Definition and Behavior of Langton's Ant in Three Dimensions

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The "virtual ant" automaton was invented by C. Langton [1]. It has an interesting behavior that has been studied in several researches. A definition of generalized ants in three dimensions, as an extension to Langton's ant, is given here. The phenomenon of periodic motion with drift, the so-called "highway," is also observed in three dimensions but occurs in very different forms. Two classification systems of three-dimensional ants according to the period length of their highways and the resources they need before building a highway are presented.

1. Introduction

1.1 Langton's ant

This paper concerns the "simple" automaton introduced by C. Langton [1] called the "virtual ant." An ant lives on a plane partitioned into squares or—as it is called here—cells. The ant moves and interacts with its environment in the following way: Starting from an arbitrary cell and heading in an initial direction the ant moves one step forward into the next cell. Depending on the state of this cell it changes its direction. The original ant of Langton has only two different cell states: *left* or *right*. The ant performs a turn either to the right or left according to the state of the cell. The ant moves one cell in the current direction and the cell vacated by the ant flips its state from right to left or *vice versa* and so on. Such an ant starting on an initialized plane with all cells in the left state shows an interesting behavior. After having performed 9977 steps in a seemingly chaotic manner it starts to build a highway; that is, a periodic, infinite motion with drift. The period length is 104 steps.

1.2 Generalized ants

Generalized ants were introduced by G. Turk and independently by L. Bunimovich and S. Troubetzkoy [2]. The exact model of generalized

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ants was introduced by J. Propp [3]. They have two or more states and their behavior is described by an n-letter string over the alphabet $\{L, R\}$. L means turn 90 degrees left, R means turn 90 degrees right. There are n different cell states now. By visiting a cell with state k the direction changes according to the kth letter counting from the left in the rule string. The ant changes the state of the cell to $(k + 1) \pmod n$. Many of the generalized ants show the same phenomenon as the original ant: they build a highway. For some it was proven that they never build a highway but instead recur to the origin infinitely often. At these times the cell configuration is symmetric. However, for some generalized ants it could not be shown that they ever start to build a highway [4–6].

2. Definition of three-dimensional ants

In this section generalized ants in three dimensions are defined. In three dimensions there are ants that build highways, too. A classification system according to the period length of highways and another one according to the resources needed by three-dimensional ants (3D ants) before starting to build highways are presented.

A 3D ant lives in a cubic world partitioned into cubic cells. The ant is defined by a rule string of n letters over the alphabet $\{L, R, U, D\}$. The state of a 3D ant is given by two parameters. The first one is the present *direction* absolute to the cartesian coordinate system of the world: positive/negative in x, y, and z-directions. The second one is the present *plane* the ant performs its left and right changes of direction on: x-y plane, *up* is positive/negative in the z-direction; x-z plane, *up* is positive/negative in the y-direction; and y-z plane, *up* is positive/negative in the x-direction. The parameter *plane* can be changed by up and down turns only. Up and down turns change both direction and plane. The new plane results from a rotation of the present plane +/−90 degrees (according to where up in the present plane is) about the axis orthogonal to the present direction. The rule of a 3D ant needs to consist of at least three different letters out of $\{L, R, U, D\}$. Otherwise it would be an ant in two dimensions (rule string only over $\{L, R\}$ or $\{U, D\}$) or a trivial ant; that is, an ant which never consumes more than eight cells (rule string only over $\{L, U\}$, $\{L, D\}$, $\{R, U\}$, or $\{R, D\}$). Therefore 3D ants have rule lengths of three or more.

Generalizations of Langton's ant in higher dimensions have been introduced and studied by L. Bunimovich [6]. The proven result concerning 3D ants is that "all vortices in three-dimensional ant models are concentrated only in two-dimensional 'vortex sheets.'" As far as that could be done by simulation, it has been confirmed.

For all experiments stated here the initial state of all cells is 0 (corresponding to the first letter from the left in the rule string).

3. Classification of three-dimensional ants according to period length of highways

A special property of 3D ants is that they build highways with big differences in period length. In the following a classification system of 3D ants according to the period length of their highways is given. There are infinitely many highway period length classes (PLC). Class 0 contains 3D ants that show a new phenomenon: They build triangles in planes that are orthogonal to an axis of the coordinate system (Figure 1). The ant oscillates between the legs and builds the triangle line-by-line. With a wider interpretation of what a "periodic motion" is we can consider this as being a highway. However we cannot determine a period length. Therefore Class 0 is defined for this type of ant. Class n ($n > 0$) contains all 3D ants that build a highway with less than 4^{n+1} steps per period and that are not contained by a previous class. This definition pays attention to the hypothesis that the frequency of 3D ants decreases exponentially with bigger period lengths. In comparison, ants in two dimensions that are in Class 1 (RRL, RRRL) or Class 2 (RL) with short rule lengths can easily be found. But it seems unlikely that there are ants with relatively short rule lengths of higher classes in two dimensions. Figure 2 shows a 3D ant of PLC 7.

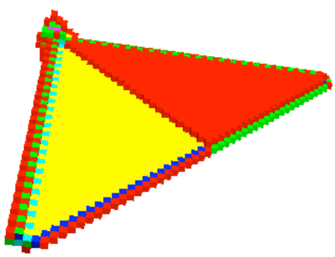

Figure 1. Ant RRLDDDULRRLLLL, PLC 0.

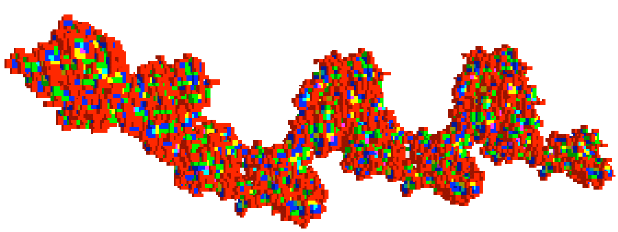

Figure 2. Ant RLRUUUL, PLC 7, the highway period is 25,436 steps.

4. Classification of three-dimensional ants according to resources consumed

This classification of 3D ants pays attention to the time and space that is needed before the ant starts to build its highway. Thus it is a more practical approach according to the simulation. There are infinitely many resource classes (RC). Class 0 contains all trivial ants. Class 1 contains all 3D ants that start to build a highway after at most 1024 steps and that are not trivial. Class n ($n > 1$) contains all 3D ants that are not contained by a previous class and that perform the first period of their highway inside a virtual cube with side length $100n$ cells. The position of the middle of this virtual cube is defined by the starting cell of the ant (actually there are four cubes and different initial directions in question, all of them are allowed). Figures 3 and 4 show 3D ants of different RC. However, due to finite resources for some 3D ants it could not be shown that they ever start to build a highway. Such ants appear in a very compact, nearly spherical group (Figure 5) or; if fewer, in a widespread space-consuming form.

Tables 1 and 2 show the RC and the PLC of all 3D ants with rule lengths three and four. The remaining 40 ants of rule length three and the remaining 88 of rule length four are either trivial or two-dimensional ants.

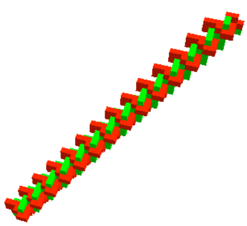

Figure 3. Ant RLU, RC 1.

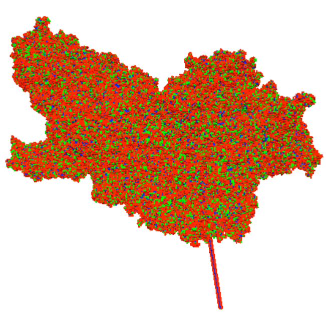

Figure 4. Ant RLUR, RC 4.

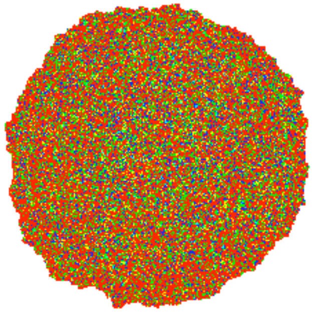

Figure 5. Ant RRRULL, after having performed about three billion steps.

Rule	Symmetric 3D ants	RC	PLC
RLU	RLD, LRU, LRD, UDR, UDL, DUR, DUL	1	3
RUL	RDL, LUR, LDR, URD, ULD, DRU, DLU	1	2
RUD	RDU, LUD, LDU, URL, ULR, DRL, DLR	1	2

Table 1. All 3D Ants of rule length three.

Rule	RC	PLC	Period length
RRLU	1	2	32
RRUL	unknown	unknown	unknown
RRUD	1	2	22
RLRU	1	3	188
RLLU	unknown	unknown	unknown
RLUR	4	2	46
RLUL	1	2	32
RLUU	1	3	102
RLUD	2	2	28
RURL	2	2	30
RURD	unknown	unknown	unknown
RULL	2	2	22
RULR	1	2	22
RULU	1	2	22
RUUL	1	2	26
RULD	1	2	22
RUUD	1	2	22
RUDR	1	1	14
RUDL	unknown	unknown	unknown
RUDU	2	3	114
RUDD	2	2	40

Table 2. All 3D Ants of rule length four. There are seven other symmetric ants for each entry. The "unkown" ants have been simulated in the virtual cube of RC 4.

5. Conclusions

The definition of three-dimensional ants (3D ants) given here results in ants that build three-dimensional highways. This definition shows that the rules of Langton's ant are elementary and in some cases universally applicable. An interesting result is that there is more than one style of highway. First, classification systems were presented. The researches stated here are only a small part of the infinite world of 3D ants. But the search space for ants grows exponentially with the rule length and big three-dimensional worlds are hard to simulate (with respect to memory, computing time, and costly visualization). Some additional questions come up. Are there 3D ants that build arbitrarily complex highways? Does every 3D ant build a highway? Is there an ant and a number of steps for every arbitrary shape? The consumption of which resource (time or space) grows faster by increasing rule length?

A simple software tool to simulate 3D ants made this work possible and will be made available by the author to those who are interested.

References

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