Conjecture 1. For any CA rule asymptotically emulating identity, the density of ones after n iterations, starting from a Bernoulli distribution, is always in the form

$$c_n \sim \sum_{i=1}^{k} a_n \lambda_i^n, \tag{64}$$

where a_i, λ_i are constants that may only depend on the initial density c_0, and where $|\lambda_i| \leq 1$.

Note that some of the λ_i can be complex, and then they come in conjugate pairs, like in rule 44 (equation (63)). When one of the λ_i is equal to 1, then $c_\infty > 0$; otherwise $c_\infty = 0$.

Rule	c_n	Proof/conjecture
13	equation (53)	conjecture
32	equation (55)	conjecture
40	equation (37)	conjecture
44	equation (63)	conjecture
77	equation (57)	conjecture
78	equation (59)	conjecture
128	equation (13)	proof [13]
132	equation (14)	proof [13]
136	equation (15)	proof [13]
140	equation (16)	proof [5]
160	equation (19)	proof
164	unknown	
168	equation (30)	proof
172	equation (61)	conjecture
232	equation (50)	conjecture

Table 2. Density of ones c_n for arbitrary initial density for elementary rules asymptotically emulating identity.

Such behavior of c_n strongly resembles hyperbolicity in finitely dimensional dynamical systems. Hyperbolic fixed points are a common type of fixed points in dynamical systems. If the initial value is near the fixed point and lies on the stable manifold, the orbit of the dynamical system converges to the fixed point exponentially fast. It can be argued that the exponential convergence to equilibrium observed in CAs described in this paper is somewhat related to finitely dimensional hyperbolicity. We suspect that the finite-dimensional map, known as the local structure map [16], which approximates the dy-

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namics of a given CA, should posses a stable hyperbolic fixed point for every CA asymptotically emulating identity. This hypothesis is currently under investigation and will be discussed elsewhere.

As a final remark, let us note that in this paper we discussed binary rules only. It would be equally interesting to consider rules with a larger number of states k, for example, $k = 3$, and check if the above conjecture applies to them as well. The authors are planning to examine this issue in the near future.

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