A Game of Life Shifted toward a Critical Point

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The Game of Life (GoL), which produces complex patterns of life, has been employed to describe biological systems through self-organized criticality and scale-free properties. This paper develops two novel GoL models. One model allows each cell to update the time for the state update following interactions with other local cells using parameter tuning. Thus, individual cells replace their behaviors from intermittent state updates with continuous ones. The system evolves unpredictably close to a critical point and occasionally close to extinction for the alive population if an adequate parameter is chosen. This event occurs with a power-law tailed time interval and presents synchronous behaviors, since individual cells modify their state-update intervals and create time continuity. The other model is the same except that the system evolves unpredictably without any parameter tuning. In the second model, actions of individual cells are tuned not by a fixed parameter but by the surrounding situation. We found that the GoL system in the second model behaved in a similar manner in the first model, which suggests that that model shifts toward a critical point autonomously.

Keywords: Game of Life, critical point, power-laws, phase transition

1. Introduction

The Game of Life (GoL) was proposed by Conway to describe the evolution of life [1]. GoL generates complex patterns even though each cell follows simple local rules. Also, it computes biological systems using its defined cellular automaton, which is related to a universal Turing machine [2, 3].

The classical GoL generates patterns as time continues; however, it no longer generates new patterns at steady states. Biological systems show self-organized criticality (SOC) as a characteristic property [4–6]. For classical GoL, adding perturbations to the system is related to the existence of the SOC in the GoL [7]. The scale-free properties also emerge around SOC [8–10], implying that the system is unpredictable and is related to the unstable evolution of the system.
Therefore, the system shows a phase transition for a certain parameter around a critical point [8, 11]. Switching between synchronous and asynchronous updates is related to phase transitions in the GoL [12–16]. However, these papers focus on tuning a parameter for the ratio of asynchronous updates, which is linked to the emergence of a phase transition. In actual living systems, however, each agent (cell) determines their actions, including their state updates, using their experiences or interacting with the environments [17–20]. Although a few studies deal with this problem in the GoL [10], no studies have reported the importance of the timing of state updates and its relations with the active processes of individual cells.

Furthermore, using simulation experiments, an experimenter must identify optimal parameter values when tuning parameter values. However, parameter tuning does not apply to the decision-making processes of individual agents in the system since they cannot take an extensive view of the entire system. This point of view can be an important problem when agents in the system do not have highly developed cognitive abilities, which inhibits them from modifying their actions according to optimal parameter values [21, 22]. As already mentioned, each agent has to determine their actions, including their state updates, using their experiences or interacting with the environments. Through the active processes of individual agents, the system will shift toward its critical point autonomously.

To this end, this paper considers two purposes using two proposed models. One produces an unstable evolution of the GoL, which is linked to a phase transition using parameter tuning. As mentioned, we focus on the importance of the timing of state updates and its relations with the active processes of individual cells. For the proposed model, each cell does not conduct state updates on every generation, but instead, it does it at appropriate intervals. Under such circumstances, each cell considers and compares a time lag for the state update between itself and the surrounding cells and coordinates its time intervals for the state update if most surrounding cells conduct the state update. By doing so, synchronization and time continuity can emerge. Also, each cell can re-coordinate its time intervals regarding the state update if a minority of surrounding cells conduct the state update. This re-coordination occurs with a probability using parameter tuning. Using these modifications, the alive population in the system evolved unpredictably and dynamically. More importantly, the alive population in the system exhibited a phase transition regarding the tuned parameter.

The other model produces a similar phenomenon without parameter tuning. In the second model, each cell re-coordinates its time intervals regarding the state update by considering a balance between the number of surrounding cells that conduct the state update and the
number of surrounding cells that do not conduct the state update. As a result, we found that the GoL system in the second model presented a shift toward a critical point autonomously.

2. Methods

2.1 Agent and Space
Cells arranged 100×100 are set on the simulation field and periodic boundaries are assumed. The Moore neighborhood was adopted. Each cell changed states from alive to dead or vice versa using the classical GoL rule. The state of a cell in the next generation is alive if three cells within its neighborhood are alive and can be dead if fewer than two or more than three neighboring cells are alive. Each trial finishes at \( t = 10000 \). However, some analyses finish each trial at \( t = 100000 \). The state of each cell is randomly selected as alive or dead with an equal probability at the beginning of each trial.

2.2 Model Description: The \( p \) Model
Each cell in the proposed model does not change its state in each generation; rather, state change occurs at every interval \((i, j)\) for cell \((i, j)\). The parameter interval \((i, j)\) is randomly selected from \(\{1, 2, 3, \ldots, n\}\) at the beginning of each trial. For example, cell \((i, j)\) replaces its state using the classical GoL rule every three generations if interval \((i, j) = 3\). In classic GoL models, interval \((i, j)\) is set to 1 for all cells. The parameter active \((t, i, j)\) can be 1 or 0 and determines whether cell \((i, j)\) uses the GoL rule for the state change at generation \(t\). For example, active \((0, i, j) = 1\) and active \((1, i, j) = 0\) if interval \((i, j) = 3\).

For the proposed model, each cell modifies its interval as follows; interval \((i, j)\) is replaced with 1 at generation \(t\) with a probability \(\varphi(i, j)\) if active \(t, i, j = 0\). Here, a parameter \(\varphi(i, j)\) is determined as:

\[
\varphi(i, j) = 1 - 1 / N_{\text{active}}(i, j),
\]

where \(N_{\text{active}}(i, j)\) is the total number of neighboring cells that satisfy active \(= 1\) at generation \(t\). If \(N_{\text{active}}(i, j) = 0\), then \(\varphi(i, j)\) can be 0 at that time.

Thus, cells change state by replacing their interval when neighboring cells are active (here, active means that cells conduct a state update). After the parameter interval \((i, j)\) is fixed to 1, cell \((i, j)\) modifies this parameter with a probability \(p\) as follows: interval \((i, j)\) is replaced with a natural number selected from a set \(\{1, 2, \ldots, n\}\) at generation \(t\) with a probability \(\Phi(i, j)\):

\[
\Phi(i, j) = 1 / N_{\text{active}}(i, j),
\]
where \( N_{\text{active}}(i, j) \) is the total number of neighboring cells that satisfy active = 1 at generation \( t \).

Thus, cells reevaluate their interval when neighboring cells are inactive. First, our simulation experiment investigated a critical value for the parameter \( p \) using the parameter \( p \), respectively. After evaluating a critical value for parameter \( p \), our simulation experiment fixed the parameter \( n = 5 \) for further investigations.

### 2.3 Model Description: The Situation Model

The situation model is the same as the \( p \) model except that the parameter \( p \) is replaced with a non-fixed value. Here, we explain the modification of the interval as follows.

For the proposed model, each cell modifies its interval as follows; interval\((i, j)\) is replaced with 1 at generation \( t \) with a probability \( \varphi(i, j) \) if \( \text{active}(t, i, j) = 0 \). Here, a parameter \( \varphi(i, j) \) is determined as follows:

\[
\varphi(i, j) = 1 - 1/N_{\text{active}}(i, j),
\]

where \( N_{\text{active}}(i, j) \) is the total number of neighboring cells that satisfy active = 1 at generation \( t \).

Thus, cells change state by replacing their interval when neighboring cells are active. After the parameter interval\((i, j)\) is fixed to 1, cell \((i, j)\) remodifies this parameter with a probability \( \Psi(i, j) \) as follows: interval\((i, j)\) is replaced with a natural number selected from a set \( \{1, 2, \ldots, n\} \) at generation \( t \) with a probability \( \Phi(i, j) \):

\[
\Phi(i, j) = 1/N_{\text{active}}(i, j),
\]

where \( N_{\text{active}}(i, j) \) is the total number of neighboring cells that satisfy active = 1 at generation \( t \).

Here, a probability \( \Psi(i, j) \) is defined as

\[
\Psi(i, j) = 1/N_{\text{inactive}}(i, j), \quad N_{\text{inactive}}(i, j) = 8 - N_{\text{active}}(i, j)
\]

where \( N_{\text{inactive}}(i, j) \) is the total number of neighboring cells that satisfy active = 0 at generation \( t \).

Please note that cells in the situation model do not use the fixed parameter \( p \) but coordinate the parameter \( \Psi(i, j) \) according to the environmental situation. Individual cells in that model therefore consider both \( N_{\text{active}} \) and \( N_{\text{inactive}} \) when they return the interval to a natural number selected from a set \( \{1, 2, \ldots, n\} \) by using two probabilities (\( \Phi \) and \( \Psi \)).

### 3. Results

First, we focused on the performance of the \( p \) model. A critical value is evaluated for parameter \( p \) using the parameter \( n = 3, 4, 5, 6, 7 \) and
8. Figure 1 demonstrates a relationship between fractions of alive population at the end of trials and parameter $p$ and shows a phase transition. This data was obtained from 100 trials. The system reaches a steady state with the alive population almost extinct when $p$ is set to low values. However, some fraction of the alive population obeys phase transition at $p$ approximately 0.80, around which some critical values also exist. This tendency can be related to a second-order phase transition [12, 14]. Importantly, the parameter $n$ does not seem to affect a relationship between fractions of alive population at the end of trials and parameter $p$, including a phase transition. Thus, we evaluate the system evolution in detail by evaluating the population dynamics using $n = 5$. Further investigations were conducted using $p$ values around 0.80 and setting 100 000 generations as one trial.

![Figure 1](https://doi.org/10.25088/ComplexSystems.32.1.57)

**Figure 1.** A phase transition regarding alive populations by tuning the parameter $p$ in the $p$ model. Data is obtained from 100 trials. (a) $n = 3$. (b) $n = 4$. (c) $n = 5$. (d) $n = 6$. (e) $n = 7$. (f) $n = 8$. 


The instability of the fraction of the alive population is shown in Figure 2(a), 2(b) and 2(c) as examples for several parameter values ($p = 0.80$, $p = 0.85$ and $p = 0.75$, respectively). From these figures, the system transits between some population densities. This tendency disappeared if each cell did not modify the parameter interval and obeyed a fixed interval value from the beginning to the end of each trial (the control model) (Figure 2(d)). In the control model, the fraction of alive cells seldom fluctuates compared with the proposed model. This is because cells are not to be dead cells even if they are under overcrowded conditions. For the proposed model, the parameter interval at 1 and a population of alive cells fluctuate dynamically. Figure 3 presents an example of alive distributions at several generations of one trial using $p = 0.80$, which shows that the system did not reach a steady state and confirms that the system shows some SOC.

**Figure 2.** Time evolution of alive populations in the $p$ model using $n = 5$. (a) $p = 0.80$. (b) $p = 0.85$. (c) $p = 0.75$. (d) The control model.
In fact, alive distributions appear to change gradually (Figure 4). Figure 4 presents the last four generations of one trial.

Investigating scale-free properties, which relate to a phase transition, we examined the generation length that satisfies the alive density < 5% at $p = 0.80$. Figure 5 shows the relationship between the

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**Figure 3.** An example of spatial patterns of the GoL system in the $p$ model using $p = 0.80$ and $n = 5$. Yellow cells are alive cells, while dark blue cells are dead cells. Patterns at 60,000, 70,000, 80,000 and 90,000 generations are shown from left to right.

**Figure 4.** An example of spatial patterns of the GoL system in the $p$ model using $p = 0.80$ and $n = 5$. Yellow cells are alive cells, while dark blue cells are dead cells. Patterns at 99,997, 99,998, 99,999 and 100,000 generations are shown from left to right.

**Figure 5.** A power-law tailed distribution in the $p$ model regarding the duration of extinctions obtained from 1 trial using $p = 0.80$ and $n = 5$. 

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generation length and its cumulative frequency. From Figure 5, a power-law tailed distribution can emerge ($n$ of data = 126, $\mu = 2.06$, AIC weights for a power-law = 1.00). This result shows that the system was almost extinct. However, the system moved past the crisis autonomously.

From that point, we investigated whether the situation model presented similar results to the $p$ model. If so, we can confirm that the situation model exhibits some SOC without any parameter tuning. This is because individual cells in the situation model coordinate their actions without using the parameter $p$, which is served as a key parameter in the $p$ model to present SOC. We set 100 000 generations as one trial. First, we examined the instability of the fraction of the alive population using the situation model. According to Figure 6, which shows some examples for several parameter values ($n = 3, 4, 5, 6, 7$ and $8$, respectively), the system transits between some population densities. Moreover, this tendency was independent from the parameter $n$ effect. As with the case of the $p$ model, a population of alive cells fluctuates dynamically and unpredictably. We also calculated alive distributions of the trial and the scale-free properties using $n = 5$. As shown in Figure 7, the system again did not reach a steady

![Figure 6](image-url) **Figure 6.** Time evolution of alive populations in the situation model. (a) $n = 3$. (b) $n = 4$. (c) $n = 5$. (d) $n = 6$. (e) $n = 7$. (f) $n = 8$. 

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state and evolved as time went on. Furthermore, according to Figure 8, which presents the last four generations of one trial, the system appears to change gradually. These results confirm that the situation model behaves in a similar manner to the critical $p$ model. Also, the situation model exhibits a power-law tailed distribution regarding the relationship between the generation length and its cumulative frequency (Figure 9, $n$ of data = 526, $\mu = 1.98$, AIC weights for a power-law = 1.00). Here, we examined the generation length that satisfies the alive density $< 7.5\%$, since the fraction of the alive population in the situation model seemed to be a little bit higher than that in the $p$ model.

Figure 7. An example of spatial patterns of the GoL system in the situation model using $n = 5$. Yellow cells are alive cells, while dark blue cells are dead cells. Patterns at 60 000, 70 000, 80 000 and 90 000 generations are shown from left to right.

Figure 8. An example of spatial patterns of the GoL system in the situation model using $n = 5$. Yellow cells are alive cells, while dark blue cells are dead cells. Patterns at 99 997, 99 998, 99 999 and 100 000 generations are shown from left to right.

Finally, to verify the similarity between these two models near the phase transition, the entropy calculation was applied to the evolution of the system [23, 24]. In fact, several methods have been proposed for studying complexity and the qualitative dynamical properties of cellular automata and other systems [25, 26]. Shannon's entropy is one of those methods [26]. Given a random variable $X$, with possible outcomes $x_1$, $x_2$, ..., $x_n$, the entropy is defined as:

$$H(X) = -\sum_{i=1}^{n} p(x_i) \log_2 p(x_i),$$

where $p(x_i)$ represents the occurrence probability for $x_i$. 

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Here, each cell changed states from alive to dead or vice versa. Therefore, possible outcomes can be $x_1$, $x_2$ (0 and 1, resp.). On each trial, each $p(x_i)$ was calculated every four time steps and the final $H$ is output by averaging individual cell’s $H$ at the end of each trial (10 000 generations). Figure 10(a) presents the average entropy $H$ for each model, obtained from 100 trials. We found that the entropy $H$ in the situation model was similar to that of the $p$ model around $p = 0.80$. In fact, according to Figure 10(b), the distribution of the final $H$ obtained from 100 trials in the situation model seems to be more similar to that in the $p$ model with $p = 0.80$ than the $p$ model with $p = 0.0$. The entropy $H$ in the $p$ model with $p = 0.0$ is lower than the other two conditions, suggesting that the system in the $p$ model with $p = 0.0$ reaches a steady state. On the other hand, the entropy $H$ in both the situation model and the $p$ model with $p = 0.80$ appears to vary and depend on the trials. These results suggest the system in the situation model did not reach a steady state and confirm that the system shows some SOC.

4. Discussion

This paper developed two Game of Life (GoL) models. In both models, each cell modified its timing regarding the state update. The system in the $p$ model exhibited a phase transition for the alive population if an adequate $p$ value was selected. Furthermore, around a critical value, the system presented a power-law tailed distribution regarding the extinction interval. These results show that the proposed model demonstrates self-organized criticality (SOC) with a scale-free evolution [7–10]. The timing of state updates in the GoL is

Figure 9. A power-law tailed distribution in the situation model regarding the duration of extinctions obtained from one trial using $n = 5$. 
important and linked to the emergence of a phase transition regarding populations or other properties [12–16]. Nevertheless, few studies paid attention to the decision processes of cells for state updates.

![Graph](https://doi.org/10.25088/ComplexSystems.32.1.57)

**Figure 10.** The average entropy $H$ in the situation model and the $p$ model using $n = 5$. (a) The average entropy $H$ in both models and its relation with a phase transition regarding alive populations by tuning the parameter $p$ in the $p$ model. Data is obtained from 100 trials. (b) Box plots indicate the average entropy $H$ for each category (the $p$ model with $p = 0.0$, $p = 0.80$ and the situation model). Black dots indicate outliers. Vertical lines represent the maximum and minimum. Horizontal lines in each box indicate the median. Light green, yellow and purple boxes represent $p = 0.0$ (the $p$ model), $p = 0.80$ (the $p$ model) and the situation model, respectively.

The situation model also deals with the decision processes of cells for state updates. An important different point of this model from the $p$ model is the evolution of the GoL system without the parameter $p$. The situation model also exhibited the SOC with a scale-free evolution. These results indicate that the system in this model evolves unpredictably toward a critical point. Contrary to choosing the parameter $p$, individual cells in the situation model consider not only a probability $1/N_{\text{active}}$ but also a probability $1/N_{\text{inactive}}$ when they re-coordinate their interval. This is because every cell considers two
possibilities: one possibility is that each cell will adjust to surrounding cells and tend to replace its interval if a minority of surrounding cells conduct the state update \((1/N_{\text{active}})\) is high). The other possibility is that each cell will regard the local situation regarding the state update of surrounding cells as something temporary. In this case, it tends to replace its interval if a majority of surrounding cells conduct the state update \((1/N_{\text{inactive}})\) is high). Thus, cells consider two local situations that are contradictory to each other, which may be a key effect to produce some SOC in GoL system autonomously [21, 22]. Agents in the system are unaware of the contradiction involved and succeed in producing the GoL evolution toward a critical point as a collective behavior [20].

In the default setting of these two models, each cell follows a unique time interval for the state update. Therefore, the timing of state updates varies depending on individual cells. Cells change their state-update intervals from intermittent updates to continuous updates, which is brought about by the pressure of local others. This event solves a contradiction for every cell, since individual cells need not consider any inconsistency regarding the updated timing between themselves and other local cells. Concurrently, a time continuity can emerge that is built by both themselves and other local cells. This phenomenon brings synchronization of time at the system level since almost all cells update their states every generation. However, this tendency disappears if some cells return their state-update intervals to intermittent updates [27]. Importantly, the proposed models obey the classical GoL updates rule, which varies the state update timing. The classical rule gives some constraint on each cell, since the states of individuals can be determined by those rules. Modifying the update timing under such a constraint can exhibit active synchronization of individual cells with others, implying that the individual cells and other local cells together create time continuity.

Finally, we would like to comment on the entropy calculation. Here, we used the entropy calculation as a method to check the similarity between two models near the phase transition. Our findings will be strengthened if some of the other methods like Compression and BDM are applied to this calculation and yield the same results [25, 26].

References


