive, the total relationship increases (becomes more friendly) as time goes on. If the initial net relationship rate is negative, it gets worse with the passage of time. I believe this to be a new, interesting, "reasonable" (and perhaps testable) result: with no change in system arms, no matter how big the arms stocks are to begin, the two, initially friendly, parties grow used to each others arms and so their friendship increases. If they are initially unfriendly, the presence of arms worsens the relationship between them with the passage of time.

Adding the two equations of equation (1), the following is obtained:

$$\dot{x}_A + \dot{x}_B = \alpha_A p_A^2 x_B + \alpha_B p_B^2 x_A - \beta_A x_A - \beta_B x_B - r_{AB} - r_{BA}. \tag{15}$$

Substituting equation (14) into equation (15), the result is:

$$\dot{x}_A + \dot{x}_B = \alpha_A p_A^2 x_B + \alpha_B p_B^2 x_A - \beta_A x_A - \beta_B x_B + \gamma(x_A + x_B) - R - R^0 t. \tag{16}$$

If the two prosperities p_A and p_B are constants, this is a simple, linear differential equation which implies *no chaos*! The solutions will just display simple linear and exponential growths and/or declines. Thus the possibility of chaos (implying crisis instability in the Saperstein paradigm), in this particular model of a nuclear arms race, depends upon time variations in national prosperity.

Since national prosperity is so important for international stability, its variation is now examined, assuming that the two nations have similar responses, that is:

$$q_A = q_B = Q; \quad \eta_A^0 = \eta_B^0 = \eta^0, \eta_A = \eta_B = \eta; \quad \sigma_{AA} = \sigma_{BB} = \sigma$$
$$\sigma_{AB} = \sigma_{BA} = \overline{\sigma}; \quad \rho_A = \rho_B = \rho; \quad \alpha_A = \alpha_B = \alpha; \quad \beta_A = \beta_B = \beta. \tag{17}$$

Adding the pair of differential equations following from equations (7) and (8) results in:

$$\dot{p}_A + \dot{p}_B = \eta^0(p_A + p_B) + \eta(r_{AB}\rho_B + r_{BA}\rho_A) + (\sigma + \overline{\sigma})(x_A + x_B) + Q[E_A + E_E + E_C - \rho(\dot{x}_A + \dot{x}_B)]. \tag{18}$$

Note that $E_A + E_B + E_C = E_T$ is the total (finite) energy of the world system. Now assume that the relationship between the two nations is reciprocal: $r_{AB} = r_{BA} = r$. Also define their average prosperity $2p = p_A + p_B$ and their average arms commitment $2x = x_A + x_B$. Equations (14), (16), and (18) become the fundamental sets:

$$\dot{p} = (\eta^0 + \eta r)p + \hat{\sigma}x + Q[E_T/2 - \rho\dot{x}] \qquad \text{(from 18)} \tag{19}$$

$$2\dot{x} = \alpha[p_A^2 x_B + p_B^2 x_A] - 2\hat{\beta}x - R - R^0 t \qquad \text{(from 16)} \tag{20}$$

$$2r + 2\gamma x = R + R^0 t \qquad \text{(from 14)} \tag{21}$$

where $\hat{\beta} = \beta - \gamma$, $\hat{\sigma} = \sigma + \overline{\sigma}$, $2p = p_A + p_B$, $2x = x_A + x_B$, $r = r_{AB} + r_{BA}$, $R^0 = r_{AB}^0 + r_{BA}^0$.

8. Sufficient, not necessary, conditions for richardsonian stability

As indicated following equation (16), the constancy of both p_A and p_B is sufficient for the absence of chaos, that is, for stability in the Saperstein paradigm. Now *assume* this stability and see what the additional requirement of richardsonian stability (RS) implies. Substituting equation (21) for r into equation (19) and forcing the time derivative of p to vanish, results in a simple first-order, linear differential equation for x:

$$0 = (\eta^0 + \eta[(R + R^0 t)/2 - \gamma x])p + \hat{\sigma} x + Q[E_\mathrm{T}/2 - \rho \dot{x}]. \tag{22}$$

This is an equation of the form

$$a\dot{x} + bx + ct + d = 0 \tag{23}$$

whose general solution, with one arbitrary constant D, determined by initial conditions, is

$$x = De^{-(b/a)t} - (c/b)t + (ac - bd)/b^2. \tag{24}$$

Everything depends upon the sign of b/a; if it is positive, the system is RS; if negative, it is Richardson unstable—there will be a run-away (exponentially increasing) arms race. Comparing equation (22) with equation (23), the system will be RS if

$$b/a = (\eta\gamma p - \hat{\sigma})/(Q\rho) > 0 \tag{25}$$

which implies, since Q is positive, that

$$\eta\gamma p > \hat{\sigma} \text{ for burn, } \rho > 0 \tag{26}$$

$$\eta\gamma p < \hat{\sigma} \text{ for bury, } \rho < 0. \tag{27}$$

If $\rho = 0$, marginally possible in the burial scenario, the only allowed solution is $x =$ constant, r proportional to t. This case is ignored.

Remember that the σ terms represent the dependence of prosperity upon the supply of arms; small numerical values imply a strongly civilian based economy, in contrast to an "iron-triangle" dominated society. If the net value of $\hat{\sigma}$ is negative, equation (26) is automatically fulfilled since η, γ, and p are each necessarily positive. The coefficient η represents the coupling between prosperity, international relationship, and the variation of prosperity, should it occur; a large numerical value is expected to be characteristic of a healthy international economic system. Finally γ represents the relation between arms acquisition rate and variation in relationship; a large numerical value implies that relationships are very sensitive to arms procurements.

Thus, from equation (26), an economically healthy international system, in which the constituent societies are prosperous, civilian based, and sensitive to each other, will be RS if the burn option is chosen, Richardson unstable if the option is for burial of the surplus pits. On the other hand, from equation (27), a system of poor, iron-triangle states, very insensitive to each other, in which prosperity is minimally coupled to international relationship, will be RS if the burial option is chosen, unstable if the surplus pits are burned. If $\hat{\sigma}$ is negative, implying that the presence of arms stocks detracts from the growth of prosperity, only the burn scenario can be RS. Note that all of these results are independent of whether the initial relations between the competing states are positive or negative!

9. Conclusions

The fundamental premise of this paper is that nations, seemingly extremely complex entities, describable, if at all, only by the use of countless variables, can actually be productively described, in-so-far as their international security behavior is concerned, with the use of very few dynamical variables. These variables evolve according to relatively simple mathematical laws describing the different possible interactions such as economic, military, cultural, and so forth between the nations. Each possible choice of variables and initial conditions, and of the equations relating them, represents a choice of "science" and "model." The dynamical laws are deterministic, as are the resultantly evolving variables, but they may, or may not, be predictive. Thus, different forms of stability can be ascribed to the different models. One possible stability, employed by Richardson, is the prohibition of exponential growth of armaments in the system. Another choice, possible only in nonlinear systems (necessary to realistically describe any real-world system), is the prohibition of the possibility of chaotic evolution of the system variables. In either case, these mathematical stabilities are assumed to represent the absence of "crisis instability" in the real-world system, an absence taken to be the goal of desirable international security policies. Given a model, the model parameters are varied until the desired model stability is attained. This choice of parameters is representative of a choice between policy options.

In this paper, the model is a Richardson arms competition model in which arms procurement depends upon national prosperity which, in turn, depends upon energy available to the nation. The availability of energy itself depends upon international competition for finite resources. Thus there is a feedback loop: armaments require energy which engenders competition which *requires* armaments. The question then becomes: is the feedback positive, leading to growing hostility and/or system unpredictability; or is there negative feedback in the loop, which means that the hostile competition remains stable? The examination of both forms of instability is used as a decision tool for the major international problem of what to do with surplus nuclear weapons.

The relatively simple energy competition model of this paper seems to give reasonable results outside of the burn *versus* bury arena: the relation between total arms stocks and interstate relationship and the "getting used to," or getting increasingly "annoyed" by, large armories if they are not changing, depending upon whether the initial interstate relation is friendly or hostile. These common sense results give some credence to the model. The model then seems capable of giving definite answers to the burn *versus* bury policy question, under fairly realistic international conditions, whereas common sense seems to get bogged down in popular emotions, and is thus seemingly unable to provide useful answers.

It thus seems reasonable to conclude that any future useful theory of international relations and security will have to be a dynamical science, incorporating possible time evolutions of the system in response to policy choices. Furthermore, the concepts of Richardson and/or stability against chaotic evolution will most likely be important aspects of that science. It does not necessarily follow that the future political science of international security will have to be mathematically expressed. Even a purely verbal science, if it is to be useful, will have to incorporate aspects of system evolution and system stability. It seems to me that inclusion of these concepts is most easily, transparently, usefully, and verifiably done using a mathematical language.

Acknowledgment

The author is grateful for the careful scrutiny given to his manuscript by the reviewer and for the many helpful suggestions resulting from this care and diligence.

References

[1] Alvin M. Saperstein, "Plutonium—Burn It or Bury It? Using the Richardson Model, and its Non-Linear Extensions, as a 'Decision Tool' in Helping to Choose Between the Use and Disposal of Surplus Nuclear Weapons Materials," *Peace Economics, Peace Science and Public Policy*, **5** (4) (Fall, 1999); http://www.crp.cornell.edu/peps.

[2] James Dougherty and Robert Pfaltzgraff, Jr., *Contending Theories of International Relation: A Comprehensive Survey*, fourth edition (Longman, New York, 1997). Discussing the works of Nazli Choucri and Robert C. North, for example, *Nations in Conflict: National Growth and International Violence* (W. H. Freeman, San Francisco, 1975), and of A. F. K. Organski, for example, *World Politics* (Knoppf, New York, 1958), and subsequent works by both.

[3] Allison Macfarlane and Adam Bernstein, "Canning Plutonium," *Bulletin of the Atomic Scientists*, May/June (1999) 66–69.

[4] Arjun Makhijani, "Technical Aspects of the Use of Weapons Plutonium as Reactor Fuel," http://www.ieer.org/sdafiles/vol_5/5-4/moxmain4.html.

[5] M. Miller and F. von Hippel, "Let's Reprocess the MOX Plan," *Bulletin of the Atomic Scientists*, July/August (1997) 15–17.

[6] NAS Panel to Review the Spent-Fuel Standard for Disposition of Excess Weapons Plutonium 1999, interim report, http://www4.nas.edu/oia/oiahome.nsf/b32612cf990294e852562fc005d1-afa/f8b2bd32c740f3a8852564b900718810?OpenDocument.

[7] OSTP 1997, "Final Report of the U. S.–Russian Independent Scientific Commission on Disposition of Excess Weapons Plutonium," http://www.whitehouse.gov/OSTP.html.

[8] William J. Weide, *Regaining Security—A Guide to the Costs of Disposing of Plutonium and Highly Enriched Uranium* (Ashgate, Aldershot, U. K., 1997).

[9] G. D. Hess, "An Introduction to Lewis Fry Richardson and his Mathematical Theory of War and Peace," *Conflict Management and Peace Science*, **14** (1) (1995) 77–113.

[10] Robert S. Norris and William M. Arkin, "World Plutonium Inventories," *Bulletin of the Atomic Scientists*, September/October (1999) 71.

[11] Christopher Flavin and Nicholas Lenssen, *Power Surge: Guide to the Coming Energy Revolution* (Norton, New York, 1994).

[12] Gordon Aubrecht, *Energy*, second edition (Prentice Hall, Upper Saddle River, NJ, 1995).

[13] Steve Fetter, "Energy 2050," *Bulletin of the Atomic Scientists*, July/August, (2000) 28–38.

[14] Alvin M. Saperstein, "Mathematical Modelling of the Effects of Capability and Intent on the Stability of a Competitive International System," *Synthese*, **100** (3) (1994) 359–378.

[15] L. F. Richardson, *Arms and Insecurity* (Boxwood, Pittsburgh, PA, 1960).

[16] L. F. Richardson, *Statistics of Deadly Quarrels* (Boxwood, Pittsburgh, PA, 1960).

[17] Alvin M. Saperstein, "Chaos—A Model for the Outbreak of War," *Nature*, **300** (1984) 303–305.

[18] Alvin M. Saperstein, *Dynamical Modeling of the Onset of War* (World Scientific, Singapore, 1999).

[19] Barbara Tuchman, *The Guns of August* (Dell, New York, 1962).

[20] Alan D. Beyerchen, "Clausewitz, Nonlinearity, and the Importance of Imagry," in *Complexity, Global Politics, and National Security*, edited by David S. Alberts and Thomas J. Czerwinski (National Defense University, Washington, D. C., 1997).

[21] Alvin M. Saperstein, "Luddites vs. Panglossians: Using Richardson to Decide Whether Nuclear Capabilities Enhance or Detract from International Stability," paper presented at the *Peace Science/ECAAR Congress*, December 16–18, 1997, Centre for Applied Economic Research, University of New South Wales, Sydney, Australia.

[22] Thomas L. Saaty, *Mathematical Models of Arms Control* (Wiley, New York, 1968).

[23] Alvin M. Saperstein, "Chaos as a Tool for Exploring Questions of International Security," *Conflict Management and Peace Science*, **13** (2) (1994) 149–177.

[24] Robert G. Muncaster and Dina A. Zinnes, "Structure and Hostility in International Systems," *Journal of Theoretical Politics*, **2** (1) (1990) 31–58.

[25] S. C. Lee, R. G. Muncaster, and D. A. Zinnes, " 'The Friend of my Enemy is my Enemy;' Modeling Triadic Internation Relationships," *Synthese*, **100** (1994) 333–358.