A Note About the Discovery of Many New Rules for the Game of Three-Dimensional Life

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1. Background and terminology

In 1987, this author introduced three-dimensional (3D) versions of the two-dimensional (2D) cellular automaton (CA) made famous by John Conway [1]. At that time, some criteria were given to validate candidate 3D CA rules as "game of life" (GL) rules. Informally, we say that a CA rule is a GL rule if (*a*) when we count neighbors of a cell, we count all touching neighbors, and they are all treated the same; (*b*) the rule supports a "glider" (a translating oscillator); and (*c*) random patterns exhibit bounded growth. These informal criteria are spelled out more formally in [2], which also gives notation for writing rules, namely: $E_1, E_2, \ldots / F_1, F_2, \ldots$ Here, the E_i and F_i are listed in ascending order; the E_i specify the number of touching neighbors required to keep a living cell alive, and the F_i give the number required to bring a nonliving cell to life. Furthermore, when we write $/F_1, F_2, \ldots$ we are implying that the E terms have not been specified.

Conway's rule is thus written 2,3/3 (and not 3,2/3). The two 3D GL rules introduced in 1987 were 4,5/5 and 5,6,7/6 [3]. There followed a series of notes in this journal wherein the discovery of two more GL rules was explored [4, 5]; the rules were 5,6/5 and 6,7,8/5. It should be noted that the rule 6,7,8/5 does not strictly adhere to the formal GL criteria given in [2]. Nevertheless it was well behaved enough to be considered, and this slight variation was mentioned in [5].

2. Recent work

In the several years that have passed since the author's previous attempts at seeking out new 3D GL rules, computer processor speed has increased dramatically. Hence it appeared to be worthwhile to renew the effort, utilizing random "symmetric soup" configurations [4, 5] to ferret out rules that might have been missed due to much slower processor speeds. Many rule combinations have been tried. Quick to be eliminated were rules of the form /5,6; /5,7; /5,8 as they produce unbounded growth. (Note that if E/F exhibits unbounded growth, then it is almost certain that adding terms to either E or F will not render unbounded growth less likely.) It appears also that certain rules of the form /6,7, though exhibiting bounded growth, may not support any gliders that can be discovered with current computing capabilities. The conclusion to date is that only rules of the form /5 or /6 contain GL rules, and that $\max(E_i) < 9$; $i \leq 3$ for all the "simplest" GL rules found to date.

3. A plethora of newly discovered game of life rules

The surprisingly long list of newly discovered 3D GL rules is given here; Figure 1 illustrates the discovered gliders for these new rules. The rules are 2,3/5; 2,5/5; 2,7/5; 3,5/5; 3,6/5; 3,7/5; 3,8/5; 4/5; 4,7/5; 5,7/6; 5,8/5; 8,5. Other figures show the full periods of the new gliders. Note that some of the rules sport more than one glider; in fact it is likely that more gliders exist for the rules given, and more new 3D GL rules of the form /5 or /6 will possibly be unearthed. It should also be observed that some gliders exist for more than one rule. When this occurs, only the simplest rule is given. Thus two of the gliders for 6,7,8/5 [5] exist also for GL rules 6,8/5; 7,8/5; and 8/5. (Figure 1 lists only 8/5.) Unless a unique glider is found for 8/5, we might say that the rule 8/5 is "contained" by the other three rules. On the other hand 4,5/5 and 4/5 appear to have no gliders in common [6].

Note also that the simplest rule is seldom the richest. Rule 6,7,8/5 contains many more elementary oscillators and interesting constructable forms than 8/5. Observe also that rule 5,7/6 supports one of the two gliders found in 5,6,7/6, which was shown to be the 3D analog of the 2D Conway game [3]. And of course we can always add terms to most rules without altering their status as GL rules. Thus 8,5 and 8,22/5,19,20 both behave rather similarly. Here we are listing only the simplest versions. Perhaps if we can add appropriate E_i and F_i terms to some GL rule without altering its GL status, we might eventually find a rule rich enough to be as interesting as Conway's original game.

Acknowledgment

Much of this work was done using a class assignment programmed by William Rollins, Matti Vanninen, Josh Dolinger, and Jamie Huenefeld. The reader is encouraged to visit www.cse.sc.edu/~bays/CAhomePage and investigate 3D CA further.

References

[1] Gardner, Martin "Mathematical Games," *Scientific American*, **223** (October 1970) 120–123.

[2] Bays, Carter, "A Note on the Game of Life in Hexagonal and Pentagonal Tessellations," *Complex Systems*, **15** (2005) 245–252.

[3] Bays, Carter, "Candidates for the Game of Life in Three Dimensions," *Complex Systems*, **1** (1987) 373–400.

[4] Bays, Carter, "A New Game of Three-Dimensional Life," *Complex Systems*, **5** (1991) 15–18.

[5] Bays, Carter, "A New Candidate Rule for the Game of Three-Dimensional Life," *Complex Systems*, **6** (1992) 433–441.

[6] Bays, Carter, "Further Notes on the Game of Three-Dimensional Life," *Complex Systems*, **8** (1994) 67–73.

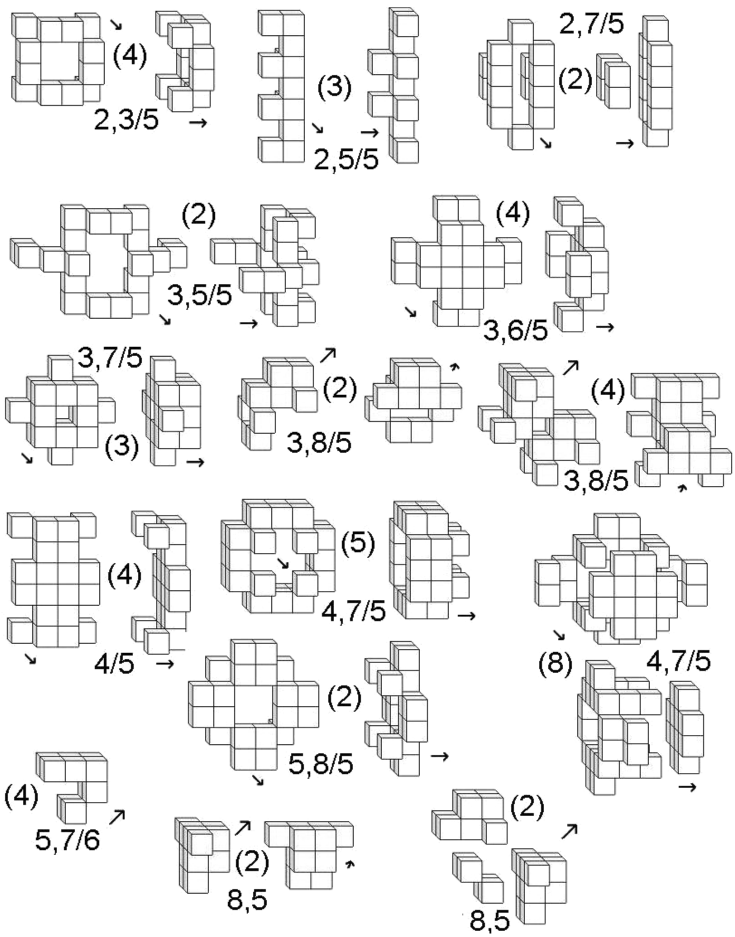

Figure 1. The new GL rules and their known gliders are shown here. The arrows indicate the direction of translation; the numbers in parentheses specify the period. The gliders shown for 5,7/6 and 8/5 are also supported by the previously discovered rules 5,6,7/6 and 6,7,8/5. Most of the illustrated gliders can be easily reproduced. However a few may present a bit of a challenge; hence their coordinates are given here. Coordinates for the period eight 4,7/5 glider are

0,3,5/0,4,5/1,3,6/1,4,6/2,2,4/2,2,5/2,2,6/2,3,2/2,4,2/2,5,4/2,5,5/ 2,5,6/3,1,6/3,2,2/3,2,6/3,3,2/3,4,2/3,5,2/3,5,6/3,6,6/4,1,6/4,2,2/ 4,2,6/4,3,2/4,4,2/4,5,2/4,5,6/4,6,6/5,2,4/5,2,5/5,2,6/5,3,2/5,4,2/ 5,5,4/5,5,5/5,5,6/6,3,6/6,4,6/7,3,5/7,4,5/.

Coordinates for the second 8/5 glider are

0,1,3/0,1,4/1,0,3/1,0,4/1,1,3/1,1,4/1,3,3/1,3,4/2,0,3/2,0,4/2,1,2/ 2,1,3/2,1,4/2,1,5/2,4,3/2,4,4/4,3,2/4,3,3/4,3,4/4,3,5/4,4,3/4,4,4/ 4,5,3/4,5,4/5,3,3/5,3,4/5,4,3/5,4,4/.

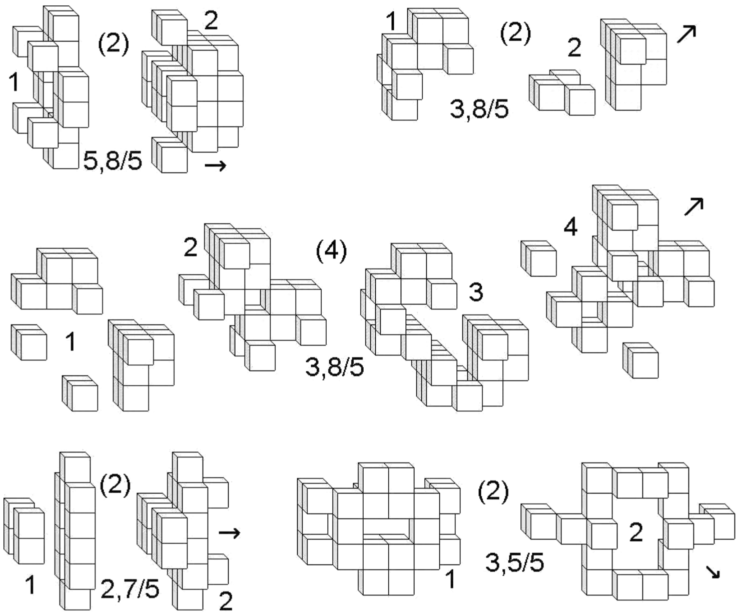

Figure 2. Figures 2 through 5 show all the states for each of the newly discovered gliders. The three in Figure 1 whose various states are not shown (5,7/6; two gliders for 8/5) were discussed in previous work [3, 5].

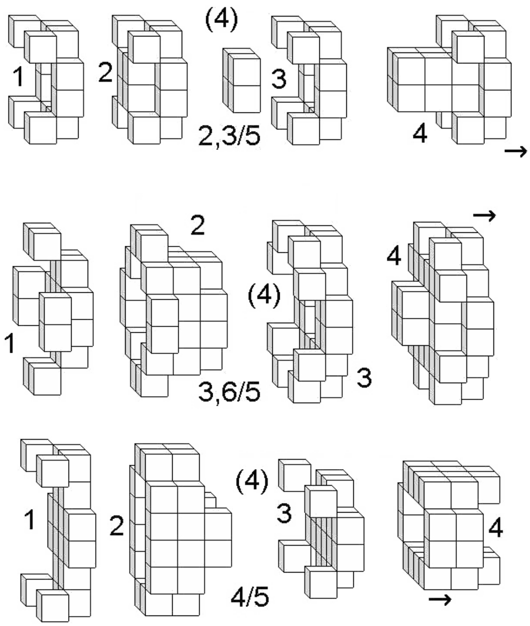

Figure 3.

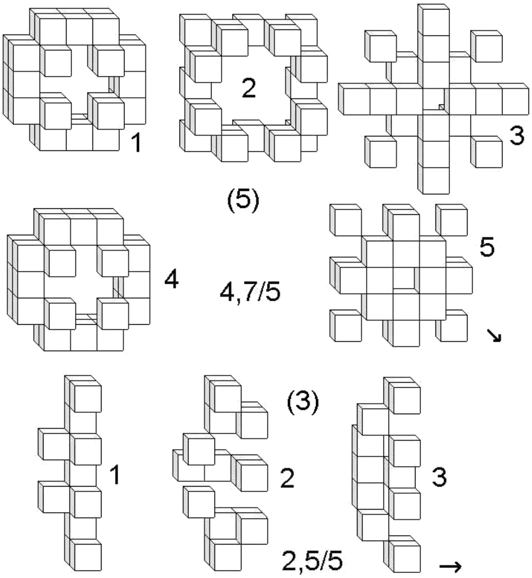

Figure 4.

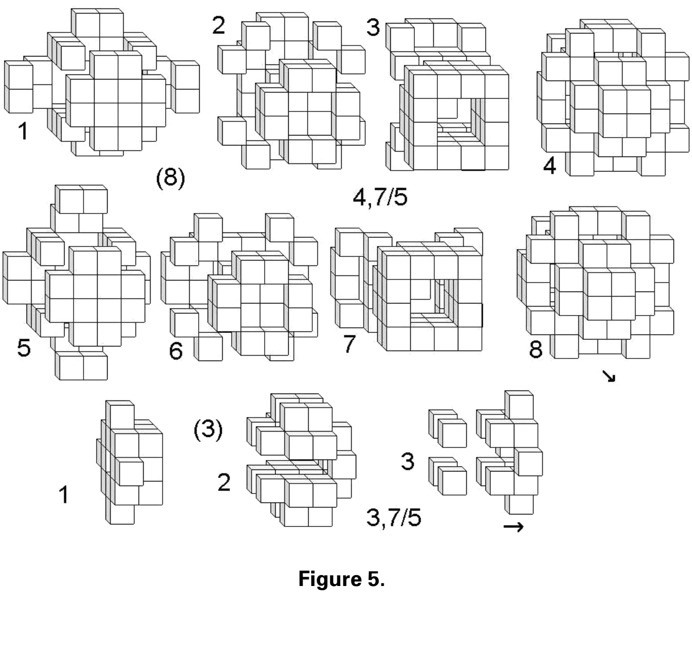

Figure 5.