Circulation Transport Phenomena Involving the Interaction between Arterial and Venous Vessel Systems Based on a Simulation Using Fractal Properties

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The blood vessel system has attracted attention as a typical biological complex adaptive system. In the present study, we investigate the properties of blood flow in terms of the fractal structure of retinal blood vessel systems, which are modeled by diffusion-limited aggregation (DLA). It is demonstrated from the study that there is a difference in the properties of blood flows between the arterial and venous vessel systems, provided that they have different fractal properties of structure. This result suggests that the difference in fractal structures between the arterial and venous vessel systems may provide a clue to gain insight into the transport phenomenon of the blood flow.

1. Introduction

The blood vessel system is a typical complex adaptive system of a living body [1]. Fractal dimensions of structures in blood vessel systems of higher animals have been observed [2–8]. In recent years, it has been reported that the fractal dimension of blood vessel systems varies with scale [7, 8]. Matsuo *et al.*, measured fractal dimensions for the patterns of vessels in the cat brain and in human retina, claiming that the vascular patterns appear to be fractals characterized by two fractal dimensions on different scales [2]. It is expected that the fractal properties of the blood vessel system are closely related to a function in the system. However,

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Figure 1. A picture of a typical retinal blood vessel system.

there have been few studies on the possible correlation between the structural and functional characteristics of blood vessel systems.

In an earlier paper [9], we evaluated the fractal dimension as structural properties of generative cell growth simulated with hormonal proliferation algorithms. In those investigations, fractal properties of generative proliferation organisms are characterized. However, it is not yet understood what role the fractal property can play when describing the transport phenomena of blood flow in living organisms.

Recently, it has been pointed out by Meakin that a nonlinear growth model constructing a fractal pattern might provide new insight into the underlying formation processes of certain biological patterns [10]. We use a nonlinear growth model that enables us to produce blood vessel patterns built by diffusion-limited aggregation (DLA).

Therefore, in the following, we investigate the relationship between properties of blood flow and fractal structures of retinal blood vessel systems based on a blood vessel system model built by DLA.

2. Materials and methods

We consider retinal blood vessels as a system having fractal structure. Figure 1 shows an image of a typical retinal blood vessel system [2]. Fractal dimensions for the arterial and venous blood vessels in the image are found to be 1.61 and 1.65, respectively according to calculation by the box-counting method. Retinal arterial and venous blood vessel patterns have different structures that can be described by fractal dimension. They are known to be very similar to those obtained from a simulation by the two-dimensional DLA [11]. In order to examine blood vessel patterns with various fractal dimensions, we use a DLA model to produce the fractal patterns for retinal blood vessels with various parameters controlling fractal dimensions. DLA-based simulations produce patterns whose random growth process is governed by Laplace's equation and appropriate boundary conditions. The DLA algorithm is described as follows.

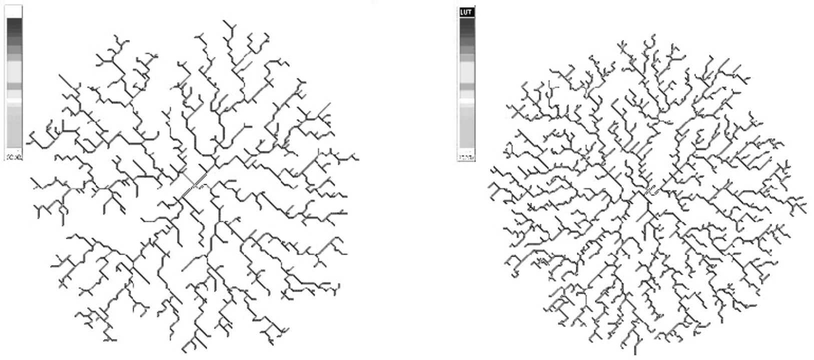
1. Put a seed particle on the origin $(x_0,\ y_0)$ of the two-dimensional lattice.
2. Put another particle on a site far from the origin and make the particle do a random walk.
3. If the particle comes to the site $(x,\ y)$ in such a way that $|x-x_0| = |y-y_0| = ds$, put the particle on $(x,\ y)$ and connect the two particles located at $(x,\ y)$ and $(x_0,\ y_0)$ with a straight line. Here ds is a distance between particles composing DLA clusters.
4. Repeat step 2.
5. If the particle comes to the site (x, y) in such a way that $|x-x_c| = |y-y_c| = ds$ where (x_c, y_c) is the site occupied by a particle composing cluster, put the particle on (x, y) and connect the two particles located at (x, y) and (x_c, y_c) with a straight line.
6. Repeat steps 4 and 5.

The pattern generated according to the algorithm shows branching structures similar to those of retinal blood vessel systems (Figure 2). Note that the distance of particles ds controls the fractal dimensions of structures. Figure 3 shows the relationship between the fractal dimension and ds, which tells us that the fractal dimension decreases monotonously with increasing ds. When $ds = 6$, the DLA pattern has a structure with fractal dimension similar to venous blood vessels, 1.65 (Figure 2(b)). On the other hand, the DLA pattern generated with $ds = 8$ has a structure with a fractal dimension of 1.61, similar to arterial blood vessels (Figure 2(a)).

Blood flows in retinal blood vessels modeled by DLA are assumed to be determined by the branching procedure: flow in a parent branch (main branch) is divided into flows in daughter branches (subbranches) at equal weight (Figure 4). We calculate blood flows in retinal blood vessel patterns modeled by DLA with various fractal dimensions according to the branching procedure. In the simulation, the magnitude of flow at the center of the pattern is set to 1.

3. Results

Figure 5 shows the histograms of blood flows for the DLA-modeled blood vessel patterns. The flows are normalized to the mean flow. Note that the x-axis is a logarithmic scale. Figures 5(a) and (b) correspond



(a) (b)

Figure 2. Retinal blood vessel patterns modeled by DLA. The pictures represent changes in blood flow by changes in gray level. (a) Pattern generated with $ds = 8$ corresponding to arterial blood vessel with fractal dimension 1.61. (b) Pattern generated with $ds = 6$ corresponding to venous blood vessels with fractal dimension 1.65.

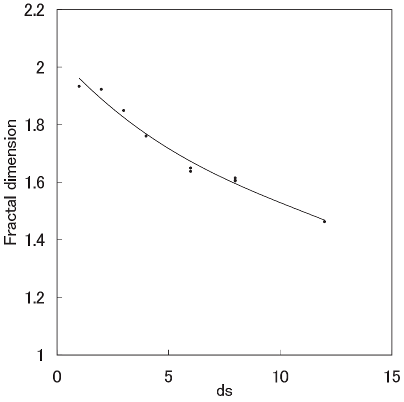


Figure 3. Relationship between fractal dimension and a distance *ds* between particles composing DLA clusters.

to the arterial and venous vessel patterns shown in Figures 2(a) and (b), respectively. The histogram of blood flows for blood vessel patterns with a fractal dimension of 1.85 is also shown in Figure 5(c). It is noted that the flows in the arterial system are seen over a range of seven digits while those of the venous system have a range of eight digits. On the other hand, the flows for blood vessel patterns with a fractal dimension of 1.85 stretch over a much wider range of 15 digits.

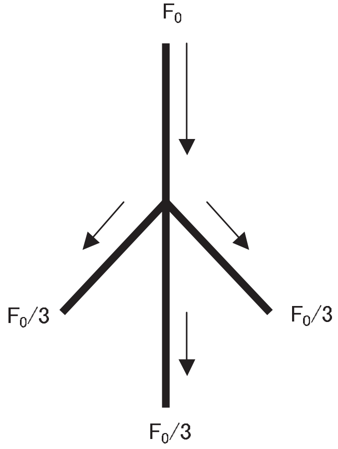
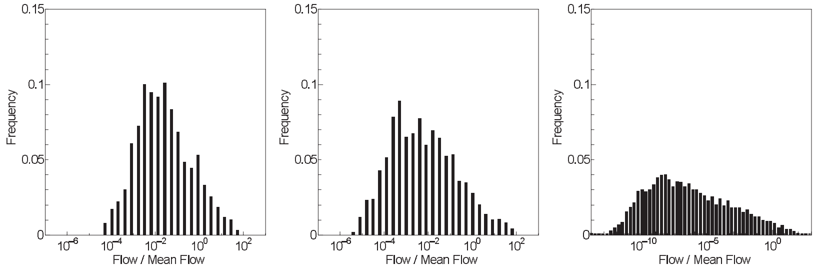


Figure 4. Branching procedure for determining flow in the blood vessel system. Flow in a parent branch is divided into flows in daughter branches at equal weight.



(a) (b) (c)

Figure 5. Distribution of flow in blood vessel patterns modeled by DLA. Flows are normalized to mean flow. The x-axis is a logarithmic scale. (a) Arterial blood vessel pattern, (b) venous blood vessel pattern, and (c) blood vessel pattern with a fractal dimension of 1.85.

Figure 6 shows the mean flow as a function of distance from the center of the pattern. It can be seen in the figure that the mean flows for the blood vessel patterns decrease exponentially with increasing distance: if the mean flow is F, with distance r, and decay constant λ, this is expressed as $F = e^{-\lambda r}$. The decay constant is obtained by the slope of the straight line on the semi-log graph. We see that the venous system has a larger decay constant than the arterial system. It is interesting to note that the blood vessel pattern with a fractal dimension of 1.85 has a much larger decay constant than the others.

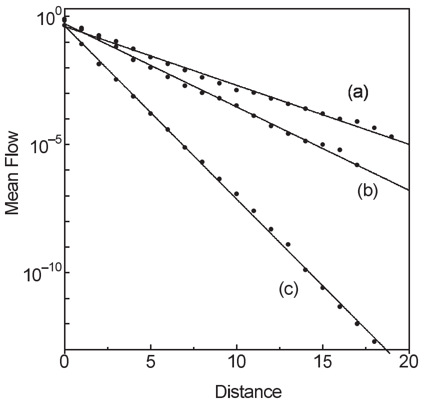


Figure 6. Mean flow as a function of the distance from the center of pattern. (a) Arterial pattern, (b) venous pattern, and (c) pattern with a fractal dimension of 1.85.

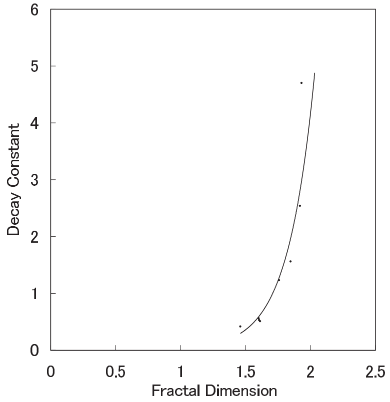


Figure 7. Relationship between decay constant and fractal dimension.

Figure 7 depicts the relationship between fractal dimension and the decay constant showing that the decay constant increases rapidly with increasing fractal dimension

4. Discussion and conclusion

We have examined blood flows in retinal blood vessel patterns with arterial and venous blood systems that have different fractal dimensions. It has been found that blood flows for vessel patterns with different fractal properties are distributed in a widening range with increasing fractal dimension. We have shown that the mean blood flow decreases exponentially with increasing distance from the center of the pattern. The decay constant of flow for vessel patterns increases rapidly with increasing fractal dimension, which explains why the decay constant for the venous system is larger than that for the arterial system.

These results suggest that the difference of fractal dimensions of blood vessel patterns are closely related to their properties of flow. On the other hand, we have a speculation that the difference in properties of spatial distribution of blood flow in vessel systems may be a cause for fluctuation in the circulation of blood. Therefore, it may follow that the fractal structure of blood vessel systems should provide us with a clue to better understand their functions, including blood circulation, that may be largely described by the difference in pressures between arterial and venous blood vessel systems. That is to say, the proximity of arterial and venous blood vessels with fractal properties provides the complex spatial relationship that leads to diffusive interaction between paired arterial and venous blood vessels. Consequently, it can be expected that the specific fractal structures observed in the blood vessel systems of higher animals has an effect on the control of the spatial distribution of blood flows adapted to the organism's distribution.

Another interesting point from these results is that optimal configuration of blood vessel systems should be realized by the difference of fractal dimensions of blood vessel patterns. The significance of an optimal configuration may be related to the fact that large and rapid changes in blood flow are sometimes required in the circulation. The geometry of branching patterns for arterial and venous blood vessel systems would then be space filling and thus provide maximum area for the exchange of oxygen and metabolites.

The growth of blood vessels is known to be controlled by a certain growth factor (angiogenic factor) which is released by the surrounding tissues according to the oxygen demand of the tissue. Since the angiogenic factor may diffuse in the plane of the retina and stimulate the growth of new blood vessels, it seems natural to conclude that such a diffusion-limited process is the physical process responsible for growth of the retinal vessel pattern. However, it is worth pointing out that the blood vessel patterns show considerable differences from the patterns modeled by DLA, where a sprout can grow almost anywhere and the resulting pattern is quite notchy. Such intractable notchy patterns are inconvenient for the blood circulation system because they would create a huge fluid resistance. Nevertheless, The DLA-based simulations described in this paper should be useful in better understanding the dynamics of blood flow because they provide blood flow distributions similar to those seen in experimental studies in terms of countinuity for blood flow.

In this paper, we have not yet considered a pulsing effect (pumping of the heart) on blood flow. This effect might be important in examining the function of blood vessel systems in terms of fractal properties. Therefore, a further study should focus on investigating the characterizations of blood flows in blood vessel systems taking into account the pulsing effect.

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References

[1] L. S. Liebovitch, *Fractals and Chaos Simplified for the Life Sciences* (Oxford University Press, New York, 1998).

[2] T. Matsuo, M. Takahashi, and M. Funata "Characterization of Bifurcating Structure of Blood Vessels using Fractal Dimensions," *FORMA*, **5** (1990) 19–27.

[3] F. Family, B. R. Masters, and D. E. Platt, "Fractal Pattern Formation in Retinal Vessels," *Physica D*, **38** (1989) 98–103.

[4] B. R. Masters, "Fractal Analysis of Normal Human Retinal Vessels," *Fractal*, **2** (1994) 103–110.

[5] A. Daxer, "Mechanisms in Retinal Vasculogenesis: An Analysis of the Spatial Branching Site Correlation," *Curr. Eye Res.*, **14**(4) (1995) 251–254.

[6] B. B. Mandelbrot (Hironaka supervision of translation), *The Fractal Geometry of Nature* (Nikkei Science, Tokyo, 1995).

[7] M. Monma, Y. Yonezawa, and T. Igarashi, *Proceedings of Advanced Workshop of Complex Functionary Systems* (Hitachi, 1998).

[8] M. Monma, Y. Yonezawa, and T. Igarashi, *Proceedings of The Fifth International Symposium on Artificial Life and Robotics* (Oita, 2000).

[9] Y. Yonezawa and K. Ohtomo, "Structure Properties of Generative Form by Hormonal Proliferation Algorithm," *FORMA*, **15** (2000) 103–107.

[10] P. Meakin, "A New Model of Biological Pattern Formation," *Journal of Theoretical Biology*, **118** (1986) 101–113.

[11] M. A. Mainster, "The Fractal Properties of Retinal Vessels: Embryological and Clinical Implications," *Eye*, **4** (1990) 235–241.