Surface Logic

Architectural Investigations into Equation-Based Surface Geometries

Andrew Saunders
Amie Nulman

Building on the preexisting deployment of equation-based surface geometries in architecture, surface logic explores the dialogue between twentieth-century pioneers of reinforced concrete and the contemporary possibilities made accessible by the instrumentation of computation. Computational modeling of equation-based surfaces opens designers to unprecedented access and design sensibilities driven by parametric variation, differential topological relationship, fabrication techniques, material analysis, and physical performance.

■ Surface Logic

Architecture has long been dominated by orthogonal Cartesian principles of design preferring two-dimensional planning and composition. Traditionally, three-dimensional surface principles, such as domes and vaults, were implemented at positions predetermined by planimetrics. Although it is possible to produce complex three-dimensional space from such principles, the guiding parameters were usually generated by orthographic projections: plans, sections, and elevations. Advancements in computation such as calculus-based non-uniform rational B-spline (NURBS) surfaces and the accessibility of three-dimensional modeling interfaces have liberated architects from two-dimensional orthogonal logics. Surface logic attempts to describe a new way of thinking for architects guided by the principles inherent to working with equation-based surfaces.

□ Surface Logic Precedent

It is important to first preface this argument with the fact that surface logics are not entirely new to architecture. One can trace the roots from complex three-dimensional principles of double-helix stairs to extreme vaulting of high Gothic cathedrals. Even though these elements are extremely complex in themselves, it was not until the work of Antoni Gaudi that surface logic truly manifested itself three-dimensionally at all levels of architectural design. Gaudi's vaults at the Guell crypt marked the first use of the hyperbolic paraboloid [1]. The linear characteristic of this surface forms a developable surface, which naturally integrates with the linear brick work of the masonry construction of the crypt. Unlike traditional vaulting, the logic of the surfaces was the primary guiding principle of architectural space. The plan of the crypt can only be read as the result of such principles, not vice versa. Gaudi continued his investigation of equation-based surfaces by exploring the principles of the catenary curve. The word "catenary" is derived from the Latin word for “chain”; it is the curve a hanging flexible wire or chain assumes when supported at its ends and acted upon by a uniform gravitational force [2]. In order to access these principles, Gaudi constructed stereostatic models using weighted chains. Acting fully in tension, the inverted catenary provides logic perfectly integral to the physics of compression for load-bearing structures. These surface logics not only provided a structural solution to the Sagrada Familia, but continue to register at every scale of the design even to surface articulation and composition.

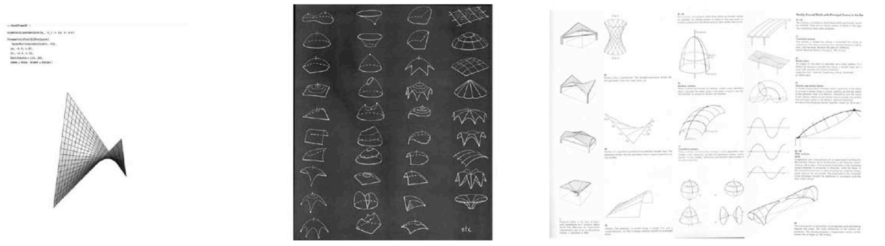


Figure 1. Typical geometry deployed in reinforced concrete.

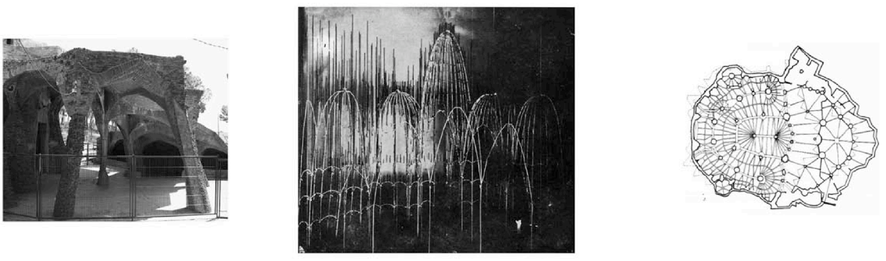


Figure 2. Antoni Gaudi's Guell crypt.

The evolution of these principles that Gaudi employed translated into the work of early twentieth-century reinforced concrete pioneers. Although reinforced concrete was a radical material departure, able to act in compression and tension simultaneously, the equation-based developable surfaces were equally integral to the new material. The geometry of the surface performed structurally as in the work of Gaudi and also provided logic for the construction of the wood formwork necessary to house the new fluid material. These principles were first deployed as singular spanning solutions to the infrastructure of bridges and viaducts, but eventually made their way into architectural projects such as the Hippodrome of Eduardo Torroja, the Orly hangers of Eugène Freyssinet, and the concrete exhibition hall of Robert Maillart. A direct relationship between the logic of the equation-based surfaces and the structural performance, constructability, material deployment, and spatial organization informs all of these works.

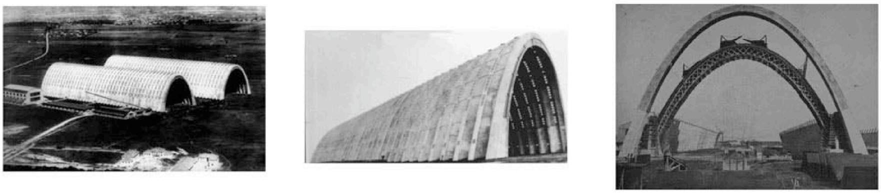


Figure 3. Eugène Freyssinet Orly hangers.

Figure 4. Robert Maillart concrete hall and bridges.

Figure 5. Eduardo Torroja hippodrome.

Figure 6. Pier Luigi Nervi sports palace.

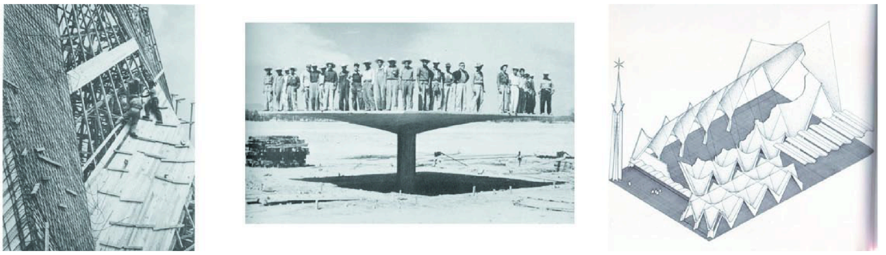


Figure 7. Work by Félix Candela.

Figure 8. Eero Saarinen TWA terminal.

Later, other concrete masters such as Pier Luigi Nervi, Félix Candela, Eero Saarinen, and Eladio Dieste exhausted such principles in countless variations. Because of the surface logic integration, it is very hard to say whether the work of such builders is that of an engineer or an architect. At the same time it is important to note that other architects, such as Erich Mendelsohn, Rudolph Steiner, and Frederick Kiesler, were making use of reinforced concrete. Unlike previous builders, their work relies on the ability of the surface aesthetics to convey notions of dynamism, religion, or sculptural space. This difference is not an attempt to dismiss these works, but for the purpose of this article it is critical to understand that they were not working within the discipline of equation-based surface logic. In return, this work employed reductive secondary logics for constructability and material performance and organization.

With the invention and standardization of prestressed concrete, the surface relation to geometry became internalized. The one-to-one relationship of geometry and structure of thin-shell concrete again transitioned to the new materials of membrane structures and pneumatics. Although the guiding principles are similar due to the lightweight nature of the material of such surfaces, these buildings rarely had the holistic organizational impact of their concrete and masonry predecessors. Conventional implementations usually followed tent typologies, allowing only roofing capabilities, returning to elementary deployment similar to that of planimetric installations of the dome or vault.

New Surface: New Accessibility

In the last 10 years, the availability of personal computers and the advance in processing power have enabled architects everywhere to generate and manipulate complex surfaces with ease in the digital realm. At first, architects integrated three-dimensional software with advanced rendering and dynamic packages from the movie industry. The resultant surfaces were usually smooth, semi-transparent, and seductively rendered products. Initially, there was an attempt to legitimate such surface generation through postmodern processes of semiotics or collage. Typical projects in this realm attempted to use and form surfaces by embedding external indexes or traditional manual art and sculpture techniques. Regardless of whether the metaphoric import was that of stock market trends, animated site flows, or expressionistic dynamism, the surface technique rarely varied. In order to create a continuous blending of these external logics, the technique defaulted to freeform lofting. Furthermore, the digital translation of such external logics usually resulted in a numerical set that allowed little more than a variety in narrative for the generation of similar forms in which global syntactical principles of the index rarely provided any internal local logics to build upon. If the projects advanced further into the physical material realm, reductive orthogonal conventions such as Cartesian sectioning were needed to provide a clear formal understanding of the logic outside of the creation of the surface. This just continued to typify the predictability of such projects. If the formal translation of surfaces generated by external logics will eventually default into Cartesian bread-slicing, then there are only two options to pursue for the surfaces to exist materially. The first is to accept the Cartesian slicing and start with it as a generator. The second is to generate surfaces by logic inherent to their formation.

Surfaces in Mathematics

In order to truly understand surface principles, it is important to learn from other disciplines that also work with surfaces. Two adjacent mathematical fields that are particularly relevant are differential topology and differential geometry. As in architecture, surface logic in mathematics developed before the use of computation. Born on the bridges of Koenigsberg in 1735, topology emerged out of the lack of an adequate language for describing forms [3]. The new field created a number of principles and tools for evaluating complex surfaces. Early twentieth-century plaster models by Ludwig Brill and later Martin Schilling can still be found exhibited for the teaching of mathematical surfaces. Recently there has been a revived interest in equation-based surfaces [4]. This is due largely to the new accessibility to complex surfaces made possible by computational programs such as *Mathematica*. Speed of computation, ease of representation, and computer-based manufacturing have allowed radical advances in both the accessibility to traditional complex surfaces as well as the development of entirely new ones. Although minimal surfaces exist in natural forms such as soap bubbles, now topologists are actually engineering new ones with the potential for applications in molecular and material design.

The following work represents a series of architectural investigations into the logics of equation-based surfaces.

■ Reinventing the George Washington Bridge Bus Station

□ Architecture Studio: Andrew Saunders

We first looked at the possibility of reinventing the George Washington Bridge Bus Station in New York City, originally designed by Pier Luigi Nervi. The studio started with common surfaces documented in *Modern Differential Geometry of Curves and Surfaces with Mathematica* by Alfred Gray. Enabled by *Mathematica*, the students were able to gain quick access to parametrically plotted surfaces. With the help of *Mathematica* notebooks from Matthias Weber of Indiana University, the students progressed into more advanced minimal surfaces. Students conducted a series of compositional diagrams and stereolithography models to understand the complex symmetries and bipolar spatial relationship brought about by the kaleidoscopic patching composition of minimal surfaces. Given these new advances in equation-based surfaces coupled with advances in manufacturing and fabrication, the studio speculated on how to evolve the vocabulary of Nervi and the other masters of reinforced concrete.

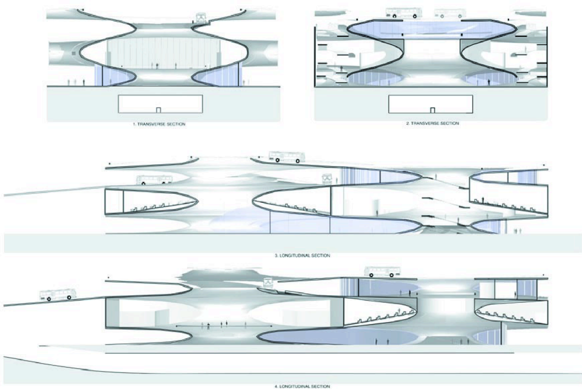


Figure 9. Sections: Ashley Hanrahan.

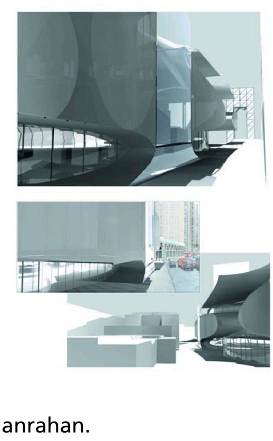


Figure 10. Renderings: Ashley Hanrahan.

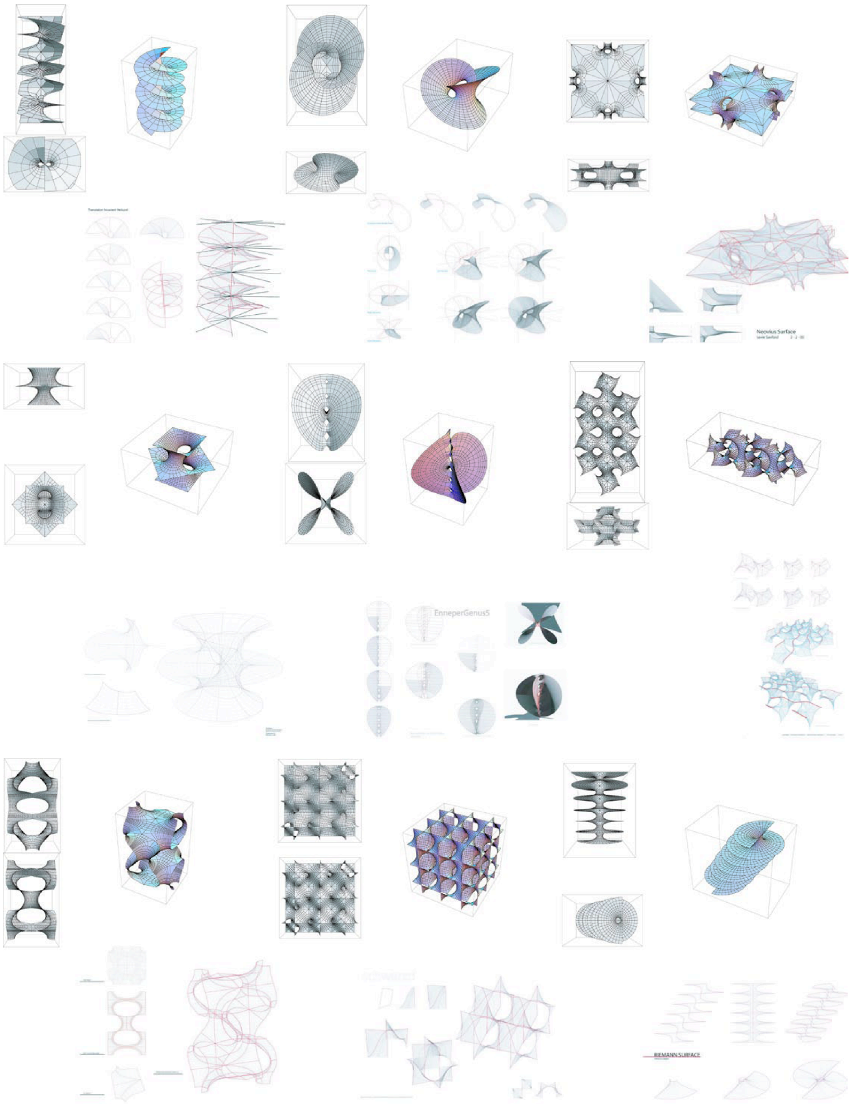


Figure 11. Compositional analysis of minimal surface from Matthias Weber: Kerstin Kraft, Lexi Sanford, Ashley Hanrahan, Douglas Samuel, Emaan Farhoud, Joe Morin, John Davi, Justin Bosy, Monzoor Tokhi, Adam LoGiudice.

■ Fabricating Differential

□ Seminar/Workshop: Andrew Saunders and David Riebe

The second project investigated the possibilities of material fabrication through differential geometry. The seminar again began with common surfaces in differential geometry from *Modern Differential Geometry of Curves and Surfaces with Mathematica*. The students started by parametrically plotting 40 variations of a common surface such as the Enneper, catenoid, helicoid, and monkey saddle. At first the manipulations of the formulas were random. Once the students analyzed the variants, certain characteristics of each of the original surfaces began to emerge. The students returned to the original and started to guide the modification in pursuit of certain formal topological signatures that could inform fabrication techniques. When the students authored parametrically a variation of the original surface, the investigation turned to the potential physical properties of the new surfaces. Students first physically modeled the surfaces with stereolithography. It is important to the studio to advance beyond the representation of the surface by using the identified topological signatures to inform material organization.

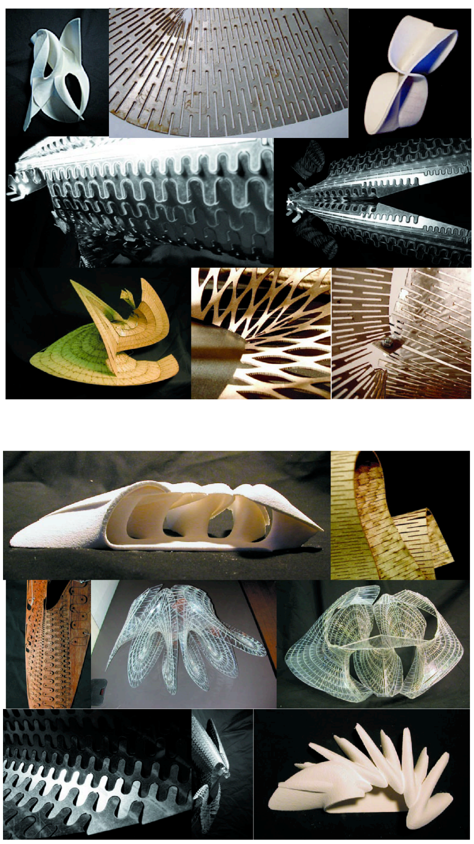


Figure 12. Fabrication models enabled by the Math Plug-In for Rhino (by Jess Maertterer): Ryan Salvas, Eric Smith, Alex Lagula, Brent Hanson.

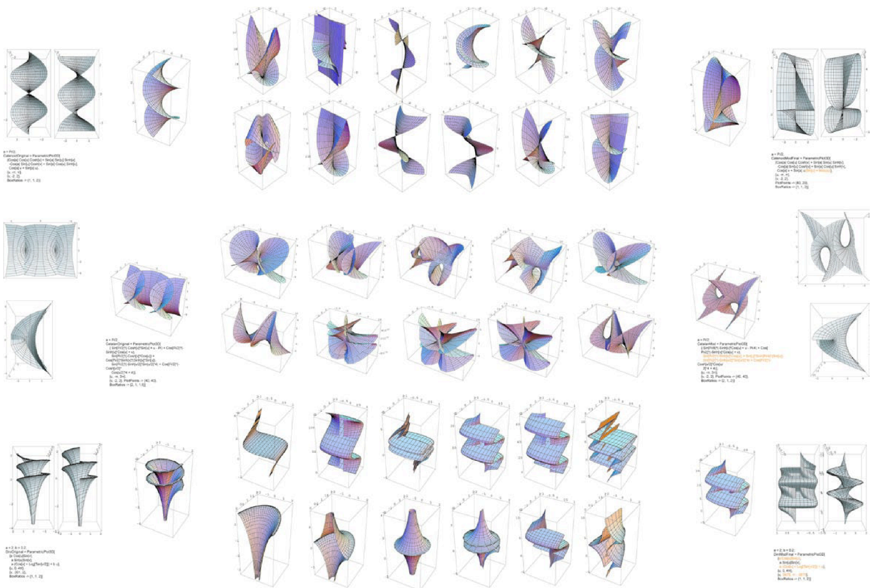


Figure 13. *Mathematica* differential hybrids: Brent Hanson, Alex Lagula, Ben Waserman.

■ Parachute Pavilion

□ Architecture: Andrew Saunders and Ted Ngai

The final project was a proposal for the Parachute Pavilion on Coney Island and the analysis of the structural performance of minimal surfaces. In this project for Coney Island, the brief asked for a pavilion to be sited at the base of the historic parachute ride. The program is composed of public viewing, dining, and retail, as well as private rental and dining facilities. The project incorporated the surface logic of the Riemann minimal surface. Topologically the surface acts as a spatial knot of circulation that negotiates the public and private programs vertically three levels through the boardwalk. This knot creates an ambiguous relationship between the iconic and figural singularities of the popular rides of the theme park and the infrastructural ground condition of the boardwalk. The kaleidoscopic patching of the minimal surface is directly translated into prefabricated local modules that mirror and rotate to form the trunk for the cantilevered pavilion.

Figure 14.

Figure 15.

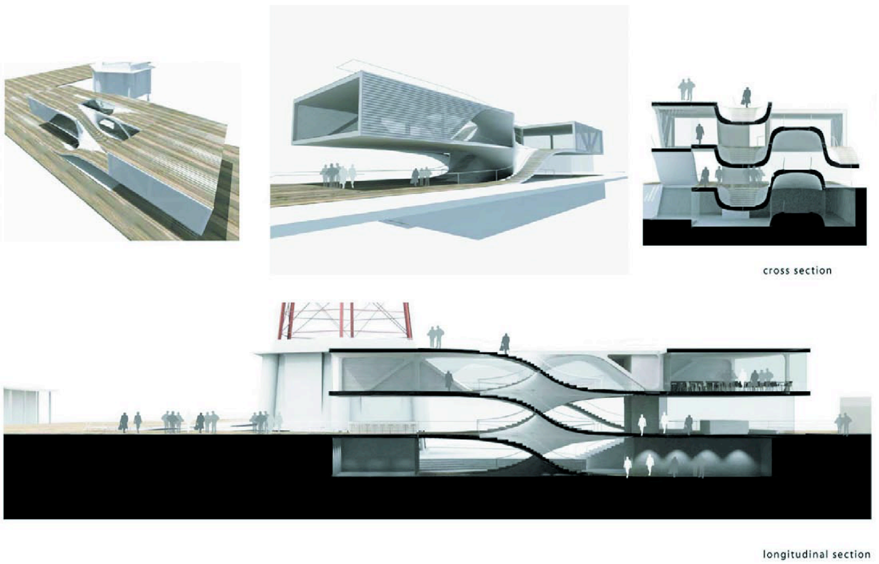
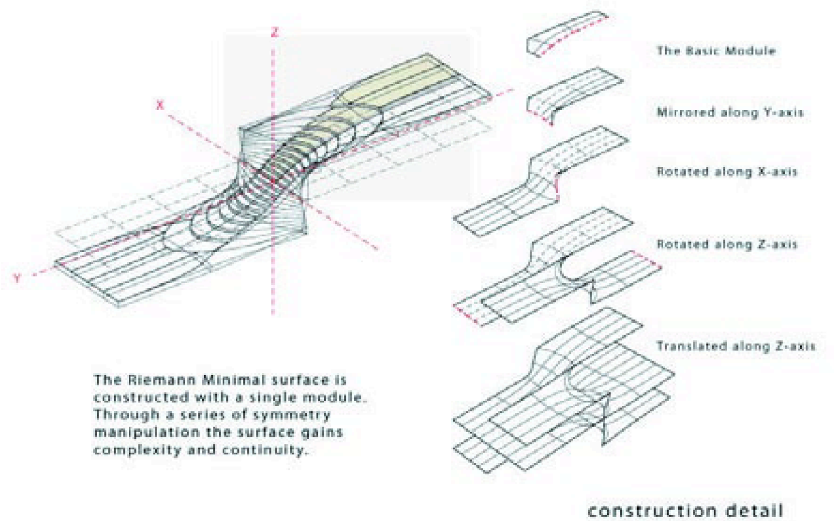

cross section
longitudinal section

Figure 16.


Z
X
Y
The Basic Module
Mirrored along Y-axis
Rotated along X-axis
Rotated along Z-axis
Translated along Z-axis
The Riemann Minimal surface is
constructed with a single module.
Through a series of symmetry
manipulation the surface gains
complexity and continuity.
construction detail

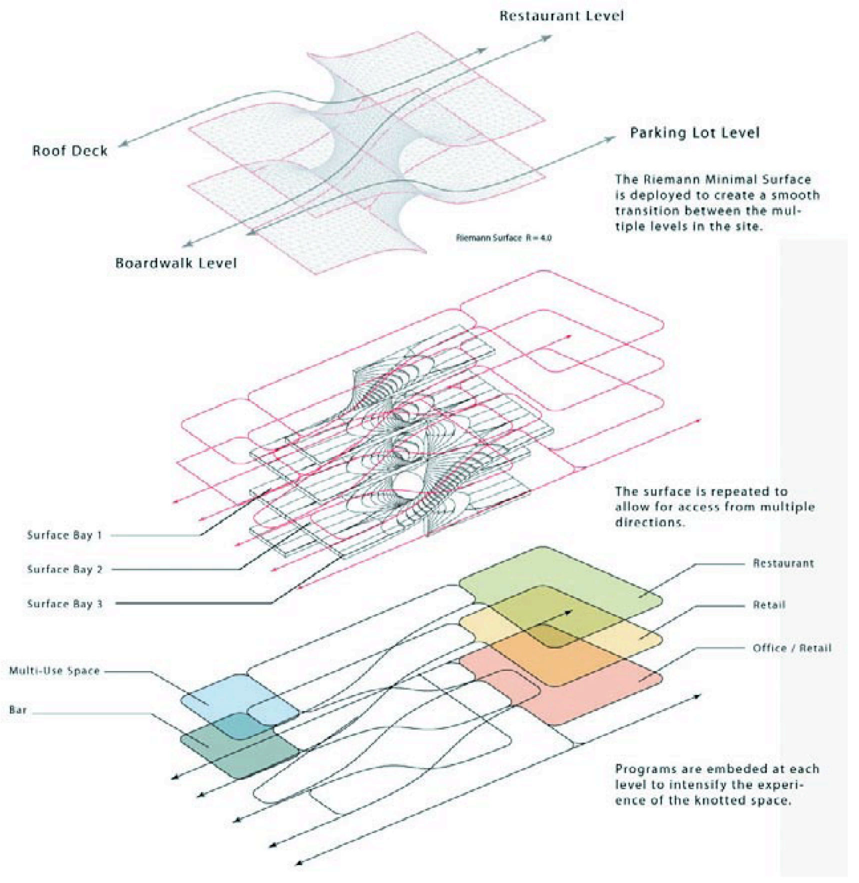


Figure 17.

■ Parachute Pavilion Structure

□ Structural Performance Analysis: Amie Nulman, Structural Engineer at Ove Arup & Partners, Ltd.

Complex curved geometrical surfaces must follow the same laws of gravity, physics, and material behavior as simple linear geometries. The primary difference for structural analysis between simple linear surfaces and complex curved surfaces is the way the surface is supported and the subsequent analysis method. Complex curved surfaces can be constructed by either treating the surface as a facade supported by a rationalized framing system or by requiring the surface itself to be a load-bearing structure.

Traditionally, curved surfaces were supported by a secondary rational framing system except for specific simple geometries, such as arches, vaults, and domes, where linear analysis approaches could be used to determine and solve the element forces, stresses, and support reactions. Continuing advancements in computer-aided analysis permit increasingly complex geometries to be analyzed and designed as independent load-bearing elements.

If a complex curved surface is supported on a secondary framing system, conventional structural analysis methods can be employed to solve stress calculations based on member orientation, spans, loading, and support conditions. For instance, a complex curved roof system supported by beams and columns would be analyzed using a single structural member for each framing element. Initial sizing for scheme design can sometimes be done using proven rules of thumb, and the final force calculations can occasionally be done by hand, without the aid of computer analysis. Still today, complex curved geometries required to span significant unsupported distances, like sports arenas and concert halls, are likely achieved by introducing a secondary system.

If a complex curved surface is treated as a self-supported load-bearing structure, engineers use the finite element method to solve element stress calculations, as that allows them to correctly capture the complicated geometry and the effects of loading and support conditions. Finite element analysis essentially transforms an unintuitive, complex form into a system of piecewise-continuous uncomplicated objects by reducing compound geometries into a series of simple shapes with deflection compatibility parameters. Once the internal forces and stresses are determined by analysis, material-specific design to determine final constructible parameters is a parallel process for structural elements, regardless of their shape.

The finite element method employed by computer programs to perform structural analysis is not a new discovery. It has been a feasible analysis alternative for a plethora of mathematical and engineering tasks for over four decades, following significant development of digital computing in the 1960s.

Recent technological advances that enable the design of complex curved surfaces in architecture include capabilities to create surfaces digitally, transfer the geometries between development and analysis programs, properly discretize the surfaces to ensure reliable analysis, and increase computer analysis capacities to analyze the resulting models. Now the decision on how to support complex curved surfaces is less predetermined and is a result of collaboration between the architect and the structural engineer.

□ Analysis Approach

The central translational corridor of the proposed Parachute Pavilion evolved from symmetrical manipulations of a basic module of the mathematically derived Riemann minimal surface. Based on the mathematical logic and the scale of the resulting minimal surface, the Pavilion surface was analyzed as an independent load-bearing structure. The remainder of the Pavilion structure would be resolved following investigation of the central surface.

A finite element analysis computer program developed by Arup, Oasys GSA, was used to analyze the surface. The GSA processor solves for surface mesh node displacements based on specified material and loading conditions and interpolates the results to compute element strains and stresses.

To create the finite element model, the Pavilion surface model is discretized into a mesh of finite elements using a preprocess program. The finite element mesh is the primary source of analysis accuracy and an efficient finite element model will balance structurally accurate results with sensible computational demands. The mesh must be refined enough to correctly capture the complex geometries of the surface but compact enough not to inhibit analysis.

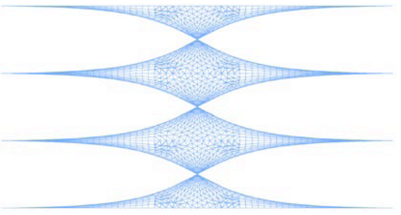


Figure 18. Finite element mesh.

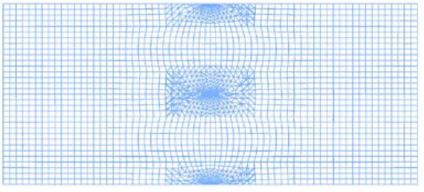


Figure 19. Finite element mesh.

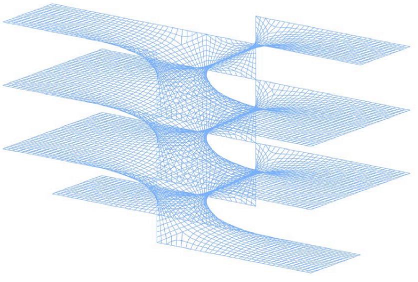


Figure 20. Finite element mesh.

The finite element method is not an exact analysis, but the results of an appropriately constructed finite element model are accurate enough for engineering purposes, especially given the safety factors and tolerances associated with structural engineering and building construction.

□ Behavior Assessment

The structural behavior of the Pavilion surface is unintuitive, so an initial run of the finite element model was performed to assess general characteristics of the shape as well as the level of mesh refinement. The Pavilion surface mesh was imported into GSA, the elements were assigned material properties (concrete shell elements, 8" uniform thickness), and the mesh was given boundary conditions to represent the proposed structure beneath the surface in the full Parachute Pavilion design.

Based on the proposed level beneath the boardwalk of the Parachute Pavilion, the first two levels of the surface have vertical and horizontal supports around the edges and are therefore relatively stiff areas with low displacements and stresses. The two levels above the boardwalk are modeled with no additional vertical or horizontal supports, and the load is thereby transferred down the structure via the continuous surface. The interior central warped segment (where the opposing surface geometries meet) is the most rigid area above ground and therefore attracts load and resists vertical and horizontal deflections.

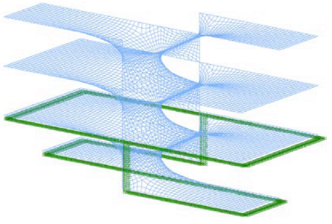


Figure 21. Applied support conditions.

The vertical deflections of the surface are largest at the extreme unsupported ends (top surface) and are symmetric about the center. The horizontal deflections of the surface are largest at the points farthest from the central rigid element and indicate that the structure is rotating—almost an identical amount at both ends—about the central stiff element.

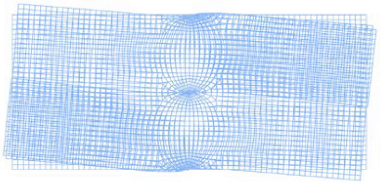


Figure 22. Vertical deflection.

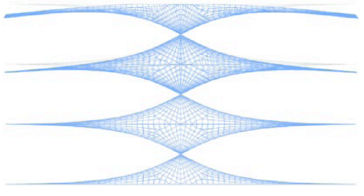


Figure 23. Rotation.

The deflections in all three directions (x, y, z) increase the farther the surface extends from applied support conditions and act symmetrically about the stiff center.

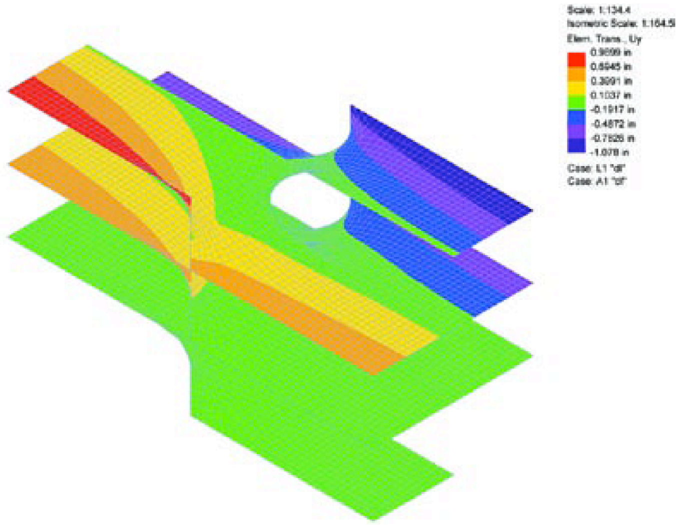


Figure 24. Horizontal deflection in long direction.

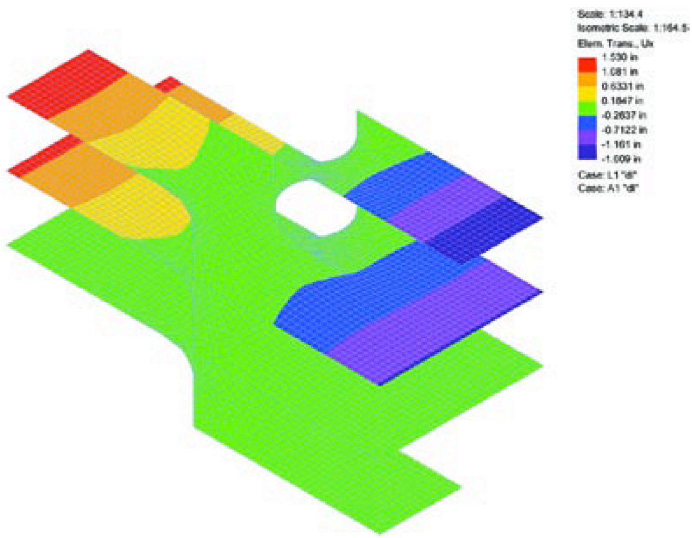


Figure 25. Horizontal deflection in short direction.

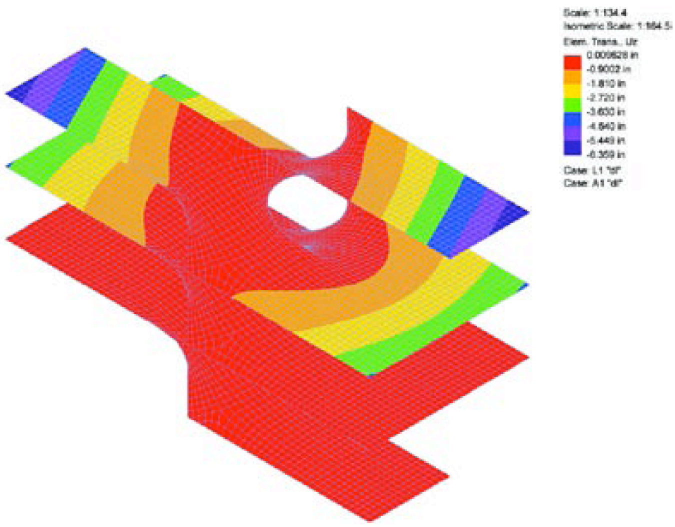


Figure 26. Vertical deflection.

Large stress concentrations formed at both the exterior and interior vertical surface faces in the central region. The stresses along the exterior vertical face result from the structural discontinuity, and therefore change in stiffness, at the point adjoining two vertical levels of the surface. The stresses along the interior vertical face are a result of the opposing cantilever structures that cause large tensile forces across the warped, stiff central support.

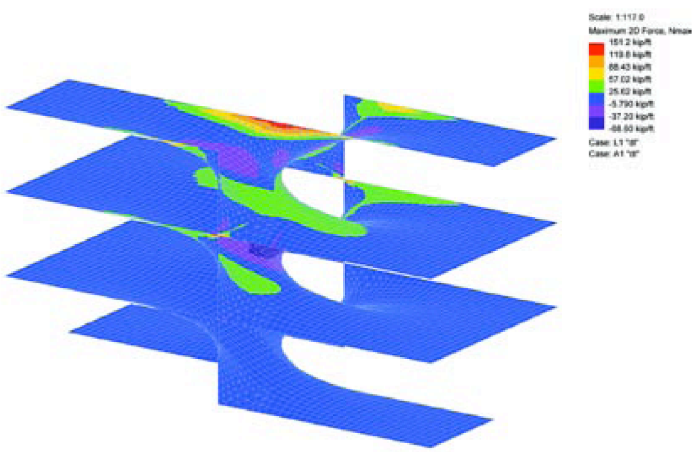


Figure 27. Axial stress.

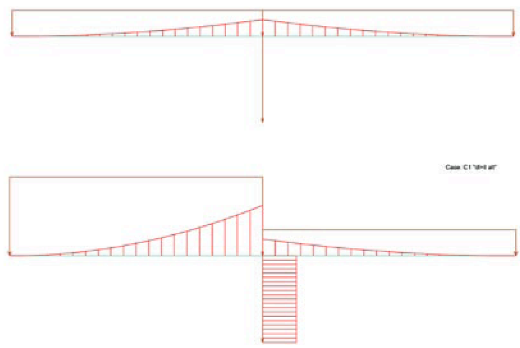


Figure 28. Uniform and nonuniform loading deflection diagram.

Figure 29. Uniform and nonuniform loading bending moment diagram.

In order to achieve a more detailed and accurate finite element analysis, the mesh was further refined to replace the original quadrilateral mesh elements with triangle mesh elements (of half the size) in the regions of high stress. This alleviates large stress gradients across elements and stress discontinuities between elements.

The deflections and stress patterns of the initial uniform surface analysis are analogous to a central vertically rigid element (column) with equivalent cantilevered horizontal elements (beams). This structural system yields maximum deflections at the ends of the cantilevers and large tensile stresses in the horizontal members across the top of the vertically rigid element (column). Deflection limits characteristically govern the required depth of the horizontal (beam) structure, and the system is particularly sensitive to unbalanced loading conditions.

Unbalanced loads result in an unbalanced moment at the column element. This unbalanced moment must be resolved by the column element because the moment is not counterbalanced with an equal and opposite moment in the opposing beam element. This condition, unbalanced loading resulting in unbalanced moment transfer into the column, requires a stiffer (larger) column element than a balanced system.

□ Structural Analogies

Following the initial assessment of the structural behavior of the Pavilion surface, a further study was done to determine the structural efficiency of the surface. To accurately model the behavior of the repeated Riemann surface module, comparisons to the entire Pavilion surface were complex and unnecessary.

The symmetry of the construction of the Pavilion surface (based on rotations and translations of the basic Riemann surface module) simplified the process of determining a smaller module to experiment with. Using analogies to deflection behavior of the Pavilion surface model, the boundary conditions for modeling a double surface element (one rotation of the Riemann surface) were developed, and this element was compared to numerous alternate structures under similar loading conditions.

The surface was compared to two different surface models, a cantilevered frame and a strict cantilever, as well as stick models of those shapes. Due to geometric differences such as longer beam elements and columns unrestrained along the vertical face similar to the Riemann surface, it is logical to have larger resultant deflections of the comparison shapes. However, the resulting displacements of the comparison shapes being four to five times greater is an unpredicted result and demonstrates the shape is clearly a more efficient structure.

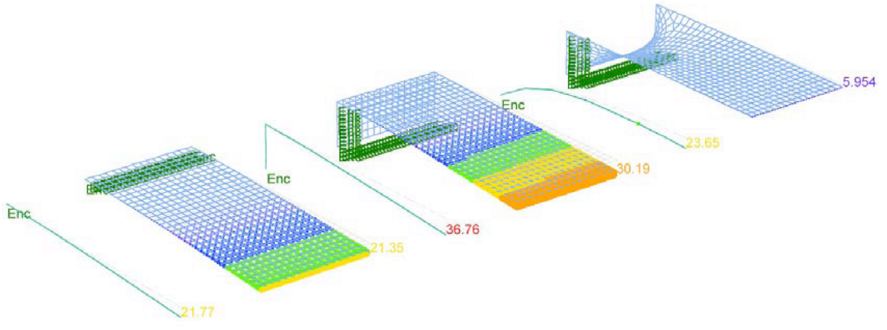


Figure 30. Deflections of analogous surface and stick models.

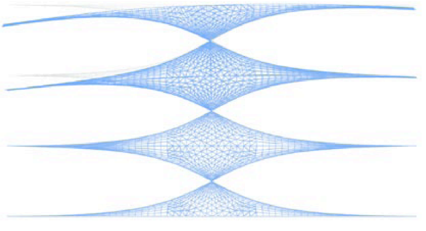


Figure 31. Deflections from unbalanced loads.

□ Structural Analysis

The structural investigation of the surface was initially focused on understanding how the shape responds to a nonzero gravity environment. Under uniform loading, the symmetry of the shape is accentuated by symmetrical deflections and element stresses. The surface behavioral characteristics can be compared to a double cantilever beam and column system, but it is evident that the geometry of the surface renders a more efficient structural form.

In order to complete the structural analysis of the Pavilion surface, a final model of the surface was developed with what had been assessed about the surface behavior combined with material properties and building code requirements (such as deflections and loadings).

Deflection criteria are determined based on a combination of building code requirements and interface of structural elements with other elements of the building. Building codes specify maximum deflection criteria, which are typically set to ensure stability of the structural design. Where structural interfaces are sensitive, for example the connection of the perimeter horizontal structure to the building facade, vertical deflection criteria will typically be imposed that are more onerous than the code requirements for stability.

In combination with the weight of the structure and any applied material loads (facade, floor finishes), building codes specify a minimum live load be applied to structures based on occupancies. The live loads are applied across the entire horizontal surface as well as to alternating spans in order to fully represent the nature of an inhabited space with respect to people flow and congregation. Finite element analysis of the structure under the various load combinations proved that alternately locating the live loads across the surface was the most onerous analysis case, as it caused unsymmetrical deflection and stresses in both the vertical and horizontal elements.

Initial surface thicknesses are classically estimated using rules of thumb based on span and boundary conditions. The initial surface thicknesses are then specified in the finite element model and adjusted, as necessary, to resolve member deflections and forces, following the first analysis run of the fully loaded model. For a strict cantilever, the initial thickness in the middle of the surface would be estimated at almost twice the thickness the model actually requires, further emphasizing the positive structural contributions of the surface shape.

A post-processor was run directly following the finite element analysis to determine reinforcing ratios for specified concrete shell thicknesses and again to check that all internal stresses and forces were within allowable ranges.

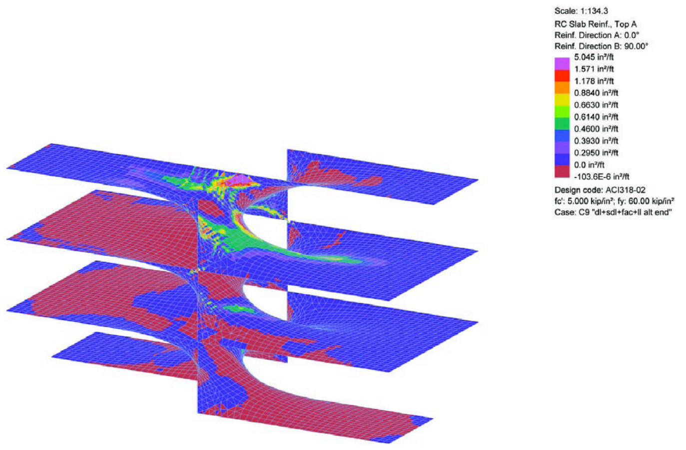


Figure 32. Concrete reinforcement quantities.

□ Additional Analysis

Although we were specifically interested in understanding the behavior of the mathematically derived Parachute Pavilion minimal surface, the next step was to explore structural rationalizations of the Pavilion to make a more efficient structure.

As the structure acts roughly like a double cantilever, the forces in the surface are largest in the central region and thus the maximum thickness of the surface is required in this area. The more load applied to the surface at locations away from the central region (at the ends of the cantilevers), the higher the resultant deflections and forces will be at the center, and subsequently, the thicker the surface must be. Therefore, one approach to rationalization can be thinning out the surface structure as it progresses from the central region, in order to minimize excessive thickness and avoid additional material weight. This approach can be seen as a departure from the pure mathematically derived surface, which would still apply if a uniformly thick surface was used. One surface of the structure, top or bottom, can maintain the characteristics of the mathematically derived surface, while the curvature of the other surface adjusts to permit the changes in thickness.

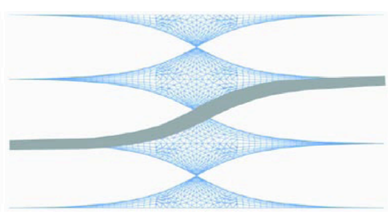


Figure 33. Varied thickness elements.

The single point at the intersection of the vertically stacked surfaces needs to be thickened to allow a vertical load path down the sides of the surface. This alternate load path would minimize the load required to transfer down the surface, which would minimize deflections and stresses in the surface. This rationalization can virtually be considered not to be a departure from the original mathematically derived surface, as it naturally occurs from the inherent structural thickness of the surface.

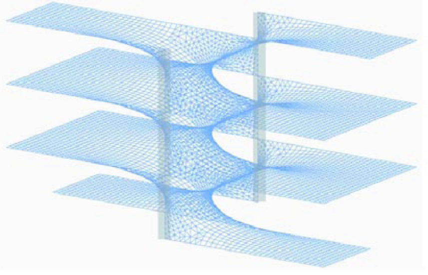


Figure 34. Vertical thickened elements.

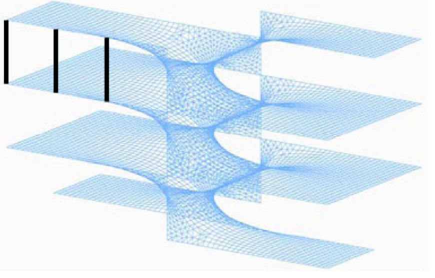


Figure 35. Edge trusses.

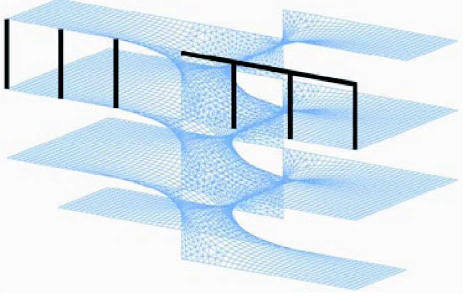


Figure 36. Edge trusses.

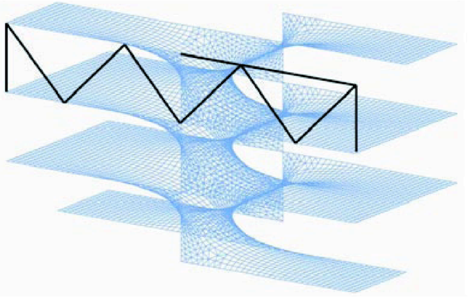


Figure 37. Edge trusses.

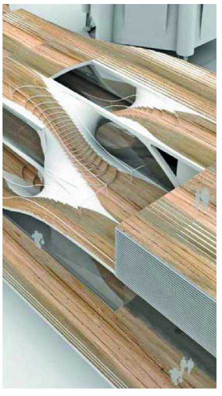


Figure 38.

Further consideration of materials and construction techniques may also provide structural rationalization without the need to compromise the original Pavilion surface. Structural engineers are currently exploring advanced analysis and new construction materials and techniques in order to promote the increasing architectural explorations of complex geometric surfaces.

Although each investigation emphasizes different deployments of surface logic, five new characteristics of design thinking consistently emerge.

□ Thinking Parametrically

All projects begin by looking at preconceived equation-based surfaces ranging from common surfaces found in differential geometry to newer minimal surfaces. Because these surfaces are determined by separate equations in the x, y, and z functions, they can easily be altered by modifying the parameters of the equation. Initially, this approach may seem extremely unintuitive for the understanding of complex three-dimensional forms, especially for those outside the field of mathematics. Because of the speed of computation, the modified calculations quickly return a variation of the original. Instead of manually modifying the surface geometry by pulling points or using transforming tools of conventional modeling programs, *Mathematica* allows unprecedented access to the internal logic of the surface equation. The slightest manipulation of code returns instant formal consequences for the replotted surface. This is a radical shift in the way architects have traditionally mastered geometry. Initial frustration from not being able to guide the form by manually sculpting the geometry turns into a revelation of inconceivable possibilities brought about from harnessing the true power of computation to inform surfaces.

Thinking Iteratively

One of the major advantages of working computationally is the extreme ease of generating and processing huge amounts of information at such a rapid pace. Traditionally, design is an iterative process. For architects, one of the most critical parts of the design process is to learn through doing. Although the design process may be presented linearly, the actual process itself is a constant reevaluation of certain premises through iterative investigation. The current speed of computation enables the iterative process to expand exponentially. It is only through these iterations that the designer can start to gain a new intuition about certain signatures and predictability within equation-based surfaces. This ability to be on the one hand prolific and on the other specific and precise is a whole new sensibility for design thinking.

Thinking Topologically

By studying the properties of geometric figures or solids that are not changed by homeomorphisms, topology puts preference on flexible formal relationships. Thinking topologically comes out of a desire to focus on the possibilities of three-dimensional relationships that exist outside of Cartesian-defined geometry. It is important that the flexibility does not substitute for precision in form making, but rather enables the exploration of the precise intricate relationships inherent in mathematical surfaces. The surfaces are not seen as the means to an end, but rather the motivating diagram for complex connections.

Thinking Beyond the Representational

Modeling is a critical part of the architectural design process. One of the major practical uses of computers in the field of architecture is for three-dimensional modeling. As in *Mathematica*, architectural modeling software proves useful in its ability to quickly generate convincing representations of complex three-dimensional forms. This form of three-dimensional representation even extends into the physical realm, enabled by digital fabrication processes. Like the earlier mathematical models of Ludwig Brill and Martin Schilling, these models are very helpful in understanding and teaching the three-dimensional consequences of surfaces derived from computation. Although these models are physical, they are not material. It is critical to move beyond a physical model that only represents the precise geometry of the surface and into the possibilities of material organization. The surface logic of equation-based surfaces provides an opportunity for both material effects and performance. These new surfaces are not reduced to flexible conduits representing foreign indexical systems or metaphoric import, but instead rely on internal logics of surface formation. These principles have the ability to provide material organization that is coherent with the surface itself. Reductive logic is no longer needed in order to fabricate the physical.

Thinking Bottom Up

Surface characteristics can be divided into two types; local and global. Local characteristics can be described by examining local neighborhoods of points. These micro relationships are not dictated by a macro organization, but in turn genetically compose the larger global characteristics, such as embeddedness, orientability, symmetry, and periodicity. As Stephen Wolfram states in *A New Kind of Science*, simple rules combine to form complex behaviors. Although these behaviors can be observed in urban conditions, architects have never consciously deployed simple local rules in the attempt to create an entirely different characteristic of global organization. The mosque hypostyle and the mat typologies do deploy algebraic relations that result in indeterminate field conditions, but they do not exhibit entirely different characteristics of organization as a whole. Understanding the global design implications from the local conditions could have many architectural consequences at every scale.

The results of these investigations into surface logic may appear motivated by purely formal or sculptural desires. Although this is a beneficial byproduct of the research that cannot be undervalued, the use of *Mathematica* truly allows architects to access the logics of equation-based surfaces in pursuit of not only new form but also new performance.

References

[1] M. Burry, *Expiatory Church of the Sagrada Familia Antoni Gaudi*, London: Phaidon Limited Press, 1993 pp. 12-14.

[2] E. W. Weisstein, "Catenary" from Wolfram *MathWorld*—A Wolfram Web Resource. mathworld.wolfram.com/Catenary.html.

[3] J-M. Kantor, "A Tale of Bridges: Topology and Architecture," *Nexus Network Journal,* **7**(2), 2005 pp. 13-21.

[4] A. Gray, *Modern Differential Geometry of Curves and Surfaces with Mathematica*, 2nd ed., Boca Raton, FL: CRC Press, 1997. library.wolfram.com/infocenter/Books/3759.

Andrew Saunders, and Amie Nulman, "Surface Logic," *The Mathematica Journal*, 2010. dx.doi.org/doi:10.3888/tmj.11.3–7.

Image References:

1. J. Joedicke, *Shell Architecture*, New York: Reinhold Publishing Corporation, 1963.
2. "Gaudiclub." (May 31, 2009) www.gaudiclub.com.
3. J. A. F. Ordonez, *Eugene Freyssinet*, Barcelona: 2c Ediciones, 1978.
4. D. P. Billington, *Robert Maillart's Bridges: The Art of Engineering*, Princeton, NJ: Princeton University Press, 1989.
5. F. Levi, M. A. Chiorino, and C. B. Cestari, *Eduardo Torroja. From the Philosophy of Structures to the Art and Science of Building*, Milan: FrancoAngeli, 2003.
6. P. L. Nervi, *Aesthetics and Technology in Building*, Boston: Harvard University Press, 1965.
7. C. Faber, *Canella the Shell Builder*, Princeton, NJ: Princeton University Press, 1988.
8. E. Stoller, *The TWA Terminal*, Princeton, NJ: Princeton Architectural Press, 1999.

About the Authors

Andrew Saunders is an assistant professor of architecture at Rensselaer Polytechnic Institute in New York. He received his master's degree in architecture from the Harvard Graduate School of Design. He has significant professional experience as project designer for Eisenman Architects, Leeser Architecture, and Preston Scott Cohen, Inc. He has taught and guest lectured at a variety of institutions, including the Cooper Union and the Cranbrook Academy of Art. In 2004 Saunders was awarded the SOM Research and Travel Fellowship to pursue his research on the relationship of equation-based geometries to early twentieth-century pioneers in reinforced concrete. His current practice and research interests lie in computational geometry as it relates to emerging technology, fabrication, and performance. He is currently working on a book using parametric modeling as an analysis tool of seventeenth-century Italian Baroque architecture.

Amie Nulman is a structural engineer in California. She is an associate at Arup in Los Angeles and has also worked in the Boston and London offices on a variety of building types, including arts and culture, education, residential, and sports venues. Recently Nulman completed the design and construction administration for Kroon Hall, the new Yale School of Forestry & Environmental Studies building designed by Hopkins Architects.

Andrew Saunders
Rensselaer School of Architecture, Troy, New York
saunda2@rpi.edu

Amie Nulman
Ove Arup & Partners