System Behaviors and Measures: Compressed State Complexity and Number of Unique States Used in Naval Weapons Elevators

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Relationships among the performance, robustness, and design of naval weapons elevators can be found in both static and dynamic measures of complexity. This paper examines two such dynamic measures: compressed state complexity and the number of unique states used by the system. Simulation results show that the mean compressed state complexity does not have a trend, due to the effect of logical evolution length. Evidence is found that the number of unique states used in a naval weapons elevator has a relationship to throughput and connectivity.

1. Introduction

This work is the second part in a multi-part series investigating relationships between complexity and optimality with respect to both performance and robustness, performed in the context of a naval weapons elevator system. Each part of the series will present an analysis of a number of static or dynamic complexity measures evaluated using a large set of naval weapons elevator system simulation results. After presenting the motivation for understanding the relationships between system complexity, adaptability, and system optimality, using cellular automata and natural systems as models, the first part [1] addressed logical and state complexity. These measures, based on the concept of algorithmic complexity, quantify the information content available in a simulation evolution by considering different forms of compressing evolution histories. Analysis of the simulation results shows a relationship between system performance and complexity for both measures. The most complex evolutions tend to correspond to configurations with the greatest adaptability to inputs, which enables higher system throughput. This part picks up where the first left off, presenting an analysis of the compressed state complexity and the number of unique states used in an evolution.

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2. Compressed State Complexity

The compressed state complexity is the ratio of the number of unique states entered to the number of logical steps in an evolution. The direct relationship between the compressed state complexity and the state complexity is evident in Figure 1, the mapping of the state complexities of evolutions in the compressed state complexity/throughput space. The mapping illustrates the similarity between the compressed state complexity and state complexity distributions and, since the measures are both normalizations of the number of unique states occurring during an evolution, the differences between the distributions are a function of the logical to temporal evolution lengths—the logical complexity.

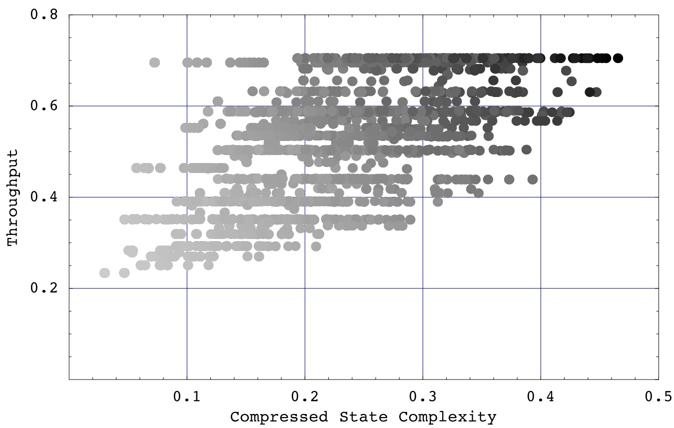

Figure 1. The mapping of state complexity values in compressed state complexity/throughput space for 3-3-3 evolutions. Darker colors indicate evolutions with greater state complexity.

2.1 Theoretical Boundaries

By the definition of state complexity and compressed state complexity, there should exist an inverse relationship between compressed state complexity and logical complexity. However, as seen in Figure 2, logical complexities are distributed throughout the compressed state complexity space along any line of constant throughput, which implies that there is no definitive inverse relationship between compressed state complexity and logical complexity. Furthermore, there exists the nearly direct relationship between state complexity and compressed state complexity.

Theoretically, the minimum compressed state complexity corresponds to an evolution with the minimum state complexity and the maximum logical complexity at a given throughput. The minimum

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compressed state complexity should therefore correspond to an evolution with low exploration of the state space and the most phase lags possible. The theoretical maximum corresponds to an evolution that explores a great deal of its state space, but has a minimum number of phase lags, meaning the carriages are essentially independent. While the conditions for the theoretical minimum are possible, the conditions for the maximum theoretical compressed state complexity are unlikely for the model used. To obtain the theoretical minimum and maximum, we have assumed that it is possible for the evolutions with a given state complexity to correspond to any logical complexity possible for a given throughput. However, because of system dynamics, this assumption is invalid, and practical limitations on the range of one complexity measure exist from the value of another complexity measure and are evident in Figure 2.

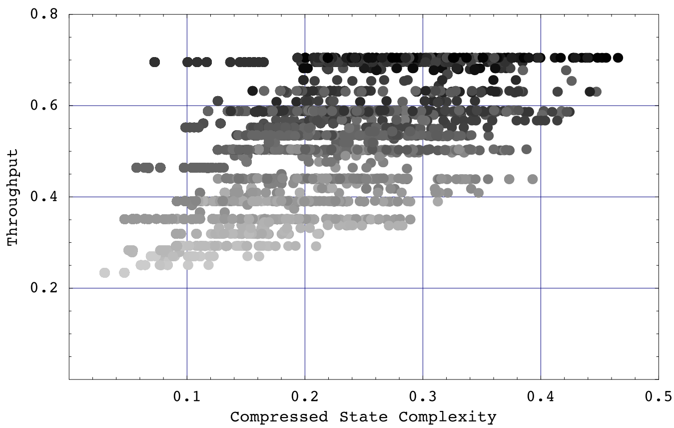

Figure 2. The mapping of logical complexity values in compressed state complexity/throughput space for 3-3-3 evolutions. Darker colors indicate evolutions with greater logical complexity.

Figure 3 is a direct comparison of the compressed state complexity and state complexity and supports the correlation evident in the mapping of state complexities into compressed state complexity space in Figure 1. However, the correlation of 0.939 supports the general trend in the two-dimensional representation of the relationship in Figure 3. The most interesting feature of Figure 3 is, however, the lines of various slopes that correspond to evolutions with constant logical complexity. Since each line extends over a limited range of the compressed state complexity/state complexity space, we can see directly that there are effective limits on the values of logical complexity with respect to both the state and compressed state complexities. Because

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the state and compressed state complexities are both based on the number of distinct states entered in the course of an evolution, the number of states must increase along a line of constant logical complexity as the state and compressed state complexities are simultaneously increased. Qualitatively, the number of states increases along the line of constant logical complexity as the state and compressed state complexities increase. The fact that lines of constant logical complexity do not extend throughout the entire space and have upper and lower limits of state and compressed state complexity implies that the number of distinct states entered is effectively bounded by the state and compressed state complexities as well as the logical complexity. The distribution of evolutions in the compressed state complexity/throughput space demonstrates that the combinations are approximately linear or at least, assuming the minimum theoretical boundaries of the state and logical complexities, are approximately linear with respect to throughput.

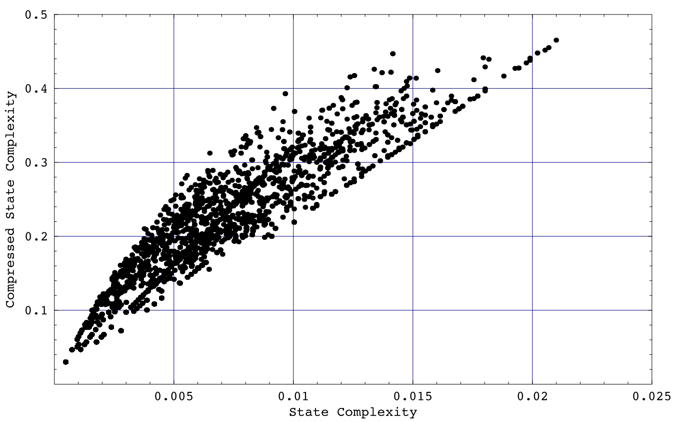

Figure 3. A comparison of the compressed state complexity to the state complexity for the set of complete 3-3-3 evolutions shows not only a general direct relationship between the measures, but also lines of constant slope that correspond to lines of constant logical complexity.

The ratio of two linear equations r at a common dependent variable y given by equation (1) always results in a nonlinear relationship with respect to the dependent variable, which is analogous to the ratio of the state complexity and the logical complexity relationships to throughput:

$$r = \frac{x_1}{x_2} = \frac{m_1}{m_2}\left(\frac{y - b_1}{y - b_2}\right). \tag{1}$$

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Rearranging equation (1) in terms of r to express y as the independent variable results in equation (2). The first- and second-order derivatives of the relationship between the dependent variable y and the ratio of independent variables r are presented in equations (3) and (4):

$$y = \frac{r b_2 - \frac{m_2}{m_1} b_1}{r - \frac{m_2}{m_1}} \tag{2}$$

$$\frac{dy}{dr} = \frac{(b_1 - b_2)\frac{m_2}{m_1}}{\left(r - \frac{m_2}{m_1}\right)^2} \tag{3}$$

$$\frac{d^2 y}{dr^2} = \frac{(b_1 - b_2)\frac{m_2}{m_1}}{\left(r - \frac{m_2}{m_1}\right)^3}. \tag{4}$$

In the context of compressed state complexity, where compressed state complexity equals r, throughput equals y, the subscript 1 denotes state complexity values, and the subscript 2 denotes logical complexity values, we are interested in the region where throughput is less than $+\infty$ and greater than the maximum of b_1 and b_2. In this region, the curve always has positive curvature, since the limits of equation (2) are finite value b_2. The slope is, however, dependent on the relative values of the intercepts of the linear equations.

Since the boundaries of the compressed state complexity asymptotically approach a limiting value, we can get an idea of the ratio of the slopes of the actual distribution of corresponding evolutions in the state and logical spaces, and we can bound one variable based on another. While the limiting values of the compressed state complexity do not indicate the absolute values of the slope of corresponding state and logical complexity relationships to throughput, we can obtain the theoretical boundaries by comparing the actual minimum and maximum boundaries of state and logical complexity. From the logical and state complexity distributions with respect to throughput for the set of complete evolutions presented along with a line defining the assumed minimum and maximum complexity boundaries shown in Figure 4, the minimum logical complexity boundary encompasses nearly all points and does not correspond to a fixed number of equivalent phase lags. It therefore includes a few points that correspond to evolutions in which carriages act independently for some fraction of the

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evolution, resulting in no phase lags. The maximum state complexity boundary is not necessarily linear. Therefore, any conclusions based on results using the maximum state complexity must be made in the context of the assumed linearization of the boundary.

The equations of linear boundaries assumed in Figure 4 are given in equations (5) to (8). The equations for the minimum and maximum theoretical boundaries of compressed state complexity, based on the assumed actual boundaries of the logical and state complexities, are described in equations (9) and (10):

$$\text{Min logical complexity: } R = 26.2255\, C_L - 0.116713 \tag{5}$$

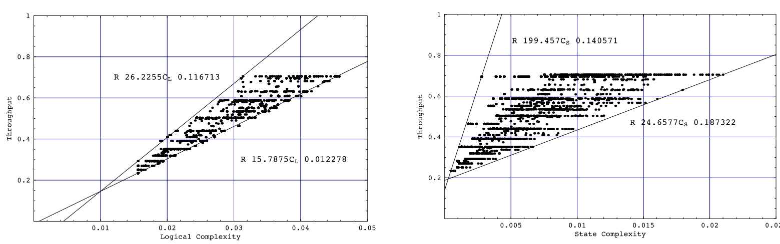
$$\text{Max logical complexity: } R = 15.7875\, C_L - 0.012278 \tag{6}$$

$$\text{Min state complexity: } R = 199.457\, C_L - 0.140571 \tag{7}$$

$$\text{Max state complexity: } R = 24.6577\, C_L - 0.187322 \tag{8}$$

$$\text{Min compressed state complexity: } R = \frac{-0.012278\, C_C - 0.0111265}{C_C - 0.0791524} \tag{9}$$

$$\text{Max compressed state complexity: } R = \frac{-0.116713\, C_C - 0.199233}{C_C - 1.06358}. \tag{10}$$



(a) (b)

Figure 4. The two-dimensional distributions of 3-3-3 evolutions in (a) logical complexity/throughput space and (b) state complexity/throughput space. The distributions include the assumed linearizations of the minimum and maximum complexity boundaries.

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The lines, along with the distribution of 3-3-3 in compressed state complexity/throughput space, are plotted in Figure 5 (the dashed line indicates the line corresponding to the ratio of the maximum state complexity and the minimum logical complexity boundary defined by a constant equivalent phase lag). By equations (9) and (10), the limiting values of the compressed state complexity for 3-3-3 evolutions are 0.079 and 1.064 for the minimum and maximum boundaries, respectively.

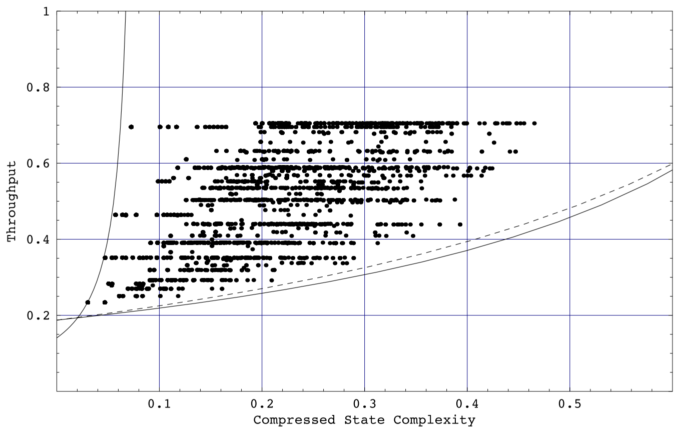

Figure 5. The distribution of 3-3-3 evolutions with respect to compressed state complexity and throughput along with the extreme boundaries defined by assumed linearizations of the minimum and maximum boundaries of the state and logical complexities.

To identify what logical complexities are possible when state complexity is maximal, and what state complexities are possible when logical complexity is maximal, we compare the maximum state complexity and maximum logical complexity and the minimum state complexity and minimum logical complexity. The curves describing these ratios are presented in Figure 6, along with the minimum and maximum compressed state complexity boundaries.

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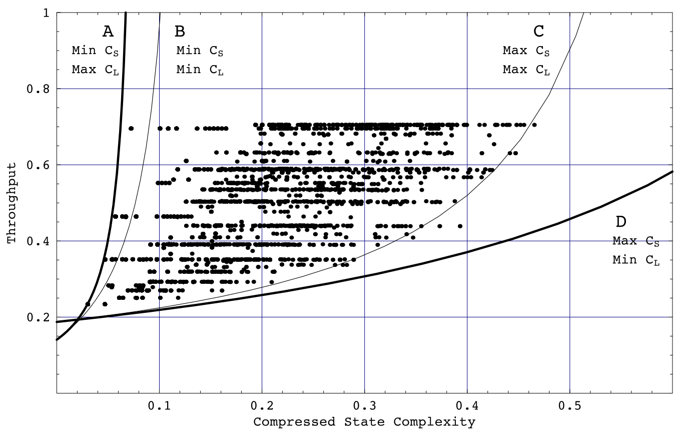

Figure 6. The distribution of 3-3-3 evolutions with the (A) minimum and (D) maximum compressed state complexity boundaries along with the curves describing (B) the ratio of minimum state complexities to minimum logical complexities and (C) the ratio of maximum state complexities to maximum logical complexities. These curves collectively indicate some limits on corresponding values of state complexity and logical complexity.

2.2 Algorithmic Complexity

For a system of a given size, a bound on the algorithmic complexity certainly exists and is equal to the maximum randomness or lack of any patterns. If we define the lack of patterns as the maximum distinct states that can logically be entered, then for even systems of modest size, the algorithmic complexity may be extremely large, but still finite and tractable. Additional patterns equate to more information required for a complete description, so increases in the number of queues, shafts, or magazines, which create more states/patterns and therefore result in greater algorithmic complexity. While the algorithmic complexity has a finite limit for a given system size, the bound is significantly greater than that for the compressed state complexity. Additionally, the bounds on algorithmic complexity increase with increases in system size, while the compressed state complexity bound remains relatively unchanged. The compressed state complexity therefore does not match the definition of algorithmic complexity with respect to boundaries. For algorithmic complexity, the evolution with the greatest throughput is also the simplest (involving all carriages throughout the evolution). For the greatest compression, the simplest evolution has the fewest patterns, and the number of states defining each pattern is minimal.

The algorithmic complexity and compressed state complexity do match at the greatest complexity/performance combination. At lower

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throughputs, compressed state complexity also matches the algorithmic complexity at both the minimum and maximum complexity boundaries. The time at which carriages halt in an evolution has little impact on the algorithmic complexity, although, if carriages halt earlier, there is less potential to explore the larger possible state space resulting from greater numbers of carriages operating simultaneously. The minimum and maximum algorithmic complexity boundaries therefore remain relatively constant over a range of throughputs, but might have a tendency to decrease as carriages halt earlier in the evolution, which leads to lower throughput. At the lowest throughput, only a single carriage is involved, which always has the fewest patterns composed of the fewest states, and the algorithmic complexity is always less than the algorithmic complexity of complete evolutions where all carriages are involved. This description matches the distribution of evolutions with respect to compressed state complexity very closely, although it is unclear if the slopes of the boundaries conform with the definition of algorithmic complexity.

2.3 Compressed State Complexity, Throughput, and Robustness

The salient differences between algorithmic complexity and compressed state complexity may result in a useful relationship between complexity and performance with respect to optimization, which is identifiable partly through the comparison of mean values of compressed state complexity and throughput for different system sizes.

Table 1 presents the mean compressed state complexity values for system sizes with relative numbers of queues, shafts, and magazines and /or consisting of sufficient numbers of evolutions to make them of interest. The table also includes the mean values for evolutions that mimic smaller systems with respect to the number of shafts. As with state complexity, it is misleading to include incomplete evolutions because of the effects of evolution length—in this case, logical evolution length—on compressed state complexity. They are, however, included to indicate how incomplete evolutions can raise the mean complexity.

Since incomplete evolutions misrepresent the mean compressed state complexity, we only consider complete evolution sets and evolutions of the most-robust configurations. Table 1 indicates that the mean compressed state complexity of all evolutions corresponding to the most-robust configurations is greater than the mean for the set of all complete evolutions for all system sizes involving more than one queue. While the greater mean compressed state complexities of the most-robust evolutions appear to indicate a correlation between complexity and adaptability on face value, especially because of the consistency in the trend across the complete range of system sizes considered, the mean values provide no indication of the number of

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System	N	C	R	UN	UC	UR	M
1-2-2	0.13500	0.13500	0.35000	0.15200	0.15200	0.15330	0.05000
1-2-3	0.16008	0.16008	0.16008	0.17117	0.18117	0.19429	0.06565
1-2-4	0.17805	0.17805	0.17805	0.19989	0.19989	0.21964	0.07760
1-3-2	0.22594	0.22594	0.22594	0.26371	0.26371	0.28533	0.09350
1-3-3	0.23631	0.23631	0.23631	0.28311	0.28311	0.32146	0.12587
1-3-4	0.25912	0.25912	0.25912	0.30849	0.30849	0.36246	0.15074
1-4-2	0.32251	0.32251	0.32251	0.38041	0.38041	0.39501	0.14882
1-4-3	0.33263	0.33263	0.33263	0.40782	0.40782	0.44056	0.20531
1-4-4	0.34868	0.34868	0.34868	0.42914	0.42914	0.47275	0.23237
2-2-2	0.09994	0.09014	0.09484	0.11270	0.09987	0.09540	0.04250
2-2-3	0.12488	0.10915	0.11630	0.14119	0.12344	0.11929	0.05554
2-2-4	0.14508	0.12399	0.13232	0.16293	0.14047	0.13516	0.06550
2-3-2	0.16296	0.15935	0.16785	0.19374	0.19316	0.21647	0.06483
2-3-3	0.18890	0.18015	0.19710	0.22979	0.22287	0.27064	0.09959
2-3-4	0.21610	0.20247	0.22566	0.26076	0.24831	0.30780	0.12261
2-4-2	0.22065	0.20416	0.21283	0.26351	0.24598	0.25595	0.10827
2-4-3	0.24603	0.23440	0.25069	0.30419	0.28839	0.29417	0.15878
3-2-2	0.08369	0.07571	0.08060	0.09429	0.08257	0.08259	0.03688
3-2-3	0.10573	0.09196	0.09999	0.11961	0.10290	0.10475	0.04796
3-2-4	0.12352	0.10483	0.11438	0.13887	0.11811	0.11997	0.05643
3-3-2	0.13812	0.13090	0.14400	0.16543	0.15947	0.17783	0.05416
3-3-3	0.16559	0.15446	0.17488	0.20284	0.19300	0.23372	0.08615
3-3-4	0.19189	0.17643	0.20071	0.23304	0.21867	0.24975	0.10730
3-4-2	0.18892	0.17802	0.18231	0.23008	0.24450	0.25560	0.08268
4-2-2	0.07193	0.06297	0.06726	0.08075	0.06770	0.06801	0.03346
4-2-3	0.09233	0.07623	0.08277	0.10437	0.08425	0.08475	0.04336
4-2-4	0.10872	0.08706	0.09398	0.12233	0.09780	0.09672	0.05092
4-3-2	0.12237	0.11678	0.13204	0.14700	0.14203	0.16470	0.04682
4-3-3	0.14760	0.13713	0.16140	0.18137	0.17212	0.21288	0.07591
4-4-2	0.16730	0.15927	0.17677	0.20403	0.20168	0.23941	0.07159

Table 1. The mean compressed state complexity for evolution subsets of different system sizes (N = nonhalting, C = complete, R = robust, UN = unique and nonhalting, UC = unique and complete, UR = unique and robust, M = mimics).

evolutions in the most-robust set and if it is significant enough to reach a conclusion regarding robustness and complexity or how values are distributed about the mean. Nor can we tell from the mean value what makes a configuration robust or restricts its robustness.

This lack of information also prohibits any conclusions regarding a relationship between complexity, robustness, and throughput, which we have seen is also consistently greater for the most-robust evolutions compared to all complete evolutions. To reach any conclusions about the relationship between complexity, robustness, and throughput, we must examine the relationships at all levels of robustness and determine what is required of an evolution for a certain level of robustness. In doing so, we must also revisit the effects of relative numbers of queues, shafts, and magazines.

We would like to investigate the mean values at all levels of robustness for all complete 2-3-4 evolutions, which is the same system size investigated in the discussion of robustness with respect to state complexity. Figure 7 presents the correlation between the level of robustness and the state complexity, compressed state complexity, logical complexity, and throughput. The maximum mean values of the state complexity, compressed state complexity, and throughput do not correspond to the most-robust configurations. For the 2-3-4 evolutions, the maximum mean values for all attributes occur at a level of robustness of 33, and all normalized mean values therefore have a value of unity at this level of robustness. The similarity in the shapes between state and compressed state complexity curves implies that correlations are present not only between each complexity measure and throughput, but also between the various complexity measures.

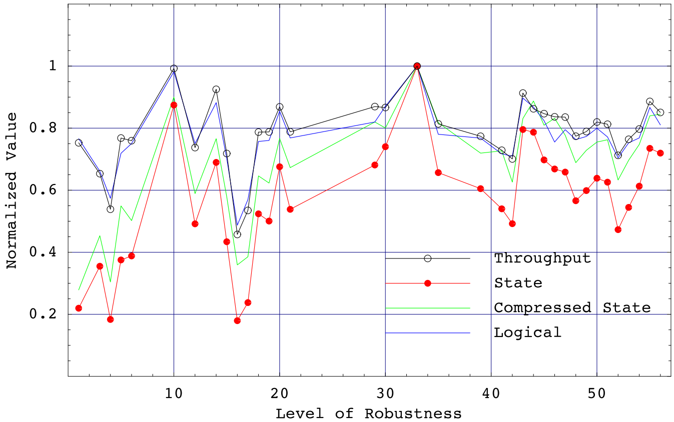

Figure 7. A comparison of normalized mean values for throughput, compressed state complexity, state complexity, and logical complexity for all complete 2-3-4 evolutions at all levels of robustness.

Figure 8 shows that the majority of evolutions (74%) define only three levels of robustness: 6, 21, and 56. Of these evolutions, 64% correspond to the most-robust configurations. The shapes of the

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curves in Figure 7 are then arguably misleading—levels of robustness composed of small number of evolutions have little averaging and may alter our perception of the relationship between robustness and complexity/throughput. Therefore, the relationship between mean values and the level of robustness should always be viewed in the context of the number of configurations or evolutions, as well as the distribution of values involved at each level, because the mean alone by no means indicates trends or correlations.

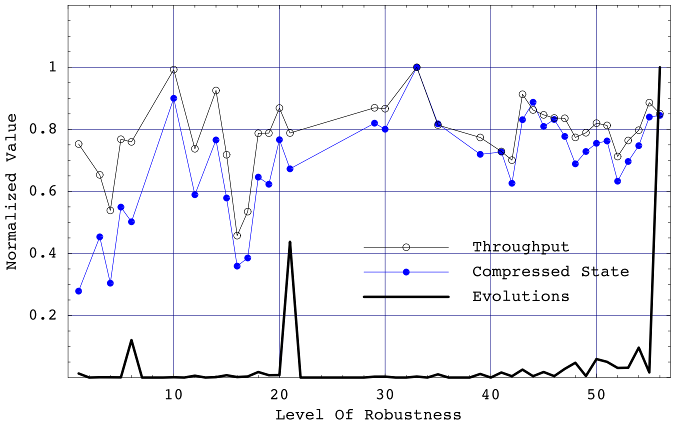

Figure 8. The throughput and compressed state complexity level of robustness curves with the normalized number of evolutions at each level of robustness.

To give us an idea of the distribution of values for complexity measures and throughput, we use the set of evolutions of the most-robust configurations for complete 2-3-4 evolutions as an example. Figure 9 presents the compressed state complexity value for each of the 10 080 evolutions comprising the most-robust set. When the evolutions are sorted according to their compressed state complexity value, we get an idea of the distribution. Figure 9 presents the compressed state complexity value for each of the 10 080 evolutions comprising the most-robust set. Figure 10 shows the distribution of sorted values relative to the mean compressed state complexity for the most-robust configurations. Figures 11 through 13 present the distributions of the throughput, state complexity, and logical complexity in the same format.

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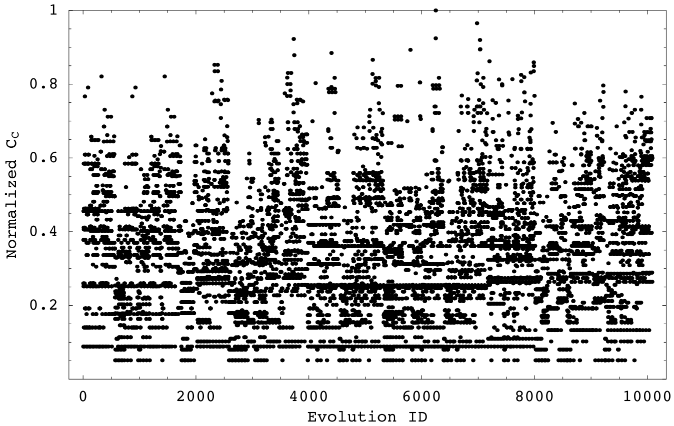

Figure 9. The distribution of compressed state complexity values for the 10 080 evolutions corresponding to the most-robust 2-3-4 configurations.

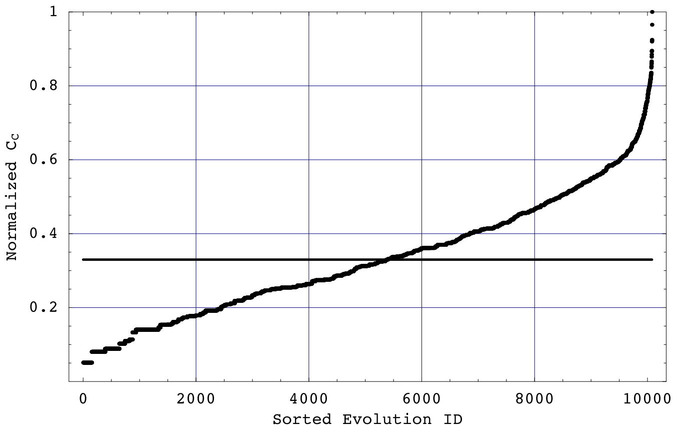

Figure 10. The distribution of compressed state complexity values for the most-robust 2-3-4 evolutions, sorted according to their compressed state complexity values.

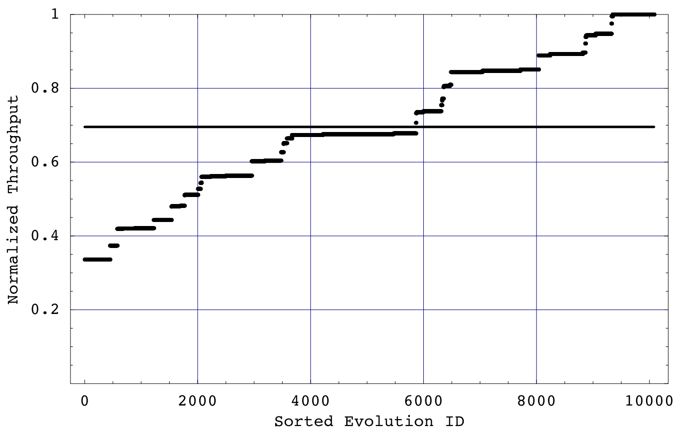

Figure 11. The distribution of sorted normalized throughput values for the most-robust 2-3-4 evolutions.

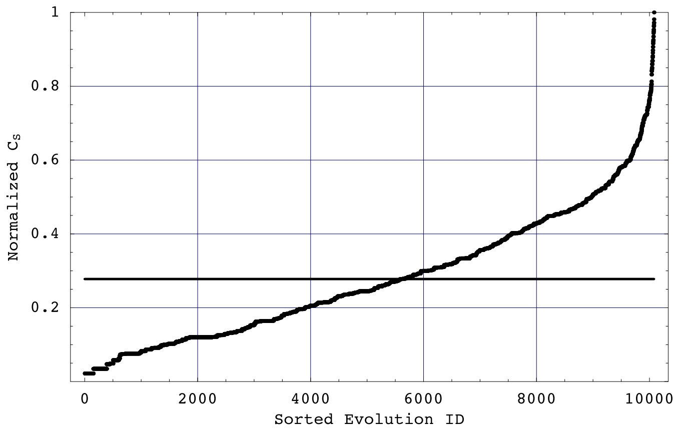

Figure 12. The distribution of sorted normalized state complexity values for the most-robust 2-3-4 evolutions relative to normalized mean value.

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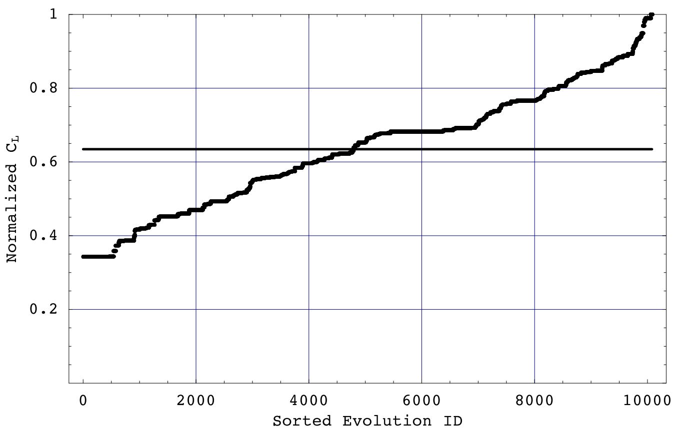

Figure 13. The distribution of sorted normalized logical complexity values for the most-robust 2-3-4 evolutions.

Figures 14 and 15 present the number of configurations at each level of robustness, normalized with respect to the corresponding maximum number of configurations, for a range of systems with both two and three shafts. The figures also include the mean throughput and compressed state complexity values for levels of robustness (not connected) at levels of robustness that are populated by configurations. Once again, the points indicating the normalized number of configurations are joined to create a curve, despite the fact that the relationship between the number of configurations and robustness is not continuous, simply to make comparisons between populations and mean attribute values and to illustrate the similarity between system sizes.

All figures have peak populations at the 1^{st}, 6^{th}, and 21^{st} levels of robustness. For systems with four magazines, a peak also exists at the 56^{th} level of robustness. As the number of queues is increased, the relative number of configurations/evolutions corresponding to lower levels of robustness increases. This trend is evident in the comparison of the graphs in any column in Figures 14 or 15. The addition of shafts also results in the presence of nonpeak populations for any size system, although these nonpeak populations are a function of the number of each type of location. It is difficult to make comparisons between distributions involving different numbers of magazines: since the levels of robustness are dependent on the number of magazines—up to the 21^{st} level of robustness—the relative numbers of configurations are strikingly similar.

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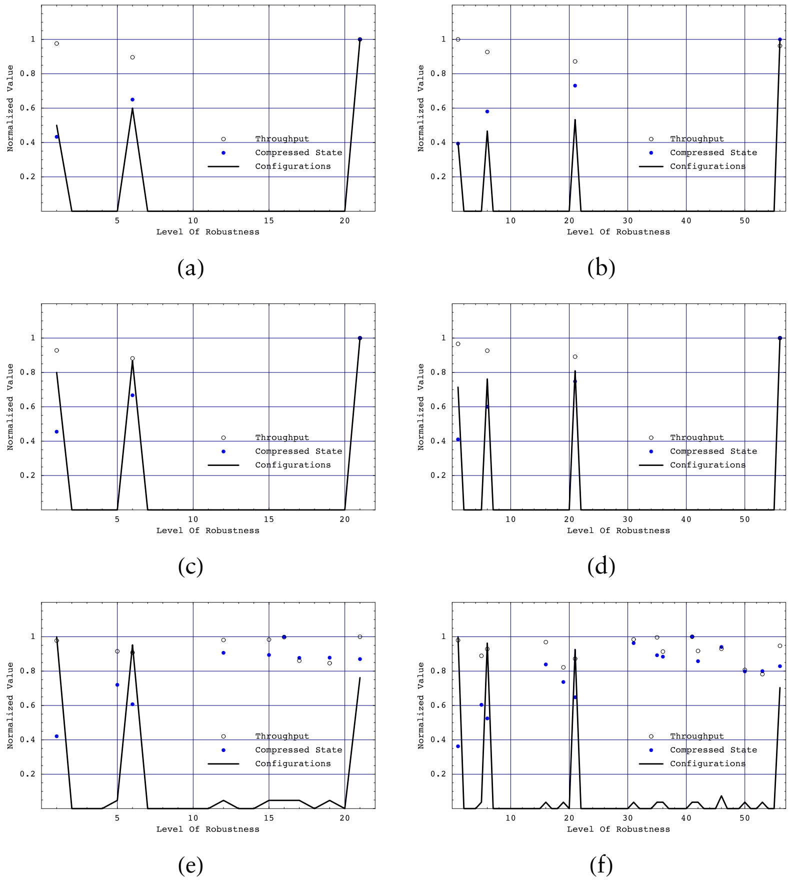

Figure 14. The mean throughputs and mean compressed state complexities compared to the curves of the number of configurations at all levels of robustness for all complete (a) 2-2-3, (b) 2-2-4, (c) 3-2-3, (d) 3-2-4, (e) 4-2-3, and (f) 4-2-4 evolutions.

The correlation between logical complexity and robustness also changes in the context of connectivity. For configurations with present connectivity for all magazines and levels of robustness greater than the level corresponding to robust mimicry of a three-magazine system, the correlation value is 0.655, which is significantly greater than the correlation value of 0.310 for mean logical complexity values for configurations at all levels of robustness. However, the correlation values between mean compressed state complexity and robustness and between mean state complexity and robustness do not change significantly relative to the connectivity considered. The correlation value between the compressed state complexity and robustness

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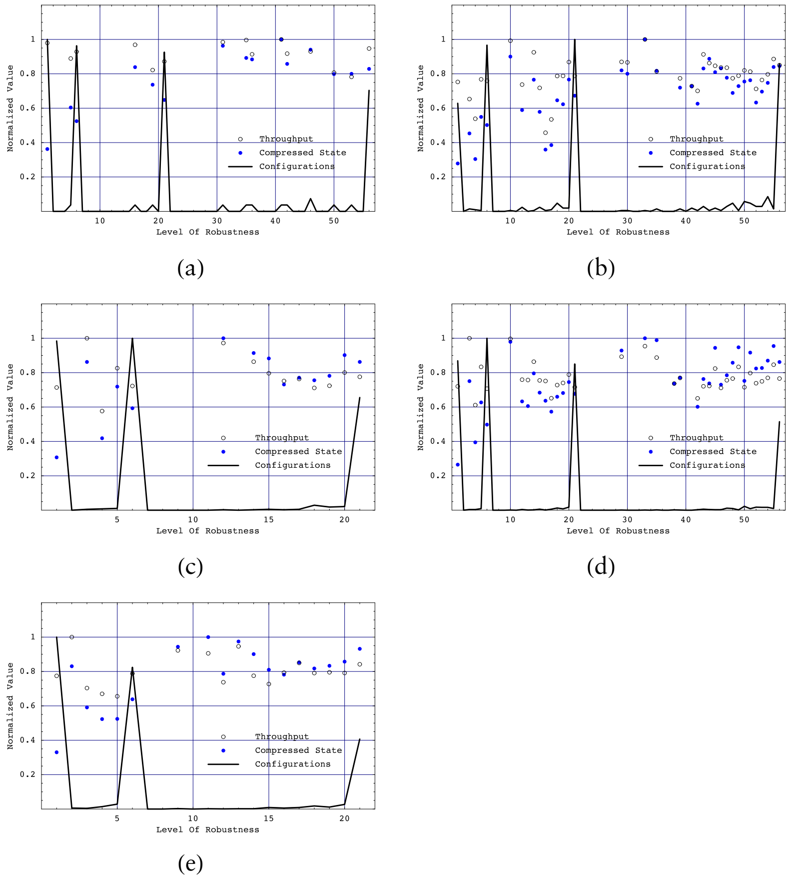


Figure 15. The mean throughputs and mean compressed state complexities compared to the curves of the number of configurations at all levels of robustness for all complete (a) 2-3-3, (b) 2-3-4, (c) 3-3-3, (d) 3-3-4, and (e) 4-3-3 evolutions.

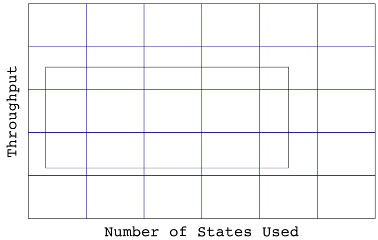
is 0.850 for configurations at all levels of robustness and 0.851 for configurations with present connectivity for four magazines and levels of robustness greater than 21. Between the 6^{th} and 21^{st} levels of robustness, which correspond to the levels for robust mimics of two- and three-magazine systems, the correlation value is also 0.851. Similarly, the correlation value between the state complexity and robustness is 0.787 for all configurations and 0.836 for configurations with present connectivity for all four magazines. Correlation values remain relatively unchanged for these complexity measures because fewer states are possible in evolutions with fewer numbers of active magazines. Therefore the cyclic relationship between robustness and mean values evident for throughput and logical complexity as the level of robustness passes through a level corresponding to a robust mimic is not present for the state and compressed state complexities.

3. Number of Unique States Used

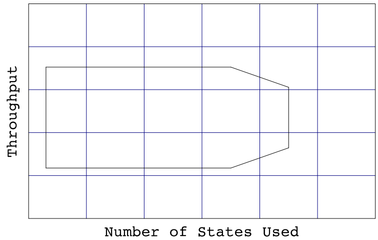
3.1 Algorithmic Complexity

In the discussion of state complexity, we saw that normalization by the temporal evolution length introduced a correlation between state complexity and throughput. This resulted in a divergence from a strict interpretation of algorithmic complexity and implies that a description of complexity solely by the number of states is a closer representation of algorithmic complexity.

However, under the assumption of a constant minimum/maximum theoretical number of states, removal of normalization, while bringing complexity conceptually closer to algorithmic complexity, removes any correlation between complexity and throughput. The theoretical distribution is presented in Figure 16(a) for intermediate throughputs under this assumption and indicates that the information required to describe an evolution at a lower throughput can theoretically equal the information required to describe an evolution at a higher throughput, if both evolutions experience the same states, regardless of the timing, assuming the lower throughput evolution can enter the equivalent states as the higher throughput evolution before halting of carriages begins.



(a)



(b)

Figure 16. Qualitative theoretical distributions of the number of unique states with respect to throughput. The minimum and maximum numbers of state boundaries in (a) are vertical, suggesting no correlation between the number of states and throughput exists because, across the range of intermediate throughput values, evolutions can theoretically enter the same number of states, but carriages may halt at different times. The boundaries can be refined in (b) at near minimum and maximum throughputs, because more information about the possible halting sequences is known.

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While the theoretical distributions offer no apparent correlations between the number of states and throughput, actual distributions, like the example distribution of 2-3-4 size evolutions in Figure 17, do exhibit correlations. The reasons are related to several factors, including the accuracy of the theoretical boundaries and the validity of the assumptions used in their creation. More importantly, the size of the potential state space and the susceptibility to halting of carriages are directly related to the connectivity of a configuration in their context of the queue distribution used.

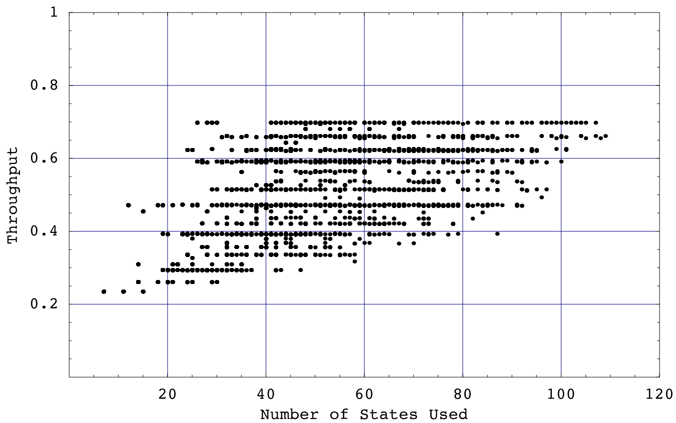

Figure 17. The distribution of complete evolutions of 2-3-4 size systems reveals a relationship between the number of unique states and throughput.

At all intermediate throughput values, we know that carriage halting must occur, but we do not know when halting occurs in an evolution. At lower and higher throughput, we do have more information regarding halting, which refines the theoretical boundaries, depending on the system size at the minimum and maximum throughputs as in Figure 16(b), to reflect the fact that not all combinations of carriage states are possible.

The cause of the overestimation of the maximum theoretical state complexity compared to the actual maximum values at any throughput is a result of the ambiguous relationship between the timing of carriage halting and throughput. The theoretical boundaries offer few clues to explain the actual relationships between the number of states and throughput evident in distributions such as that in Figure 17, simply because the minimum and maximum boundaries of the number of distinct system states assume all possible state combinations for a given throughput. The set of all possible state combinations is a significant overestimation of the actual number of states, because it as-

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sumes nondeterminism in evolutions and therefore includes unobtainable system states.

3.2 Qualitative Characterizations

The relationship between variety in queue distribution, connectivity, the number of states, and throughput is evident when we examine specific evolutions using, for example, the 2-3-4 system size distribution presented in Figure 18.

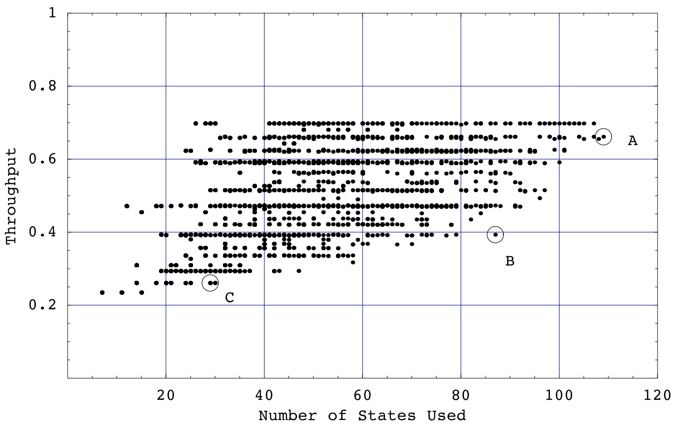

Figure 18. Points along the maximum number of unique states boundary illustrate the relationships between connectivity and variety in the queue distribution and establish a correlation between the number of unique states and throughput.

The evolution corresponding to point A is configuration 227 261, with a queue distribution of 20-40-20-20. The throughput of this evolution is slightly less than the maximum because the second carriage halts two cycles prior to the end of the evolution, and the third carriage halts one cycle prior to the end of the evolution. Since halting occurs, the system has the opportunity to enter more potential states, further illustrating the maximum complexity at submaximal throughput previously seen with respect to state and compressed state complexity. However, the more important factor in the exploration of the state space is the variety of item types in the queue distribution.

The connectivity of configuration 227 261 is presented in Figure 19. When the queue distribution is most uniform, the number of states is typically near maximal. Only for one of the most uniformly distributed queue distributions (40-20-20-20) is the number of distinct states relatively low. This queue distribution corresponds to an

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evolution with maximum throughput, where all carriages remain operational throughout the evolution.

$$(SQ) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \qquad (S\,M) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix} \qquad (Q\,M) = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 2 & 2 & 3 \end{pmatrix}$$

Figure 19. The incidence matrices for configuration 227 261 2-3-4.

For the configuration corresponding to point B in Figure 18, configuration 251 787 with a 20-40-20-20 queue distribution, fewer states are possible for a queue distribution with maximum variety because of a decrease in connectivity. Figure 20 presents the connectivity for configuration 251 787. This connectivity limits the state space entered because, while the first carriage enters all possible states except those corresponding to the first magazine, the second carriage only enters states related to the first magazine, and the third carriage never enters the second queue. At the same time, the limited connectivity also results in the halting of carriages two and three, since they can only carry certain items loaded in particular locations. Therefore, this connectivity, for this particular queue distribution, results in less diversity in system states and lower throughput, which establishes a correlation between the number of states and performance.

$$(SQ) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} \qquad (S\,M) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix} \qquad (Q\,M) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 2 \end{pmatrix}$$

Figure 20. The incidence matrices for configuration 251 787 2-3-4.

To extend this concept of the association between sparse connectivity, limitation on the number of system states, and the early onset for carriage halting to its logical limit, we can imagine a configuration that is capable of complete delivery of all queue distributions and either minimal valid connectivity in the *SM* or *SQ* matrix, within the definition of valid configuration. For the 2-3-4 system size, an example configuration is 94 495, point C in Figure 18, which has connectivity shown in Figure 21.

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$$(SQ) = \begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \qquad (S\,M) = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \qquad (Q\,M) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 3 \end{pmatrix}$$

Figure 21. The incidence matrices for configuration 94 495 2-3-4.

When the greatest diversity of item types is present in the queues, the first and second carriages still only experience four distinct states each. The majority of system states therefore result from the evolution of the third carriage. The first and second carriages halt relatively early in the evolution, after they exhaust all items bound for the fourth magazine in the second queue. The remaining number of system states is then a function of the variety of individual carriage states entered by the third carriage, which is dependent on the variety of items destined for the first, second, and third magazines.

The example evolutions demonstrate that connectivity has a direct effect on both the number of unique states entered by a system and when carriages halt in evolution, and therefore connectivity introduces a direct relationship between the number of states and throughput. However, placing the connectivity in the context of the wrong environmental conditions is key to the relationship between connectivity, the number of states, and throughput. In each of the example evolutions located along the maximum number of states boundary in Figure 18, the queue distributions have all had the greatest variety of item types. That is, items corresponding to all magazines are present in some numbers in the queue. The absolute fraction of item types is important to the number of states entered and the throughput, even if maximum variety exists, and can significant affect these attributes, depending on the connectivity of a configuration.

The association between maximum variety in the queue distribution and the number of unique system states is that more item types result in potentially more magazines that a carriage can enter, and the loading, transit, and unloading states that follow. The relationship between the item variety, connectivity, and the number of states is shown in Figure 22, which presents a comparison between the distribution of complete evolutions corresponding to the four queue distributions in which all item types are present and the distribution of all complete evolutions.

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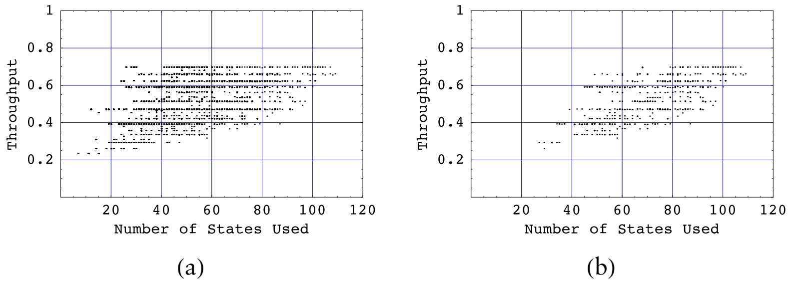

Figure 22. A comparison of the distributions of (a) all complete 2-3-4 evolutions and (b) complete evolutions resulting from queue distributions with maximum variety shows that maximum queue variety tends to increase the number of unique states entered in an evolution.

Figure 23 presents the distributions with zero to three item types absent and indicates that, as variety decreases and configurations essentially mimic systems with fewer magazines, the minimum and maximum boundaries on the number of unique states in an evolution decrease. The item variety effect is also sloped, indicating that item variety also affects both the number of states and throughput. While it is clear from the distributions in Figure 23 that item variety is strongly related to the number of unique states, the distributions do not themselves indicate any relationship between item variety and throughput since they are two-dimensional, and the ranges of throughput values are constant for all sets of evolutions with various degrees of item variety. However, when we look at the mean throughput values for evolutions with different item variety, which are present in Table 2 along with the mean number of states, we see that the mean throughput tends to increase when more item types are present in an evolution.

As with the qualitative description of the effects of connectivity on the number of states and throughput, the slope of the item variety effect is unclear. However, based on the slopes of curves in the individual distributions of evolutions at various degrees of item variety in Figure 23 and the slope of the curve describing the relationship between the mean number of states and throughput corresponding to various degrees of item variety, the variety of item types appears to have a more significant effect on the number of states than on throughput, and we therefore draw a shallower slope than that for the connectivity.

From the discussion, it is apparent that greater connectivity offers greater adaptability and therefore performance in the face of variable and unknown queue distributions.

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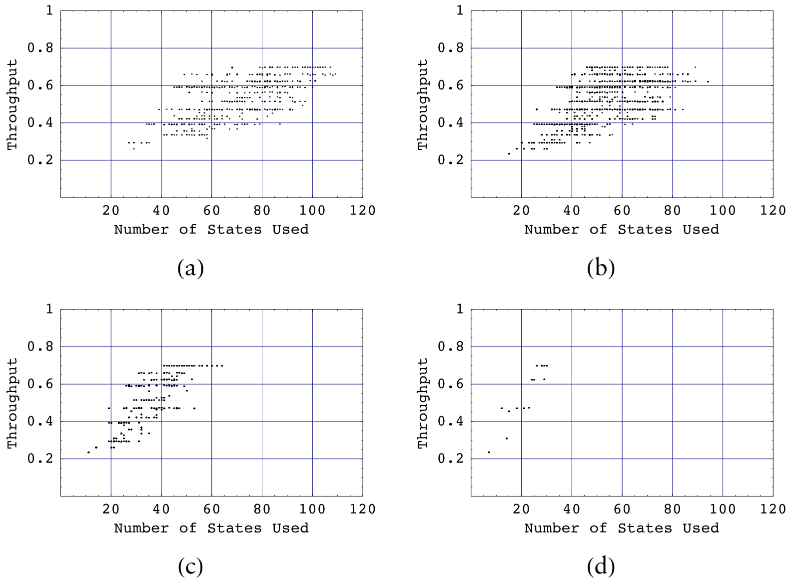

Figure 23. The effective bounds on the number of unique states in an evolution for (a) the greatest variety of item types, (b) when a single item type is absent, (c) when two item types are absent, and (d) when a single item type is present in a queue distribution.

	Mean No. of States	Mean R
Greatest item variety	66.2	0.511
Single item type absent	47.5	0.487
Two item types absent	29.9	0.459
Single item type	14.0	0.419
All complete	36.2	0.467

Table 2. The mean number of unique states and throughput as a function of the amount of variety in the queue distribution, along with the mean value of all complete evolutions.

3.3 Robustness

Although throughput is only one factor in identifying a suitable configuration, we need to identify whether we can use the number of unique states as a tool for identifying candidate configurations with respect to robustness in the same manner as with respect to throughput. Figure 24 presents the mean number of unique states at various levels of robustness for 2-3-4 size systems, normalized by the maximum

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mean number of unique states. The shape of the normalized mean number of states curve is quite similar to the state complexity and compressed state complexity curves and, to a lesser extent, the logical complexity and throughput curves presented in the discussion of compressed state complexity.

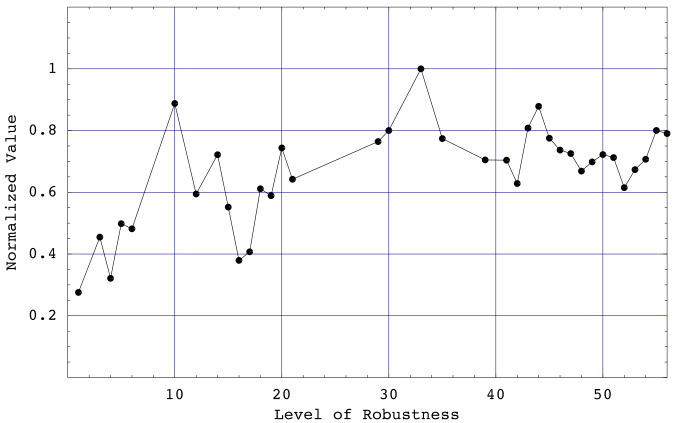

Figure 24. The normalized mean number of unique states curve across possible levels of robustness.

The similarity in shapes indicates not only a relationship between the number of unique states and other complexity measures, but also provides further evidence of a correlation between the number of states and throughput. The similarity in shapes of the curves also implies that any conclusions drawn from the curves of other complexity measures also apply to the number of unique states—in particular, we are interested in the reasons behind trends in the curves. That is, how connectivity is related to robustness and therefore how robustness and the number of states are related through a common association with connectivity.

Figure 25 presents the same curve relating the mean number of states over the range of robustness values, but superposed with the normalized number of configurations at each level of robustness. The 1^{st}, 6^{th}, 21^{st}, and 56^{th} levels—those corresponding to the most robust mimics—dominate the population. Comparison of the highlighted mean numbers of unique states at these levels is more meaningful, since the number of configurations at these levels is on the same order of magnitude. When we account for populations and comparable averaging, a correlation between the number of unique states and robustness is apparent.

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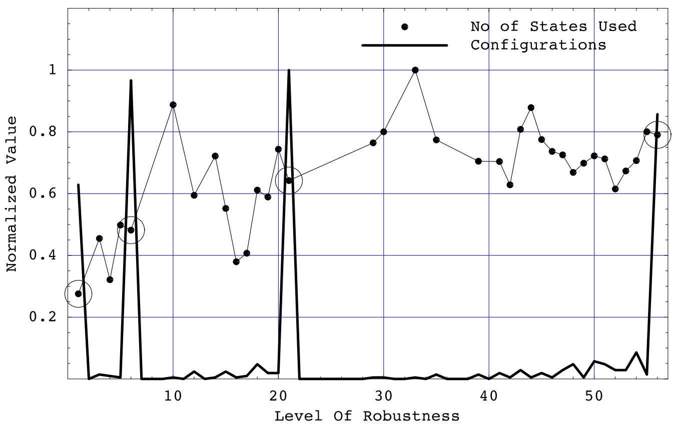

Figure 25. The normalized mean number of unique states at all levels of robustness, superposed with the number of configurations comprising each level for the set of 2-3-4 evolutions.

For queue distributions containing items corresponding to magazines with absent connectivity to queues, the evolutions can only be incomplete. Therefore, for the set of complete evolutions, most configurations exist at the most-robust levels of robustness for various numbers of magazines (the first level for a single magazine, the 6^{th} level for two magazines, the 21^{st} level for three magazines, and 56^{th} level for all magazines with present connectivity to all queues). While the correlation between the mean number of unique states and levels of robustness is 0.599, the correlation between the mean number of unique states and levels of robustness corresponding to the various dominant levels of robust mimicry that are highlighted in Figure 25 is significantly greater at 0.907. This strong correlation for the more meaningful, refined set of configurations suggests that configurations with evolutions that enter, on average, a greater number of unique states also tend to demonstrate adaptability with respect to variations in the queue distributions.

In the analysis of the number of unique states as a complexity measure, we have shown that, although relationships between the number of states and throughput are present, the number of unique states as a complexity measure does not violate the definition of algorithmic complexity, which does not exhibit any relationship to performance. The inherent relationships between the number of unique states and throughput is strongly related to connectivity, which, when placed in the context of the amount of item variety on queue distributions, leads to a qualitative mapping of the number of states/throughput

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space. Connectivity was also shown to play an important role in the relationship between the number of unique states and robustness, for both all complete and unique complete evolution sets. The importance of physical connectivity to various complexity measures, robustness, and throughput leads us to examine the use of connectivity as a predictive measure of complexity, performance, and adaptability.

4. Conclusion

Although the mapping of state complexity values in compressed state complexity/throughput space suggests evidence that a system with high state complexity has high compressed state complexity, there are some evolutions having high state complexity with low compressed state complexity. This kind of behavior also appears in the mapping of logical complexity values in compressed state complexity/throughput space. Due to the effect of logical evolution length, the mean compressed state complexity for evolution subsets of different system sizes, including incomplete evolutions, is different from that of logical or state complexities. Since the mean value itself cannot provide any evidence of what makes a configuration robust or restricts its robustness, we must revisit the effects of relative numbers of queues, shafts, and magazines to conclude the relationship between complexity, robustness, and throughput. From the investigation of the mean values at all levels of robustness for all complete 2-3-4 evolutions, the correlations between the level of robustness and the state complexity, compressed state complexity, logical complexity, and throughput indicate that correlations are present not only between each complexity measure and throughput, but also between the various complexity measures.

In the discussion of the number of unique states used, we can clearly see the strong relationship between connectivity and the inherent relationship between the number of unique states used and throughput. Furthermore, connectivity has an influence on the relationship between the number of unique states used and robustness. However, the measures of complexity presented (logical, state, and compressed state complexities), as well as the number of unique states used, are only dynamic measures of complexity. Static measures of complexity must be considered to describe the potential complexity of a naval weapons elevator system, that is, the total number of possible valid states the system can enter, the physical connectivity, and the logical connectivity of the system. All these remaining measures will be presented in our future work.

Acknowledgments

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Reference

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