Uncertain Density Balance Triggers Scale-Free Evolution in Game of Life

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Since Conway proposed the Game of Life, it has attracted researchers' attention due to complex "life" evolutions despite simple rules. It is known that the Game of Life exhibits self-organized criticality, which might be related to scale-free evolutions. Despite the interesting phenomenon of self-organized criticality, the Game of Life turns to steady states within several generations. Here, we demonstrate a new version of the Game of Life in which cells tried to stay "alive" even though neighboring sites were over- or underpopulated. These rule changings enabled the system to show scale-free evolutions for many generations.

1. Introduction

The Game of Life (GoL) was proposed by Conway [1]. The GoL describes the complexity and the evolution of "life" using simple local rules. Despite its simplicity, the GoL generates complex patterns and therefore has attracted a lot of attention. Biological systems computed algorithmically can be described using the GoL, since cellular automata defined by the GoL are related to a universal Turing machine [2, 3].

Biological systems exhibit self-organized criticality (SOC) [4–6]. Bak and his colleagues showed the existence of SOC in the GoL [7]. In their research, perturbations were added after the system reached "rest"—the steady states in simple periodic states. Thus, the classical GoL reaches steady states, which no longer generate new patterns. Several versions of the GoL were performed. For example, stochastic components and reversibility were introduced [8, 9]. However, there are few studies showing spontaneous pattern generations for long periods with scale-free properties.

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Scale-free properties may be related to SOC [10, 11]. Systems that are "scale free" can be adaptive thanks to their hierarchical transitions [12]. Although biological systems employ local interactions, many SOC systems demand global information [13, 14]. In this paper, we constructed a new version of the GoL in which each cell estimated global information using only local information. Cells sometimes tried to remain "alive" even if their neighboring cells were over- or underpopulated, to prevent the system from being a low-density system. As a result, the system could evolve with power-law properties without reaching steady states.

2. Materials and Methods

We proposed a new version of the Game of Life called Rule-Changed Game of Life (RCGoL).

This algorithm initially follows the rules of the classical GoL.

On each time step, each cell labeled (i, j) will change its state based on the current rule. However, if each cell experiences the following situations, then it changes the current rule:

If $\text{state}_{i,j}^{t} = 1$ && $\text{presumed\_state}_{i,j}^{t} = 0$ && $\text{sum}_{i,j}^{t} - \text{presumed\_sum}_{i,j}^{t} < 0$,

then $F_{i,j}(1, \text{sum}_{i,j}^{t}) = 1$ with probability P_1.

Else if $\text{presumed\_state}_{i,j}^{t} = 1$ && $\text{sum}_{i,j}^{t} - \text{presumed\_sum}_{i,j}^{t} > 0$,

then all rules are reset to initial conditions (the rules of the classical GoL) with probability P_2,

where

$\text{state}_{i,j}^{t}$ represents the current state; "alive" (1) or "dead" (0);

$\text{presumed\_state}_{i,j}^{t}$ represents temporary next-generation $\text{state}_{i,j}^{t}$ estimated from the current state and the rules;

$\text{sum}_{i,j}^{t}$ represents the total number of "alive" neighboring sites;

$\text{presumed\_sum}_{i,j}^{t}$ represents temporary next-generation $\text{sum}_{i,j}^{t}$ estimated from the current state and the rules.

The rules for each cell are defined as follows:

$$F_{i,j}\left(\text{state}_{i,j}^{t}, \text{sum}_{i,j}^{t}\right) = 1 \text{ or } 0.$$

For example, the rules of the classical GoL are described as follows (also see Figure 1):

$$F_{i,j}(0, 0) = 0 \qquad F_{i,j}(1, 0) = 0$$
$$F_{i,j}(0, 1) = 0 \qquad F_{i,j}(1, 1) = 0$$
$$F_{i,j}(0, 2) = 0 \qquad F_{i,j}(1, 2) = 1$$
$$F_{i,j}(0, 3) = 1 \qquad F_{i,j}(1, 3) = 1$$
$$F_{i,j}(0, 4) = 0 \qquad F_{i,j}(1, 4) = 0$$
$$F_{i,j}(0, 5) = 0 \qquad F_{i,j}(1, 5) = 0$$
$$F_{i,j}(0, 6) = 0 \qquad F_{i,j}(1, 6) = 0$$
$$F_{i,j}(0, 7) = 0 \qquad F_{i,j}(1, 7) = 0$$
$$F_{i,j}(0, 8) = 0 \qquad F_{i,j}(1, 8) = 0$$

On each time step (generation), the current state is updated as follows:

$$\text{state}_{i,j}^{t+1} = F_{i,j}\left(\text{state}_{i,j}^{t}, \text{sum}_{i,j}^{t}\right).$$

Note that $\text{state}_{i,j}^{t+1} \neq \text{presumed\_state}_{i,j}^{t}$. The $\text{presumed\_state}_{i,j}^{t}$ is just a temporary state and used for changing the rules. The actual next-generation state ($\text{state}_{i,j}^{t+1}$) is determined using "changed rules."

We simulate RCGoL on finite lattices of size 100×100. Periodic boundary conditions are chosen. We set 10 000 time steps (generations) as the calculation length of one trial. However, we run 100 000 time steps (generations) when evaluating scale-free properties. Random initial configurations are assumed. The probabilities P_1 and P_2 are set to 0.25, 0.75, respectively; $(P_1, P_2) = (0.25, 0.75)$. Later, we will discuss the relation between these two parameters.

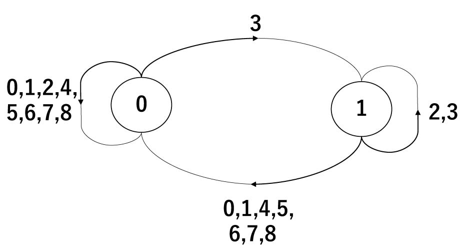
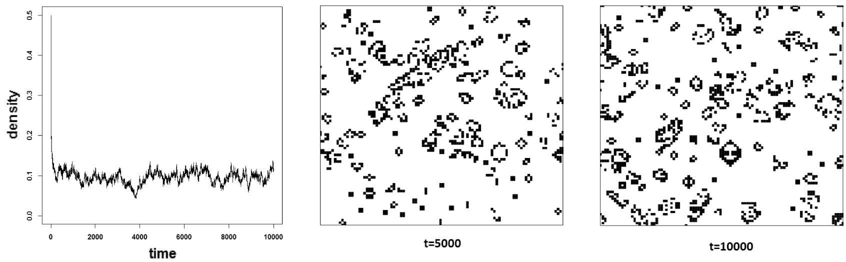


Figure 1. Cell state transition of the classical GoL. The transition rules are described in a classical automaton state transition diagram. Nodes are states of a cell. Arcs indicate cell state transitions, which are labeled by the numbers of neighbors.

3. Results

Figure 2(a) represents one example of the relationship between density and time evolution. In this example, initial density was set to 0.50. Even after one trial, new patterns emerged (Figures 2(b) and 2(c)). Actually, we conducted 100 trials and evaluated whether or not the last configuration (10 000th generation) matched any previous configurations. The RCGoL generated new patterns after 10 000 time steps, which was never observed in the classical GoL (N of matching trials = 0 (RCGoL) versus 100 (classical GoL); chi-squared test, $P < 1.0E - 15$).



(a) (b) (c)

Figure 2. Examples of pattern evolutions when initial density was set to 0.50. (a) The relationship between density and time evolution. (b) Life patterns at $t = 5000$. (c) Life patterns at $t = 10\,000$.

Next, we examined the existence of the scale-free property in the RCGoL system by evaluating the duration of high-density phases. Here, the duration of any high-density phase was defined as the time interval between two consecutive low-density phases. We conducted one trial regarding each initial density (0.30, 0.50, 0.70). We defined the high-density phase as when the density was more than 0.05. As shown in Figure 3, we found power-law distributions for each initial density (initial density = 0.70: n of data = 38, $\mu = 1.43$, AIC weights of power-law against exponential law = 1.00; initial density = 0.50: n of data = 35, $\mu = 1.44$, AIC weights of power-law against exponential law = 1.00; initial density = 0.30: n of data = 29, $\mu = 1.36$, AIC weights of power-law against exponential law = 1.00).

Finally, we compared the $(P_1, P_2) = (0.25, 0.75)$ version with the $(P_1, P_2) = (0.40, 0.90)$ and $(P_1, P_2) = (0.75, 0.25)$ versions. Figure 4 indicates the relationship between initial density and final density after 10 000 time steps for each version. Averaged data was shown from 10 trials for each initial density $(0.10, 0.20, \ldots, 0.10)$. In every condition, the RCGoL appeared to maintain certain densities after many generations. However, final densities were higher in the (0.75,

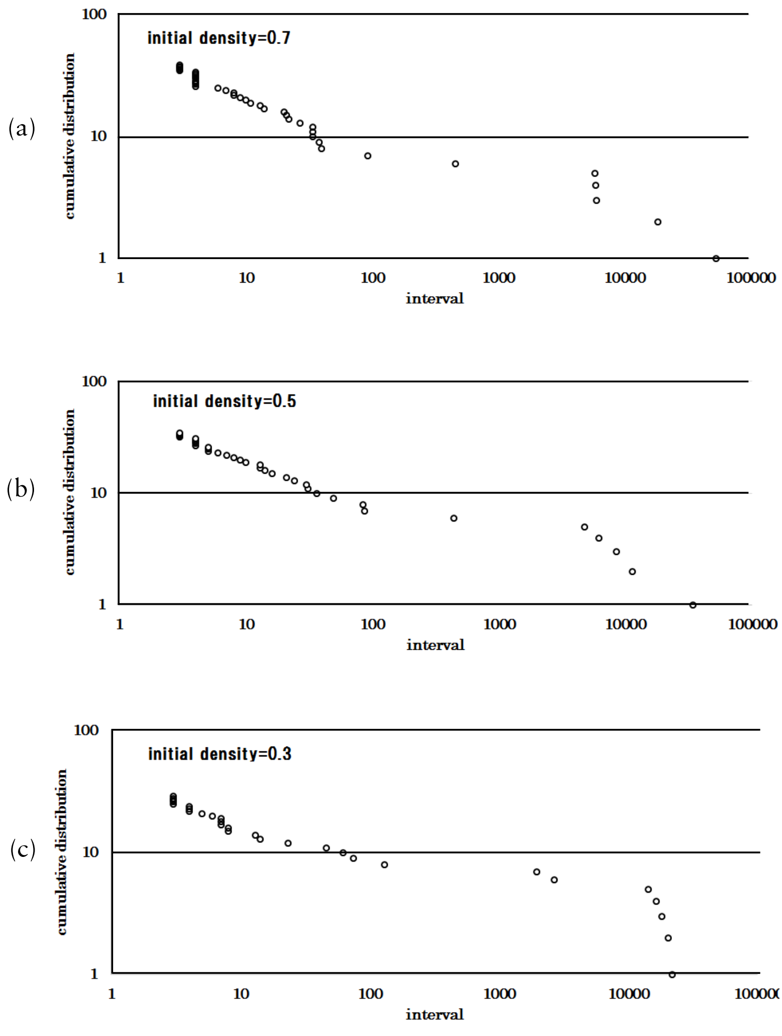


Figure 3. The relationship between the duration (interval) of the high-density phase and cumulative distribution. (a) Initial density = 0.70. (b) Initial density = 0.50. (c) Initial density = 0.30. Note that calculations were conducted for 100 000 time steps (generations).

0.25) version than other versions, suggesting that relative relations between P_1 and P_2 might determine density evolutions of the RCGoL. A scale-free distribution was not found when $(P_1, P_2) = \left(0.75, 0.25\right)$ but was when $(P_1, P_2) = \left(0.40, 0.90\right)$ (Figure 5) ($(P_1, P_2) = \left(0.40, 0.90\right)$: initial density = 0.50, n of data = 83, $\mu = 1.40$, AIC weights of power-law against exponential law = 1.00;

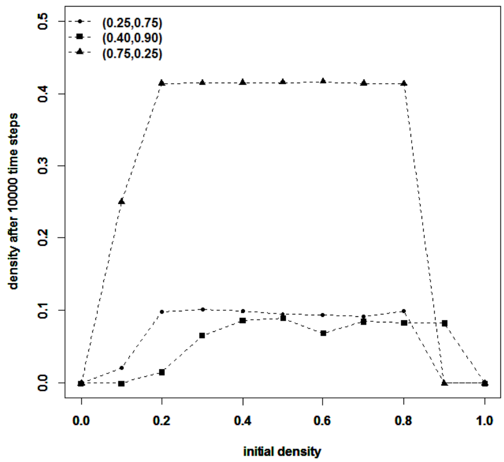


Figure 4. The relationship between initial density and final density after one trial. Averaged data from 10 trials is shown. $(P_1, P_2) = (0.25, 0.75)$, $(P_1, P_2) = (0.40, 0.90)$ and $(P_1, P_2) = (0.75, 0.25)$ are plotted, respectively.

$(P_1, P_2) = (0.75, 0.25)$: initial density = 0.50, n of data = 349, $\lambda = 0.0035$, AIC weights of power-law against exponential law = 0.00). Note that we defined the high-density phase as having a density more than 0.05, 0.40 when $(P_1, P_2) = (0.40, 0.90)$, $(0.75, 0.25)$, respectively. These results again suggest that the relative relations between P_1 and P_2 might influence the time evolutions of the RCGoL. It appears to be necessary for P_2 to be higher than P_1 in order to maintain scale-free properties.

4. Discussion

Our results clearly show that the Rule-Changed Game of Life (RCGoL) generates various patterns for long periods. This system also exhibits scale-free distributions, which might be related to self-organized criticality [10, 11]. Spontaneous pattern generations for long periods without rest are not observed in the classical Game of Life (GoL), even though it exhibits self-organized criticality [7]. In the RCGoL, the rules are changed according to local situations. When the total number of "alive" neighboring sites will decrease using current rules, the rules are changed in order to maintain the local density. The important point is that each cell can only detect local neighbors. It evaluates the global density using the local density. To prevent density decreasing, each cell sometimes remains "alive" by changing the rules even when almost all of the neighboring sites are "dead." This event implies that cells will not turn "dead" even if local situations are over- or underpopulated. At the same time, there is always ambiguity about whether the local density condition is inconsistent with the global density condition. Therefore, changed rules return to default rules when the local density increases.

The RCGoL might be evolved with scale-free properties based on uncertain relationships between local and global situations. Scale-free properties that accompany biological evolutions would enable the system to be adaptive due to large-scale time evolutions without facing extinctions [10].

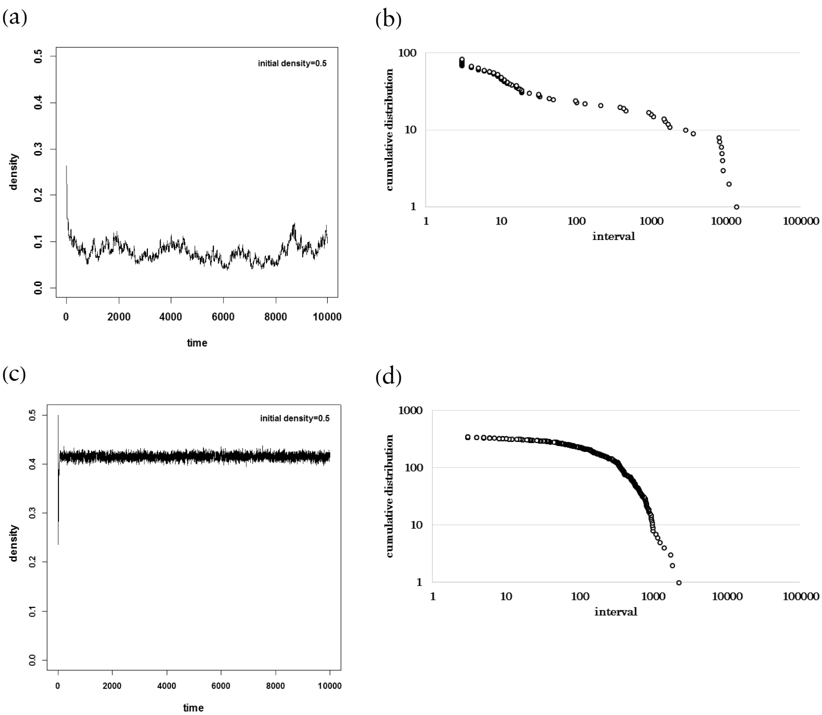


Figure 5. Density-time evolution and a distribution of the duration of the high-density phase when $(P_1, P_2) = (0.40, 0.90)$ or $(P_1, P_2) = (0.75, 0.25)$. Initial density was set to 0.50. (a) The relationship between density and time evolution: $(P_1, P_2) = (0.40, 0.90)$. (b) The relationship between the duration of the high-density phase and cumulative distribution: $(P_1, P_2) = (0.40, 0.90)$. (c) The relationship between density and time evolution: $(P_1, P_2) = (0.75, 0.25)$. (d) The relationship between the duration of the high-density phase and cumulative distribution: $(P_1, P_2) = (0.75, 0.25)$. Note that calculations were conducted for 100 000 time steps (generations) when evaluating scale-free properties ((b) and (d)).

References

[1] M. Gardner, "The Fantastic Combinations of John Conway's New Solitaire Game 'Life'," *Scientific American*, **223**(4), 1970 pp. 120–123.

[2] A. Adamatzky, ed., *Game of Life Cellular Automata*, New York: Springer, 2010.

[3] A. Adamatzky, ed., *Collision-Based Computing*, London: Springer, 2002.

[4] D. R. Chialvo and P. Bak, "Learning from Mistakes," *Neuroscience*, **90**(4), 1999 pp. 1137–1148. doi:10.1016/S0306-4522(98)00472-2.

[5] P. Bak and D. R. Chialvo, "Adaptive Learning by Extremal Dynamics and Negative Feedback," *Physical Review E*, **63**(3), 2001 031912. doi:10.1103/PhysRevE.63.031912.

[6] S. Bornholdt and T. Röhl, "Self-Organized Critical Neural Networks," *Physical Review E*, **67**(6), 2003 066118. doi:10.1103/PhysRevE.67.066118.

[7] P. Bak, K. Chen and M. Creutz, "Self-Organized Criticality in the 'Game of Life,' " *Nature*, **342**, 1989 pp. 780–782. doi:10.1038/342780a0.

[8] L. S. Schulman and P. E. Seiden, "Statistical Mechanics of a Dynamical System Based on Conway's Game of Life," *Journal of Statistical Physics*, **19**(3), 1978 pp. 293–314. doi:10.1007/BF01011727.

[9] D. Bleh, T. Calarco and S. Montangero, "Quantum Game of Life," *Europhysics Letters*, **97**(2), 2012 20012-1-20012-5.

[10] R. V. Solé and S. C. Manrubia, "Extinction and Self-Organized Criticality in a Model of Large-Scale Evolution," *Physical Review E*, **54**(1), 1996 pp. 42–45. doi:10.1103/PhysRevE.54.R42.

[11] J. Vandermeer, I. Perfecto and S. M. Philpott, "Clusters of Ant Colonies and Robust Criticality in a Tropical Agroecosystem," *Nature*, **451**, 2008 pp. 457–459. doi:10.1038/nature06477.

[12] G. M. Viswanathan, E. P. Raposo and M. G. E. da Luz, "Lévy Flights and Superdiffusion in the Context of Biological Encounters and Random Searches," *Physics of Life Reviews*, **5**(3), 2008 pp. 133–150. doi:10.1016/j.plrev.2008.03.002.

[13] P. Bak, C. Tang and K. Wiesenfeld, "Self-Organized Criticality: An Explanation of the 1/f Noise," *Physical Review Letters*, **59**(4), 1987 pp. 381–384. doi:10.1103/PhysRevLett.59.381.

[14] P. Bak and K. Sneppen, "Punctuated Equilibrium and Criticality in a Simple Model of Evolution," *Physical Review Letters*, **71**(24), 1993 pp. 4083–4086. doi:10.1103/PhysRevLett.71.4083.