

Empirical Evidence of Some Stylized Facts in International Crude Oil Markets

Ling-Yun He*

*College of Economics and Management,
Chinese Agricultural University,
Beijing 100083, P. R. China
and
School of Management,
University of Science and Technology of China (USTC),
Hefei 230026, P. R. China*

Feng Zheng†

*School of Business,
Beijing Jiaotong University (BJU),
Beijing 100088, P. R. China*

In this paper, based on the time series of Brent and WTI crude oil prices (daily spot), some stylized facts such as autocorrelation and scaling/multiscaling features are investigated as observed in international crude oil price markets.

1. Introduction

Research on the dynamic behavior of crude oil prices has become a hot issue in recent years. Currently the study of petroleum prices is largely based on the mainstream literature of financial markets whose fundamental assumption is that returns of stock prices follow a normal distribution and price behaviors obey a so-called random walk hypothesis. This notion was first introduced by Bachelier in 1900 [1], since then it has become the essence of many asset pricing models. However, daily financial time series also provide empirical evidence that there exist fundamentally different ubiquitous properties called “stylized facts,” such as fat-tailed distribution, volatility clustering, and scaling/multiscaling features [2–12]. Another important context in this domain is the efficient market hypothesis (EMH) proposed by Fama in [13] which states that stock prices already reflect all available information useful in evaluating their value. These hypotheses have been widely criticized in the financial literature.

*Electronic mail address: lyhe75@yahoo.com.cn, arnold@mail.ustc.edu.cn.

†Electronic mail address: zhengfengyuan@gmail.com.

Empirical evidence supports that the time series of actual prices in crude oil markets do not observe the so-called random walk hypothesis. Crude oil price systems are nonlinear complex issues with chaotic [14–16] and multiscaling features [17–19]. Many factors exist that impact the dynamics and behavior of pricing systems such as the supply and demand of crude oil, the fundamental value of crude oil in itself, and the behavior and psychological factors of heterogeneous agents in the markets.

In this paper we employ the methodology of complexity science to the study of energy price dynamics. We treat the international crude oil price systems as complex adaptive systems (CAS) and find empirical evidence for the existence of stylized facts in daily time series of crude oil prices (returns), which is seldom discussed in current crude oil price studies.

2. Model

2.1 Scaling/multiscaling analysis

The R/S analysis process of crude oil price systems is as follows [19, 20].

Let $P(t) = \{p(t_1), p(t_2), \dots, p(t_n)\}$ be the actual price time series. We define τ -returns as

$$r(\tau) = \log \frac{p(t_i + \tau)}{p(t_i)}, \quad i = 1, 2, \dots, \tau. \quad (1)$$

After obtaining the new time series of τ -returns it is divided into M subseries with length N :

$$E_{N,k}(\tau) = \{r_{1,k}(\tau), r_{2,k}(\tau), \dots, r_{N,k}(\tau)\}, \quad k = 1, 2, \dots, M. \quad (2)$$

From the time series of τ -returns, the deviation $D_{N,k}(\tau)$ can be defined directly from the mean of returns $\bar{r}_{N,k}(\tau)$ as

$$D_{N,k}(\tau) = \sum_{d=1}^N (r_{d,k}(\tau) - \bar{r}_{N,k}(\tau)), \quad k = 1, 2, \dots, M, \quad (3)$$

where

$$R_{N,k}(\tau) = \max\{D_{1,k}(\tau), D_{2,k}(\tau), \dots, D_{N,k}(\tau)\} - \min\{D_{1,k}(\tau), D_{2,k}(\tau), \dots, D_{N,k}(\tau)\}. \quad (4)$$

Thus the hierarchical average value $(R/S)_N(\tau)$ that stands for the rescaled/normalized relation between $R_{N,k}(\tau)$ and $S_{N,k}(\tau)$ becomes

$$\left(\frac{R}{S}\right)_N(\tau) = \frac{1}{M} \sum_{k=1}^M \frac{R_{N,k}(\tau)}{S_{N,k}(\tau)} \propto N^{H(\tau)}, \quad (5)$$

where

$$S_{N,k}(\tau) = \sqrt{\frac{1}{N} \sum_{d=1}^N (r_{d,k}(\tau) - \bar{r}_{d,k}(\tau))^2}. \quad (6)$$

$H(\tau)$ in equation (5) stands for the Hurst exponent [21, 22].

Hurst exponents can be obtained by linear regression [14, 21–24] using

$$\log\left(\frac{R}{S}\right)_N = \log(c) + H(\tau) \cdot \log(N). \quad (7)$$

In order to investigate the multifractal properties for tick data, Barabasi *et al.* [25] have studied the multifractality of self-affine fractals, the multi-affine function, and the multifractal spectrum. The q th-order price–price correlation function $F_q(\tau)$ can be expressed as

$$F_q(\tau) = \langle |p(t_i + \tau) - p(t)|^q \rangle \propto \tau^{qH_q}, \quad (8)$$

where H_q denotes the generalized q th-order Hurst exponent and the angular brackets denote an average over time t . A nontrivial multi-affine spectrum occurs if H_q varies with q , and then the large fluctuation effects in the dynamical behavior of the price $P(t)$ can be explored using equation (8).

2.2 Two-time autocorrelation functions

Let $p(t) = \{p_0, p_1, p_2, \dots, p_n\} (n \rightarrow \infty)$ be the time series of prices. We define returns $r(t)$ and volatility $\sigma(t)$ as

$$r(t_i) = \log \frac{p(t_i)}{p(t_{i-1})}, \quad i = 1, 2, \dots, n, n \rightarrow \infty; \quad \sigma(t) = |r(t)|. \quad (9)$$

The two-time autocorrelation functions of such quantities are of particular interest in econophysics, we define them as

$$C_r(\tau) = \frac{\langle r(\tau)r(t+\tau) \rangle - \langle r(\tau) \rangle^2}{\langle r^2(\tau) \rangle - \langle r(\tau) \rangle^2}, \quad (10)$$

$$C_\sigma(\tau) = \frac{\langle \sigma(\tau)\sigma(t+\tau) \rangle - \langle \sigma(\tau) \rangle^2}{\langle \sigma^2(\tau) \rangle - \langle \sigma(\tau) \rangle^2}. \quad (11)$$

3. Discussion

3.1 Data source and preprocessing

In this paper, we induce the price time series of the Europe Brent Spot Price FOB (dollars per barrel) (abbreviated as “Brent”) and the WTI

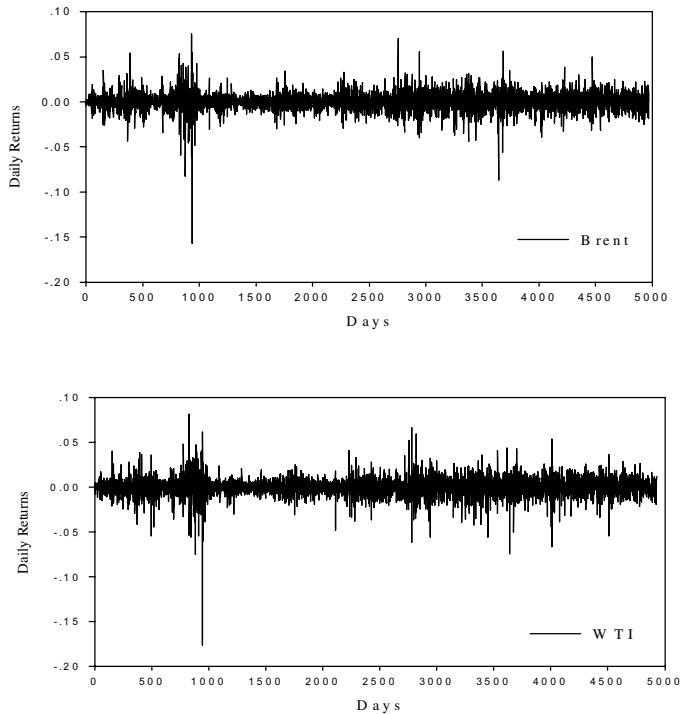


Figure 1. Daily returns of Brent and WTI crude oil prices.

Spot Price FOB (dollars per barrel) (abbreviated as “WTI”), dating from May 20, 1987 to November 21, 2006 (Figure 1). There are 4928 data points from WTI and 4970 data points from Brent. (Data source: U. S. Department of Energy, Energy Information Administration, 2006, <http://tonto.eia.doe.gov/oog/ftparea/wogirs/xls/psw14.xls>.)

■ 3.2 Scaling/multiscaling features

We preprocess the τ -returns ($1 \leq \tau \leq 2500$) according to equation (1). Seasonal effects, including weekends and holidays, were eliminated so that the daily records are described in business time. Nevertheless, the time units are said to be in business days (b-day for short): a week often has five b-days, a month about 20, a quarter about 60, a half-year about 120, and a year about 250.

In this paper we compute Hurst exponents for all τ -returns ($1 \leq \tau \leq 2500$), including some important characteristic time scales, for example, $\tau = 1, 5, 20, 60, 120, 250$, representing daily, weekly, monthly, quarterly, semi-yearly, and yearly returns. Our results are demonstrated in Tables 1 and 2 and are plotted in Figures 2 and 3.

	$H(\tau)$		H_q	
Brent	$H(1) = 0.5242$	$H(60) = 0.7369$	$H_1 = 0.8667$	$H_4 = 0.5765$
	$H(5) = 0.5685$	$H(120) = 0.8121$	$H_2 = 0.6507$	$H_5 = 0.5703$
	$H(20) = 0.6376$	$H(250) = 0.8884$	$H_3 = 0.5947$	$H_6 = 0.5660$
WTI	$H(1) = 0.5148$	$H(60) = 0.7050$	$H_1 = 0.9080$	$H_4 = 0.5727$
	$H(5) = 0.5622$	$H(120) = 0.7823$	$H_2 = 0.6880$	$H_5 = 0.5584$
	$H(20) = 0.6341$	$H(250) = 0.8454$	$H_3 = 0.5997$	$H_6 = 0.5472$

Table 1. Numerical results of R/S analysis under characteristic time scales.

$H(\tau)$ ($1 \leq \tau \leq 10$)		
τ	Brent	WTI
1	0.5242	0.5148
2	0.5370	0.5234
3	0.5461	0.5377
4	0.5580	0.5526
5	0.5685	0.5622
6	0.5772	0.5711
7	0.5836	0.5785
8	0.5895	0.5856
9	0.5950	0.5916
10	0.6016	0.5971

Table 2. Hurst exponents at short-term time scales ($1 \leq \tau \leq 10$).

As is well known, systems with different Hurst exponents exhibit different dynamical behaviors [21, 22]: when $0 \leq H(\tau) < 0.5$ the system has antipersistence features; when $H(\tau) = 0.5$ the time series is uncorrelated and indicates a Gaussian or gamma white-noise process. Stochastic processes with $H(\tau) = 0.5$ are also referred to as fractional Brownian motions. The price behaviors exhibit so-called random walks, while the system’s memory is a Markov chain. When $0.5 < H(\tau) \leq 1$ the systems under study are persistent and characterized by long-term memory that affects all time scales. The time series is initially value-dependent and has chaotic characteristics, thus it is hard to predict future trends. The system has long-term memory of historical information.

According to our study on Hurst exponents and generalized Hurst exponents at different time scales we note the following.

1. We observed that for all τ there exists $0.5 < H(\tau) \leq 1$ for all time series. That is, the crude oil price systems are persistent and autocorrelated and exhibit long-term memory features. Furthermore, we observed a nontrivial multifractal spectrum in that H varies with q , and there are large fluctuation effects in the dynamical behavior of the price $P(t)$.
2. The behaviors of Brent daily returns and WTI exhibit distinct persistence and inherent long-term memory. Although the dynamic behaviors of

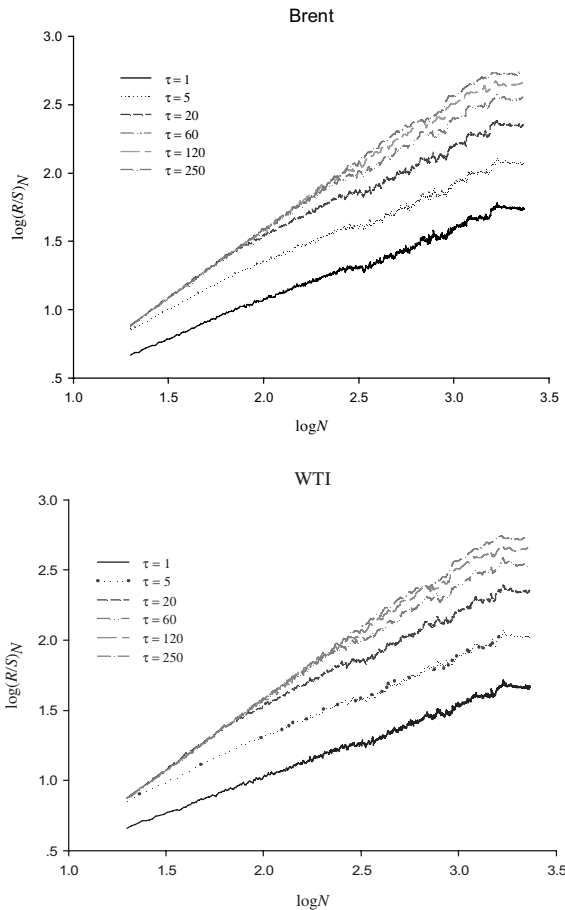


Figure 2. Numerical results of R/S analysis under characteristic time scales ($1 \leq \tau \leq 2500$).

daily returns of Brent and WTI are close to a Brownian motion, a long-term memory mechanism emerges in the two systems as time scales are increased. The Hurst exponents of daily returns of Brent and WTI are approximately 0.5, which implies the existence of noise in the systems. At the same time, the Hurst exponents of yearly returns are greater than 0.8, which implies that much less noise affects the system dynamics in long-term transaction behaviors.

■ 3.3 Long-term memory effects

The main purpose of inducing and computing V statistics is to find nonperiodic cycles by observing a relative map of the statistics. If the curve of the V statistics is horizontal, the time series under study is a

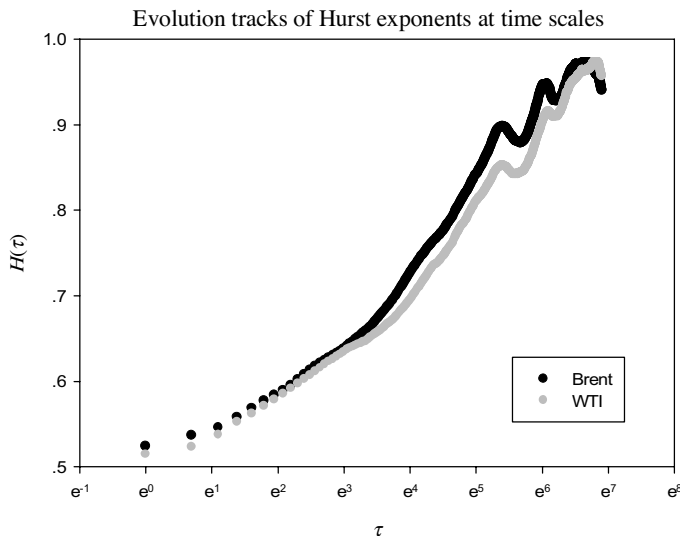


Figure 3. Evolution tracks of Hurst exponents on different time scales ($1 \leq \tau \leq 2500$).

random one which follows random walks, otherwise, there exists long-term memory in the time series. If any critical points exist, the N of the $\log N_s$ of the critical points is the length of the nonperiodic cycles, which we are seeking, namely, the memory of system information will be lost in the system after N days [21, 22]. V statistics can be computed by

$$V_N = \frac{\left(\frac{R}{S}\right)_N}{\sqrt{N}}, \quad (12)$$

where N is the number of observations (days). Our numerical results are demonstrated in Table 3 and are plotted in Figures 4 through 6. The V statistics of the characteristic returns of Brent and WTI demonstrate obviously distinctive uprising trends. By the turning points of the curves of V statistics, we obtain corresponding nonperiodic cycles (see Table 3, the numbers listed are $\log N$, where N stands for the number of observations (days)). For example, the curve of the V statistics of the quarterly returns of Brent has the turning points at $\log N \approx 2.8344$, meaning that its nonperiodic cycle is about 683 b-days.

We find that there exist nonperiodic cycles with different lengths, which is strong evidence for long-term memory in the two crude oil price systems. We can draw some relative conclusions from our results of analyzing V statistics.

1. The V statistics of the daily returns of Brent and WTI demonstrate no distinctive uprising trends, which implies that the price behaviors follow

	The nonperiodic cycles					
	$\tau = 1$ (b day)	$\tau = 5$	$\tau = 20$	$\tau = 60$	$\tau = 120$	$\tau = 250$
Brent	2.4014	2.4014	2.3997	2.8344	2.7959	2.8102
WTI	2.4031	2.4133	2.3997	2.8357	2.8267	2.7973

Table 3. The nonperiodic cycles of long-term memory in the Brent and WTI price systems.

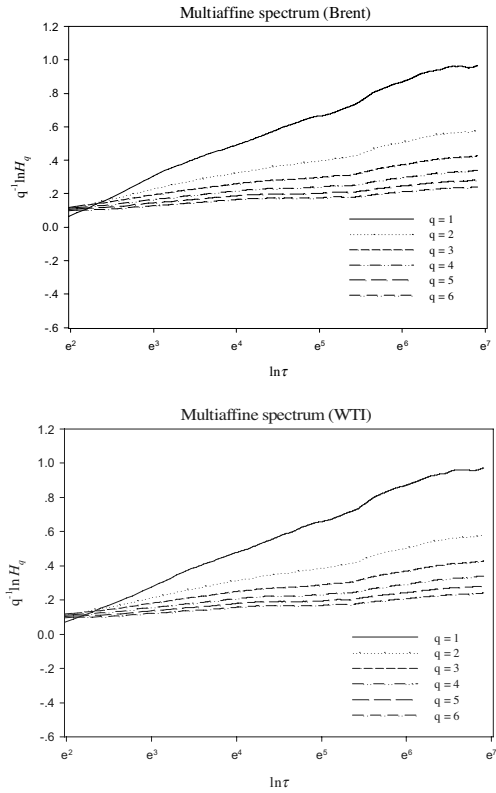


Figure 4. Multi-affine spectra of Brent and WTI price systems ($1 \leq \tau \leq 2500$).

- random walks or fractal Brownian motions, and which coincidentally authenticate our results of H exponents.
- Comparatively, the average of nonperiodic cycles of WTI is longer than that of Brent, which implies that historical information will be remembered longer or will have a longer effect on the price dynamics.

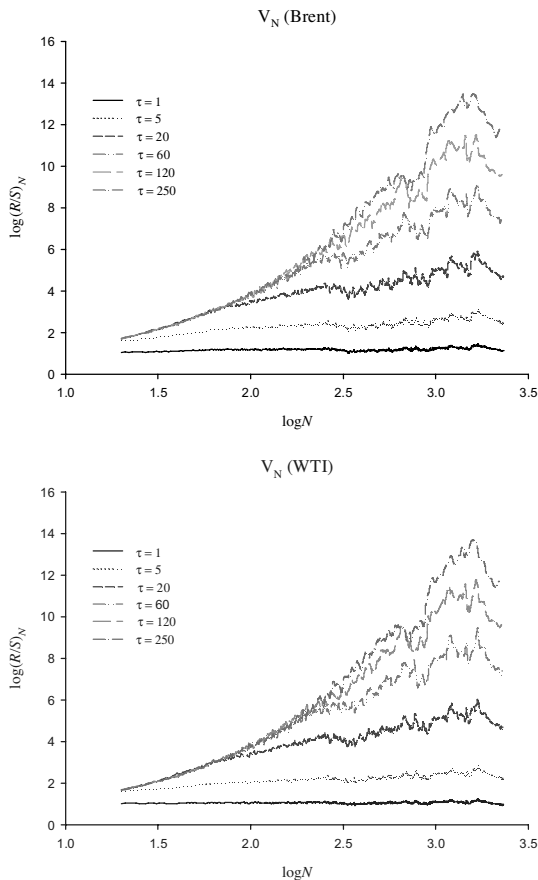


Figure 5. V statistics at different time scales ($1 \leq \tau \leq 2500$).

■ 3.4 Two-time autocorrelation

We obtain the two-time autocorrelation functions according to equations (10) and (11). In Figure 7 we exhibit the absolute values of $C_r(\tau)$ and $C_\sigma(\tau)$. The well-known exponential decay for small values of t can be found in the autocorrelation function of returns. A satisfactory agreement with the expected empirical behavior for the correlation function $C_r(\tau)$, that is, exponential decay for small τ (i.e., daily to weekly data ticks) and noisy behavior for larger values of τ (i.e., monthly or above data) is obtained in Figures 7(c) and 7(d). However, in regards to the autocorrelation function of returns Figures 7(a) and 7(b) both display simple noise patterns, which is in disagreement with the empirical results.

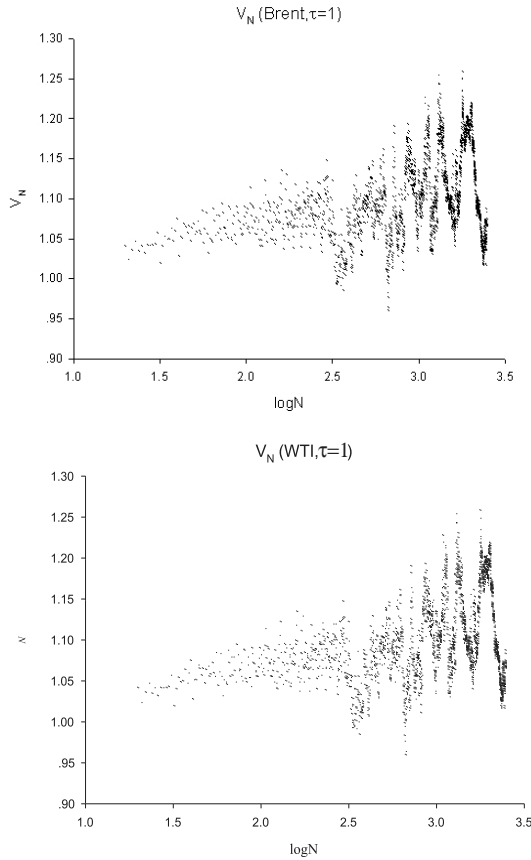


Figure 6. V statistics ($\tau = 1$).

4. Conclusion

Some functions are introduced for empirically exploring the price dynamics and behavior in crude oil prices. We found that on short time scales oil prices effectively show Brownian noise and that there exist nontrivial self-affine and multi-affine spectra in the Brent and WTI price systems. Furthermore, we discussed the long-term memory mechanism and numerically analyzed the nonperiodic cycles in the two price systems. Finally, we calculated the two-time autocorrelation functions and found exponential decay for small values. All numerical results support the existence of stylized facts in crude oil price systems.

References

- [1] L. Bachelier, *Theory of Speculation* (Gauthier-Villars, 1900).

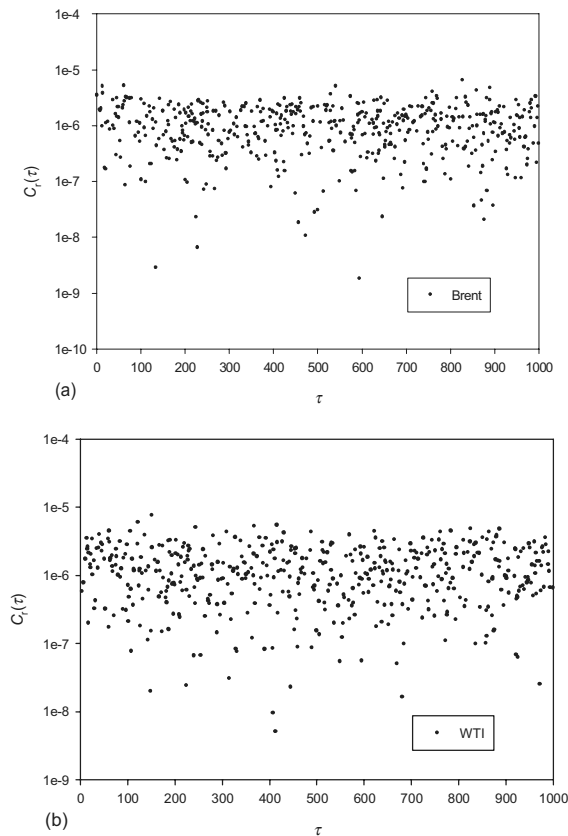


Figure 7. Absolute values of the autocorrelation functions of returns $C_r(\tau)$ and $C_\sigma(\tau)$.

- [2] B. Mandelbrot, "The Variation of Certain Speculative Prices," *Journal of Business*, **36** (1963) 394–419.
- [3] B. Mandelbrot, "The Variation of the Prices of Cotton, Wheat, and Railroad Stocks, and of Some Financial Rates," *Journal of Business*, **40** (1967) 393–413.
- [4] B. Mandelbrot, "When Can Price be Arbitraged Efficiently? A Limit to the Validity of Random Walk and Martingale Models," *Review of Economic Statistics*, **53** (1971) 225–236.
- [5] M. Ausloos and K. Ivanova, "Multifractal Nature of Stock Exchange Prices," *Computer Physics Communications*, **147** (2002) 582–585.
- [6] A. Z. Gorski, S. Drozd, and J. Speth, "Financial Multifractality and Its Subtleties: An Example of DAX," *Physica A: Statistical Mechanics and its Applications*, **316** (2002) 496–510.

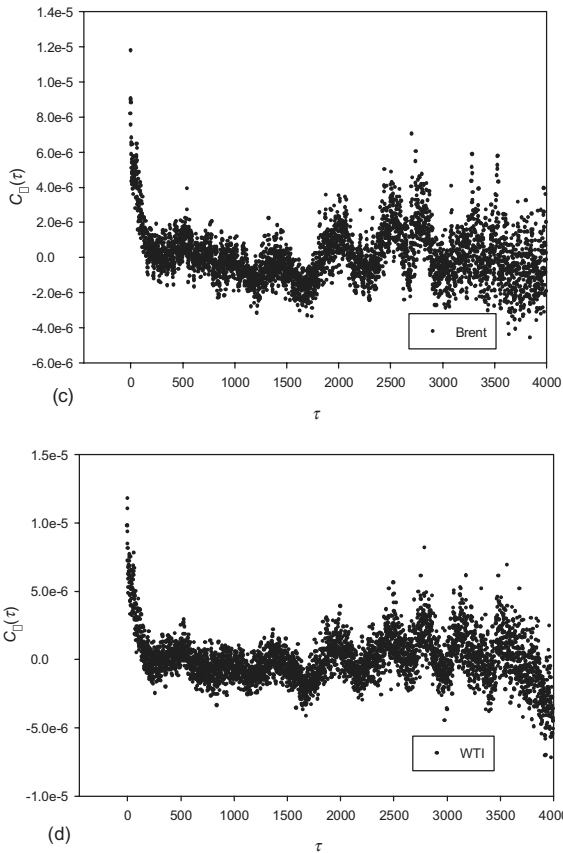


Figure 7. (continued).

- [7] K. Kim and S.-M. Yoon, “Multifractal Features of Financial Markets,” *Physica A: Statistical Mechanics and its Applications*, **344** (2004) 272–278.
- [8] A. Turiel and C. J. Perez-Vicente, “Multifractal Geometry in Stock Market Time Series,” *Physica A: Statistical Mechanics and its Applications*, **322** (2003) 629–649.
- [9] J.-P. Bouchaud and M. Potters, “More Stylized Facts of Financial Markets: Leverage Effect and Downside Correlations,” *Physica A: Statistical Mechanics and its Applications*, **299** (2001) 60–70.
- [10] G. A. Christodoulakis and S. E. Satchell, “Correlated ARCH (Corr-ARCH): Modelling the Time-Varying Conditional Correlation between Financial Asset Returns,” *European Journal of Operational Research*, **139** (2002) 351–370.

- [11] T.-H. Kim and H. White, "On More Robust Estimation of Skewness and Kurtosis," *Finance Research Letters*, **1** (2004) 56–73.
- [12] A. Pagan, "The Econometrics of Financial Markets," *Journal of Empirical Finance*, **3** (1996) 15–102.
- [13] E. F. Fama, "Efficient Capital Markets: A Review of Theory and Empirical Work," *Journal of Finance*, **25** (1970) 383–417.
- [14] J. Alvarez-Ramirez, M. Cisneros, C. Ibarra-Valdez, and A. Soriano, "Multifractal Hurst Analysis of Crude Oil Prices," *Physica A*, **313** (2002) 651–670.
- [15] J. Alvarez-Ramirez, A. Soriano, M. Cisneros, and R. Suarez, "Symmetry/Anti-Symmetry Phase Transitions in Crude Oil Markets," *Physica A*, **322** (2003) 583–596.
- [16] E. Panas and V. Ninni, "Are Oil Markets Chaotic? A Non-linear Dynamic Analysis," *Energy Economics*, **22** (2000) 549–568.
- [17] J. Alvarez-Ramirez, M. Cisneros, C. Ibarra-Valdez, and A. Soriano, "Multifractal Hurst Analysis of Crude Oil Prices," *Physica A: Statistical Mechanics and its Applications*, **313** (2002) 651–670.
- [18] J. Alvarez-Ramirez, A. Soriano, M. Cisneros, and R. Suarez, "Symmetry/anti-symmetry Phase Transitions in Crude Oil Markets," *Physica A: Statistical Mechanics and its Applications*, **322** (2003) 583–596.
- [19] L.-Y. He, Y. Fan, and Y.-M. Wei, "The Empirical Analysis for Fractal Features and Long-run Memory Mechanism in the Petroleum Pricing Systems," *International Journal of Global Energy Issues*, **27**(4) (2007) 492–502.
- [20] J. Alvarez-Ramirez, M. Cisneros, C. Ibarra-Valdez, and A. Soriano, "Multifractal Hurst Analysis of Crude Oil Prices," *Physica A*, **313** (2002) 651–670.
- [21] E. E. Peters, *Fractal Market Analysis: Applying Chaos Theory to Investment and Economics* (John Wiley & Sons, Inc., New York, 1994).
- [22] E. E. Peters, *Chaos and Order in Capital Markets: A New View of Cycles, Prices and Market Volatility* (John Wiley & Sons, Inc., New York, 1996).
- [23] A. W. Lo, "Long-Term Memory in Stock Market Prices," *Econometrica*, **59** (1991) 1279–1313.
- [24] K. Kim and S. M. Yoon, "Multifractal Features of Financial Markets," *Physica A*, **344** (2004) 272–278.
- [25] A.-L. Barabasi, P. Szepefalussy, and T. Vicsek, "Multifractal Spectra of Multi-Affine Functions," *Physica A: Statistical and Theoretical Physics*, **178** (1991) 17–28.