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Cosmology with very large gauge models

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Several theoretical principles suggest the existence of large numbers of very massive particles. Such particles have negligible effect in the present universe, but may have been important in the very early universe. It is shown that under some circumstances their presence could completely change the equation of state and expansion rate of the very early universe, and could have important effects on baryon-number generation. Possible cosmological constraints on the complexity of grand unified gauge models are discussed.

I. INTRODUCTION

The simplest grand unified gauge model is based on the gauge group SU(5), and contains Higgs bosons in the $\underline{5}$ and $\underline{24}$ representations.[1] To avoid inconsistency with the measured light-fermion-mass spectrum,[2] and to allow generation of the observed cosmological baryon asymmetry in the early universe,[3,4] this minimal model must be supplemented by an additional $\underline{5}$ or $\underline{45}$ Higgs-boson representation. The resulting model involves at least 100 particle degrees of freedom. Low-energy phenomenology potentially provides a few indications that this model should be extended. Avoidance of instanton-induced *CP* violation in QCD without necessity for an observable axion suggests doubling the $\underline{24}$ Higgs multiplet, and requires an additional $\underline{5}$ Higgs multiplet.[5] Observation of small neutrino masses or oscillations would support models based on the gauge group SO(10).[6] Explanations of results from present or anticipated experiments do not appear to require models more complicated than these. However, explicit implementations of several general theoretical schemes do appear to involve significantly more complicated models, containing many more particle degrees of freedom. Supersymmetric models suggest at least a doubling in the number of fundamental fields, and often require additional fields to obtain the required pattern of symmetry breaking.[7] Models with "hypercolor" require large numbers of particles to arrange for appropriate mixings and mass spectra.[8] For example, the SO(10)[5] model of Ref. 9 involves at least 225 gauge bosons, 305 fermions, and 11 485 Higgs bosons, giving a total of 12 015 fundamental fields. Models with subquark structure imply many effectively pointlike particles, including radial and orbital excitations. Models based on reduction from above four dimensions yield an infinite sequence of "particle" states corresponding to quantized excitations in the compactified dimensions.[10]

Low-energy phenomena are for the most part unaffected by the presence of large numbers of very heavy particles. Only in the early universe are sufficient energies available such that the presence of heavy particles may be significant. The purpose of this paper is to consider the consequences of very complicated gauge models for the evolution of the early universe, and to use existing cosmological observations to infer constraints on the particle content of these models. In most cases, the overall structure of complicated models is defined by general theoretical principles, but details such as values of Higgs-boson couplings and other parameters are left undetermined. We attempt, therefore, to obtain results which depend only on gross properties of models, and are insensitive to their details. In some cases, we resort to a statistical analysis, and extract general

features from averages over statistical ensembles of models with distributions of values for undetermined parameters.

This paper assumes the standard "hot big bang" model for the early universe. Most of its results nevertheless also apply in the "hot" phases of "inflationary" models.

II. EXPANSION OF THE UNIVERSE

Assuming isotropy and homogeneity (and zero or negligible cosmological constant), the Robertson-Walker scale factor R of the universe expands according to (e.g., Ref. 11)

$$\dot{R}/R = \left[\frac{8\pi\rho}{3m_P{}^2}\right]^{1/2}, \tag{1}$$

where ρ is the energy density of matter in the Universe, and $m_P \simeq 10^{19}$ GeV is the Planck mass. In the standard cosmological model, it is usually assumed that at sufficiently high temperatures all matter can be approximated as a weakly interacting gas of ultrarelativistic particles in thermal equilibrium. The resulting energy density is then linearly proportional to the number of particle degrees of freedom ξ in equilibrium at a particular temperature; in a very large gauge model, the expansion rate of the early universe thus increases as $\sqrt{\xi}$. Such an increase in the expansion rate must be matched by an increase in interaction rates if processes are to remain in thermal equilibrium (see, for example, comments on baryon-number generation below).

The approximation of a weakly interacting ultrarelativistic gas leads to an equation of state $p=\rho/3$ for the pressure p in the early universe. As discussed at length elsewhere, this equation of state may be modified by phase transitions associated with the restoration of spontaneously broken symmetries.[12] A component with an equation of state $p=-\rho$ corresponding to an effective cosmological term is then introduced. The usual $p=\rho/3$ equation of state can also be modified in complicated gauge models by the presence of effective couplings which become strong at high energies and which lead to strong interactions in the gas.

The behavior of effective couplings is governed by a set of coupled renormalization-group equations. In the leading-logarithm approximation, all couplings typically exhibit Landau singularities (e.g. Ref. 13) at either high or low momenta (or temperatures). For example, QCD in the leading-logarithm approximation yields a Landau singularity at low energies ($Q \simeq \Lambda \simeq 0.5$ GeV), but is asymptotically free at high energies. The leading-logarithm approximation becomes inaccurate near the Landau singularity. Nevertheless, beyond the region of the Landau singularity, a region of strong coupling is reached. QED also exhibits a Landau singularity, but at very high energies $\sim m_e e^{1/\alpha}$. Grand unified models may under some circumstances also exhibit Landau singularities at high energies, yielding strong couplings and potentially modifying the equation of state for matter and thus the expansion rate of the early universe.

The effective temperature-dependent gauge coupling $g^2(T^2)$ for a grand unified model in the leading-logarithmic approximation takes the form

$$g^2(T^2) \simeq \frac{g^2(T_0{}^2)}{1-g^2(T_0{}^2)\beta_0 \ln(T^2/T_0{}^2)}, \tag{2}$$

where

$$\beta_0 = -(11K_V/3 - 2K_F/3 - K_S/6)/(16\pi^2)$$

is the lowest-order coefficient in the β function and $K_{V,F,S}$ are, respectively, the total combinatorial weights associated with all vector, two-component spinor, and real scalar-boson representations whose masses are much less than T. T_0 is the temperature at which the coupling is normalized. For a group with an adjoint representation of dimension N_A, the K_i of a particular representation is given in terms of its dimensionality N_i and quadratic Casimir invariant C_i by $K_i = C_i N_i / N_A$. Thus, for example, with the gauge group SU(5), the $\underline{5}$ representation has $K=1$, the $\underline{10}$ has $K=3$, the $\underline{24}$ (adjoint) has $K=10$, the $\underline{45}$ has $K=24$, and the $\underline{75}$ (popular in some recent supersymmetric models) has $K=50$ (e.g., Ref. 14). The effective gauge coupling of Eq. (2) ceases to be asymptotically free at high temperatures if $\beta_0 > 0$ and it exhibits a Landau singularity at the temperature for which its denominator vanishes. Equation (2) shows that these effects occur if the number of fermions and Higgs scalar bosons becomes large compared to the number of gauge vector bosons. The number of vector bosons is, however, always given by the dimensionality of the adjoint representation of the gauge group, and is fixed for a particular gauge group. The number of fermions and particularly Higgs bosons may however usually be chosen in an apparently arbitrary manner.

For an SU(5) model with three families, asymptotic freedom is lost in Eq. (2) when $K_S \gtrsim 175$, which is attained with 4 or more $\underline{45}$ representations of Higgs scalar bosons or 87 $\underline{5}$ representations. With $\alpha = g^2/(4\pi) = \frac{1}{40}$ at $T = 10^{15}$ GeV,[1] the SU(5) effective gauge coupling exhibits a Landau singularity below the Planck mass $m_P \simeq 10^{19}$ GeV if $K_S \gtrsim 340$, corresponding to 7 or more $\underline{45}$ representations with mass $\simeq 10^{15}$ GeV. When many Higgs bosons have masses $\ll 10^{15}$ GeV, as in models involving "intermediate" mass scales, the effective coupling may

start to increase at lower temperatures, and a Landau singularity below the Planck mass is achieved with fewer Higgs bosons. For example, in the SO(10) model of Ref. 15, a Landau singularity appears below m_P for some choices of parameters even with the "minimal" complement of three 126, a 54, and a complex 10 representation of Higgs bosons. Asymptotic freedom may be lost not only through the presence of large numbers of Higgs bosons, but also by the effects of many fermions. Models based on SO(n) have the special feature that if fermions are restricted to the spinor representations, their number grows like $2^{n/2}$, while the dimensionality of the vector-boson adjoint representation grows only like n^2 so that the contributions of fermions alone destroy asymptotic freedom if $n \gtrsim 22$.

The variation of effective fermion Yukawa couplings and Higgs self-couplings with temperature is analogous to that given in Eq. (2) for the effective gauge coupling. Typically, however, these couplings are smaller then the gauge coupling, so that any Landau singularity appears at the lowest temperature in the gauge coupling. Nevertheless, if there exist sufficiently massive quarks [$\gtrsim 200$ GeV (Ref. 16)] the fermion Yukawa couplings are sufficiently large that they alone yield a Landau singularity below the Planck mass. Such massive quarks are, however, supposedly forbidden by the stability of the effective potential.[17]

If a Landau singularity is reached, the matter in the Universe would become strongly interacting at high temperatures, and its equation of state would presumably deviate from the ideal-gas form $p = \rho/3$. There are, however, few reliable indications of the possible equations of states attained. For classical gases with short-range interactions (and potential energy always much less than kinetic energy) one finds $0 \leq p \leq \rho/3$.[18] These inequalities are preserved if electromagnetic or other conformally invariant interactions are introduced.[19] However, in general it appears that the inequalities may be violated[20] (first-order corrections might be obtained by a virial expansion, but relativistic and hence particle production effects must be included), although there are indications that in most cases $p = c^2\rho \leq \rho$, where c is the velocity of sound.[20]

One possibility is that the equilibrium statistical properties of a strongly interacting system of elementary particles may be approximated by those of a weakly interacting gas of bound states of these particles.[21] The resulting effective system always yields $p \leq \rho/3$. The relation between energy and temperature depends on the level density for the bound states. A power-law density is obtained from simple (essentially nonrelativistic) potential models, and is to some extent supported by low-energy hadron spectroscopy. With such a density, ρ increases with temperature faster than T^4, leading to a lower increase of temperature at early times after the "big bang." An exponentially rising level density is suggested by the Veneziano model and statistical bootstrap approaches to strong interactions.[21] Such a level density would lead to a maximum temperature,[21,11] beyond which any increase in energy generates particles with higher rest masses, rather than increasing particle kinetic energies. Degrees of freedom which become significant only above the maximum temperature would not appear in the early universe. (Phase transitions associated with restoration of symmetries at higher temperature would not occur, and their associated magnetic monopoles would thus not be produced.) In addition, the expansion of the early universe would be more rapid.[21,11]

Many proofs in general relativity assume either the "strong" energy condition $p \geq -\rho/3$ or the "dominant" energy condition $|p| \leq \rho$.[22] If the strong energy condition is violated (as by an effective cosmological constant giving $p = -\rho$), then the expansion of the early universe is greatly slowed, and no particle horizons survive, so that causal dynamical processes could give rise to the observed homogeneity and small density fluctuations (galaxies). In general, with an equation of state of the form $p = (\gamma - 1)\rho$, the Robertson-Walker scale factor of the Universe expands with time according to $R \sim t^{2/3\gamma}$.

"Stiff" equations of state with $p > \rho/3$ appear to have little direct effect on the early universe, although it appears that they may lead to a significant increase in the rate of primordial black-hole formation.[23]

Regardless of strong coupling, the presence of many particle species increases the density of the Universe, and reduces the interaction length for all particle species. Ultimately, the interaction length may become shorter than the thermal Compton wavelength of each particle (typically when the number of species exceeds $1/\alpha$), leading to the possiblity of important collective quantum-mechanical effects.

III. BARYON-NUMBER GENERATION

Grand unified models generically involve very massive gauge and Higgs bosons whose couplings violate baryon-number (B) conservation. Exchanges of such bosons should lead to proton decay. In the very early universe their presence would cause any baryon number introduced as an initial condition to be destroyed (unless this is prevented by other absolute conservation laws), so that the apparent baryon

excess in the present universe must have been produced through subsequent *CP*-violating *B*-violating processes.

Baryon asymmetry may be generated through the decays of heavy (usually Higgs) bosons, but tends to be destroyed by the corresponding inverse decays and by *B*-violating two-to-two scattering reactions. These processes are represented by the three terms on the right-hand side of the Boltzmann transport equation for the time development of the baryon-number density relative to photon-number density B:

$$\dot{B} \simeq \sum_X \langle \Gamma_X \rangle [\epsilon_X (X - X^{\rm eq}) - B X^{\rm eq}] - B n_\gamma \langle \sigma v \rangle \ . \tag{3}$$

Here X represents the number density (relative to the photon-number density n_γ) of each species of *B*-violating bosons, with total decay width Γ_X, *CP*-violation parameter ϵ_X, and equilibrium number density $X^{\rm eq}$. $\langle \sigma v \rangle$ gives the cross section for *B*-violating $2 \to 2$ scattering reactions.

Consider first the simple case (discussed in Sec. 4.4 of Ref. 24) of N identical *B*-violating bosons all with mass m and decay width $\Gamma = \lambda_1 a m/(8\pi)$. If the back reactions represented by the second and third terms in Eq. (3) were absent, the final asymmetry generated in this case would increase linearly with N. However, larger N also leads to larger back-reaction effects, tending to reduce the final baryon number. The importance of these back reactions depends critically on the magnitude of the $2 \to 2$ scattering cross section. This cross section is obtained as a sum over amplitudes for the exchange of each of the N bosons. If all these amplitudes were of the same sign, they would add coherently to give a total cross section $O(N^2)$; if they carried random signs, they would add incoherently to give a result $O(N)$. In practice, the amplitudes for exchanges of many bosons in a single irreducible representation of the gauge group are related by Clebsch-Gordan coefficients. The cross section relevant for Eq. (3) is summed over all possible initial and final states, weighted with the square of the baryon-number difference between them.[24] Ignoring this weighting factor (and thus considering the total cross section) one may sum over exchanged bosons and possible initial and final states by standard methods. For the t-channel exchange diagram, one obtains a combinatorial factor (e.g., Ref. 25) $K_f^2 N$, where K_f is related to the quadratic Casimir invariant for the fermion representation, as in Eq. (1). The s-channel exchange diagram yields the same result, while the u-channel diagram gives $K_f F^3/N^2$, where F is the total number of initial and final fermions. Diagrams involving interference between exchanges of bosons in different irreducible representations vanish after summing over all possible initial and final states. These results suggest that the total cross section obtained with N bosons grows roughly linearly with N. We shall thus assume in Eq. (3) a cross section $\sigma v \simeq 96\pi\lambda_2 \alpha^2 N T^2/m^4$ for $T \lesssim m$. Numerical results using this form were given in Ref. 4. For large N, a simple analytical approximation to Eq. (3) yields for the final baryon number the result

$$B \simeq \frac{N_\epsilon}{\xi} \exp\left[-\frac{m_P}{m\sqrt{\xi}} N (0.002\alpha\lambda_1 + 7\alpha^2\lambda_2) \right] , \tag{4}$$

where ξ is the total number of particle species (whether *B*-violating or not) contributing to the expansion rate of the universe, as in Eq. (1). λ_1 and λ_2 are model-dependent parameters, and typically $\lambda_1 \lesssim 1$ and $\lambda_2 \gtrsim 10$ [for example, in the SU(5) grand unified model, the relevant Higgs-boson couplings give $\lambda_1 \simeq 1$ and $\lambda_2 \simeq 14$, while in the SO(10) grand unified model (with symmetry broken to $SU(3) \otimes SU(2) \otimes U(1)$) $\lambda_1 \simeq 0.13$ and $\lambda_2 \simeq 16$ Ref. (4)]. Equation (4) manifests the exponential decrease of the final baryon number for large N as a consequence of increased back-reaction effects. Notice that the small numerical coefficient in the first term of Eq. (4) typically renders inverse decay unimportant compared to $2 \to 2$ scattering processes. Since present observations imply $B \gtrsim 10^{-10}$, Eq. (4) potentially provides an important constraint on N. In actual grand unified gauge models, however, one must account for the finite spread in *B*-violating boson masses. Ignoring the resulting modifications for now, one may make some simple estimates based on Eq. (4). Taking $\lambda_2 \geq 10$ and neglecting the term involving λ_1, Eq. (4) implies $N \lesssim 40$ for $\alpha^2 m_P/m = 0.1$ and $N \lesssim 1000$ for $\alpha_2 m_P/m = 0.01$, assuming that the *CP*-violation parameter ≤ 0.01 and taking $\xi \simeq 160 + N$. The largest Higgs-boson couplings (responsible for the t quark mass) are expected to give $\alpha^2 m_P/m \gtrsim 5 \times 10^{-2}$, yielding a significant constraint on N.

The derivation of constraints for actual grand unified models requires more detailed consideration of the mechanisms of baryon-number generation. The results of Refs. 3, 4, and 24 show that gauge-boson decays do not usually involve sufficient *CP* violation to yield significant baryon asymmetry: all baryon asymmetry must therefore come from Higgs-boson decays, with *CP* violation arising from one-loop diagrams involving exchange of a *B*-violating Higgs boson or in some cases a gauge boson. The contribution to *CP* violation in the decay of a boson X_1 from exchange of a heavier boson X_2 decreases as m_{X_1}/m_{X_2}. The sign of the contribution depends on

the Clebsch-Gordan coefficients involved. If a large number N of bosons with approximately equal masses and couplings of random sign are exchanged, then their net contribution decreases like $1/\sqrt{N}$ for large N. In addition, asymmetry arising from CP violation generated by X_2-exchange corrections to X_1 decays is cancelled by the asymmetry from X_1 exchange in X_2 decay if X_1 and X_2 have nearly equal masses and couplings. Net asymmetries can arise only if the bosons have different couplings or are not degenerate.

The results of Ref. 4 indicate that in most cases the final net baryon asymmetry is dominated by the decays of the longest-lived B-violating species: effects of shorter-lived species are typically eradicated by back reactions involving the longer-lived species (except when additional conserved or partially conserved quantum numbers exist, as mentioned below). For an adequate final baryon number to survive, it is therefore necessary that back reactions to the decays of the longest-lived species be suitably small. The magnitude of the back reaction depends on the couplings of the decaying species, usually Higgs bosons. In contrast with the case of proton decay, the rate of B-violating Higgs-boson interactions in the early universe is determined by the largest Higgs-Yukawa coupling, presumably $\sim g(m_t/m_W) \gtrsim \frac{1}{10}$, rather than by their couplings to the lightest fermion flavor. With such a coupling, back reactions lead to an asymptotically exponential damping of baryon number produced by a single decaying species when its mass is below about 5×10^{14} GeV.[24] When more than about a hundred B-violating bosons with these couplings and all with masses of roughly 5×10^{14} GeV are present, back reactions reduce the final baryon number by a factor $\gtrsim 10^6$, leaving an inadequate asymmetry. If the boson masses are $\simeq 10^{15}$ GeV, only about 20 boson species are permitted if the observed baryon asymmetry is to be accounted for. Because of the exponental dependence of back reactions on the number of contributing species, these bounds are much more stringent than those obtained solely from multiple B-violating exchanges in proton decay.

The magnitude of back reactions, and thus the importance of large numbers of B-violating species, depends crucially on the couplings of the relevant Higgs bosons. Two effects may increase these couplings over the estimates used above. First, if sufficient Higgs bosons or fermions are present for asymptotic freedom to be lost, as discussed in Sec. II, then the effective Higgs-Yukawa couplings will increase as described by the appropriate renormalization-group equation. Second, in many models some of the Higgs bosons which couple to fermions do not attain vacuum expectation values, so that the magnitudes of their couplings are not constrained by the values of the light-fermion masses.

While gauge-boson decays rarely generate baryon asymmetry, gauge-boson exchanges contribute back reactions which destroy baryon number. As discussed in Sec. II, however, the number of gauge bosons (which is determined by the dimensionality of the adjoint representation of the gauge group) is typically much smaller than the number of Higgs bosons. In addition, gauge-boson couplings connect only fermions of the same chirality in a single irreducible representation of the gauge group; Higgs-boson couplings may connect fermions in different irreducible representations and must flip the fermion chirality. [For SU(n) gauge groups, reducible fermion representations are required for axial anomaly cancellation. With other gauge groups, a single irreducible fermion representation usually suffices.] Thus gauge-boson reactions may neither affect asymmetries between different fermion irreducible representations [e.g., $\underline{5}$ and $\underline{10}$ in SU(5)], nor destroy the total asymmetry in fields of a given chirality in an irreducible representation. One "zero mode" exists in the Boltzmann transport equations for each irreducible fermion representation when only vector-boson exchanges are included.[26] Thus gauge-boson exchanges alone are usually insufficient to destroy all asymmetries which have been generated.[4,27]

The discussion above indicates that for an adequate baryon asymmetry to survive, a few B-violating Higgs bosons must be significantly lighter than the rest. In general, Higgs-boson masses are determined by minimization of the effective Higgs potential and diagonalization of the resulting mass matrices. Some models exhibit approximate symmetries, broken on scales $\ll 10^{14}$ GeV, which lead to approximately degenerate sets of Higgs bosons corresponding to irreducible representations of the approximate symmetry. Models with large approximate symmetry groups may therefore be unable to account for the observed baryon asymmetry. In the absence of approximate symmetries, one must usually resort to a purely statistical treatment, taking the boson masses as random variables. In the simplest case, one takes the boson masses to be independent random variables, distributed over a definite, say, unit, interval. With 5 particles, the average spacing of the lowest two masses is then $\simeq 0.16$; the spacing decreases to $\simeq 0.09$ for 10 particles, to $\simeq 0.04$ with 20, and to $\simeq 0.03$ with 50. With more than 20 particles, the probability that the lowest two particles are separated by more than 10% in mass falls below 0.1. An alternative and perhaps more realistic approach takes the elements of the boson-mass matrices to be

independent Gaussian random variables (with, say, zero mean and unit variance). [Note, however, that the Clebsch-Gordan coefficients on which the mass matrix elements depend are not in fact Gaussian distributed at least in the case of SO(3).[28]] The distribution of eigenvalues for such matrices becomes semicircular when the dimension n of the matrix exceeds about 5.[29] Monte Carlo simulation shows that the mean spacing between the lowest masses, normalized by the total range of masses, falls roughly linearly with n, taking on a value ~ 0.1 for $n=10$. For $n \gtrsim 20$, the probability for a spacing larger than 10% between the lowest masses again falls below 0.1.

IV. DISCUSSION

The implementation of general theoretical principles in grand unified gauge models often appears to require the introduction of very large numbers of particles. This paper has discussed the effects of such very large gauge models on the very early universe. It has shown that in many cases, the evolution of the very early universe can be significantly altered. In some cases, the generation of an adequate baryon asymmetry appears to place quite stringent constraints on the complexity of gauge models. The precise form of these constraints should be investigated in detail for any specific very large grand unified gauge model.

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