Elementary Cellular Automata along with Delay Sensitivity Can Model Communal Riot Dynamics

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This paper explores the potential of elementary cellular automata to model the dynamics of riot. Here, to model such dynamics, we introduce probabilistic loss of information and delay perturbation in the updating scheme of automata to capture sociological parameters—presence of anti-riot population and organizational presence of communal forces in the rioting society, respectively. Moreover, delay has also been incorporated in the model to capture the nonlocal interaction of neighbors. Finally, the model is verified by an event of riot that occurred in Baduria of West Bengal, India.

Keywords: elementary cellular automata (ECAs); delay; probabilistic loss of information; phase transition; riot dynamics

1. Introduction

Riots and their dynamics have been a popular topic for sociologists and historians [1–9]. In a parallel journey, computer scientists and mathematicians have found interest in the study of riots, with a target of mathematically modeling their dynamics [10–19]. The most popular approach to developing such models is to adopt an epidemiological framework [10–13, 17]. For example, 1960s riots: Los Angeles (1965), Detroit (1967), Washington D.C. (1968) [10]; 2005 French riot [11]; 2011 London riots [12] and others have been modeled using this approach. Recently, nonlocal interactions along with neighborhood dependency of elements of the system have been introduced in

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the models of riots [11, 13, 16, 20]. It is argued that due to globalization and the advent of communication technology, long range, that is, nonlocal, communications among elements are necessary to better model the dynamics of riots.

In this scenario, we undertake this research to show that the simplest one-dimensional, two-state, three-neighborhood elementary cellular automata (ECAs) [21] that rely only on local neighborhood dependency can efficiently model the complex societal dynamics of riots if the neighborhood dependency is *delay sensitive*. In particular, to model riot dynamics by ECAs, we introduce "probabilistic loss of information perturbation" and "delay perturbation" in the updating scheme of the automata. We observe that due to this updating scheme, the ECAs show a new kind of dynamical behavior, which suggests to us that some ECAs can be better models of a riot. Finally, to validate our claim, we take into consideration a riot event of 2017 that happened in Baduria of West Bengal, India. Since media reports do not always reflect the ground realities of riots, we organized an extensive field study in Baduria to get insight about rioting dynamics.

Here, in the proposed elementary cellular automaton (ECA)–based model, probabilistic loss of information perturbation rate is related to sociological factors such as the presence of *anti-riot* population in the rioting society. Similarly, the presence of communal elements in society, which plays a role in regenerating rioting spontaneity in the rioting society, indicates the physical implication of delay in the system. However, an inherent property of the cellular automaton (CA) is local interaction, which contradicts the recent trends of considering nonlocality in the age of globalization [11, 13, 16, 20]. The delay passively induces a nonlocality in the environment. To illustrate, the updated state information of a cell at time t reaches its neighboring cell at time step $t+n$, where n depicts delay for the cell and its neighboring cell. This implies nonlocal information from distance n reaches the corresponding neighboring cell. The presence of communal organization in society, which physically indicates delay in the system, induces this nonlocality to regenerate the rioting spontaneity.

2. Delay-Sensitive Cellular Automata

Here, we work with simple one-dimensional, three-neighboring, two-state cellular automata (CAs), which are commonly known as ECAs [22]. The next state of each CA cell is determined as $\mathcal{S}_i^{t+1} = f(\mathcal{S}_{i-1}^t, \mathcal{S}_i^t, \mathcal{S}_{i+1}^t)$, where f is the next state function, and $\mathcal{S}_{i-1}^t$, $\mathcal{S}_i^t$, $\mathcal{S}_{i+1}^t$ are the present states of left, self and right neighbor of the i^{th} CA cell at time t, respectively. The local transition function

$f : \{0, 1\}^3 \to \{0, 1\}$ can be expressed as eight arguments of f. The decimal counterpart of eight next states is referred to as a "rule." Each rule is associated with a "decimal code" w, where $w = f(0, 0, 0).2^0 + f(0, 0, 1).2^1 + \cdots + f(1, 1, 1).2^7$, for naming purposes. There are 2^8 (256) ECA rules, out of which 88 are minimal representative rules and the rest are their equivalent [23]. Classically, all the cells of a CA are updated simultaneously. In the last decade, researchers have explored the dynamics of CAs under asynchronous updating schemes [24–31].

Classically, in ECAs, delay and probabilistic loss of information during information sharing among the neighboring cells are not considered. In traditional CAs, if a cell updates its state at time t, then that state information is available to a neighboring cell at time $t + 1$. To define the delay involved in sharing of information for two neighboring cells i and j ($i \neq j$), we introduce a non-negative integer function $\mathrm{D}(i, j)$. In the proposed system, $\mathrm{D}(i, j) = \mathrm{D}(j, i) \geqslant 1$ for any pair of neighboring cells i and j. To illustrate, $\mathrm{D}(i, j) = n$ in the system implies that if cell i updates its state at time t, then the updated state information is available to cell j at time $t + n$. In the proposed system, the delays are nonuniform in space; that is, $\mathrm{D}(i, j)$ may be different from $\mathrm{D}(i', j')$, where i and j; i' and j' are neighboring cells; however, delay between two neighboring cells does not change with time. Practically, the delay perturbation parameter $d \in \mathbb{N}$ assigns the maximum possible delay in the proposed CA system. Every pair of neighboring cells is randomly initialized with delay between 1 and d following a uniform distribution. For the loss of information, we can consider that the delay is ∞ (infinity). Here, ι ($0 \leq \iota \leq 1$) indicates the probabilistic loss of information perturbation rate.

Now, for introducing probabilistic loss of information and delay in the system, each cell has to maintain state information of neighbors to get a view of a neighbor's state. In the proposed system, each cell has a *view* of the states of its neighbors, which may change from time to time depending on the arrival of state information about neighbors. However, the cells act depending on the current state information about neighbors at that time. In this context, the state set is distinguished into two parts—the actual state (self) of a cell and a vector of a neighbor's view state. The state set can be written as $\mathcal{S}' = \mathcal{S} \times \mathcal{S}^2$. Therefore, for a cell c, configuration at time t is distinguished into two parts—a_c^t and $\mathbf{v}_c^t$, where $a_c^t \in \mathcal{S}$ is the actual state and $\mathbf{v}_c^t \in \mathcal{S}^2$ is the vector of the view state of the left and right neighbors. Note that the actual state set $\mathcal{S}$ is sufficient to represent traditional CAs. Here, in the proposed CA system, the local transition function is also subdivided into two parts—in the first *state update* step, a cell changes its actual state depending on the actual state of the self and the view states of its neighbors; and, in the second *information sharing* step, the cell shares its updated actual state with its neighboring cells. Now, the local transition function can be written as $f' = f_u \circ f_s$, where f_u is the state update function and f_s is the information-sharing function. Here, the operator "$\circ$" indicates that the functions are applied sequentially to represent the actual update.

To illustrate, Figure 1 depicts a simple three-cell ECA, where $D(i-1, i) = D(i+1, i-1) = 1$ and $D(i, i+1) = 2$. In Figure 1, each cell has a view of the states of its neighbors, that is, the left and right neighbors for each cell. For every time step, the first step (dotted line) shows the state update function and the second step (straight line) shows the information sharing function. Here, the information about the state change of cell i (resp. cell $i+1$) at the first time step reaches cell $i+1$ (resp. cell i) at the third time step due to delay perturbation. In Figure 1, the information about the state change of cell i at the first time step does not reach cell $i-1$ at the second time step due to probabilistic loss of information perturbation.

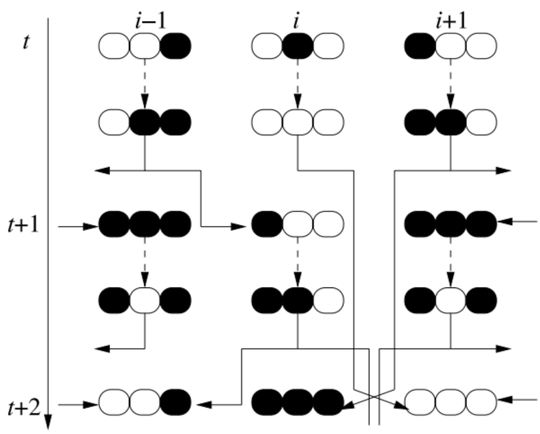


Figure 1. Example of delay and probabilistic loss of information perturbation updating scheme. The applied rule is ECA 50.

To sum up, the proposed CA system depends on the following two parameters: (*i*) the delay perturbation parameter $d \in \mathbb{N}$ indicates the maximum delay limit of the system; (*ii*) the probabilistic loss of information perturbation rate ι ($0 \leq \iota \leq 1$) indicates the probabilistic loss of information during information sharing.

3. Modeling of Riots

3.1 Dynamic Behavior

To model riot dynamics, we investigate the generative behavior of the proposed ECA system. In this paper, we start with the smallest possible seed pattern as the initial configuration, where a single cell is in state 1, that is, $\langle \cdots 0\,001\,000 \cdots \rangle$. From the modeling and theoretical research point of view, investigating the dynamics of seed patterns starting with a single cell in state 1 is a well-established research approach [32–34]. From the physical point of view, the initial seed with a single cell in state 1 represents the triggering event of riots.

Here, we study the qualitative behavior of the system starting from a single seed, where we need to look at the evolution of the configuration, that is, spacetime diagrams, by inspection over a few time steps. Though this is not a formal method, this approach can provide a good comparison. Note that this generative behavior study does not include 29 odd ECAs, out of 88 minimum representative ECAs, that have local transition function $f(0, 0, 0) = 1$. For odd ECAs, an empty background configuration, that is $\cdots 000 \cdots$, evolves to $\cdots 111 \cdots$, which is unable to produce the generative behavior of the system. Therefore, for the remaining 59 even rules, ECAs depict the following behaviors: (*i*) *evolution to zero*: after one time step, the seed cell in state 1 has vanished; (*ii*) *constant evolution*: the initial seed remains unchanged during the evolution of the system; (*iii*) *left evolution*: the seed shifts or grows to the left side; (*iv*) *right evolution*: the seed shifts or grows to the right; (*v*) *growth behavior*: the seed cell develops into a pattern for both left and right sides. Figure 2 depicts left evolution for ECA 14, right evolution for ECA 60 and growth behavior for ECA 30. Finally, all the minimal representative even ECA rules have been explored, and Table 1 shows the classification of ECAs depending on the generative behavior.

Here, the target of this simple classification is to identify the ECAs that develop into a pattern for both left and right sides, that is, growth behavior. Here, we make a sensible simple assumption that the riot propagation affects every neighbor, that is, both left and right for ECAs. Therefore, 12 ECAs, out of 88, with growth behavior are our target for modeling riot dynamics. In this context, note that Redeker et al. [32] classifies the behavior of traditional synchronous CAs starting from a single seed into "evolution to zero," "finite growth," "periodic patterns," "Sierpinski patterns" and "complex behavior," which have no clear equivalence to Wolfram's classes [35]. Only evolution to zero class shows similarity to this study. Here, the stable structure gets quickly destroyed in the presence of delay and probabilistic loss of information perturbation. As evidence, in

Figure 2, fractal-like Sierpinski patterns [32] are destroyed for ECA 18 under the proposed system. Therefore, the target of this paper is to identify candidate ECAs from 12 growth behavior ECAs to model riot dynamics.

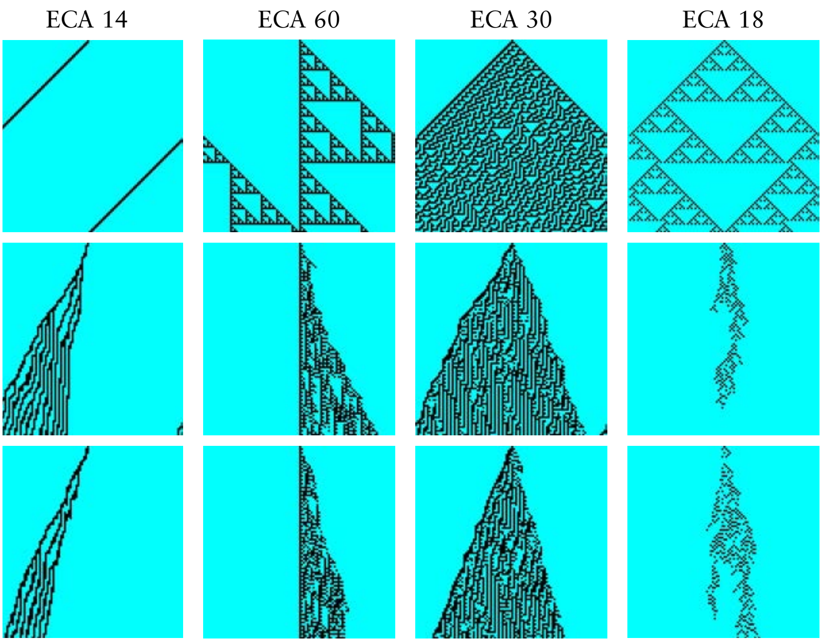


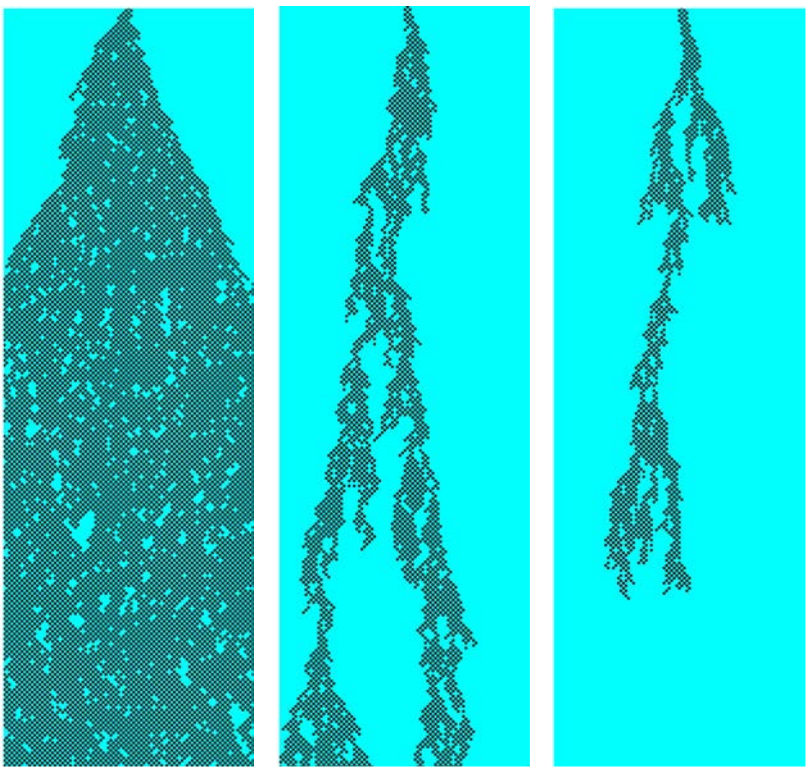
Figure 2. The samples of spacetime diagrams for the proposed updating schemes: (top) $d = 1$, $\iota = 0.0$; (middle) $d = 1$, $\iota = 0.5$; (bottom) $d = 2$, $\iota = 0.5$.

Evolution to zero:	0	8	32	40	72	104	128	136
	160	168	200	232				
Constant evolution:	4	12	36	44	76	108	132	140
	164	172	204					
Left evolution:	2	6	10	14	34	38	42	46
	74	78	106	130	134	138	142	162
	170							
Right evolution:	24	28	56	60	152	156	184	
Growth behavior:	18	22	26	30	50	54	58	90
	146	150	154	178				

Table 1. Classification of ECA rules depending on the generative behavior.

3.2 Candidate Elementary Cellular Automata for Modeling Riots

Let us assume that riot dynamics have two phases: spreading phase and diminishing phase. So we choose four candidate ECAs, out of 12, that show phase transition under the proposed updating scheme, to model riot dynamics (as an example, see ECA 18 in Figure 2). For these four ECAs—18, 26, 50 and 146—out of 88 minimal representative rules, there exists a critical value of probabilistic loss of information perturbation rate that distinguishes the behavior of the system in two different phases—passive phase (i.e., the system converges to a homogeneous fixed point of all zeros) and active phase (i.e., the system oscillates around a nonzero density). As an example for ECA 50, Figure 3 (left) depicts the active phase for probabilistic loss of information perturbation rate 0.3 ($\iota = 0.3$); however, a phase transition from active to passive phase is observed in Figure 3 (right) where $\iota = 0.5$.



(a) (b) (c)

Figure 3. The samples of spacetime diagrams depicting phase transition: (a) $d = 1, \iota = 0.3$; (b) $d = 1, \iota = 0.4$; (c) $d = 1, \iota = 0.5$.

As a physical implication with respect to rioting dynamics, the diminishing phase is not observed without the presence of a certain percentage of anti-riot population, that is, probabilistic loss of information perturbation rate. However, the presence of a certain percentage of anti-riot population in the society leads to the passive phase in the diminishing rioting dynamics. According to [25], this phase transition belongs to the directed percolation universality class. In the literature of rioting dynamics research, the idea of critical threshold was also discussed in [13, 36] for understanding the level of social tension to start a riot and a sufficiently large number of protests to start a revolution, respectively. Here, this study considered only initial seed with a single cell in state 1. However, the given phase transition dynamics of ECAs 18, 26, 50 and 146 are also observed for an initial seed with multiple (random) start states [25]. Therefore, note that the proposed CA model is also valid for initial seed with multiple start states.

To understand the quantitative behavior of these candidate rules, we let the system evolve through 2000 time steps and average the density parameter value for 100 time steps. Note that for a configuration $x \in \mathcal{S}^{\mathcal{L}}$, the density can be defined as $d(x) = \#_1 x \,/\, |x|$, where $\#_1 x$ is the number of ones in the actual state for the configuration and $|x|$ is the size of the configuration. Figure 4 shows the plot of the

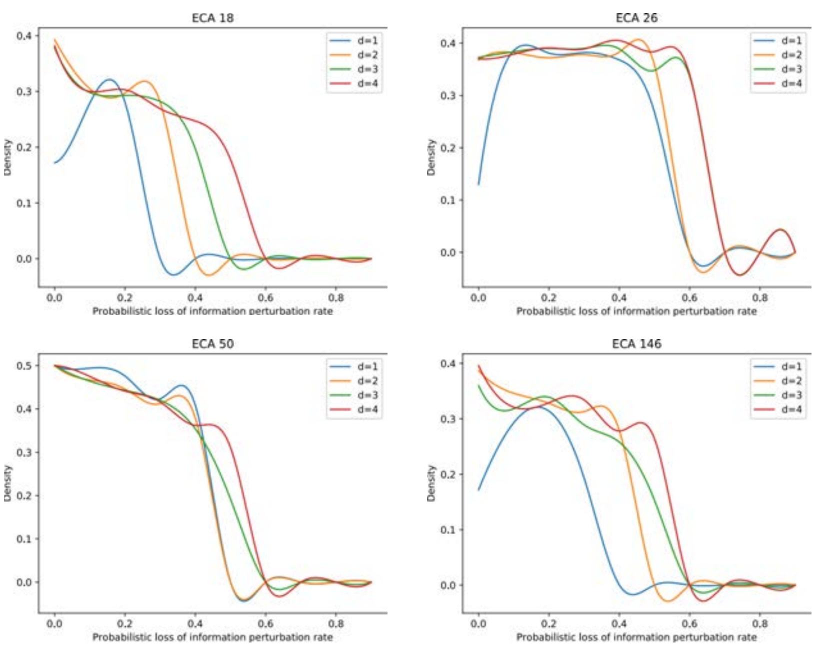


Figure 4. The plot shows the profile of the density parameter as a function of the probabilistic loss of information perturbation rate with a fixed d parameter for ECA rules.

profile of the density parameter starting from a single "1" seed as a function of the probabilistic loss of information perturbation rate with a fixed d parameter for ECA rules that depict phase transition behavior. Table 2 depicts the critical value of probabilistic loss of information perturbation rate for phase transition associated with these ECA rules where $\iota^c_{d=k}$ indicates the critical value with d parameter value k. Note that the critical value for phase transition increases when the updating scheme is also associated with delay perturbation; see Table 2 for evidence. Moreover, the critical value of probabilistic loss of information perturbation rate for phase transition proportionally increases with increasing value of delay. Table 2 justifies that the diminishing phase of riot needs more percentage of anti-riot population in the presence of sociological factor delay. Note that this phase transition result is not observed for only the delay perturbation updating scheme. To sum up, ECAs 18, 26, 50 and 146 are the final candidate rules for modeling rioting dynamics. Therefore, the target is to identify the best candidate rule among those ECAs for validation of Baduria riot dynamics. In this scenario, the next section depicts the case study comprising the Baduria riot's dataset.

ECA	$\iota^c_{d=1}$	$\iota^c_{d=2}$	$\iota^c_{d=3}$	$\iota^c_{d=4}$
18	0.27	0.38	0.46	0.48
26	0.51	0.54	0.63	0.67
50	0.48	0.50	0.53	0.55
146	0.32	0.43	0.46	0.50

Table 2. The critical value for phase transition of ECAs 18, 26, 50 and 146.

4. Baduria Riot and the Proposed System

4.1 Baduria Riot Dataset

Attracting nationwide media attention, the Baduria riot is the most well exposed among recent riot events of West Bengal, an Indian state [37]. The triggering event of the Baduria riot took place following a social media religious post by a 17-year old student in the village Baduria of West Bengal on July 2, 2017. This social media post was seen as objectionable and went viral in Baduria. Starting with this, violent clashes were triggered between two religious communities in the subdivisions Baduria, Basirhat, Swarupnagar, Deganga, Taki and Haroa [37].

Here, we base our analysis on reported incidents in the media during the time of the Baduria riot. After scraping of chosen newspaper websites, a topic modeling technique [38] has been used with some tuning. Some manual inspection was required to zoom into the topics that discussed news pertaining to communal riots. The authenticity of media report data is cross-verified with field study. We extract riot-like events, for example, "attack on police vehicles," "serious injuries," "group clashed" and so on, from 20 media reports to build the dataset. In the literature, the traditional methodology for quantifying rioting activity is to study the daily crime reports of police data to analyze the riot dynamics [11, 12]. In this incident, arrest records, though available [39], do not indicate communal riot as the explicit reason for the arrest. However, police records [39] available in the official website are used to corroborate the incidents through manual intervention.

Here, we adopt two simple methodologies for quantifying the rioting activity: First, we define as a single event any rioting-like act, as listed in the media reports after ambiguity checking, depending on its intensity. Thus, "rail blockades at 3 places" counts as three events, "three police vehicles have been torched" indicates three riot-like events. We thus get a dataset composed of the number of riot-like events for every day from July 2 to July 9, 2017. It should be noted that no event appears more than once in the dataset. Second, we quantify rioting activity from area (by square km) affected by riot on a day-to-day basis from media reports. Figure 5 reflects the spatial propagation of riot dynamics. It is not possible to quantify some important riot events, like (number of) group clashes, road blockades, market/shop/school closures, from media reports. Therefore, we quantify those rioting events by area affected due to riots day by day. Figure 6 shows the number of attack events on: (a) police (vehicles, stations, persons); (b) religious place, home, rail line and serious injuries; (c) affected area (in square km) over the course of time. In this context, recall that the authenticity of media reports data is cross-verified with field study. Specifically, the target of the field study is to fill the gap of the quantitative media data. Here, the field study mainly follows the methodology of "unstructured interview" [40, 41] along with "participant observation" [42] and "focus group discussion" [43]. Following are the identifiers for the cited unstructured field interviews—GM: gender male, GF: gender female, RH: religion Hindu, RM: religion Muslim, A1: age between 20–45, A2: age between 45–60 and A3: age above 60. Since the nature of the study is not to find fault with the communities, rather measure simply the spatiotemporal progress and regress of the riots, it has been possible to collect some reliable datasets for cross-verification of quantitative media data.

Now, we calculate the summation of percentage of intensity per rioting day, out of total intensity, for the rioting event datasets of Figure 6(a), (b) and (c). Hereafter, the percentage of intensity per rioting

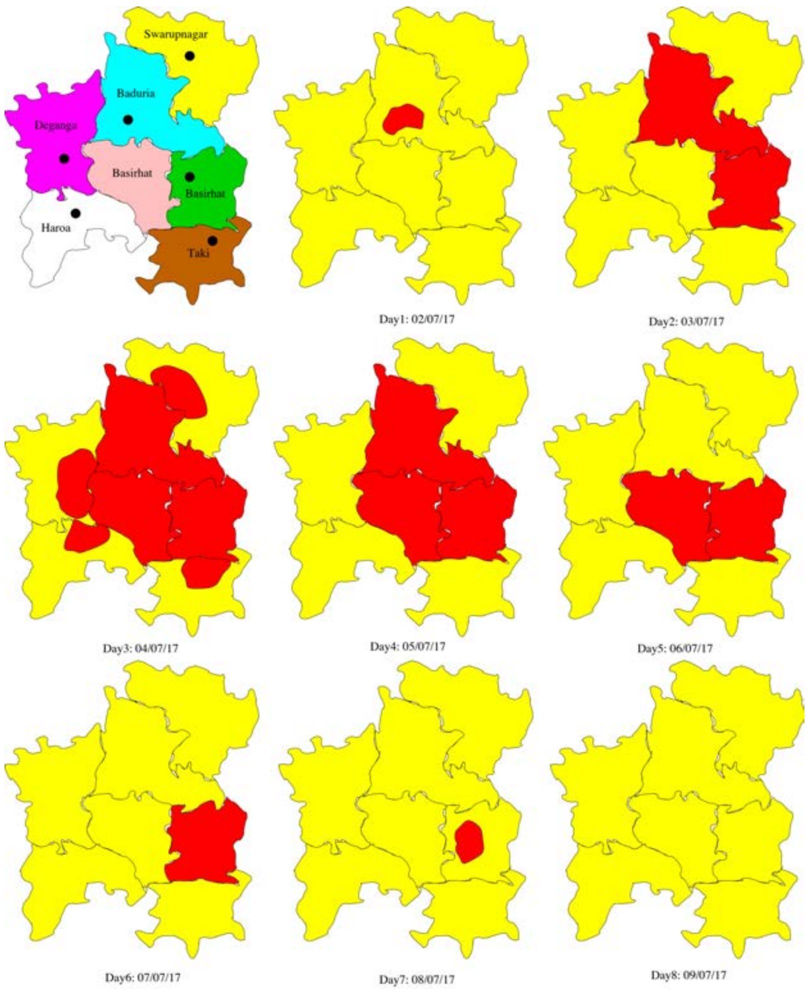


Figure 5. Graphical riot propagation dynamics of the Baduria riot.

day of this summarized intensity indicates the *normalized overall intensity* of the Baduria riot that is reflected in Figure 6(d). Here, we work with this normalized overall intensity to understand the rioting dynamics, which show simple growth (up) and shrink (down) dynamics. These simple up and down dynamics, without any rebound, were also observed for the 2005 French riots [11] and US ethnic riots [10]. With a contradiction, dynamics with up and sudden down (i.e., rise for four days and suddenly down on fifth day) were found in the 2011 London riots [12, 13].

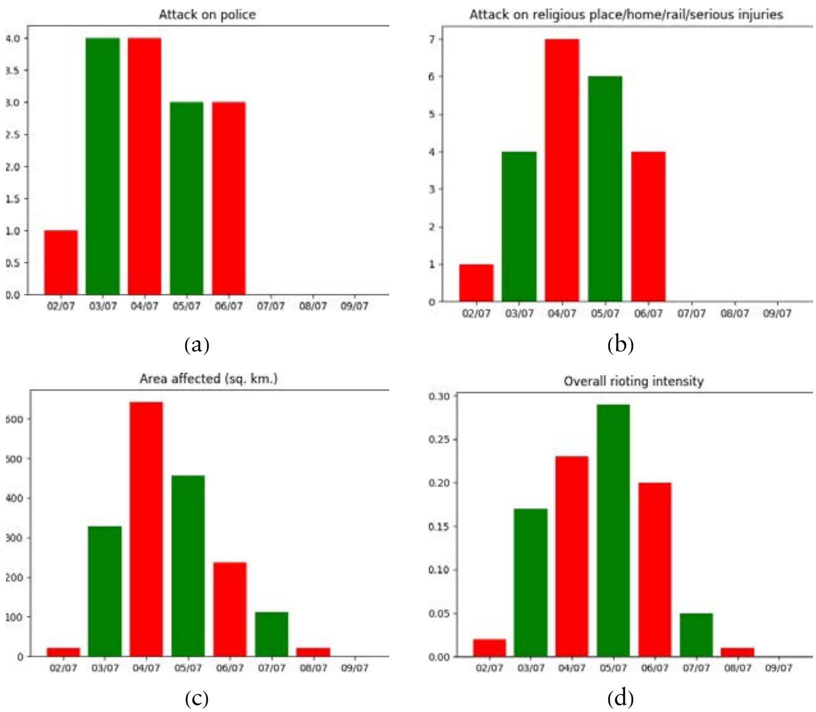


Figure 6. The plot shows quantified rioting activity for every day from July 2 to July 9, 2017 of: (a) attack on police (vehicles, stations, persons); (b) attack on religious place, home, rail line and serious injuries; (c) affected area in riot (by square km); (d) normalized overall intensity.

From a different internal dynamics perspective, the propagation dynamics of the Baduria riot depend on the local communication of violent events [44] and local rumor propagation [45]. However, during the riot event a parallel journey of religious harmony was also reported in the media [46, 47], which finally converted the dynamics of the event to an early-diminishing riot event. The field data also reflects this evidence [48].

In this context, the anti-riot population of society does not participate in this rumor propagation. Moreover, they play an important role in the early-diminishing dynamics of the riots. Here, the term "anti-riot" population is composed of the following population—first, the secular population of society [46, 47]; second, not "purely" secular population, however, due to economic dependency, they play an anti-riot role during the time of the riot [49].

4.2 Verification

This finding is mapped with the best candidate rule among ECAs 18, 26, 50 and 146 for modeling the Baduria rioting dynamics. To compare the CA dynamics and Baduria riot dynamics, we let the system evolve starting from a single state "1" seed and average the density parameter value for every 100 time steps, which defines one single time unit ($\therefore$1 time step $\approx$15 minutes). Therefore, the *normalized density parameter* and *normalized overall intensity* of the Baduria riot are the parameters for comparison in this study. The normalized density parameter is also calculated, following the similar procedure of calculating the normalized overall intensity parameter. Figure 7 shows the profile of the normalized density parameter or normalized overall intensity of the Baduria riot as a function of the time series. According to Figure 7, ECAs 26 and 146 show similar dynamics with the Baduria riot; however, ECAs 18 and 50 come up with "late" convergence dynamics for critical probabilistic loss of information perturbation rate (ι^c) where $d = 1$. We identified the best-fitting CA model using the standard *root mean square deviation*:

$$\sqrt{\frac{\sum_{t=1}^{T}\left(x_{1,\,t} - x_{2,\,t}\right)^2}{T}}$$

where $x_{1,t}$ and $x_{2,t}$ depict the time series normalized overall intensity of the Baduria riots and the density parameter value of the CA model, respectively. Here, ECA 26 ($d = 1$ and $\iota^c = 0.51$) shows the best-fitting attitude with the Baduria riot data where the root mean square deviation is 0.064. ECAs 18, 50 and 146 are associated with root mean square derivation values 0.129, 0.138 and 0.071, respectively. The best-fitting CA with $d = 1$ depicts the evidence of no major presence of organized communal force in the rioting society. The field data also reflects this absence. Figure 8 depicts the evidence associated with "late" convergence with increasing value of the delay perturbation parameter for these ECA rules. In the absence of organized communal force, the rioting society reacts spontaneously and a simple up

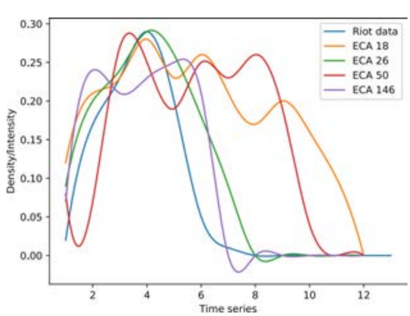


Figure 7. The plot compares normalized overall intensity of the Baduria riot and normalized density of ECA rules as a function of time series. Here, $d = 1$ and $\iota = \iota^c$.

and down dynamics are reflected. However, increasing rebound dynamics are observed for a rise in the value of the delay perturbation parameter; see Figure 8 as evidence. As an insight, the rebound dynamics indicate that organized communal force plays a role in regenerating the spontaneity of rioting in the society.

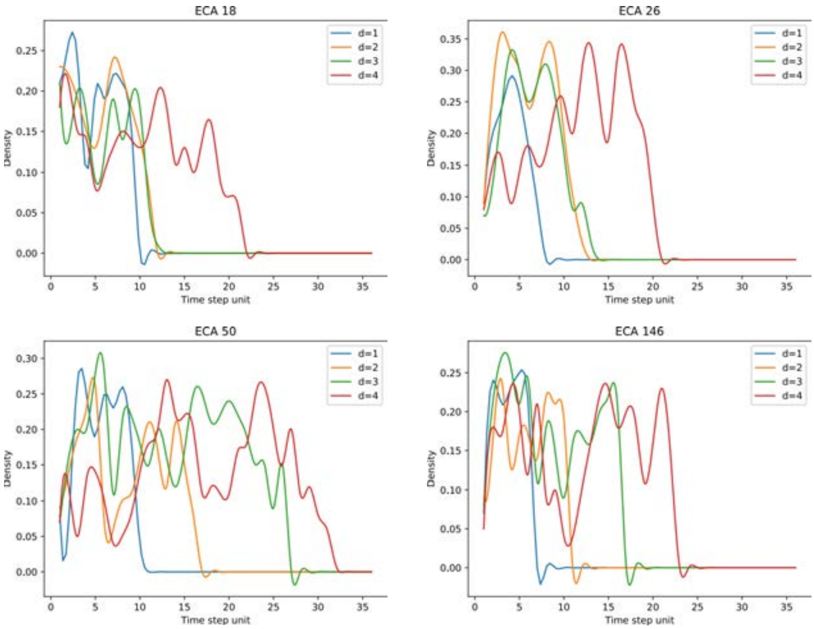
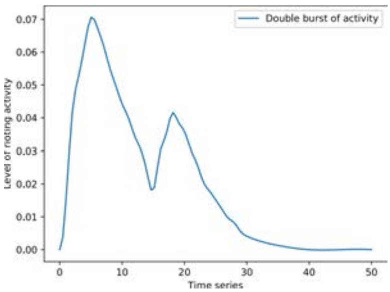
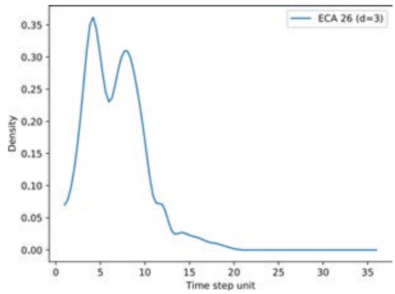


Figure 8. The plot shows the profile of the density parameter as a function of time steps with changing delay parameter for ECA rules. Here, the plot works with critical probabilistic loss of information perturbation rate ι^c.

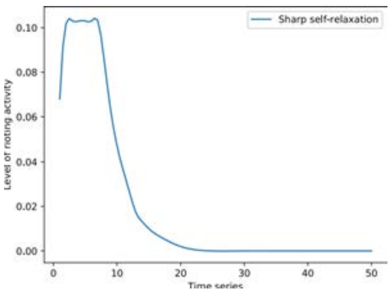
The proposed model only verifies the rioting dynamics of the West Bengal (India) event. However, as a discussion, the proposed model also depicts similar dynamics with the 2005 French riot [11], the 2011 London riot [12] and the effect of exogenous and endogenous factors in riot dynamics [13]. For evidence, recall that simple up and down dynamics, without any rebound, were observed for the 2005 French riot. In this context, Figure 9(c1) shows normalized row data and calibrated model data for the 2005 French riot following the work of Gahot et al. [11]. According to [13], the behavior of the 2005 French riot depicts slow relaxation of rioting activity. Figure 9(c2) shows the normalized simulation result of slow relaxation behavior following the work of [13]. ECA 26 depicts similar slow relaxation dynamics of the 2005 French riot; see Figure 9(c3).



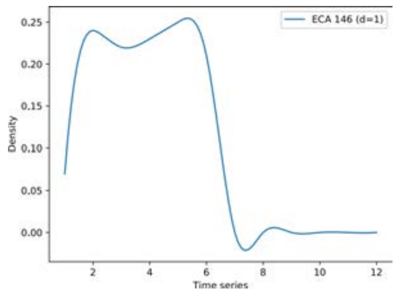
(a1)



(a2)



(b1)



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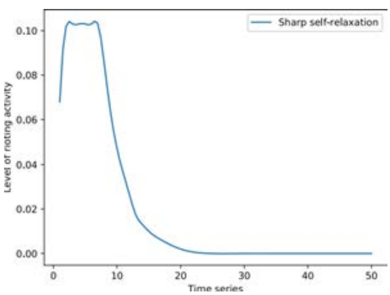
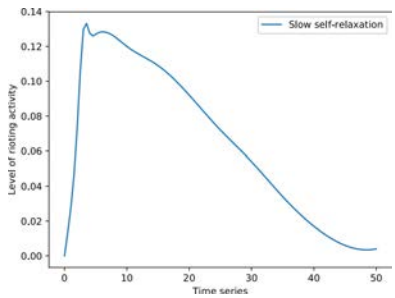


(c1)



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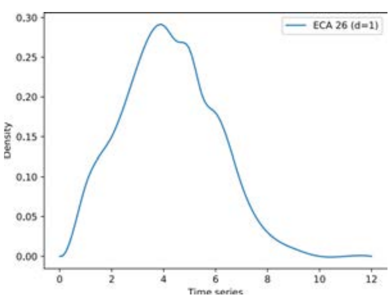

(c3)

Figure 9. (a) Two bursts of rioting activity: (a1) normalized simulation result of [13] showing two bursts of rioting activity where $z_0 = 10$, $w = .3$, $A_1 = 6$, $A_2 = 3$, $\theta = .4$, $p = 1$, $\beta = 1$, $a = 6$ [13]; (a2) similar two bursts of rioting

dynamics with ECA 26 where $d = 3$ with critical probabilistic loss of information rate. (b) 2011 London riot: (b1) normalized simulation result of [13] showing sharp self-relaxation rioting activity where $z_0 = 10$, $w = .2$, $A = 6$, $\theta = .1$, $p = 1$, $\beta = 1$, $a = 6$, $t_1 = 0$ (According to [13], this type of relaxation was observed during the 2011 London riot.); (b2) similar relaxation dynamics with ECA 146 where $d = 1$ with critical probabilistic loss of information rate. (c) 2005 French riot: (c1) normalized raw data and calibrated model data for the 2005 French riot of [11]; (c2) normalized simulation result of [13] showing slow self-relaxation, similar to the 2005 French riot, where $z_0 = 10$, $w = .2$, $A = 5$, $\theta = .1$, $p = 1$, $\beta = 10$, $a = 6$, $t_1 = 0$, $t_2 = 12$; (c3) similar relaxation dynamics with ECA 26 where $d = 1$ with critical probabilistic loss of information rate.

As a different example, according to [12, 13], the 2011 London riot depicts fast relaxation of rioting activity. Figure 9(b1) shows the normalized simulation result of fast relaxation of rioting activity following the model of [13]. Here, as evidence, ECA 146 depicts similar fast-relaxation (sudden-down) dynamics of the 2011 London riots; see Figure 9(b2). Moreover, the work of [13] analyzed two bursts of rioting activity where the first shock was strong enough to lead to a burst of activities that settled down before a second shock occurred. According to [13], this type of dynamic was observed in the recent protests in Ferguson, Missouri. Here, Figure 9(a1) shows the normalized simulation result of two bursts of activity following [13]. From the proposed CA model point of view, ECA 26 with high delay perturbation (i.e., $d = 3$) shows a similar two bursts of activity; see Figure 9(a2). In this direction, Berestycki et al. [13] have also analyzed sudden spike dynamics to represent strong exogenous factors and slower but steady increase to reflect endogenous factors in rioting dynamics. ECAs 18 and 50 show similar sudden spike and steady growth dynamics, respectively; see Figure 8 for evidence. However, proper understanding about this similar signature behavior of the proposed CA system and exogenous-endogenous factors is still an open question for us. Depending on the wide variety of results, the study strongly indicates that this model is relevant for other rioting dynamics.

5. Discussion

To sum up, the study reflects the modeling aspects of the internal convergence rioting dynamics with respect to the sociological factors—presence of anti-riot population as well as organizational presence of communal forces. One may argue about the presence of other sociological factors in the rioting dynamics; however, our target is to propose a simple model that is able to capture complex rioting dynamics. To validate our argument, we quote from Burbeck et al. [10], which is the pioneering work on epidemiological mathematical riot modeling.

> "First efforts at model building in a new field necessarily encounter the risk of oversimplification, yet if the models are not kept as simple as is practical they tend to become immune to falsification."

Moreover, this simple local interaction model can capture nonlocality using delay perturbation. Here, the proposed CA system introduces nonuniformity with respect to information-sharing aspects; however, the system is associated with uniform rule. Several nonuniform internal dynamics are also reflected in Baduria incident; that is, here, three types of internal dynamics (first: looting incident; second: local resident versus refugees; third: local people versus police) are reflected from the field data. The current construction of a CA model with uniform rule is unable to address these microscopic dynamics. Therefore, this research can be extended to explore the dynamics of the CA model with nonuniform rule [50].

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Data Availability

The dataset used for this work is available from the corresponding author on reasonable request, and under the condition of proper referencing.

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