Changing the States of Abstract Discrete Systems

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The four classes or states of discrete systems—chaos, complexity, and two types of order—have been established by S. Wolfram in [1]. We describe how the ratio d/c of two parameters, differentiation and centrality, explains changes among these states. Both parameters were developed in the social sciences and have been used to explain changes of state in social systems, which are also discrete systems [2]. Differentiation is the variety within the structure of a discrete system. Centrality measures the variety of outside information presented to a discrete system. Although these ideas can be applied to cellular automata (CAs), the range of these two variables is very limited for a given CA. In this paper we use the idea of a global cellular automaton (GCA), as developed by Wolfram in [3], and a GCA network (GCAN), as developed by S. Chandler in [4], to show how changes in the state of abstract discrete systems are related to changes in the ratio d/c.

1. Introduction

For his book *A New Kind of Science* (NKS), S. Wolfram [1] studied billions of discrete systems composed of many individual cells that evolve over time based on a set of rules. This immense amount of work established that the behavior of such systems over time falls into one of four classes: classes 1 and 2 are ordered, class 3 is chaotic, and class 4 is complex. These classes are sometimes called the *states* of the system. Social systems are discrete systems and later we introduce the term *type of focusing*, where focusing refers to the output of social systems in terms of the four classes or states. Based on Wolfram's studies, we argue that there are only four possible states or types of focusing: two kinds of order, chaos, or complexity.

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Our purposes in this paper are twofold.

1. Introduce two concepts from social science, centrality and differentiation, as the parameters that together determine the resulting class or state of the system. Differentiation is the variety within the structure of a discrete system. Centrality measures the variety of outside information presented to a discrete system.
2. Show how the centrality and differentiation parameters can be applied to abstract discrete systems as successfully as we have used them to study social systems. In particular, we apply these parameters to abstract discrete systems that are in some ways more complicated than those studied by Wolfram. The systems studied are global cellular automata (GCAs), suggested by Wolfram in [3], and GCA networks (GCANs), developed by S. Chandler in [4].

A GCA is a one-dimensional CA that contains two or more rules. A GCAN is a network connecting many GCAs. At each time step a GCA selects which rule to use based on global information from the network. Increasing the number of rules in the GCA increases differentiation (d).

Centrality (c) includes the initial conditions for any CA, and for GCANs, the information flowing from the network to each GCA. Centrality can be lowered by simplifying this information.

The experiments vary d and c independently. The results confirm that increasing d/c moves chaotic GCAs toward order and vice versa, decreasing d/c tends to move ordered GCAs closer to chaos.

With the CAs and many other discrete systems studied by Wolfram, the system outcome is changed by using different initial conditions and/or by applying a different rule. We argue that the initial conditions and the rules are instances of the broader parameters, c and d respectively, that we are proposing. The GCAs as studied here offer additional insight into how the variables c and d affect the behavior of discrete systems.

First, we introduce the variables and explain how the ratio of differentiation divided by centrality (d/c) predicts which class of output will appear in social systems. We next show how the output state corresponds to another concept from social systems, the focusing of output. We then relate all three variables—centrality, differentiation, and social focusing—to what happens in more abstract discrete systems such as CAs.

2. The d/c Ratio as a Predictor of Social Focusing

We now introduce a new theory from the social sciences that seems to explain many types of changes in social systems. The theory states, in brief, that the ratio of differentiation to centrality, the d/c ratio, predicts the state of a social system. The implication is that by manipulating the two variables c and d, it is possible to "tune" a social system so that it moves from state to state (for a more extensive discussion of d/c, see [2]).

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We begin by defining each variable as it is used with social systems; then, since we wish to apply this theory to abstract discrete systems, we relate the variables of our theory to what happens in a CA.

The term *centrality* comes from Bavelas's classic study of communication in small groups [5]. He was interested in how access to other people in a group affected the behavior and attitude of each individual, as well as the overall effectiveness of group problem solving. So, for example, the individual or node that is in touch with all other members has high centrality while an individual who is connected to only one other member has low centrality. In general, Bavelas discovered that individuals with high centrality have higher morale than those with low centrality. At the same time, systems with unequal centrality, that is, where some individuals have very high centrality and most have low, are not as efficient when creative problem solving is required.

Since Bavelas's initial studies of real social groups, the idea of centrality has been applied to a vast range of social networks, from interlocking corporate boards to trade patterns among nations. As well, graph theory has been used to define more precisely the different types of centrality. *Degree* centrality, for example, is the total number of links to any one node, while *betweenness* centrality refers to links that join two subsets of nodes, and *closeness* centrality takes account of direct and indirect links [6].

For our purposes we define centrality more abstractly as the variety, not volume or quantity, of information coming from outside that affects the behavior of a node or, in general, any system. A node or system with high centrality is subject to a wide variety of outside information, while low centrality systems receive information that has little variety. Centrality can be measured indirectly by counting the number of links to other systems, or directly by counting the varied types of information crossing the boundaries of the system.

For social systems, increasing the level of centrality tends to move a system from a state of order to one closer to chaos. The new incoming information makes the output less predictable.

What, then, corresponds to centrality in a CA? It must be the initial conditions: the value of each cell at time zero. This is the variety of information with which the system has to work.

For a CA, the centrality is presented only once, at time zero, through the initial conditions. The lowest possible centrality, beyond the trivial case of all initial cells being the same, is a single different cell, say one black cell, with all the rest white. Centrality rises as we add variety to the initial conditions, say in the form of an arrangement of a few black cells. The highest centrality is a random distribution of black and white cells as the initial condition. Wolfram explored this wide range of initial conditions for a huge number of CAs and many other discrete systems. For a given rule, changing the initial conditions—centrality—may change the output state, although the results vary depending upon the specific rule. For GCAs as nodes

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in GCANs, as discussed later, centrality becomes more complicated, and we will discuss it further at that point.

The concept of *differentiation,* the "d" in the d/c ratio, comes originally from biology where it refers to the variety, not quantity, of specialized internal structures that appear as the organism develops. As used in the social sciences, differentiation refers to the internal structure of any social system: the variety of specialized occupations and skills found in a social system such as a city or organization. Differentiation can be measured by counting the variety of internal subsystems such as institutions or trades.

A more elaborate measure of differentiation takes account of how the variety is organized, that is, the structure of internal connections. Hierarchical arrangements of different occupations behave differently than a cluster of different skills that interact in a more random way. For a business corporation, differentiation is both the variety of specialized sections as well as the way in which the managers organize those specialists.

In social systems, the behavior becomes more predictable as differentiation rises, both from a simple increase in the number of specialized subsystems, and with a more organized structure. More predictable means more orderly, so more highly differentiated systems appear more ordered than less differentiated ones.

What corresponds to differentiation in CAs? CAs with the same initial conditions have very different outputs depending upon the rule used. Hence, for CAs, differentiation must be related to the internal rules that organize the output. At some deeper level, some rules must represent a higher level of differentiation, since they lead to a more ordered output, but it is not clear how specific rules are related to the level of internal variety, or differentiation, in the system.

Hence, in a simple CA operating with a single rule, any changes in behavior can only be controlled through changes in centrality (the initial conditions). Thus, for CAs with a single rule, there is no ability to change differentiation in a systematic way that corresponds to changing the variety of skills in a social system, and how those skills are managed.

As we show later, by definition a GCA is a system that contains more than one rule and, therefore, it is possible to measure the internal variety (the level of differentiation) by counting the number of rules available. We will demonstrate that for GCAs as for social systems, increasing the number of rules (the level of differentiation) increases the likelihood of order.

We now turn to *focus,* the variable in social systems that corresponds to the output state of the system.

Just what is it that changes when social systems undergo change? What is it that outsiders see when they observe a system such as a small community? Social scientists interested in social change have several words to describe the eventual state of the system after a period of change. Solidarity versus conflict is one of the dichotomies used to

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describe the extreme overall state of a system such as a society or a small group. Solidarity represents greater order, while conflict corresponds to more chaotic behavior.

Since we wish to generalize our ideas to all systems, social and nonsocial, we use the more general term of "focus" to describe the final state of any system. One can imagine systems that are sharply focused or ordered, that is, in a clearly identifiable state that remains unchanging. At the other extreme are systems that keep changing and do not come into focus; they are more chaotic. Thus focusing can vary from high to low focus, or in Wolfram's terms, from order to chaos.

On the basis that Wolfram's findings apply to all discrete systems we have combined his terms for the four classes with the term focus. So, *fixed ordered focusing* refers to systems that change little over time; they are class 1 ordered systems, sharply focused. The term *repetitive ordered focusing* describes systems that repeat the same pattern over and over; they are class 2 ordered systems. *Chaotically focused* systems are unpredictable and change constantly; they represent class 3 systems. Finally, *complex focused* systems are a mixture of some ordered patterns along with some unpredictable variation in patterns; they are class 4 systems.

Using the three variables centrality, differentiation, and focus, we now present the theory of the d/c ratio.

It makes intuitive sense that the final state of a system will be related to centrality, or to the incoming variety of information to which it is exposed. We would expect the final state of focusing to be more chaotic for systems exposed to an increased variety of incoming information—to higher centrality. So, the higher the centrality, the less focused the final pattern should be. There is an inverse relationship between centrality and focusing.

For some CAs, the greater the variety contained in the initial conditions, the more likely the output is to be less focused, to be chaotic, which supports the relationship between higher centrality and more chaos. Many CAs, however, do not change focus with higher centrality, which must be related to the other parameter, the level of differentiation.

In terms of differentiation, we might expect that the more differentiated the internal structure, the more likely it is to end up with ordered focusing, implying a direct relationship between differentiation and focusing. The system is able to "handle" more incoming variety because the new information is slowed down and organized by the complicated internal structure.

For CAs, the differentiation is contained in the particular rule applied. Hence, for a given centrality, initial condition, the behavior of a CA depends entirely on the rule. Since the rule does not change over time for CAs, the output focusing does not change, either. Again, as discussed later, GCAs draw upon a number of rules and we would expect there to be more ordered focusing in GCAs containing more rules.

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These two variables, then, centrality and differentiation, form the d/c ratio and our hypothesis is that this determines which of the four classes of focusing is obtained. Very ordered focusing happens when the system has relatively low centrality and relatively high differentiation, a high d/c ratio. Conversely, relatively high centrality and low differentiation, a low d/c ratio, should lead to a more chaotically focused output.

We now present two theories from the social sciences that support these intuitive conclusions: Ashby's law of requisite variety, and Young's theory of community solidarity (again, these are discussed in greater detail in [2]).

R. Ashby developed his law of requisite variety [7, pp. 202–218] to help him understand the science of cybernetics (how to control systems). He speculated that all such systems could be reduced to models showing the variety of information entering the systems as inputs, processed through thruputs to produce outputs. His law, then, stated that

$$\frac{\text{Variety of Input}}{\text{Variety of Thruput}} = \text{Variety of Output.}$$

He gives, among other examples, the history of war, where new weapons (input variety) must be met with a greater variety of defenses (variety of thruput) to keep the state of the system constant (variety of output low). For Ashby, systems whose output is too unpredictable are more unstable and do not survive. Another of his examples is the survival of life. The output of organisms, measured by such indicators as temperature and blood pressure, must be kept within certain limits of variety for the organism to stay alive. To do this, organisms need to use internal thruput structures to resist the impact of such external inputs as varying temperatures and diseases.

Young's theory of community solidarity [8, 9] is closely related to Ashby's law. It uses the variables of differentiation and centrality to "explain" why some human communities are more unified, or have higher solidarity in the terminology of the social sciences. His theory can be stated as the inverse of Ashby's law of requisite variety, where thruput corresponds to differentiation, input to centrality, and low output variety to high solidarity:

$$\frac{\text{Differentiation}}{\text{Centrality}} = \text{Solidarity.}$$

In his research Young discovered that communities that had been bypassed by a railway, that is, those that had lost centrality, became more unified. They had the same variety of skills but there was no longer as great a demand for these skills with their lowered centrality, so residents had more time to talk to each other, about their loss of business, for example. The resulting interaction among the overly differentiated residents led to higher unity or solidarity.

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On the other hand, when a new road was built into an isolated community, the sudden increase in centrality overwhelmed the low differentiation, and the community became more chaotic.

Whether we use Ashby's law or its inverse, Young's theory, we predict that the class or focusing state of a discrete system will be determined by the ratio d/c. As differentiation increases with constant centrality, we move toward order and as centrality increases with constant differentiation, we move toward chaos.

Figure 1 illustrates the effect of increasing d/c on the state of the system as it moves from chaos to order.

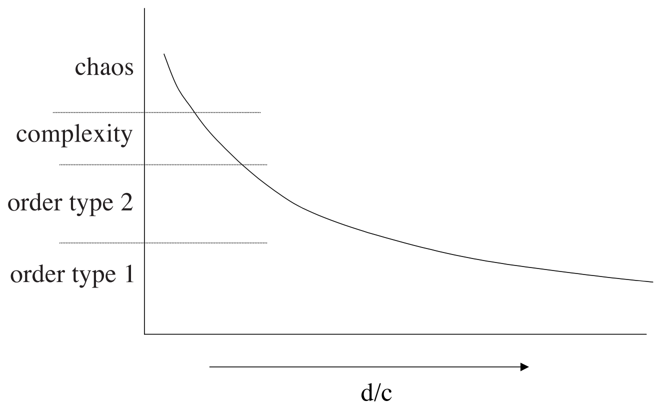

Figure 1. The relationship of CA class and d/c.

Elsewhere, we show how the principle of d/c can explain changes in social systems ranging from small groups [10] to cycles in art history [11], as well as business cycles for corporations, social movements, and other examples of social change [2].

In what follows we first apply d/c to simple CAs and subsequently to the more complicated GCAs arranged in a GCAN to show how discrete systems may be changed from one class to another by altering d/c.

3. Simple Cellular Automata and d/c

For simple CAs (e.g., two-color cells where rules apply to neighbors only) there are only two parameters that can alter the patterns: the initial conditions, the arrangement of the black and white cells on the first line (the zero time step), and the rules that govern the next step. It seems apparent that the initial conditions must correspond to centrality, the external information for the CA, and that the rules represent differentiation, the internal variety.

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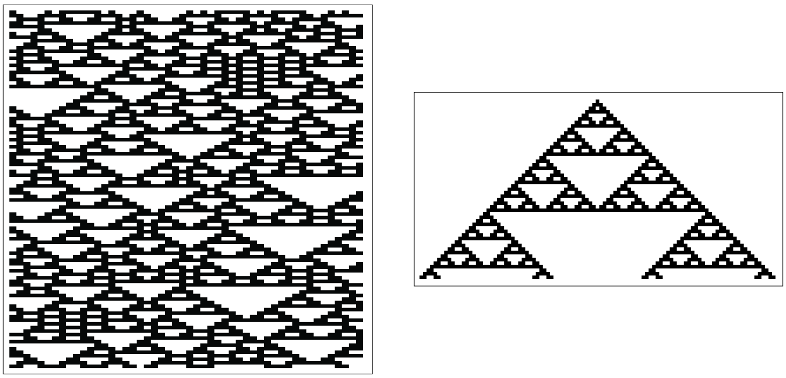
3.1 Centrality as Initial Conditions

We can change centrality for simple CAs by changing the initial conditions. These initial conditions can range from a high variety of centrality by randomly assigning the two colors to the cells to low centrality with a single black cell. As Figure 2 shows, the output is chaotic for rule 126 with random initial conditions (higher centrality). In contrast, with an initial condition of one black cell, representing low centrality, the output is type 2 order. In this case, decreasing centrality does make the output pattern more ordered.



(a) (b)

Figure 2. Rule 126 comparing (a) random and (b) single black cell initial conditions.

For many of the 256 rules of simple CAs, however, changes in initial conditions do not change the class of the CA. For example, rule 4 maintains its type 1 order state regardless of the initial conditions, as shown in Figure 3 by comparing the high centrality on the left with random initial conditions to low centrality on the right. Similarly, rule 30 remains chaotic even with a single black cell as its initial condition.

It could be argued that the kind of order produced with the high centrality initial conditions (Figures 3(a) and 3(c)) is more complicated, that is, it has more variety in it, hence it is less focused. Nevertheless, even the extremely high centrality of random initial conditions is incapable of moving the rule 4 system out of type 1 order. The differentiation level of rule 4 is high enough to continue to produce order type 1 even with this high centrality. On the other hand, rule 30 displays chaos even with the low centrality of the simplest initial condition indicating it has relatively low differentiation.

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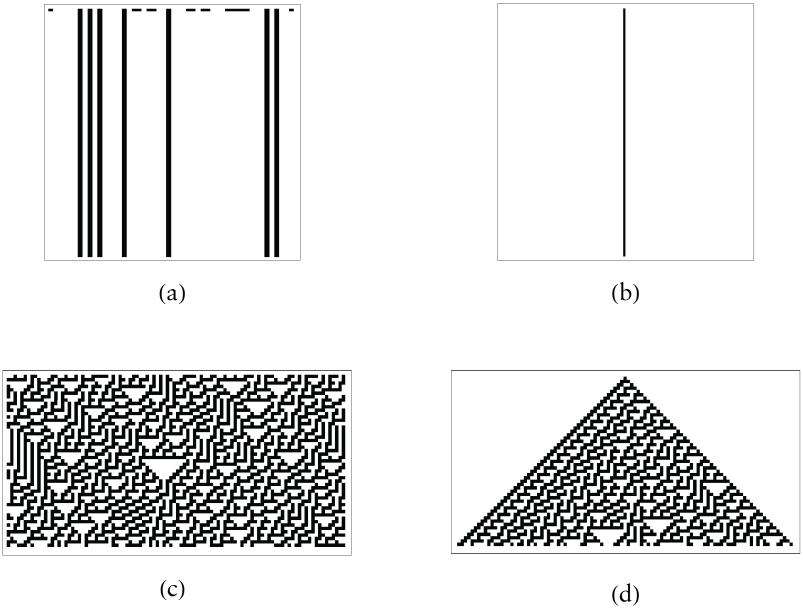

Figure 3. (a) and (b) Rule 4, (c) and (d) rule 30 comparing random and a single black cell initial conditions.

Wolfram has studied the relationship of initial conditions to output class for a very large number of CAs. This fascinating detail may be found at [1, p. 948 ff]. In all cases the effects of centrality and differentiation, as discussed here, are confirmed.

We conclude that for simple CAs changing from a random initial condition to a single cell, representing a major decrease in centrality, may or may not be sufficient to result in a change in class for the CA. The results seem to depend upon the differentiation inherent in the rule being applied. In the majority of the simple CAs this differentiation prevents a change in class even with higher centrality. Furthermore, it is difficult to demonstrate that such behavior is a function of both differentiation and centrality, the d/c ratio, with a CA controlled by a single rule.

3.2 Differentiation as Rules

For simple CAs, the differentiation must be the rule itself since, like differentiation in social systems, the rule represents both internal variety and the way in which that variety is organized. Hence, the only way we can change differentiation is by changing the rule for the CA. This, of course, is what Wolfram did with his extensive study of CAs, demonstrating that each rule leads in almost all cases to one of the four classes, where the specific type of output is determined by the rule.

A number of researchers including Langton [12] and Miller and Page [13] have explored the "rule space" to determine if one could define what Miller and Page called a "complexity dial" that would allow predicting rule behavior, but concluded that the underlying micro structure of rules makes this impossible even for simple CAs. So, for a CA running under a single rule we have no measure of the level of this micro structure so that we can test whether increasing the variety of the internal structure will produce more ordered output. We know the output focus is related to the rules but we are unable to show a systematic relationship between rule differentiation and the class of output.

4. Global Cellular Automata, Global Cellular Automata Networks, and d/c

To fully explore how centrality and differentiation affect the class or state of discrete systems requires a system that allows wider variation of these two parameters. One such system is a GCAN as developed by S. Chandler [4].

In Chandler's GCANs each cell (or node) follows a GCA rule with network connectivity between each cell. Each GCA is a single discrete system where the evolution of each cell within the GCA is a function not only of the values of the cell's neighbors, as in the simple CAs already discussed, but also depends upon certain global features of the entire GCA structure. An example of an arbitrary global feature that can be used to decide which of two rules should be applied for the next time step is to determine whether the length of the longest run of either ones or zeros in the current time step is an odd or even number. The first rule will be chosen if the length of the longest run, of either ones or zeros, is an even number and the second rule if it is odd. These runs in a time step are determined by the rule being applied in that time step as well as the cell values in the preceding time steps, hence representing a global feature of the entire GCA.

In Sections 4.1 and 4.2 we relate the parameters of GCAs to centrality and differentiation.

4.1 Centrality in a Global Cellular Automata Network

A GCAN consisting of a number of GCAs provides an opportunity to vary centrality in the d/c ratio. It is important here to note that we are using the term "centrality" in a slightly different way than we used it for simple CAs. There, centrality referred to the initial conditions at the beginning of the process. Here, centrality refers not only to the initial conditions but also to the external information that each GCA receives at each time step that is utilized to decide which rule to use at the next step.

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Perhaps an analogy to a business corporation may help make this subtle distinction. Management, representing the differentiation structure within the business system, decides which specialists and procedures are used at any given time. That process of choosing procedures and carrying them out refers to the variable of differentiation, discussed in Section 4.2.

As well, however, management also must deal with the circumstances presented at each instant in time (the time step), and this second process is also related to centrality. When the decision making of the manager is affected and determined by outside forces, such as demands for different products or government regulations, then that external information represents centrality, or increased input variety, for the company.

So, to vary centrality in our experiment, each GCA is presented with new outside information at each time step that the GCA utilizes to decide which rule is to be used at the next step. To summarize, for the GCAN in our experiment, we are not changing centrality by substituting different types of initial conditions in each GCA (they are all random), as we suggested for simple CAs. We are, instead, altering centrality by presenting each GCA with new incoming information from the GCAN at each time step, which it utilizes to choose the rule for the next step. That varying input information is, by definition, centrality.

Each GCA in the GCAN, then, uses the input from other connected GCAs to decide which rule is to be used at the next time step. This centrality from external sources can be varied in several ways.

- Centrality can be varied by changing the number of connections to each GCA. Increasing the number of connections increases centrality since it increases the variety of information available to determine the rule used.
- Centrality can be changed by varying the variety of information (the degree of difference in the information provided by each connection). If all the connected GCAs start from the same initial condition and employ the same rules, then all connections will provide the same information. If each connected GCA has a different initial condition, then centrality is increased as different information is provided from each connection, again, utilized by each GCA to decide on the next rule chosen.
- Centrality is even further increased if not only initial conditions but also the rules used are different at each of the connected GCAs.

One arbitrary way of deciding which rule each GCA uses is to look at the middle cells in the latest time step of each of the GCAs connected to it and decide which of two rules it should apply based on the relative number of ones or zeros among these middle cells.

There are other procedures that could be used by the GCA to make choices among the rule set but they perform in a similar manner. The important point is that we are presenting a variety of external information from the output of other GCAs in the GCAN to be utilized by the GCA in its rule choice. We are able to set centrality so that it varies from a lower to a higher variety.

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4.2 Differentiation in a Global Cellular Automaton

Within a GCA, the most direct way to increase differentiation is by providing a choice from more than two rules that the GCA can use at each time step. With larger rule sets, the internal variety, differentiation, increases.

There is another way to alter differentiation within the GCA. The particular GCA we are studying can be provided with a more complicated process when using the output of the connected GCAs to determine which rule applies at the next time step.

Using ones and zeros provided by the other GCAs, as described in Section 4.1, is adequate for selecting between two rules, but we wish to test the use of many rules. Therefore, we use a technique developed by Chandler and use the ones and zeros as the initial state of what we call a *processing* CA operating under a certain rule. That CA is then run a sufficient number of steps to involve all the cells. We then use the values of the first cells in the final time step of this processing CA as a basis for selecting which rule the GCA will use.

One arbitrary way to apply the values of the first cells is to convert them to a binary number since the cells are a sequence of ones and zeros. The number of these first cells selected is such that the maximum binary value they could yield is the number of rules minus one in the GCA. Since the binary numbers start from 0, we add 1 to get the rule number. For example, if the GCA needs to choose from 16 rules, we use four cells. The maximum value is when all four cells are ones (1, 1, 1, 1), the binary equivalent of 15, hence we would choose rule $15 + 1 = 16$ in the rule set.

This process that the GCA uses to select the next rule is the management structure or process that the GCA uses to decide which rule, its own differentiation, should be chosen to handle the incoming centrality. We could call this its *differentiation management process*, and it is clearly connected to the differentiation of the GCA we are interested in.

We can vary the differentiation management process by selecting different rules for this CA. For much of our work we use rule 30, the well-known class 3 rule for generating chaos. Running under other rules may affect the differentiation level, although, as we report in our results, not in a simple way.

5. The Experimental Strategy

The behavior of simple abstract discrete systems and of social systems is predominately type 1 or type 2 order. That is to say, these discrete systems appear to have a level of differentiation sufficient to handle the centrality presented to the system. For the simple CAs, the total complex and chaotic outcomes are less than 20%. As the number of colors and/or the number of neighbors consulted by each cell in the CA are increased, the fraction of chaotic outcomes in abstract systems increases to more than 50%.

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Each node on a GCAN is a GCA and the individual GCAs are the systems under study. We wish to determine the effect of changing centrality and differentiation on the behavior of these individual GCAs in the GCAN. A simple 10 node network with one to five connections per node is shown in Figure 4. Typically, we use 300 nodes in a GCAN. Connections to each GCA are usually about 10 except when we specifically use one or two connections.

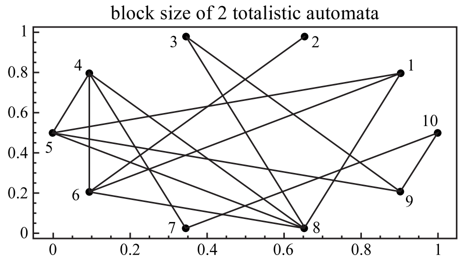

Figure 4. An example of a 10 node network with one to five connections per node.

The CA rules used in all GCANs studied in these experiments are the Boolean rules from Wolfram's original 256 simple CAs.

5.1 Varying Centrality to Change Focus

We begin by testing the hypothesis that decreasing centrality and increasing the d/c ratio with constant differentiation leads to more order. To keep differentiation constant, we use GCAs with the minimum two rule set. Still, even with the minimum differentiation of a two rule set, the number of possible permutations of unordered pairs using 256 rules is a large number (65 536) so we take a sufficient number of random samples to give us a statistically relevant number of two rule sets.

We first do this with the maximum amount of centrality that we can present to a GCA within a GCAN. Therefore, we use the 300 node GCAN with about 10 connections to each GCA. Each of these connections is to another GCA that has a different two rule set and each such GCA starts from a different initial condition. This presents the highest centrality possible for an individual GCA in a GCAN.

As with other simpler discrete systems, most of the GCAs produce ordered output even with this highest centrality, but some show chaotic behavior. We then select those systems that demonstrate chaotic behavior. Using these chaotic systems, we examine the effects of reducing centrality to determine if more focused ordered behavior appears with reduced centrality, that is, as d/c increases.

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To reduce centrality we can make all GCAs have the same two step rule. We can further reduce centrality by having all GCAs start from the same initial conditions. Finally, we can reduce the 10 connections per GCA to one. Our basic test for the effect of centrality is to determine if reductions in centrality produce order.

5.2 Varying Differentiation to Change Focus

For the second part of our experiment, we keep centrality constant but vary differentiation to see if this changes focus in the predicted direction; we want to establish if increasing differentiation reduces chaotic output. The two rule sets studied are the lowest possible differentiation for a GCA. We increase differentiation by increasing the number of rules available to the GCA. In this discussion, a two rule set means a pair of rules selected from the set of all possible combinations of two rules, a four rule set means four rules selected, and so on.

Again, for each rule set of a certain size the number of possible combinations of rules is very large. For a given rule set of a certain size, we wish to determine the fraction of all possible rule combinations that will have chaotic behavior. Then, we increase the rule set size to see what fraction of those is now chaotic. By increasing the rule set size we are increasing differentiation, and the number of chaotic outcomes should decline, since d/c is larger.

To determine how many of each rule sets of a certain size are chaotic, we establish a GCAN with maximum centrality, about 10 connections per GCA. We also maximize simple centrality by assigning to each GCA its own random initial conditions. Although each GCA for a given rule set has the same number of rules, each has a different combination of the rules in its rule set. Thus, we are using the same GCAN, the same connections, and the same initial conditions at each GCA for all rule sets. The only change made is to increase the size of the rule set available for each GCA; we are increasing differentiation then measuring the effect on output focus.

The GCAN is constructed to have sufficient GCAs to give a statistically significant sample of the population of all possible rule combinations for a given size of rule set. At each stage, the rule sets of a certain size are assigned randomly to each GCA from the total population of all combinations of rules for that rule set.

In summary, our objective for the second set of experiments is to determine if increasing the number of rules in the GCAs of our constant high centrality GCAN causes a reduction in the fraction of chaotic outcomes. We are testing the hypothesis that when the d/c ratio increases, the system tends to move closer to order.

We now have described the techniques used for varying centrality in the entire GCAN and to vary differentiation within each GCA in the network. The results of such changes in the d/c ratio are discussed next.

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6. The Experimental Results

Before experimenting with altering centrality and differentiation, we wanted to see if there were inherent relationships between the outcome and the selection of rule sets. Simple CAs, once started, develop output patterns depending only upon the initial conditions and the rule used. In the case of GCAs, each step changes as a result of which rule is selected for the next step. We might expect the results to be chaotic no matter what rule was chosen at each step, but, in fact, the general output behavior of GCAs resembles that of other discrete systems by having output patterns of all four classes of outcomes. Since Wolfram found that each rule produces a certain class of output, one might suspect that there is some relatively simple relationship between the output classes of the rules used in the experiment and the output class of a GCA.

This is not the case. One cannot predict the outcome based on the rule sets employed. The best test of this is to study many two rule GCAs, some consisting of two ordered rules, some with two chaotic rules, and some mixed.

Wolfram discovered that for the 256 simple CAs, about 222 produced order when started with random initial conditions, or roughly 87%. For the 300 pairs used in our test, about 75% of the pairs contained two of these ordered CA rules.

To determine if there is any simple relationship between the Wolfram class of different combinations of CA rules in the rule pair and the output focus of the GCA, we ran a GCAN of 300 GCAs each with about 10 other GCAs connected to it, each with a separate rule pair and its own random initial conditions.

When we used ordered rule pairs, only 70% produced types 1 or 2 ordered outcomes. The remaining 30% of these ordered pairs produced chaotic outcomes, counting chaos and complexity together. Of those pairs where one is chaotic and one ordered, 63% produced chaotic behavior and the remainder produced order. Only four pairs contained two chaotic rules; two produced order and two produced chaos.

The essential point is that GCA outcomes are a complicated function of the rules used, the input from connected GCAs, and the process used to select the rule for each time step. Chaotic or ordered behavior can arise from all types of pairings in two rule sets. Thus, behavior is the result of a complicated deterministic process in the same way as for all CAs and, indeed, all abstract discrete systems. Wolfram calls this computational irreducibility. While we will show that both centrality and differentiation, as we have defined them here, affect output patterns in somewhat predictable ways, the relationship is not linear.

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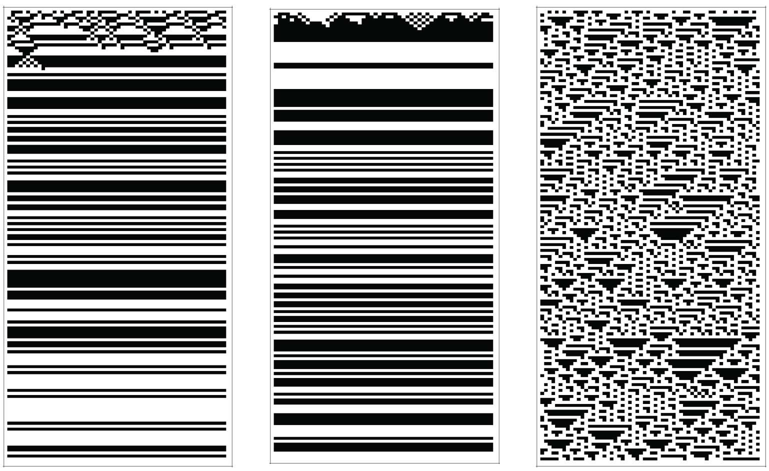
6.1 Effects of Changing Centrality on the Output Focus

To study the effects of changing centrality we use the rule set with a minimum differentiation—the two rule set—and then alter centrality from high to low by going from random to homogeneous initial conditions, and by using many and then fewer connections. We have examined many different pairs of rules.

We begin by running a two rule GCAN with the highest centrality, that is, each of the 300 GCAs (nodes) operates with a randomly selected pair of simple CA rules. Each GCA in the GCAN has 10 other GCAs connected to it and each GCA begins with a different random initial condition. Thus, for each GCA, each of the GCAs connected to it has a different rule set and each has a different initial condition. This is the high centrality condition. Figure 5 shows results for three typical nodes.



(a) (b) (c)

Figure 5. Three typical nodes from the set of 300 for the two rule pairs: (a) {160, 27} order, (b) {122, 249} order, and (c) {145, 71} chaos.

More than half of the GCAs in this 300 GCA GCAN set produce ordered output patterns that appear to contain sufficient differentiation to keep d/c high even with this high level of centrality. The number of chaotic and complex nodes is about 115 (±7.9 with a 95% confidence interval). We can choose any of these chaotic GCAs and repeat the experiment to examine the effect of reducing centrality—we expect increasing order.

Figure 6 shows what happened when we selected the rule pair {46, 90} that had chaotic output in the high centrality condition, and then reduced centrality by using the same initial condition for each GCA. We ran a GCAN with every GCA in the 300 nodes using just this pair

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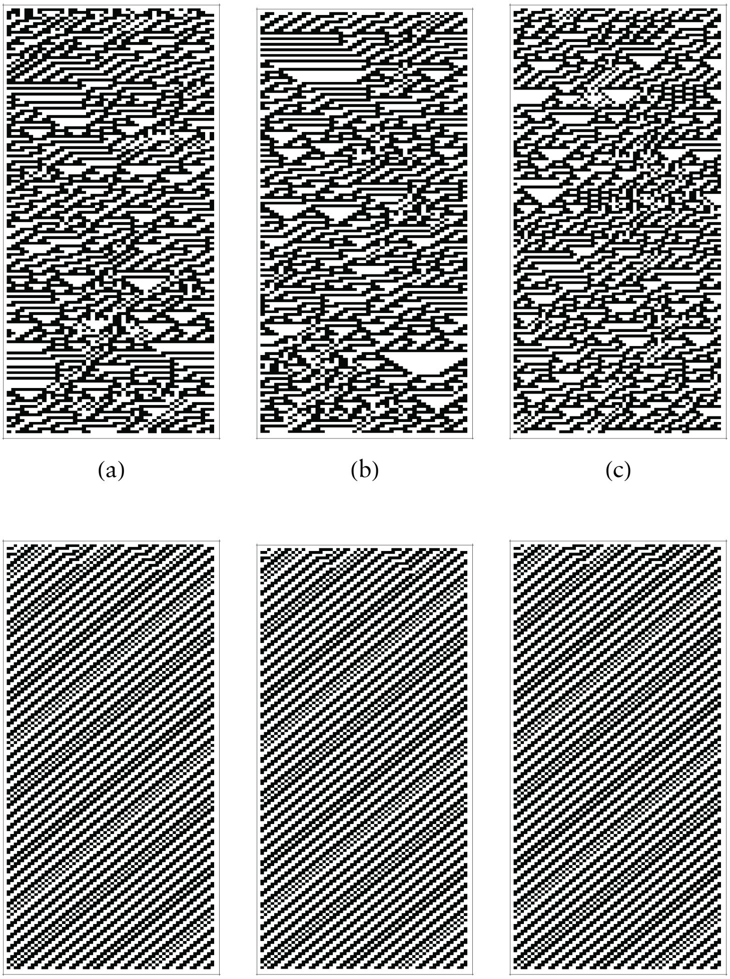
of rules and about 10 connections per node. We compared the GCAs with the original high centrality, that is, with each GCA having unique random initial conditions (top row of Figure 6) to each GCA having identical random initial conditions and therefore lower centrality (the bottom row of Figure 6). The results are striking as all the GCAs in case one show chaotic behavior and all GCAs in case two show order, as the d/c ratio would predict.



(a) (b) (c)

(d) (e) (f)

Figure 6. (a) through (c) Typical chaotic GCAs for the rule pair {46, 90} with unique random initial conditions at each GCA. (d) through (f) Each GCA has the same initial conditions.

These same results occurred with the majority of the chaotic pairs tested; while all of these selected pairs showed chaotic behavior under high centrality, most produced order with lower centrality from using the same initial conditions. True, a few pairs continued to show chaotic behavior even with this reduced centrality, but overall the results support the hypothesis that lowering centrality with fixed differentiation moves the system toward order.

For a GCAN where each GCA has the same rule pair and different initial conditions, reducing centrality by reducing the number of connections per GCA to two does not make chaotic behavior more ordered. But, with one connection per GCA, there is a change to ordered focusing, although the output patterns are not identical because we allow different initial conditions for each GCA.

The first set of studies on the effects of reducing centrality, the variety of information received, involved going from random initial conditions to giving all GCAs, each with the same rule pair, in the GCAN the same initial conditions. This lowering of centrality changed chaotic GCAs in the GCAN into order. The second set of studies showed the effect of reducing centrality by changing to one connection, while using different initial conditions at each node shows order for all GCAs. Two connections restored chaotic behavior to the GCAs. Together, the two sets of studies show that reducing centrality tends to change chaotic GCAs to ordered ones.

6.2 Effects of Changing Differentiation on Output Focus

We begin the second part of the experimental results on the effect of changing differentiation on output by looking at what happens when each GCA uses a different processing CA to process the incoming information from other GCAs.

Recall that in our experiment we used what we call a processing CA as a scheme for managing differentiation, changing the way in which we choose the rule once the external information is received by the GCA. Usually, we use rule 30 as a processing CA. What is the effect of using different processing CAs?

With rule 4, normally order type 1, as the processing rule in a GCAN with high centrality, we see a mix of ordered and chaotic patterns, that is, complex focusing as in Figure 7.

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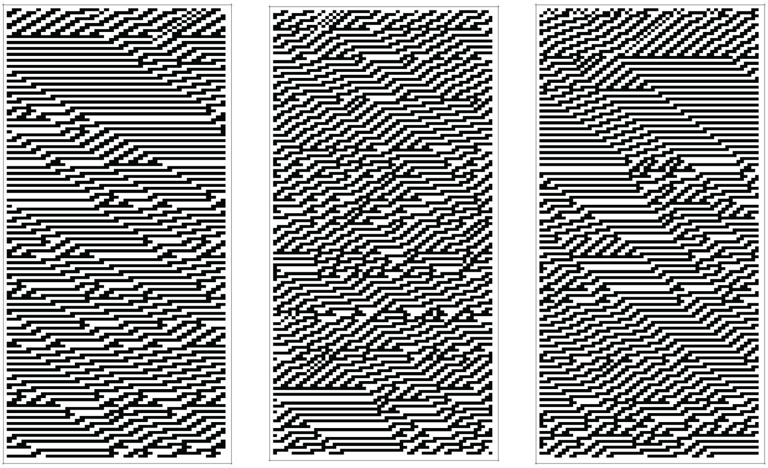
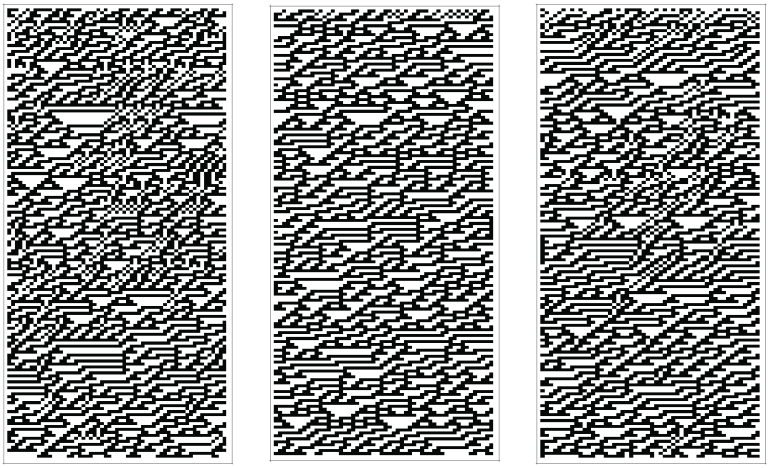


(a) (b) (c)

Figure 7. Using rule 4 (order type 1) as the processing CA shows some complex behavior.

If we use rule 50 (order type 2) as in Figure 8 we again get chaos, similar to the results of using rule 30 as the processing rule.



(a) (b) (c)

Figure 8. Using rule 50 (order type 2) as the processing CA shows chaotic behavior.

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Thus, varying the rule for the processing CA (the differentiation management process) does not appear to have a consistent effect. Using a chaotic rule 30 yields chaos, as does the type 2 ordered rule 50. Using the most extreme type 1 ordered CA, such as rule 4, changes some chaotic GCAs to complex behavior, appearing to generate somewhat more order.

To see if these results were biased by the way in which we processed incoming centrality to affect the choice of rules, we ran the system with an entirely different process. Instead of using a separate processing CA, we examined the middle cells of the GCAs connected to each GCA to determine the longest run of either ones or zeros. If the length of this run was even, we used rule 1 in the next time step; if odd, we used rule 2. The results were similar to those found when using a processing CA to manage the centrality: sometimes chaotic, sometimes more ordered.

Much more research needs to be done on the management structure of differentiation. In the meantime, we will continue to use rule 30 as the processing rule for our experiments on varying differentiation.

At this point, we attempt to change differentiation more directly by increasing the size of the rule set.

Differentiation is represented in the GCAN as the number of rules available to cope with centrality, the information presented as input to each GCA. This allows us to examine the effect on output focus of increasing differentiation.

In this experiment, we established maximum centrality within the GCAN and kept that constant during the experiment on the effects of increasing differentiation. Thus, for the 300 GCAs in the GCAN we used about 10 connections per GCA, each with a unique initial condition.

We then selected rule sets over the range 2, 4, 8, 16, 32, and 64 so that differentiation increased. To give us a large sample of results, we assigned these rule sets randomly to different GCAs. Since we used 300 GCAs in the GCAN we had 300 individual random rule sets of the same number of rules in this sample from the very large population of combinations of rules.

We then determined the fraction of GCAs in the GCAN that show chaotic behavior. Table 1 lists the results and the relationship is plotted in Figure 9.

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Rule Set	Chaotic Fraction	Range (95% Confidence Interval)
2 rules / GCA	0.383	0.082
4 rules / GCA	0.347	0.053
8 rules / GCA	0.19	0.044
16 rules / GCA	0.043	0.023
32 rules / GCA	0.023	0.017
64 rules / GCA	0.02	0.015

Table 1.

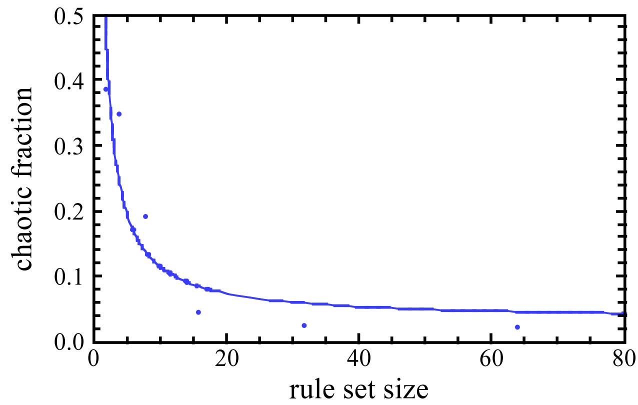

Figure 9. GCAN chaotic fraction and rule set size.

It is apparent that with constant centrality, increasing the size of the available rule set—increasing differentiation—reduces the amount of chaotic behavior. This supports the d/c hypothesis.

7. Conclusions

For the abstract discrete systems known as simple cellular automata (CAs), it is possible to show that by increasing the variety of initial conditions, the external variety of centrality, it is sometimes possible to change the class of output from order to chaos, the direction predicted by the ratio of differentiation over centrality, the d/c ratio.

It is, however, difficult to demonstrate that changes in differentiation for simple CAs, the internal rule, will change the output class, because a single rule governs behavior, and it is not clear how each rule is related to differentiation, defined as the internal structure of variety.

A more recent modification of CAs known as global CAs (GCAs) connected into networks (GCANs) allow more flexibility in changing both centrality and differentiation.

These results demonstrate that for the GCAs within GCANs, decreasing centrality by presenting a smaller variety of information does create order, as the d/c ratio predicts. Specifically, when each GCA has the same rule set, and the initial conditions in the connected GCAs are the same (representing lower centrality), the reduced variety of external information often does produce an ordered pattern of output. Similarly, when the number of connections joining each GCA in the network is reduced all the way down to one, the reduced variety of external information produces an ordered pattern of output.

When differentiation in GCANs is raised by using larger rule set sizes at each GCA, again, more order is obtained and chaotic behavior is greatly reduced. Combined with the results for altered centrality, increasing the ratio of differentiation over centrality does seem to move these abstract discrete systems into more ordered output states.

This study demonstrates that the parameters of centrality and differentiation can directly control the behavior of abstract discrete systems just as they do for social systems. We are in the process of testing our results using greater than two-color CAs. For three-color systems, centrality is higher since the possible combinations of colors presented as initial conditions or by connected cells are greatly increased. Therefore, the proportion of chaotic GCAs in the high centrality GCAN case is greater for three-color GCAs and even more for four-color GCAs. Increasing differentiation by increasing rule set size reduces the proportion of chaotic outcomes but not to the same low level as with two-color GCAs. Much more work needs to be done with these more complicated GCAs.

The next step, of course, is to fine tune the values of our parameters to investigate shifts into the boundary layer between order and chaos, the region of complexity. We believe that we have shown that the d/c ratio may eventually solve the problem of how to develop a "complexity dial" to control change in abstract discrete systems over the four Wolfram states.

If the d/c ratio applies to all systems, then it should also work with agent-based models. One of our colleagues, in fact, is using d/c to move the output of such systems among the four classes of chaos, complexity, and the two kinds of order.

We look forward to more discussion and applications of our approach.

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