Predicting the Large-Scale Evolution of Tag Systems: Code Supplement

Carlos Martin
School of Engineering and Applied Science
Columbia University
New York, New York

We present Wolfram Language code that predicts the large-scale evolution of a two-tag system from its production rules in terms of the length and density of symbols in the queue. For more information, please see the original paper [1], which presents a mathematical derivation of this method.

First, we define the production function. This function specifies the production that is to be appended to the tag system queue for each possible input pair of symbols.

```
getProduction[{0, 0}] := {1, 1, 1}
getProduction[{0, 1}] := {0, 1}
getProduction[{1, 0}] := {1, 1}
getProduction[{1, 1}] := {0}
```

This function returns the distribution of productions from the distribution of input pairs (according to a production function).

```
getProductionDistribution[pairDistribution_, getProduction_] :=
  Map[Composition[Total, Map[pairDistribution]],
    GroupBy[Keys[pairDistribution], Function[pair, getProduction[pair]]]]
```

This function returns the input pair distribution for the next epoch from the production distribution for the current epoch.

```
getPairDistribution[productionDistribution_] := (
    productions = Select[Keys[productionDistribution],
        Function[production, Length[production] > 0]];
    pairDistribution = Association[];

    (* Consider a possible production *)
    Map[Function[production,
        Map[
            Function[pair,
              If[Not[KeyExistsQ[pairDistribution, pair]],
                pairDistribution[pair] = 0];
              pairDistribution[pair] += productionDistribution[production];
            ],
            Partition[production, 2, 1]
          ];
```

https://doi.org/10.25088/ComplexSystems.26.1.93

```
(* Consider each second production that could follow the first *)
        Map[Function[nextProduction,
              pair = {Last[production], First[nextProduction]};
              If[Not[KeyExistsQ[pairDistribution, pair]],
                pairDistribution[pair] = 0];
              pairDistribution[pair] += productionDistribution[production] *
                  productionDistribution[nextProduction];
            ], productions];
        ], productions];

  Normalize[pairDistribution, Total]
  )
```

This function returns the expected density of symbols in the queue at the beginning of an epoch.

```
getDensities[pairDistribution_] := (
    densities = Association[];
    Map[Function[pair,
        Map[Function[symbol,
            If[Not[KeyExistsQ[densities, symbol]], densities[symbol] = 0];
            densities[symbol] += pairDistribution[pair] / Length[pair]
          ], pair]
      ], Keys[pairDistribution]];
    densities
  )
```

This function returns the expected growth of the queue at the beginning of an epoch.

```
getGrowth[pairDistribution_, getProduction_] := Total[Map[Function[pair,
          Length[getProduction[pair]] * pairDistribution[pair]
        ], Keys[pairDistribution]]] - 2
```

We are now able to predict the growth and evolution of the specified tag system over several epochs. This is done by setting the initial distribution of symbols in the queue and deriving the subsequent distribution of string productions. From this distribution of productions, we can derive the distribution of symbols in the queue for the next epoch and iterate this process.

```
epochs = 10;
epoch = 0;
length = 10 000;
pairDistribution = <|{0, 0} → .25, {0, 1} → .25, {1, 0} → .25, {1, 1} → .25|>;
Do[
    epoch += 1;
    Print[StringForm["\nBeginning of epoch ``", epoch]];
    Print[StringForm["Length: ``", length]];
    densities = KeySort[getDensities[pairDistribution]];
    Map[
      Function[symbol,
        Print[StringForm["Density of ``: ``", symbol, densities[symbol]]];
      ], Keys[KeySort[densities]]
    ];
```

```
    growth = getGrowth[pairDistribution, getProduction];
    length *= (1 + growth / 2);
    productionDistribution =
      getProductionDistribution[pairDistribution, getProduction];
    pairDistribution = getPairDistribution[productionDistribution];
    ,
    epochs
  ];
```

```
Beginning of epoch 1
Length: 10000
Density of 0: 0.5`
Density of 1: 0.5`

Beginning of epoch 2
Length: 10000.`
Density of 0: 0.25`
Density of 1: 0.75`

Beginning of epoch 3
Length: 7500.`
Density of 0: 0.5`
Density of 1: 0.5`

Beginning of epoch 4
Length: 7500.`
Density of 0: 0.25`
Density of 1: 0.75`

Beginning of epoch 5
Length: 5625.`
Density of 0: 0.5`
Density of 1: 0.5`

Beginning of epoch 6
Length: 5625.`
Density of 0: 0.25`
Density of 1: 0.75`

Beginning of epoch 7
Length: 4218.75`
Density of 0: 0.5`
Density of 1: 0.5`
```

https://doi.org/10.25088/ComplexSystems.26.1.93

```
Beginning of epoch 8
Length: 4218.75`
Density of 0: 0.25`
Density of 1: 0.75`

Beginning of epoch 9
Length: 3164.0625`
Density of 0: 0.5`
Density of 1: 0.5`

Beginning of epoch 10
Length: 3164.0625`
Density of 0: 0.25`
Density of 1: 0.75`
```

Reference

[1] C. Martin, "Predicting the Large-Scale Evolution of Tag Systems," Complex Systems, 25(2), 2016 pp. 79–107.
wac .36f4.edgecastcdn.net/0036F4/pub/complex-systems/25-2-1.pdf.