Self-Organizing Traffic Lights as an Upper Bound Estimate

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Self-organizing traffic lights (SOTL) are considered a promising instrument for the development of more adaptive traffic systems. In this paper we explain why some well-promoted results obtained with the use of SOTL should be scrutinized and carefully reviewed. Current computational research projects based on SOTL should be reviewed too.

1. Introduction

Self-organizing traffic lights (SOTL) methodology is highly popular and is the subject of much discussion (e.g., [1–7]). Zhung et al. [1] (based on constructions presented in [2]) claim that by using SOTL "overall better performance and higher capacity can be achieved" in comparison to the systems normally used by traffic authorities. "The idealized control scheme SOTL, which is designed to uniformize the network density distribution, always results in a higher MFD compared to the commonly used SCATS system. SOTL increases the network capacity and produces higher flows in the congested regime." [1, Section 7].

In contrast, our analysis of the methodology and results in [1] has shown that, although SOTL provides a way of building the plot of an upper bound estimate for flow, this upper bound estimate may not be realistic. The corresponding traffic control schedules may in fact be impractical and unachievable. The practical schedule (i.e., the best solution for real traffic control operations) could have its plot slightly below the SOTL-based plot but be realistic and achievable. Hence, the previous statements about SOTL outperforming the Sydney Coordinated Adaptive Traffic System (SCATS) are not correct. The statement in [1] that the "upper bound estimate plot based on SOTL is higher than the plot related to the particular model of one commonly used traffic control system" is not supported by evidence.

The purpose of this paper is neither a full-scale analysis of [1] nor the search for mathematical or logical inconsistencies in it. It is just a demonstration that the SOTL model [1] can provide upper bound esti-

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mates that are too high and unrealistic. On the other hand, the SCATS model [1] is less flexible and less complex than it should be. It provides results that are misleadingly low, although in real traffic systems SCATS performs better than presented in the model. Hence the comparison between two unrealistic models presented in [1] should not be used as a guide for the future development of traffic systems. The performance of traffic systems should not be measured via this kind of analysis.

To show that the models presented in [1] cannot be said to be realistic, we studied the traffic patterns and movements of vehicles corresponding to SOTL upper bound estimates and low traffic density (i.e., the part of the plot that is very close to the origin). We demonstrated that in this particular situation, when the traffic density is low, the actual movement of vehicles is also very low. In this critical situation, the optimal solution behavior for any traffic control system is to stop regulating, stop switching red-green-yellow-red-…, and stop dedicating time intervals for particular directions and phases. Standard SCATS in this situation is switched to flashing yellow in all directions, and such settings work well. The few vehicles on the roads would easily avoid any collision and naturally consume the capacity of the network at the optimal level. This optimal highest level of traffic flow for low traffic density is represented by the corresponding left portion of the SOTL plot. What seems to be missing from [1] is recognition that for this traffic situation the corresponding part of the SCATS plot should exactly match one for SOTL. Unfortunately, the plot presented in [1] for SCATS does not have such a feature, as it is positioned below the plot for SOTL.

We can explain how an incomplete model of SCATS may have come about. The SOTL model in [1] naturally includes the pattern/settings where traffic lights are not operating at all (or are switched to flashing yellow). However, the SCATS model used in [1] only allows for cycles red-green-yellow-red-⋯ but not flashing yellow for all directions when traffic is low. In other words, the SCATS model used is incomplete, limited, and does not fully reflect the actual performance of a SCATS traffic control system in a situation of low traffic density. This limitation of the SCATS model leads to a computational solution still within a domain of cycles red-green-yellow-red-… traffic control patterns. Correspondingly, the SCATS plot is too low to reflect how such a system performs in a real situation. Therefore, the comparison of SOTL and SCATS presented in [1] should be revised.

As a methodological solution, we suggest not accepting SOTL results automatically in the future, but carefully checking the corresponding traffic pattern and vehicle movements, verifying what optimization problems and conditions are actually applied to both

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models, and using the correct state of SCATS operations (or using all states of SCATS operations and making it possible for them to play their roles at the right time). If optimization settings are different, one cannot compare results. Although we understand the complexity of such an approach, it is critical to ensure comparisons of traffic management systems are made using valid modeling.

We would like to note that [1] is well written and clearly presented, which allowed these issues to be identified and discussed.

We analyzed [1] as an example of the typical use of SOTL as a research methodology. We studied properties of the macroscopic fundamental diagram (MFD) that was constructed as an outcome of the computational procedure based on SOTL, presented as a plot in [1, Figure 4(c)]. We concluded that this particular MFD has some good properties not because of the power of SOTL, but due to unrealistic assumptions regarding traffic and not very practical modeling of the control system.

We analyze Figures 4(c) and 4(d) as presented in [1]:

> Figure 4(a–c) shows MFDs of SCATS-F, SCATS-L, and SOTL, at hours 1, 2, … , 6, for a network with isotropic and time-independent boundary conditions, and no internal sources/sinks. Figure 4(d) shows a comparison of the stationary MFDs for the three signal systems. Error bars corresponding to one standard deviation are shown, but are often smaller than the symbol size of the data point.

As stated in [1, Section 3.2], network density in Figures 1 and 2 is dimensionless and its quantity lies in the interval [0, 1]. Also, the network flow is measured in units of vehicles/second. We used exactly the same approach to density and flow so we could directly compare our results with those of [1].

We carried out a sort of "reverse engineering" to analyze these two plots and the application of SOTL. We constructed several simple examples of traffic (i.e., pattern of distribution of vehicles and their travel speed and time profiles) with particular values of network density and network flow. These examples were done analytically, without computer simulation or use of cellular automata modeling. We also found what the corresponding optimal schedules of the traffic control system were. This gave the values of density and flow ready for comparison with those presented in [1, Figure 4(c)]. Once data points matching those of the SOTL plot were obtained, we could make some conclusions regarding the SOTL model.

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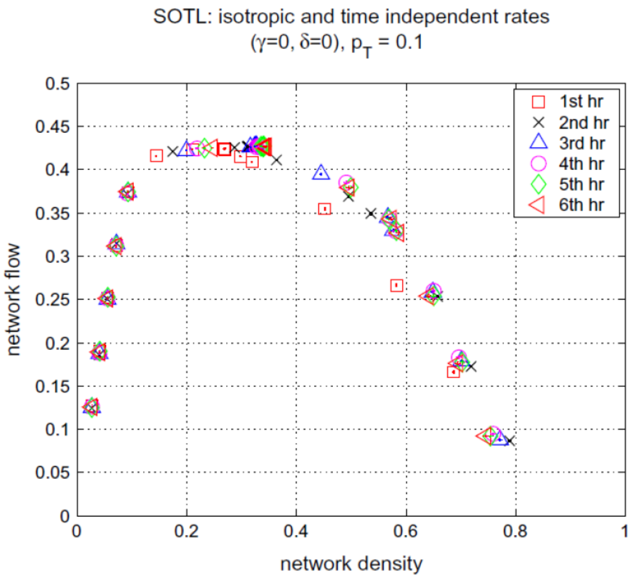
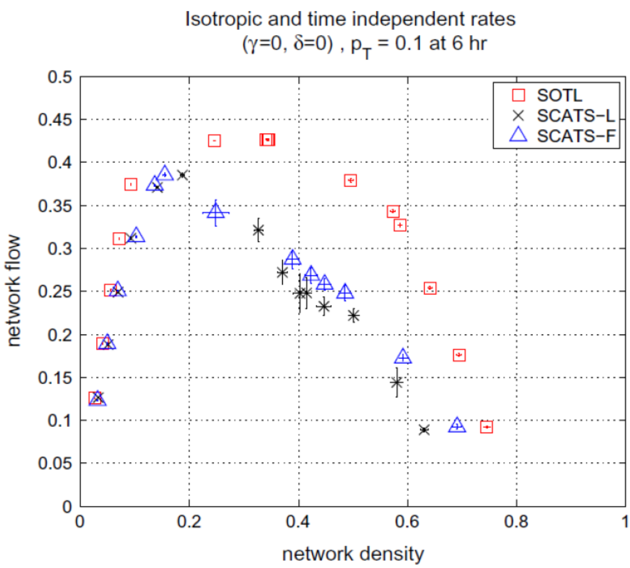


(c) Time evolution of MFDs: SOTL

Figure 1. MFD of cellular automata modeling for SOTL [1, Figure 4(c)].



(d) Stationary MFDs

Figure 2. Comparison of the stationary MFDs for the three signal systems (i.e., SOTL and two models for SCATS) [1, Figure 4(d)].

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2. Example

Vehicles have length H (7.5 meters as in [1]), they travel with constant speed V in one line (hence, without changing lanes, see Figure 3). Or they travel over several parallel lanes with exactly matching travel patterns and without turns.

The network is rectangular, highly symmetric, and even, like in [1]. Obviously we only need to study one intersection like A, presented in Figure 4. The gap between vehicles is $G = gH$, where g is a relatively

Figure 3. Regular uniform traffic (peloton) where vehicles have length H and gaps between vehicles $G = gH$. Vehicle speed is also fixed at $V = V_{\max} = 20\,H / 9$ per second.

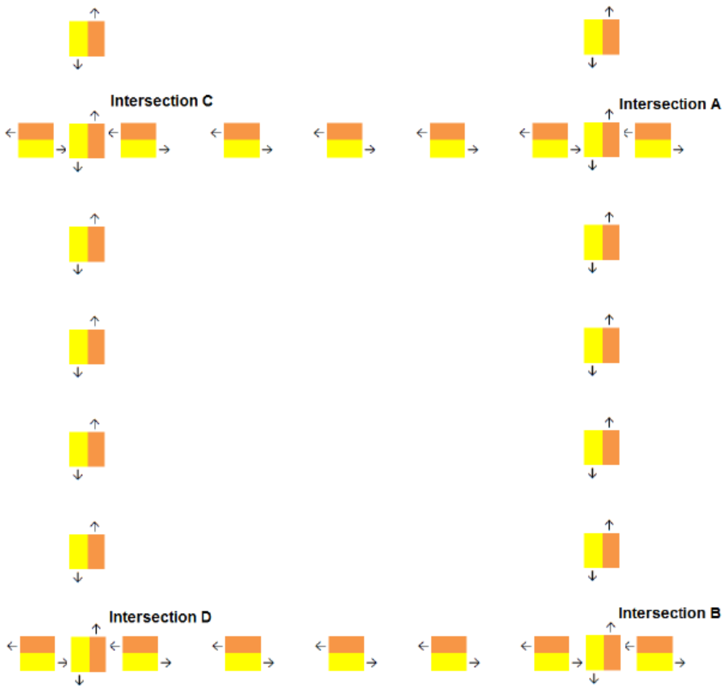

Figure 4. Regular uniform network and regular uniform traffic (peloton) where vehicles have length H and gaps between vehicles $G = gH$. Vehicle speed is also fixed at $V = V_{\max} = 20\,H / 9$ per second.

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big integer. *Network density* (or *network space occupancy*) is $H/(H+G) = 1/(1+g) \approx 0$. Typical maximum speed (60 km/h) corresponds to $(20/9) \cdot H$ per second. We consider low density where $g = 15, 31, 63$, and so on. With such a big gap and the ability of SOTL to review the traffic every 1 second (see [1]), we do not need to stop vehicles and use red lights in any direction. Road users in this system would experience no red traffic lights at all, or SOTL would very quickly switch the light to green when a vehicle is approaching the intersection.

We can consider that a kind of rigid peloton of vehicles and gaps is moving with constant speed; the up-down peloton and the right-left peloton are not impacting each other because the gaps are big and there is plenty of time and space for vehicles to cross the intersection exactly when the other (conflicting peloton) has its gap there (see again Figure 4).

Each vehicle of length H is moving through the intersection with speed $V_{\max}$ and time $t_1 = H / V_{\max}$. After that we see a big gap of length $g \cdot H$ moving with the same speed, so it takes time $t_2 = (g \cdot H) / V_{\max} = g \cdot t_1$. Formally, our example satisfies SOTL and could potentially be a part of the computational solutions that built the SOTL plot in Figures 1 and 2.

First let $g = 15$. With maximum speed of $V_{\max} = (20/9) \cdot H/$second and density $nd = 1/16 = 0.0625$, we have 20 lengths of vehicle passing in 9 seconds through the intersection in one direction and so $160 \cdot H$ in 72 seconds. Each group of $16 \cdot H$ gives one actual vehicle pass. In other words, it takes exactly $1/16$ of the total time for vehicles passing. That makes 10 vehicles in total through the intersection in one direction per 72 seconds. This gives network flow $nf = (10 \cdot 2)/72 = 5/18 = 0.27777\ldots$ for two directions: up-down and right-left (see again Figure 4).

For $g = 31$ we have density $nd = 1/32 = 0.03125$ and 5 vehicles per 72 seconds; that gives a network flow $nf = (5 \cdot 2)/72 = 5/36 = 0.13888\ldots$.

It is easy to see that we can make similar calculations for any g of the form $g = 16 \cdot 2^k - 1$ and get flow $nf = (5/18) \cdot /(k+1)$ and density $nd = (1/16)/(k+1)$, where $k = 0, 1, 2, \ldots$.

The corresponding points (density, flow) are well placed, matching the left portion of the SOTL plot. This means that our example in Section 2 contributes to the SOTL plot in Figures 1 and 2 (and so [1, Figures 4(c) and 4(d)]). In Figure 5 we put the whole straight line (related to Section 2) over the SOTL plot. The construction of our example and the unique properties of the corresponding SOTL schedule (i.e.,

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no control at all, like a roundabout or a switch to green for each individual vehicle approaching the intersection) shows that, perhaps, the corresponding SOTL schedule for traffic light operations is impractical—it is not safe to allow free passage of vehicles without the ability to stop them and to avoid collision (as a red light to one direction prevents collision). Of course, the chance of collision for very sparse low-density traffic is minor, but even a rare collision should be avoided.

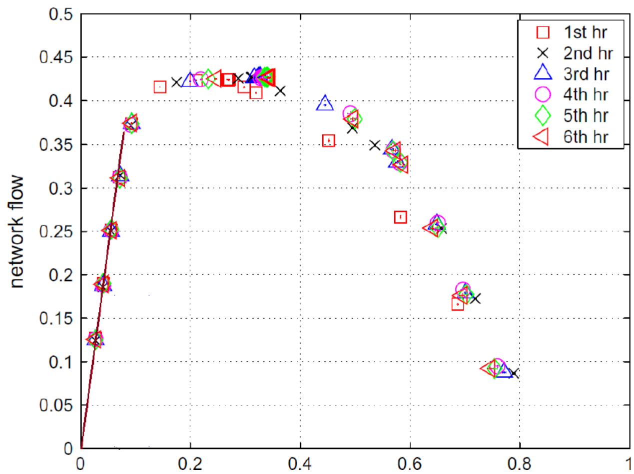

Figure 5. MFD of cellular automata modeling for SOTL and new points (black straight line) that came from Section 2 [1, Figure 4(c)].

3. Discussion and Conclusion

We modified Figure 1 by adding a new straight line related to points with coordinates $nd = (1/16)/(k+1)$ and $nf = (5/18)\cdot/(k+1)$, where $k = 0, 1, 2, \ldots$. This gives Figure 5. As mentioned earlier, we used exactly the same approach to density and flow so we could directly compare our results with those of [1].

How can we explain the perfect matching of our black straight line (i.e., points from Section 2) and the best self-organizing traffic lights (SOTL)-based points for lower density? One possible answer is that an optimal SOTL-based solution (i.e., best operational schedule for traffic lights) is the schedule considered by us in Section 2: the lights are very quickly switched from red to green and back, so all vehicles never stop or slow down. This is almost equivalent to the situation when traffic lights are not operating at all: flashing yellow everywhere or a roundabout situation. For low-density traffic (i.e., huge gaps be-

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tween vehicles) it is possible to have smooth flow under such conditions.

It also means that the "best solution" for low-density traffic produced through SOTL-based optimization is not very practical. Due to safety reasons, no traffic authority will allow cities to be without proper green-red schedules even for low-density periods of time. Any realistic, practical traffic control system should have a corresponding part of its plot a little lower than presented in Figure 1.

Another question is, if SOTL-based optimization produces an impractical solution for low density, should SOTL-based results for higher density be trusted? We would suggest establishing a procedure where each solution is scrutinized, carefully checked, and possibly rejected if it is not practical.

We are not suggesting to abandon SOTL as a methodology but only that the results should not be accepted without comprehensive scrutiny. Without proper verification, SOTL could lead into dangerous waters.

As our final conclusion we summarize the statements from Section 1:

1. For low traffic density situations, the optimal state of any traffic control system is to show flashing yellow lights in all directions, rather than cycling through red-green-yellow-red patterns. This setting (with no regulation by traffic lights) allows the few vehicles on the roads to travel freely and easily, avoid any collision, and highly efficiently use the capacity of the network. Although this fact was not recognized in [1], their SOTL model is flexible and complex enough to provide the correct computational solution (i.e., one that is related to the setting of flashing yellow for all directions). On the contrary, the Sydney Coordinated Adaptive Traffic System (SCATS) model in [1] does not include this very important feature of real traffic system performance. Hence the corresponding computational solution leads to a plot that is too low to reflect real performance. We expect both plots (SOTL and SCATS) to match for low traffic density. Consequently, the difference in behavior of the SOTL and SCATS plots presented in [1] should be revised.

2. SOTL-based results and plots [1] for low traffic density are, in fact, merely upper bound estimates for traffic flow, but these upper bound estimates may not be realistic. The corresponding traffic control schedules could be impractical and unachievable.

3. The practical schedule (i.e., the best solution for real traffic control operations) could have its plot slightly below the SOTL-based plot but be realistic and achievable. The model of SCATS used in [1] does not include all possible features/states of SCATS operations, is too rigid, and is not complex enough; hence, it resulted in a plot that is too low.

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4. We expect that for low traffic density, more realistic computational models (once they are developed) for SOTL and SCATS would result in plots very close to each other.
5. We also expect that for higher traffic density, more realistic computational models would also produce completely different plots for both SOTL and SCATS.

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