Simulating Self-Regeneration and Self-Replication Processes Using Movable Cellular Automata with a Mutual Equilibrium Neighborhood

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This paper deals with the issue of model construction of the self-regeneration and self-replication processes using movable cellular automata (MCAs). The rules of cellular automaton (CA) interactions are found according to the concept of equilibrium neighborhood. The method is implemented by establishing these rules between different types of cellular automata (CAs). Several models for two- and three-dimensional cases are described, which depict both stable and unstable structures. As a result, computer models imitating such natural phenomena as self-replication and self-regeneration are obtained and graphically presented.

Keywords: bio-like systems; computer simulation; movable cellular automata; self-regeneration; self-replication

1. Introduction

Simulating bio-like processes is one of the most relevant and promising areas of research. The mechanisms of many bioprocesses are unknown nowadays, unlike those of physical phenomena. Thus, one of the tasks that is solved through simulation is the search for answers to the questions that are still open. These questions include, in particular, the processes of self-organization and the evolution of living matter (origin of life); and mechanisms of synergistic cooperation in the colonies of unicellular organisms and their organization in the form of multicellular organisms, accompanied by cell differentiation (acquisition of specific functions by individual cell types within a single organism).

The first researcher of artificial life was John von Neumann; he was studying the possibilities of implementing self-replicating

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structures. Von Neumann showed and described [1] the possibility of constructing a discrete deterministic self-replicating automaton, which can be considered the certain analog of the Turing machine. It was capable of copying the contents of a control program and fragments of the automaton structure.

The simplest cellular automaton (CA) self-replicating model is the one proposed by Chris Langton [2]. In a Langton machine, a cell can be in one of the eight possible states. The state of the cell at a subsequent time is determined by its state and the states of its four neighbors at the present moment. The automaton is a signal strip located between two sides. The signal strip carries the information that is necessary to create a copy of the machine, which can be obtained in 151 steps after launch. One of the most well-known continuum models is the reaction-diffusion model proposed by Alan Turing in 1952. It is based on differential equations for calculating the change of the continuous parameters of the system depending on time. Various modifications of this model, including the Gray–Scott model and others, are used today to simulate self-replication processes, embryogenesis, and so on [3].

The computer simulation of chemical processes is a separate area of research on the spontaneous generation and evolution of bio-like structures. It also can lead to similar dynamics as those mentioned above (artificial chemistry) [4, 5]. In particular, the authors of [5] managed to build a self-replication model of cell-like structures (the so-called "Los Alamos bugs") that possess not only elementary metabolism, but also hereditary genetic information. The model assumes the functional interdependence of the membrane and the metabolism and also the self-replication of the genetic biopolymer. In this case, researchers argue that the model lays the foundations for the first integrated spatially distributed computer simulation of the entire protocellular life cycle. The method for simulation dynamics of three-dimensional dissipative particles was used as a tool for constructing a fairly simple model of coupled diffusion, self-assembly and chemical reactions.

The author of another interesting paper [6] considers the possibility of simulating self-replication in a dynamic chemical reaction environment. Hutton applied the transition rules determined in his study. One of the examples is Ralph Hartley's cell reproduction sequence. In this challenge, a foreign body is put in the reaction chamber, with the property that when it bumps into an atom it will break all of its bonds unless the atoms are of type "a," thus requiring the cell contents to be contained within a membrane of "a"-types. This solution builds a special "mouth" structure that ingests a particular atom only if it is not disturbed by the caustic agent. It achieves this by detecting if the "food" particle is still bonded as intended or has been unbonded. The caustic agent may unbind the atom at this point but this will be detected by the cell. After ingesting the necessary atoms, the atom sequence is copied, the two copies attach to the membrane and the cell division process starts. Eventually the cells separate completely into two similar parts. It should be noted that the author applies the CA method where the cells can move around, which is similar to the method applied by us. However, the repeated replicating process is possible only with some modification.

In [7], the authors constructed a hybrid two-dimensional CA model with a hexagonal grid. Their model displayed self-reproduction in a cell-like shape with few states in transition rules. To reduce the number of transition rules, they considered not only the state of transition rules but also the concentration diffusion in the Gray–Scott model, in which the self-reproduction phenomenon emerges with certain parameters. The authors have developed a model for the simulation of cellular self-reproduction in a two-dimensional CA and have demonstrated that the following three functions can be realized: formation of a border similar to a cell membrane, self-replication while maintaining carrier-containing information, and division of the cell membrane while maintaining the total structure.

2. The Simulation Method

The tools that are used to build models of biological objects and their dynamics are quite diverse—from describing investigating processes in terms of the theory of differential equations and then finding their numerical solution, to building imitational complex models that reflect the individual components of the corresponding processes or systems. Using movable cellular automata (MCAs) as a simulating tool is attractive due to the possibility of creating quite simple models of rather complex systems.

It should be noted that the basis of the self-replication process mentioned above is also that of self-organization, which means the absence of a single control center. In contrast to the centralized management approach, each element of self-organizing systems acts by itself, interacting with only a small number of neighboring elements, but this is enough to streamline chaotic structures. Therefore, this is another reason to choose the movable cellular automaton (MCA) method as a simulation method. To be more accurate, we chose for our research the asynchronous stochastic MCA method with the symbolic alphabet, which corresponds to the states set of the MCAs. This method is a further development of deterministic synchronous automata [8], which are used to simulate physical-mechanical processes in solid deformable bodies.

The MCA method should be essentially considered as a kind of hybridization of the automaton approach and the method of molecular dynamics or discrete elements. To construct an elementary model of the self-replication process of a cell-like structure, the latter should be decomposed into separate constituent elements. Each discrete element is represented by an MCA with an appropriate behavior, which depends on the type of element and the state of the environment, that is, the state of the neighboring MCAs. This artificial structure is also called "animat" (from artificial animal). The advantages of using MCA to create animats is given next. First, this method allows the possibility of a fairly flexible modeling of the arbitrary morphology of the artificial cellular structure and its organelles and also simplifies the simulating of composition and decomposition of individual fragments. Second, it gives the possibility of determining a wide range of functional properties of these fragments, which form the set of the corresponding parts of the structure (membranes, fibers, cytoskeleton, cytoplasm, etc.). And finally, the automata approach allows us to realize the change of complex molecular states more simply and thus simulate processes like the blocking or activation of individual genes that initiate certain regulatory mechanisms in cells.

3. The Example of Simulation

First we consider the two-dimensional model with hexagonal neighborhood scheme. The determination of the automata interactions rules is the main problem in constructing a model of an arbitrary phenomenon or system using the CA method. In our case, to simulate a self-regenerating structure, we propose to form two different types of MCAs and establish equilibrium neighborhood rules between them. These rules presuppose the existence of desirable particular types of MCAs as neighbors of specific types of MCAs. In such cases, the potential of the MCA is minimal, and the structure they constitute is robust. If the MCAs are not in positions with their equilibrium environment, then the structure is unstable, and therefore it can be arranged to move to equilibrium. There are two possible options: either the MCAs drift toward the potential decrease, and thus a self-assembly process is implemented, or the MCAs initiate the formation (synthesis) of the equilibrium environment, and thus the self-regeneration process is realized. The combination of these two options also can be considered.

As an example, we can consider the equilibrium interaction of two types of MCAs, one of which (S_1) must be surrounded by six other

MCAs (S_2) in an equilibrium state. However, for the S_2 type of MCA, there must be three equilibrium neighbors, one S_1 and two S_2. These equilibrium neighborhood rules can be expressed in tabular form as shown in Table 1.

Number of Neighbors	Type of Cells	S_1	S_2
6	S_1	-	6
3	S_2	1	2

Table 1. Equilibrium neighborhood rules for MCAs.

The essence of the MCA interaction algorithm, which leads to the arbitrary alignment of structures and their self-regenerative stability, is as follows. Each type (state) of an MCA S_i is associated with certain sets of types $\Omega_i = \{S_j, S_k, \ldots, S_m\}$; the presence of that in related relations is most preferable. In terms of physical analogies, it can be said that a fragment of the structure consisting of S_i with the environment Ω_i has the minimum value of a certain potential energy $U_{\min}$. Any other configurations are a deviation from the specified equilibrium neighborhood and the structure energy increases. Thus, it is possible to realize the self-building process of structures with a criterion of the minimum of potential energy. Energy will be estimated by counting the number of mismatches M_r^i surrounded by S_i:

$$U_i = \sum_{r=1}^{N_i} M_r^i,$$

$$N_i = \max(N_{\text{eq}}, N_{\text{ex}}),$$

$$M_r^i = \begin{cases} 0, & \text{if } s_r \in \Omega_r^i \circ \Omega_{r+1}^i = \Omega_r^i - S_r, \\ 1, & \text{if } s_r \notin \Omega_r^i. \end{cases}$$

Here N_{eq} is the number of S_i connected neighbors in the equilibrium state, N_{ex} is the number of existing connected neighbors, and S_r is the selected neighbor. If S_r is detected in the environment Ω, it is removed from this set, in order not to be taken into account in the next step of the inconsistency calculation cycle.

Next, we implement a certain analog of the gradient descent algorithm to search for configurations with a minimum of potential energy. In this case, we use the operations of local perturbations (p_i):

- p_1, adding a random neighbor of arbitrary type
- p_2, deleting a randomly selected neighbor

- p_3, changing the type of randomly selected neighbor
- p_4, establishing connection with a random MCA, located within the circumference of R
- p_5, breaking of connection with randomly selected MCAs
- p_6, exchanging one of the connections with a randomly selected MCA

Applying these perturbation operations to some MCA S_i, we estimate the new potential energy U_i^{t+1} of the structure. If this potential (the total number of inconsistencies) is less than the potential of the previous structure U_i^t, then a perturbation in the form of a corresponding modification (mutation) remains in the structure. Otherwise, when $U_i^{t+1} > U_i^t$ there is a rollback, and the modification is not accepted.

It should be noted that MCAs are also subject to rules; as a result of that they strive to be uniformly distributed relative to their nearest neighbors, with whom they have strong ties. These rules are described by the interaction formula:

$$F = \begin{cases} f_1, & \text{if } d < 2 * r, \\ f_2, & \text{if } d > 2 * r + \delta_{\max}. \end{cases} \tag{1}$$

Here r is the radius of the MCA; $d = \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2\Big)}$ is the distance between centers of MCAs with coordinates (x_i, y_i) and $\left(x_j, y_j\right)$; and f_1 is an operation that corresponds with the repulsion of two MCAs when they overlap. This simulates the incompressibility of the medium. Each MCA has a radius r corresponding to it, and when approaching a distance shorter than $2 * r$, the MCAs must be pushed away. f_2 is an operation that attracts two MCAs. It simulates the condensed state of the medium and prevents the appearance of voids. When the distance between centers of MCAs is more than $2 * r + \delta_{\max}$, they should be attracted; here $\delta_{\max} > 0$ is the maximum possible distance between two MCAs. The following pseudocode shows the essence of repulsion and attraction operations.

Repulsion:

```
d =sqrt ((xi - xj)^2 + (yi - yj)^2)
delta = (ri + rj) - d
if delta>0 then
   cos(alpha) = (xi - xj)/d
   sin(alpha) = (yi - yj)/d
   xi = xi + delta*rj/(ri + rj)*cos(alpha)
   yi = yi + delta*rj/(ri + rj)*sin(alpha)
```

```
    xj = xj delta*ri/(ri + rj)*cos(alpha)
    yj = yj delta*ri/(ri + rj)*sin(alpha)
  endif
```

Attraction:

```
  d = sqrt((xi - xj)^2 + (yi - yj)^2)
  delta = d - (ri + rj + deltamax)
  if delta>0 then
    cos(alpha) = (xi - xj)/d
    sin(alpha) = (yi - yj)/d
    xi = xi - delta*rj/(ri + rj)*cos(alpha)
    yi = yi - delta*rj/(ri + rj)*sin(alpha)
    xj = xj + delta*ri/(ri + rj)*cos(alpha)
    yj = yj + delta*ri/(ri + rj*sin(alpha)
  endif
```

Demonstration of the structure stability formed on the principles of equilibrium neighborhood by the MCA method is shown in Figure 1. The deviations lead to a nonequilibrium state. At the next moments of time there is an arbitrary comeback to the state of equilibrium and the initial structure is restored. That is, there is a process of self-regeneration.

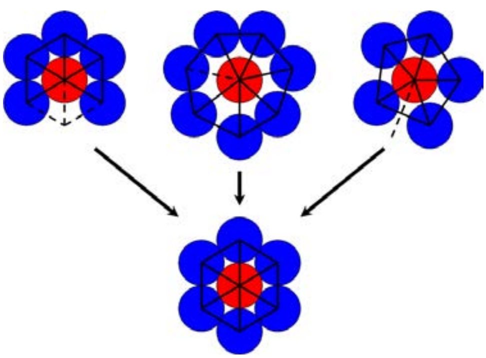

Figure 1. Schematic demonstration of the stability of a structure formed on the principles of equilibrium neighborhood described in Table 1.

Let us consider some other examples of structures with equilibrium neighborhooding.The rules described in Table 2 lead to the absolutely random formation of a chaotic structure. Moreover, it is stable, since there are no conflicts and inconsistencies from the point of view of the equilibrium neighborhood.

The result of random construction of the structure is shown in Figure 2. Due to the rules that any type of element can be surrounded by any combination of existing types, there will be growth on the bound of the structure and previously created fragments will not be changed.

Number of Neighbors	Type of Cells	S_1	S_2	S_3
6	S_1	6	6	6
6	S_2	6	6	6
6	S_3	6	6	6

Table 2. Equilibrium neighborhood rules for MCAs. The colors of the circles correspond to the colors of the table cells marking the appropriate type of automaton.

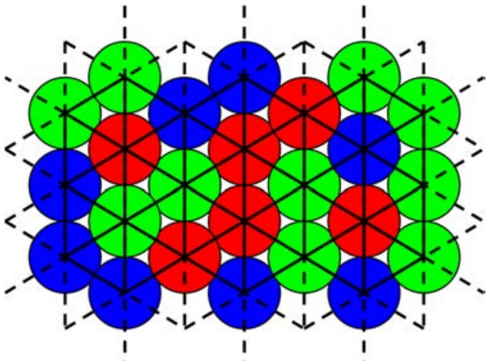

Figure 2. Schematic demonstration of the stability of a structure formed on the principles of equilibrium neighborhood described in Table 2. The colors of the circles correspond to the colors of the table cells marking the appropriate type of automaton.

Tables 3 and 4 describe the formation of the infinite periodic structure.

Number of Neighbors	Type of Cells	S_1	S_2	S_3
6	S_1	1	4	1
6	S_2	2	2	2
6	S_3	1	4	1

Table 3. Equilibrium neighborhood rules for MCAs.

Number of Neighbors	Type of Cells	S_1	S_2	S_3
6	S_1	2	4	-
6	S_2	2	2	2
6	S_3	-	4	2

Table 4. Equilibrium neighborhood rules for MCAs.

The result of construction of the periodic structure is shown in Figures 3 and 4.

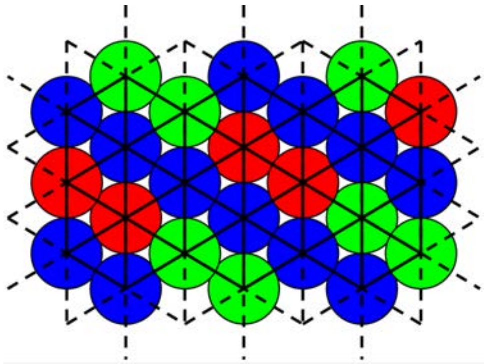

Figure 3. Schematic demonstration of the stability of a structure formed on the principles of equilibrium neighborhood described in Table 3. The colors of the circles correspond to the colors of the table cells marking the appropriate type of automaton.

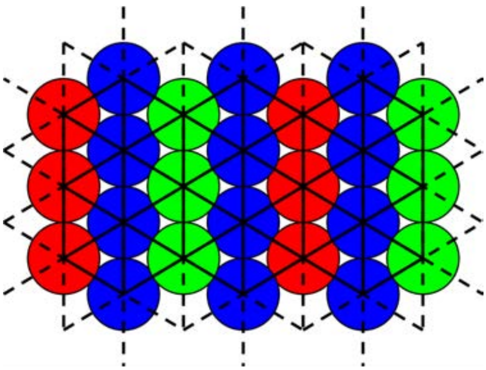

Figure 4. Schematic demonstration of the stability of a structure formed on the principles of equilibrium neighborhood described in Table 4. The colors of the circles correspond to the colors of the table cells marking the appropriate type of automaton.

By experimenting with the rules of the equilibrium state, you can also get examples in which there is no mutual support. We can detect this from the equilibrium neighborhood matrix (Table 5). In this case, during the self-constructing of the structure, inconsistencies (conflicts) occur. For example, in Figure 5, gray circles show the positions where the addition of the environment of some elements immediately leads to contradictions in the equilibrium neighborhood for other neighboring elements. Moreover, in this case of movable cells, in order to minimize the potential of deviations from the equilibrium state, the algorithm will constantly create intermediate nodes that destroy the geometry of the structure and lead to its unstable stochastic infinite growth.

Number of Neighbors	Type of Cells	S_1	S_2	S_3
6	S_1	1	2	3
6	S_2	2	2	2
6	S_3	3	2	1

Table 5. Equilibrium neighborhood rules for MCAs.

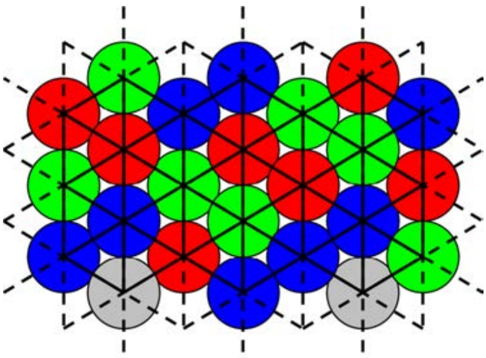

Figure 5. Schematic demonstration of the instability of the structure formed on the principles of the equilibrium neighborhood described in Table 5. The colors of the circles correspond to colors of the table cells marking the appropriate type of automaton.

In the case of classical frame structure of CAs, it would be possible to organize the existence of a specific cell that would mark inconsistencies in the structure (marked in gray, for example), but in the case of MCAs according to the algorithm the connections will be constantly under construction and destroyed. These examples are similar to the destruction of structures during the deformation of crystals. The algorithm can also be modified by implementing the feedback between the behavior of the structure and the interaction rules. For example, if conflict situations are detected in the structure, the algorithm will change (mutate) the rules. As a result, the algorithm will arbitrarily seek the rules for building consistent structures based on harmonious, mutually balanced elements.

4. The Example of Self-Replicating Structure

Rules of the MCA interactions can be supplemented by the possibility of activation of different subsets that describe transitions from one type to another, depending on some continuous parameters (analog of supplying nutrients). In this way, an arbitrary periodic dynamic will be realized, of which self-replicating is the partial case. In addition, the algorithm of the method can be supplemented by the mechanism of random addition of new types of MCAs to a modeled cell-like structure (analog of mutations), which can lead to increased stability of its dynamics, or vice versa—counteract replication. Finally, by creating the conditions for natural selection, it is possible to organize the evolution of the studied self-replicating cell-like objects.

Let us consider an example of the elementary self-replicative dynamics of some CA structure. The rules of equilibrium neighborhood of elements describing the self-replicating process are represented in Table 6.

Number of Neighbors	Type of Cells	S_1	S_2	S_3	S_4	S_5	S_6
6	S_1	-	6	-	-	-	-
3	S_2	1	2	1	1	1	1
4	S_3	-	2	-	2	-	-
6	S_4	-	3	2	1	-	-
5	S_5	-	2	-	-	1	2
6	S_6	-	4	-	-	2	-

Table 6. The rules of equilibrium neighborhood describing the self-replicating process.

Here for a type S_2 it is possible to see the peculiar ambiguity in sets of equilibrium neighborhoods. This suggests that any combination of three neighbors that corresponds to the possible variations from Table 6 is in equilibrium. For example, there may be different neighbors for S_2: either one S_1 and two S_2; or one S_3 and two S_2; or one S_1, one S_3 and one S_4, and so on.

The stages of the process of self-replication of the CA structure are depicted in Figure 6.

This process is accompanied by the transitions of the MCA from one type to another, in particular: $S_1 \to S_4 \to S_6 \to S_1$. Such transitions simulate the growth phases and the division of cell-like objects. Immediately after changing the type of the corresponding MCA, the structure will be in a nonequilibrium state, which will initiate the process of going to equilibrium, during which the spatial organization of the neighborhood is rebuilt and a new MCA appears (self-generated).

You can observe the similarity of this process with *Desmidium* algae division (Figure 7).

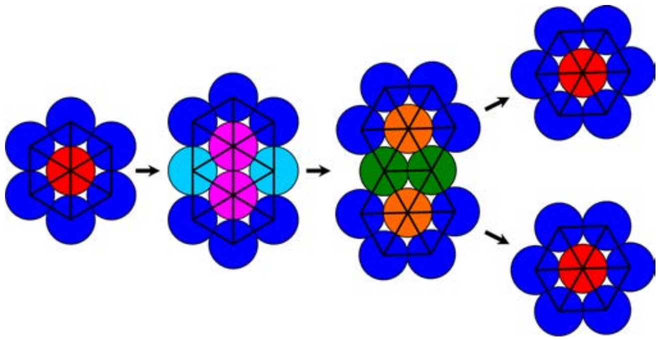

Figure 6. Demonstration of the self-replication process. The colors of the circles correspond to the color of the cells in Table 6 marking the appropriate type of automaton.

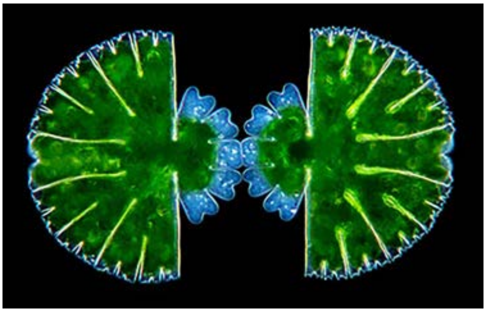

Figure 7. *Desmidium* algae division. The source of the image is www.microscopy-uk.org.uk.

5. The Example of Self-Replicating Structure in Three-Dimensional Space

By modifying the interaction functions, we can describe a similar model for the case of three-dimensional space. As an example, we can consider the equilibrium interaction of two types of MCAs, one of which (S_1-type) must be surrounded by eight other MCAs (S_2-type) in an equilibrium state (cubic neighborhooding). For the S_2-type MCA there must be four equilibrium neighbors, one S_1-type and three S_2-type. In parallel, let us consider a similar structure in which the automaton (S_1-type) must be surrounded by 12 other MCAs (S_2-type) in an equilibrium state (icosahedral neighborhooding). These equilibrium neighborhood rules are expressed in tabular form in Tables 7 and 8.

The rules of the equilibrium neighborhood of elements describing the self-replicating process in three dimensions are represented in Tables 9 and 10.

Number of Neighbors	Type of Cells	S_1	S_2
8	S_1	-	8
4	S_2	1	3

Table 7. Equilibrium neighborhood rules for MCAs in three-dimensional space in the case of cubic neighborhooding.

Number of Neighbors	Type of Cells	S_1	S_2
8	S_1	-	12
6	S_2	1	5

Table 8. Equilibrium neighborhood rules for MCAs in three-dimensional space in the case of icosahedral neighborhooding.

Number of Neighbors	Type of Cells	S_1	S_2	S_3	S_4	S_5	S_6
8	S_1	-	8	-	-	-	-
4	S_2	1	3	1	1	1	1
6	S_3	-	2	2	2	-	-
9	S_4	-	4	4	1	-	-
6	S_5	-	2	-	-	2	2
8	S_6	-	4	-	-	4	-

Table 9. The rules of the equilibrium neighborhood describing the self-replicating process in three-dimensional space in the case of cubic neighborhooding.

Number of Neighbors	Type of Cells	S_1	S_2	S_3	S_4	S_5	S_6
12	S_1	-	12	-	-	-	-
6	S_2	1	5	2	1	2	1
8	S_3	-	2	4	2	-	-
12	S_4	-	6	5	1	-	-
5	S_5	-	2	-	-	2	1
11	S_6	-	6	-	-	5	-

Table 10. The rules of the equilibrium neighborhood describing the self-replicating process in three-dimensional space in the case of icosahedral neighborhooding.

The stages of the process of self-replication of the CA structures are depicted in Figures 8 and 9.

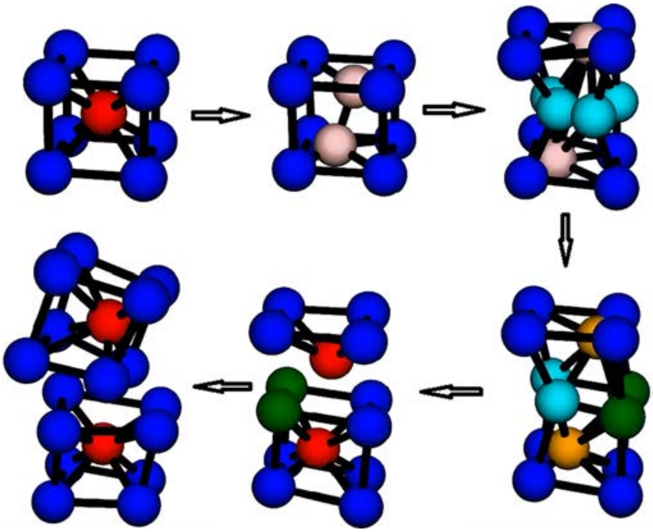

Figure 8. Demonstration of the self-replication process in three-dimensional space in the case of cubic neighborhooding. The colors of the circles correspond to the colors of the table cells marking the appropriate type of automaton.

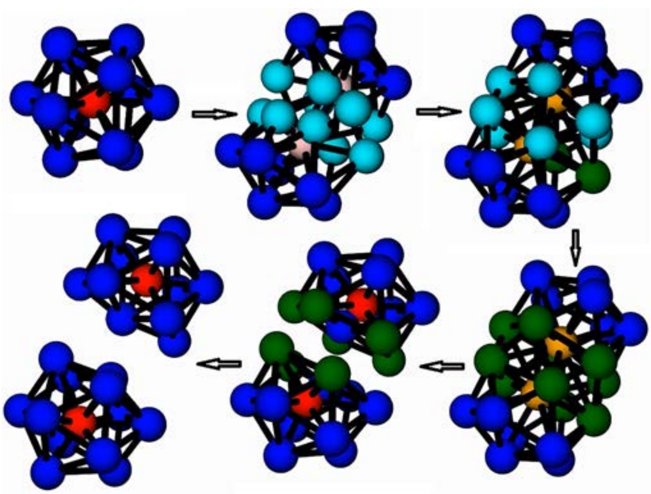

Figure 9. Demonstration of the self-replication process in three-dimensional space in the case of icosahedral neighborhooding. The colors of the circles correspond to the colors of the table cells marking the appropriate type of automaton.

6. Conclusion

In this paper the problem of the construction of a model of the self-replication process was considered. The paper gave an overview of available techniques used for the problem and outlined the arguments in favor of a cellular automaton (CA) method. The basic approach and general methodology for the development of models using movable cellular automata (MCAs) was examined. The method was implemented by establishing the rules of equilibrium neighborhood between different types of cellular automata (CAs).

Computer models simulating certain natural phenomena (self-regeneration and self-replication) were obtained. Results of calculations were graphically presented. It should be noted that the models are qualitative, not quantitative, and allow you to demonstrate the fundamental possibility of the movable cellular automaton (MCA) method for such modeling. The novelty of the research consists in the use of a substantially new approach to modeling. By changing the rules of equilibrium neighborhood (see Tables 7 and 9), it is possible to form various stable structures with arbitrary morphology, examples of which are shown in Figure 10. Moreover, it should be noted that the rules of the neighborhood may be asymmetric; that is, some MCAs can establish a connection of the neighborhood with the k^{th}, whereas k^{th} may not perceive them as its neighbor. Nevertheless, the structures will be stable, because the balance is not disturbed.

These structures are analogs of natural elementary microorganisms like radiolaria or diatoms (Figure 11). Simulation of such complex structures, as well as the study of their stability, evolution and so on, is the subject of our further research.

Number of Neighbors	Type of Cells	S_1	S_2	S_3	S_4	S_5	S_6	S_7
4	S_1	1	1	2	-	-	-	-
3	S_2	1	-	-	2	-	-	-
4	S_3	2	-	-	-	-	2	-
3	S_4	-	1	-	-	2	-	-
3	S_5	-	-	-	1	-	-	2
3	S_6	-	-	1	-	-	-	2
3	S_7	-	-	-	-	1	1	2

Table 11. The rules of equilibrium neighborhood for diatom-like structure.

Number of Neighbors	Type of Cells	S_1	S_2	S_3	S_4	S_5
5	S_1	-	5	-	-	-
6	S_2	1	2	2	-	1
7	S_3	-	2	-	1	4
6	S_4	-	-	1	-	5
5	S_5	-	1	1	1	2

Table 12. The rules of equilibrium neighborhood for radiolaria-like structure.

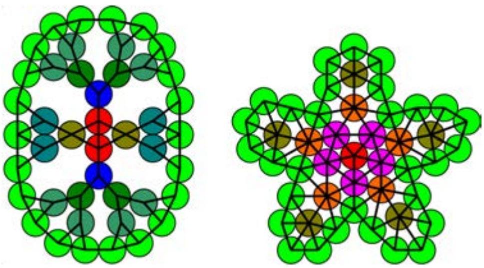

Figure 10. Examples of structures with arbitrary morphology, formed on the principles of MCA equilibrium neighborhood. The colors of the circles correspond to the cell colors in Tables 7 and 9 marking the appropriate type of automaton.

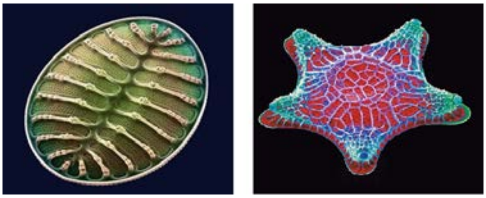

Figure 11. Elementary microorganisms: diatom and radiolaria. The source of the image is www.microscopy-uk.org.uk.

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