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Cellular automata as models of complexity

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Natural systems from snowflakes to mollusc shells show a great diversity of complex patterns. The origins of such complexity can be investigated through mathematical models termed ‘cellular automata’. Cellular automata consist of many identical components, each simple, but together capable of complex behaviour. They are analysed both as discrete dynamical systems, and as information-processing systems. Here some of their universal features are discussed, and some general principles are suggested.

It is common in nature to find systems whose overall behaviour is extremely complex, yet whose fundamental component parts are each very simple. The complexity is generated by the cooperative effect of many simple identical components. Much has been discovered about the nature of the components in physical and biological systems; little is known about the mechanisms by which these components act together to give the overall complexity observed. What is now needed is a general mathematical theory to describe the nature and generation of complexity.

Cellular automata are examples of mathematical systems constructed from many identical components, each simple, but together capable of complex behaviour. From their analysis, one may, on the one hand, develop specific models for particular systems, and, on the other hand, hope to abstract general principles applicable to a wide variety of complex systems. Some recent results on cellular automata will now be outlined; more extensive accounts and references may be found in refs 1-4.

Cellular automata

A one-dimensional cellular automaton consists of a line of sites, with each site carrying a value 0 or 1 (or in general 0,..., k - 1). The value \( a_i \) of the site at each position \( i \) is updated in discrete time steps according to an identical deterministic rule depending on a neighbourhood of sites around it:

\[
\begin{align*}
\phi[\{a_{i-r}, a_{i-r+1}, \ldots, a_{i-r+k}\}] = a_{i+1}
\end{align*}
\]

(1)

With even \( k = 2 \) and \( r = 1 \) or 2, the overall behaviour of cellular automata constructed in this simple way can be extremely complex.

Consider first the patterns generated by cellular automata evolving from simple ‘seeds’ consisting of a few non-zero sites. Some local rules \( \phi \) give rise to simple behaviour; others produce complicated patterns. An extensive empirical study suggests that the patterns take on four qualitative forms, illustrated in Fig. 1:

1. disappears with time;
2. evolves to a fixed finite size;
3. grows indefinitely at a fixed speed;
4. grows and contracts irregularly.

Patterns of type 3 are often found to be self-similar or scale invariant. Parts of such patterns, when magnified, are indistinguishable from the whole. The patterns are characterized by a fractal dimension; the value log \( 3 = 1.59 \) is the most common. Many of the self-similar patterns seen in natural systems may, in fact, be generated by cellular automaton evolution.

Figure 3 shows the evolution of cellular automata from initial states where each site is assigned each of its \( k \) possible values with an independent equal probability. Self-organization is seen: ordered structure is generated from these disordered initial states, and in some cases considerable complexity is evident.

Different initial states with a particular cellular automaton rule yield patterns that differ in detail, but are similar in form and statistical properties. Different cellular automaton rules yield very different patterns. An empirical study, nevertheless, suggests that four qualitative classes may be identified, yielding four characteristic limiting forms:

1. spatially homogeneous state;
2. sequence of simple stable or periodic structures;
3. chaotic aperiodic behaviour;
4. complicated localized structures, some propagating.

All cellular automata within each class, regardless of the details of their construction and evolution rules, exhibit qualitatively similar behaviour. Such universality should make general results on these classes applicable to a wide variety of systems modelled by cellular automata.

Applications

Current mathematical models of natural systems are usually based on differential equations which describe the smooth variation of one parameter as a function of a few others. Cellular automata provide alternative and in some respects complemen-
Mathematical approaches

Rather than describing specific applications of cellular automata, this article concentrates on general mathematical features of their behaviour. Two complementary approaches provide characterizations of the four classes of behaviour seen in Fig. 3.

In the first approach, cellular automata are viewed as discrete dynamical systems (see ref. 14), or idealizations of partial differential equations. The set of possible (infinite) configurations of a cellular automaton forms a Cantor set; cellular automaton evolution may be viewed as a continuous mapping on this Cantor set. Quantities such as entropies, dimensions and Lyapunov exponents may then be considered for cellular automata. (The analogue of equation (2) for a sufficiently wide patch of sites yields a topologically-invariant entropy for the cellular automaton evolution.)

\[ d^{(s)} = \lim_{X \to \infty} \frac{1}{X} \log_k N(X) \]  

where \( N(X) \) gives the number of distinct sequences of \( X \) site values that appear. For the set of possible initial configurations, \( d^{(s)} = 1 \). For a limit set containing only a finite total number of configurations, \( d^{(s)} = 0 \). For most class 3 cellular automata, \( d^{(s)} \) decreases with time, giving \( 0 < d^{(s)} < 1 \), and suggesting that a fractal subset of all possible configurations occur.

A dimension or limiting entropy \( d^{(l)} \) corresponding to the time series of values of a single site may be defined in analogy with equation (2). (The analogue of equation (2) for a sufficiently wide patch of sites yields a topologically-invariant entropy for the cellular automaton mapping.)

\[ d^{(l)} = 0 \]  

and its analogue, where \( p_i \) are probabilities for possible length \( X \) sequences. These measure dimensions may be used to delineate the large time behaviour of the different classes of cellular automata:

1. \( d^{(s)} = d^{(l)} = 0 \)
2. \( d^{(s)}>0, d^{(l)} = 0 \)
3. \( d^{(s)}>0, d^{(l)}>0 \)

As discussed below, dimensions are usually undefined for class 4 cellular automata.

Information propagation

Cellular automata may also be characterized by the stability or predictability of their behaviour under small perturbations in initial configurations. Figure 2 shows differences in patterns generated by cellular automata resulting from a change in a

**Fig. 2** Evolution of small initial perturbations in cellular automata, as shown by the difference (modulo two) between patterns generated from two disordered initial states differing in the value of a single site. The examples shown illustrate the four classes of behaviour found.

Information on changes in the initial state almost always propagates only a finite distance in the first two classes, but may propagate an arbitrary distance in the third and fourth classes.
Fig. 3 Evolution of various cellular automata from disordered initial states. In many cases, ordered structure is seen to be generated. The first row of pictures show examples of the four qualitative classes of behaviour found. (The rules shown are the same as in Fig. 1.) The lower two rows show examples of cellular automata with $k = 5$ (five possible values for each site) and $r = 1$ (nearest neighbour rules). Site values 0 to 4 are represented by white, red, green, blue and yellow squares, respectively. (The rules shown have totalistic codes 10175, 566780, 570090, 580020, 383330, 672900, 5694390, 59123000.) The 'orange' discoloration is a background, not part of the pattern.

Fig. 4 Evolution of multiple phases in cellular automata. Pairs of sites are shown combined: 00 is represented by white, 01 by red, 10 by green and 11 by blue. Alternate time steps are shown. Both rules simulate an additive rule (number 90) under a blocking transformation. In the first rule (number 18), the simulation is attractive: starting from a disordered initial state, the domains grow with time. In the second rule (number 94), the simulation is repulsive: only evolution from a special initial state yields additive rule behaviour; a defect is seen to grow, and attractive simulation of the identity rule takes over.

Fig. 5 Examples of the evolution of a typical class 4 cellular automaton from disordered initial states. This and other class 4 cellular automata are conjectured to be capable of arbitrary information processing, or universal computation. The rule shown has $k = 3$, $r = 1$, and takes the value of a site to be 1 if the sum of the values of the sites in its three-site neighbourhood is 2 or 6, to be 2 if the sum is 3, and to zero otherwise (totalistic code 792).

Fig. 6 Persistent structures generated in the evolution of the class 4 cellular automaton of Fig. 5. The first four structures show have periods 1, 20, 16 and 12 respectively; the last four structures (and their reflections) propagate: the first has period 32, the next three period 3, and the last period 6. These structures are some of the elements required to support universal computation.
single initial site value. Such perturbations have characteristic
effects on the four classes of cellular automata:
(1) no change in final state;
(2) changes only in a finite region;
(3) changes over an ever-increasing region;
(4) irregular changes.

In class 1 and 2 cellular automata, information associated
with site values in the initial state propagates only a finite
distance; in class 3 cellular automata, it propagates an infinite
distance at a fixed speed, while in class 4 cellular automata, it
propagates irregularly, but over an infinite range. The speed of
information propagation is related to the Lyapunov exponent
for the cellular automaton evolution, and measures the degree
of sensitivity to initial conditions (see ref. 16). It leads to different
degrees of predictability for the outcome of cellular automaton
evolution:
(1) entirely predictable, independent of initial state;
(2) local behaviour predictable from local initial state;
(3) behaviour depends on an ever-increasing initial region;
(4) behaviour effectively unpredictable.

Information propagation is particularly simple for the special
class of additive cellular automata (whose local rule function
$\phi$ is linear modulo $k$), in which patterns generated from arbitrary
initial states may be obtained by superposition of patterns gener­
at by evolution of simple initial states containing a single
non-zero site. A rather complete algebraic analysis of such
 cellular automata may be given^1. Most cellular automata are
not additive; however, with special initial configurations it is
often possible for them to behave just like additive rules. Thus,
for example, the evolution of an initial configuration consisting
of a sequence of 00 and 01 digrams under one rule may be
identical to the evolution of the corresponding 'blocked' con­
figuration consisting of 0 and 1 under another rule. In this way,
one rule may simulate another under a blocking transformation
(analogous to a renormalization group transformation). Evo­
lution from an arbitrary initial state may be attracted to (or
repelled from) the special set of configurations for which such
a simulation occurs. Often several phases exist, corresponding
to different blocking transformations: sometimes phase bound­
daries move at constant speed, and one phase rapidly takes
over; in other cases, phase boundaries execute random walks,
nihilating in pairs, and leading to a slow increase in the
average domain size, as illustrated in Fig. 4. Many rules appear
to follow attractive simulation paths to additive rules, which
correspond to fixed points of blocking transformations, and thus
exhibit self similarity. The behaviour of many rules at large
times, and on large spatial scales, is therefore determined by
the behaviour of additive rules.

**Thermodynamics**

Decreases with time in the spatial entropies and dimensions of
equations (2) and (3) signal irreversibility in cellular automaton
evolution. Some cellular automaton rules are, however, reversible,
so that each and every configuration has a unique pre­
decessor in the evolution, and the spatial entropy and dimension
of equations (2) and (3) remain constant with time. Figure 7
shows some examples of the evolution of such rules, constructed
by adding a term $-\sigma^{(r)}$ to equation (1) (ref. 26 and E. Fredkin,
personal communication). Again, there are analogues of the
four classes of behaviour seen in Fig. 3, distinguished by the
range and speed of information propagation.

Conventional thermodynamics gives a general description of
systems whose microscopic evolution is reversible; it may, there­
fore, be applied to reversible cellular automata such as those of
Fig. 4. As usual, the 'fine-grained' entropy for sets (ensembles)
of configurations, computed as in equation (3) with perfect
knowledge of each site value, remains constant in time. The
'coarse-grained' entropy for configurations is, nevertheless,
almost always non-decreasing with time, as required by the
second law of thermodynamics. Coarse graining emulates the
imprecision of practical measurements, and may be imple­
mented by applying almost any contractive mapping to the
configurations (a few iterations of an irreversible cellular
automaton rule suffice). For example, coarse-grained entropy
might be computed by applying equation (3) to every fifth site
value. In an ensemble with low coarse-grained entropy, the
values of every fifth site would be highly constrained, but
arbitrary values for the intervening sites would be allowed. Then
in the evolution of a class 3 or 4 cellular automaton the disorder
of the intervening site values would 'mix' with the fifth-site
values, and the coarse-grained entropy would tend towards its
maximum value. Signs of self-organization in such systems must
be sought in temporal correlations, often manifest in 'fluctu­
ations' or metastable 'pockets' of order.

While all fundamental physical laws appear to be reversible,
macroscopic systems often behave irreversibly, and are
appropriately described by irreversible laws. Thus, for example,
although the microscopic molecular dynamics of fluids is revers­
ible, the relevant macroscopic velocity field obeys the irreversible
Navier–Stokes equations. Conventional thermodynamics does
not apply to such intrinsically irreversible systems: new general
principles must be found. Thus, for cellular automata with
irreversible evolution rules, coarse-grained entropy typically
increases for a short time, but then decreases to follow the
fine-grained entropy. Measures of the structure generated by
self-organization in the large time limit are usually affected very
little by coarse graining.

**Formal language theory**

Quantities such as entropy and dimension, suggested by infor­
mation theory, give only rough characterizations of cellular
automaton behaviour. Computation theory suggests more com­
plete descriptions of self-organization in cellular automata (and
other systems). Sets of cellular automaton configurations may
be viewed as formal languages, consisting of sequences of sym­
ols (site values) forming words according to definite gram­
atical rules.

The set of all possible initial configurations corresponds to a
trivial formal language. The set of configurations obtained after
any finite number of time steps are found to form a regular
language^7. The words in a regular language correspond to the
possible paths through a finite graph representing a finite state
machine. It can be shown that a unique smallest finite graph
reproduces any given regular language (see ref. 15). Examples of
such graphs are shown in Fig. 8. These graphs give complete
specifications for sets of cellular automaton configurations
(ignoring probabilities). The number of nodes $\Xi$ in the smallest
graph corresponding to a particular set of configurations may

**Fig. 7** Evolution of some cellular automata with reversible rules. Each configuration is a unique function of the two previous configurations.
(Rule numbers 4, 22, 90 and 126 are shown.) As initial conditions, each site in two successive configurations is chosen to have value 1 with
probability 0.1.
be defined as the 'regular language complexity' of the set. It specifies the size of the minimal description of the set in terms of regular languages. Larger \( \Xi \) correspond to more complicated sets. (Note that the topological entropy of a set is given by the logarithm of the algebraic integer obtained as the largest root of the characteristic polynomial for the incidence matrix of the corresponding graph. The characteristic polynomials for the graphs in Fig. 7 are \( 2 - \lambda \left( \lambda_{\text{max}} = 2 \right), 1 - \lambda + 2\lambda^2 - \lambda^3 \left( \lambda_{\text{max}} = 1.755 \right) \) and \( 1 + \lambda - \lambda^2 + 2\lambda^3 - 4\lambda^4 - 2\lambda^5 - 5\lambda^6 - 3\lambda^7 - 3\lambda^8 + 5\lambda^{10} - 6\lambda^{11} + 4\lambda^{12} - \lambda^{13} \left( \lambda_{\text{max}} = 1.732 \right) \).)

The regular language complexity \( \Xi \) for sets generated by cellular automaton evolution almost always seems to be nondecreasing with time. Increasing \( \Xi \) signals increasing self-organization. \( \Xi \) may thus represent a fundamental property of self-organizing systems, complementary to entropy. It may, in principle, be extracted from experimental data.

Cellular automata that exhibit only class 1 and 2 behaviour always appear to yields sets that correspond to regular languages in the large time limit. Class 3 and 4 behaviour typically gives rise, however, to a rapid increase of \( \Xi \) with time, presumably leading to limiting sets not described by regular languages.

Formal languages are recognized or generated by idealized computers with a 'central processing unit' containing a fixed finite number of internal states, together with a 'memory'. Four types of formal languages are conventionally identified, corresponding to four types of computer:

- **Regular languages**: no memory required.
- **Context-free languages**: memory arranged as a last-in, first-out stack.
- **Context-sensitive languages**: memory as large as input word required.
- **Unrestricted languages**: arbitrarily large memory required (general Turing machine).

Examples are known of cellular automata whose limiting sets correspond to all four types of language (L. Hurd, in preparation). Arguments can be given that the limit sets for class 3 cellular automata typically form context-sensitive languages, while those for class 4 cellular automata correspond to unrestricted languages. (Note that while a minimal specification for any regular language may always be found, there is no finite procedure to obtain a minimal form for more complicated formal languages: no generalization of the regular language complexity \( \Xi \) may thus be given.)

### Computation theory

While dynamical systems theory concepts suffice to define class 1, 2 and 3 cellular automata, computation theory is apparently required for class 4 cellular automata. Examples of the evolution of a typical class 4 cellular automaton are shown in Fig. 5. Varied and complicated behaviour, involving many different time scales is evident. Persistent structures are often generated; the smallest few are illustrated in Fig. 6, and are seen to allow both storage and transmission of information. It seems that the structures supported by this and other class 4 cellular automata may be combined to implement arbitrary information processing operations. Class 4 cellular automata would then be capable of universal computation: with particular initial states, their evolution could implement any finite algorithm. (Universal computation has been proved for a \( k = 18, r = 1 \) rule \(^2\), and for two-dimensional cellular automata such as the 'Game of Life' \(^{22,23} \).) A few per cent of cellular automaton rules with \( k = 2 \) or \( r > 1 \) are found to exhibit class 4 behaviour: all these would then, in fact, be capable of arbitrarily complicated behaviour. This capability precludes a smooth infinite size limit for entropy or other quantities: as the size of cellular automaton considered increases, more and more complicated phenomena may appear.

Cellular automaton evolution may be viewed as a computation. Effective prediction of the outcome of cellular automaton evolution requires a short-cut that allows a more efficient computation than the evolution itself. For class 1 and 2 cellular automata, such short cuts are clearly possible: simple computations suffice to predict their complete future. The computational capabilities of class 3 and 4 cellular automata may, however, be sufficiently great that, in general, they allow no short-cuts. The only effective way to determine their evolution from a given initial state would then be by explicit observation or simulation: no finite formulae for their general behaviour could be given. (If class 4 cellular automata are indeed capable of universal computation, then the variety of their possible behaviour would preclude general prediction, and make explicit observation or simulation necessary.) Their infinite time limiting behaviour could then not, in general, be determined by any finite computational process, and many of their limiting properties would be formally undecidable. Thus, for example, the 'halting problem' of determining whether a class 4 cellular automaton with a given finite initial configuration ever evolves to the null configuration would be undecidable. An explicit simulation could determine...
Orientation of \textit{in situ} stresses in the oceanic crust

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Two \textit{in situ} measurements of principal stress directions have been made in DSDP Holes 504B, south of the Costa Rica Rift on the Nazca plate, and 597C, west of the East Pacific Rise on the Pacific plate. In both cases, the orientations of \textit{in situ} principal stresses determined from borehole breakouts are consistent with the stress directions inferred from intraplate earthquakes located near the sites.

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