A QCD MODEL FOR e^+e^- ANNIHILATION

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Received 19 April 1982
(Revised 4 October 1982)

A QCD model for e^+e^- annihilation is presented, and its consequences are compared with experimental data. The model involves production of a shower of partons described by a simple approximation to QCD perturbation theory, and decay of colour singlet clusters of produced partons into hadrons through a simple phase space process. The model reproduces most known theoretical features of QCD, and, with certain choices of parameters, appears to correspond well with experimental results.

This paper summarizes preliminary results of an extensive investigation of the consequences of QCD for hadron production in high-energy e^+e^- annihilation. The absence of initial hadrons makes e^+e^- annihilation the simplest process to consider. While QCD purports to provide a complete precise theory for strong interactions, exact calculations of measurable quantities remain beyond the capabilities of existing mathematical methods, and approximations must be made. QCD provides few rigorous indications as to the accuracy of such approximations. In devising a good approximation scheme, one must attempt to identify and account for the dominant physical phenomena, and merely hope that the results so obtained will lie close to eventual exact calculations. Here we discuss a sequence of approximation schemes and compare their consequences with experimental results. The necessity for approximation precludes precise quantitative tests of QCD.

In the original parton model, a hadronic e^+e^- event occurs through decay of the virtual photon γ* into a quark and antiquark, which eventually evolve into two independent jets of hadrons with fixed limited transverse momenta [7]. The Field–Feynman model [8] provided a simple parametrization for these jets, in which each

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1 Work supported in part by the US Department of Energy under contract no. DE-AS-05-81-ER 40008.
2 Work supported in part by the US Department of Energy under contract no. DE-AC-03-81-ER 40050 and the Fleischmann Foundation.
* Some results have previously been reported in refs. [1, 2]. Additional results and derivations are given in refs. [1, 3, 5, 6]. A complete treatment will be given in a future publication. (See also ref. [22].)
final hadron in turn is emitted independently from the jet with an exponentially-damped transverse momentum spectrum and a fixed distribution in the available longitudinal momentum. The Field–Feynman model gives an adequate fit to properties of e^+e^- final states at c.m. energies \( Q \) between \( \sim 4 \) and \( \sim 10 \) GeV [9]. (Energy conservation removes exact independence of successive hadron emissions, and requires modifications in the model for jets with energies below a few GeV.) A fit to experimental data at \( Q \gg 10 \) GeV with the Field–Feynman model apparently requires a broadening in the transverse momentum spectra of the emitted hadrons [10–14], in contradiction with the original parton model.

QCD purports to supercede the original parton model. QCD perturbation theory implies that \( q \) and \( q \bar{q} \) produced in \( \gamma^* \) decay should emit gluons. The cross sections for the processes \( \gamma^* \rightarrow q\bar{q}G \) and \( \gamma^* \rightarrow q\bar{q}GG \) assuming on-shell final partons have been computed explicitly in perturbation theory. Processes in which larger numbers of gluons are emitted may be treated using a simple approximate model. The kinematics of the emissions are conveniently specified by assigning to each radiating parton an invariant mass \( \mu \). The invariant masses of radiated partons are kinematically constrained to be less than those of their parents. The original \( q \) and \( q \bar{q} \) have a distribution of invariant masses extending up to the kinematic limit \( \mu \sim Q \). When the \( q \) and \( q \bar{q} \) radiate gluons, their invariant masses are reduced, being converted into the transverse momentum of the radiated gluons. The radiated gluons may themselves radiate more gluons, generating a shower of partons, as illustrated in fig. 1. QCD perturbation theory suggests that at each step, the radiated partons tend to have much smaller invariant masses than their parents, and thus retain an energy of the same order as their parents (decreasing logarithmically with \( \mu \)). A parton with invariant mass \( \mu \) travels a proper distance \( O(1/\mu) \) between successive interactions or radiations. The wavelength of a parton is governed by its energy. While the energy remains \( O(Q) \), the wavelength becomes much shorter than the distance \( O(1/\mu) \) between successive interactions, so that interference between their amplitudes may be neglected. Throughout this stage of parton shower development, the “decay” of each parton may be approximated as independent of its parent, and the spectra of its decay products are well described by an independent classical probability distribution. In this approximation, the probability for production of a parton of type 0 with invariant mass \( \mu \) which decays into partons of types 1 and 2 carrying fractions \( ~z \) and \( ~(1 - z) \) of its energy is given by

\[
\frac{\alpha_s(\mu^2)}{2\pi\mu^2} P_{0 \rightarrow 12}(z) \, dz \, d(\mu^2),
\]

where \( \alpha_s \) is the QCD effective coupling constant, and \( P(z) \) is the appropriate Altarelli-Parisi distribution [15]. Eq. (1) may be derived by consideration of Feynman diagrams for gluon production in an axial gauge where each gluon polarization is taken transverse to the original \( \gamma^* \) momentum. The parameter \( z \) is identified as the \( E + |p| \) fraction of the radiated parton with respect to its parent, evaluated in
the $\gamma^*$ rest frame*. (2z - 1 is also roughly the cosine of the angle between the spin of the decaying parton and the momenta of its decay products in the rest frame of the decaying parton.) Eq. (1), together with kinematic constraints, provides a complete "leading pole" approximation for parton shower development. In limiting cases, its consequences may be found by simple analytical methods [6, 16]; in general they are most conveniently investigated by Monte Carlo methods [4, 17].

Even in perturbation theory, the free parton emissions described by eq. (1) cannot continue unchecked forever: when the invariant masses of partons become small, back reactions in which emitted partons reinteract with their parents or with other partons should become increasingly important. The free emission approximation (1) is expected to fail for partons with invariant masses below a critical value $\mu_c$, presumably of the order of the renormalization group invariant mass $\Lambda$ which characterizes the QCD coupling (and determines the sizes of hadrons)**. Eventually, interactions between produced partons, together perhaps with phenomena not visible in perturbation theory, should cause the system of partons to condense

* A cutoff $\mu > \mu_c$ thus implies a cutoff $\mu_c^2/Q^2 < z < 1 - \mu_c^2/Q^2$.

** $\mu_c^2$ is always assumed to be timelike.

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Fig. 1. Spacetime development of typical parton showers initiated by decay of a virtual photon with invariant mass $Q = 200$ GeV, traced until each parton has invariant mass below a critical value $\mu_c = 1$ GeV. Parton emission probabilities were derived from the "leading pole approximation" discussed in the text. The QCD renormalization group invariant mass was taken as $\Lambda = 0.5$ GeV. The identification of virtual parton momenta was made in a symmetrical axial gauge [1]; the "distances" before decay were taken as $1/(E - |p|)$. The events were generated by the Monte Carlo procedure described in ref. [1].
into colour singlet hadrons. So long as the process of hadron formation respects certain locality constraints (discussed below), suitably coarse features of the parton state at $\mu > \mu_c$ should be preserved.

Assuming a sharp cutoff on parton production below $\mu = \mu_c$, one may use eq. (1) and related kinematics to perform approximate calculations of the structure of parton "final states". In the results described below, all intermediate partons were constrained to have invariant masses above $\mu_c$, but the "final" produced partons were taken massless. This prescription avoids the manifest violation of gauge invariance associated with non-zero final parton invariant masses. (Ref. [17] uses an alternative prescription in which final partons do carry non-zero invariant masses.)

The angle between a pair of partons produced by the decay of a parent with invariant mass $\mu$ and energy $E$ is $\sim \mu/E - \mu/Q$: observables which probe the final state angular energy distribution with low angular resolution should thus be sensitive only to the first few gluon emissions occurring when $\mu \sim Q$. Fig. 2 shows the $Q$ dependence of $\langle H_2 \rangle$ and $\langle H_8 \rangle$, where

$$H_i = \sum_{ij} \frac{|p_i||p_j|}{Q^2} P_i(p_i \cdot p_j),$$

$P_i$ is a Legendre polynomial and the sum runs over all particle pairs [5]. For $\gamma^* \rightarrow q\bar{q}$

![Fig. 2](image-url)

Fig. 2. Mean values of the shape parameters $H_2$ and $H_8$ [2] (defined in text) for parton systems generated from the decay of a virtual photon with invariant mass $Q$. Dashed curves were obtained from $O(\alpha_s)$ perturbation theory, including one gluon processes, and taking all final partons to be massless and on shell. Solid curves include multiple gluon emissions according to a leading pole approximation, requiring all partons to have invariant masses above a cutoff $\mu_c$. In all cases, the QCD renormalization group invariant mass $\Lambda$ is taken as 0.5 GeV. As $Q \rightarrow \infty$, all events tend to a perfect two-jet form, with $H_2 = H_8 = 1$. Deviations at finite $Q$ reflect transverse spreading of the jets as a result of gluon emission. $H_6$ probes angular structure in the parton system on smaller scales than $H_2$, and is thus more affected. The proximity of dashed and solid curves at large $Q$ demonstrates the dominance of the first emitted gluon in determining event shapes. For small $Q$, the cutoff $\mu_c$ prevents any gluon emission in the leading pole approximation, leaving a pure $q\bar{q}$ final state with $H_2 = H_8 = 1$. The effects of the cutoff die away slowly with increasing $Q$. $H_i$ distributions for hadron final states are shown in fig. 12.
Fig. 3. Mean fraction of total available energy in decay of a virtual photon with invariant mass \( Q \) carried by gluons. Multiple gluon emission cross-sections were taken from a leading pole approximation. Gluon emissions were permitted only above the cutoff invariant mass \( \mu_c \). The percentage of events for which this cutoff prevented any gluon emission (leaving a pure \( qq \) state) is marked. The line marked "collinear approximation" gives results obtained in the usual "leading logarithm approximation" obtained from the leading pole approximation by neglecting transverse momenta imparted by gluon emissions. The slow convergence of this and the solid curves emphasizes the need for exact kinematics. The dashed curve shows the small effect of a modification (discussed in the text) to the parton emission effective coupling constant in the leading pole approximation. In all cases \( \Lambda = 0.5 \text{ GeV} \).

alone, \( H_{2l} = 1, H_{2l+1} = 0 \), while for an isotropic final state \( H_l = 0 \). The \( H_L \) are coefficients in a Legendre expansion of the two-point energy correlation function, and are typically sensitive only to structure on angular scales \( \gg 1/l \). Fig. 2 shows that direct \( \mathcal{O}(\alpha_s) \) calculations agree well with results obtained from eq. (1) to all orders in \( \alpha_s \) when \( Q \gg \mu_c \). Corrections associated with the cutoff are \( \mathcal{O}(\mu_c/Q) \).

The "leading logarithm approximation" for parton shower development is obtained by using eq. (1) but making the kinematic approximation that all radiated partons are collinear with their partons, but nevertheless formally allowing the invariant mass of each parton to run up to the mass of its parent. This approximation is used to derive the standard logarithmic "scaling violations" in the moments of parton \( z \) distributions. Fig. 3 shows the average total energy carried by gluons (related to \( \langle z \rangle \)) as a function of \( Q \). Results obtained in the leading log ("collinear") approximation are found to approach results based on eq. (1) with complete kinematics only at very high \( Q \). This behaviour reflects the importance of low-energy gluons for which the collinear approximation is inadequate.

A parton of invariant mass \( \mu \) yields a relative transverse momentum \( k_T^2 = (1-z)\mu^2 \) between its decay products. If the transverse momentum distribution in individual decays had finite variance, the total \( k_T \) distribution would be gaussian (according to the central limit theorem). However, the \( d\mu/\mu \) form of eq. (1) implies a power-law tail in the average single parton \( k_T \) distributions measured with respect to the initial \( q, \bar{q} \) directions, as illustrated in fig. 4. "Double logarithm" analytical approximations to the small \( k_T \) behaviour of these distributions give very inaccurate
Fig. 4. Transverse momentum distributions for partons produced in the decay of a virtual photon with invariant mass \( Q \), obtained using a leading pole approximation truncated at \( \mu_c = 1 \text{ GeV} \), and with \( \Lambda = 0.5 \text{ GeV} \). \( k_T \) is measured with respect to the initial \( qq \) axis. The distributions are roughly gaussian for small \( k_T \), but exhibit a power-law tail at large \( k_T \) associated with hard gluon emission. Corresponding results for final hadrons are shown in fig. 9.

results even at \( Q \sim 100 \text{ GeV} \): the necessary kinematic approximations are valid only at much higher energies.

Fig. 5 shows the mean multiplicity of partons produced above the \( \mu_c \) cutoff as a function of \( Q \). As expected, the results depend sensitively on \( \mu_c \). Most of the

Fig. 5. Mean multiplicity of partons produced in the decay of a virtual photon with invariant mass \( Q \) by radiation from virtual partons with invariant masses above a critical value \( \mu_c \). The multiplicity asymptotically rises exponentially, but remains sensitive to the value of \( \mu_c \). The dashed line gives the multiplicity of quarks and antiquarks, and indicates that very few secondary \( qq \) pairs are produced. The parton production cross sections were estimated by a leading pole approximation, with \( \Lambda = 0.5 \text{ GeV} \).

The results may be compared with those for final hadrons shown in fig. 8.
produced partons are gluons. The softer behaviour of the $G^+ \rightarrow q\bar{q}$ Altarelli-Parisi distribution yields only a very small number of secondary $q\bar{q}$ pairs. Just as for fig. 4, analytical estimates of $\langle n \rangle$ based on approximate kinematics do not quantitatively reproduce the results of fig. 5.

Explicit calculations through $O(\alpha_s^2)$ may be used to test the accuracy of eq. (1) in perturbation theory. The calculations indicate that eq. (1) provides a good approximation to the kinematic structure of the final state (so that direct three-body parton decay processes may be ignored), but reveals large $O(\alpha_s^2)$ corrections to the overall normalization of the decay probability. Corrections $O(k! \alpha_s^2)$ (associated with the Landau singularity in the effective coupling constant) may appear even in low orders of perturbation theory. One must merely hope that all such corrections may be absorbed in the overall normalization of $\alpha_s$, and do not affect the kinematic structure of eq. (1)*. The optimal mass scale $\bar{\mu}^2$ on which to evaluate the effective QCD coupling $\alpha_s(\bar{\mu}^2)$ in eq. (1) should be chosen to absorb the largest part of higher-order corrections. Gluons emitted in, for example, the process $q^* \rightarrow qG$ may themselves emit further gluons. The possibility of such processes introduces $O((a_s \log (\mu_{\text{max}}/\mu_{\text{min}}))^k)$ corrections to the $q^* \rightarrow qG$ rate. The sum of these logarithmic corrections yields an effective coupling constant whose mass scale is governed by the maximum invariant mass of the emitted gluon. Imposing the cutoff $/L_e$ on all gluon invariant masses modifies the argument of the logarithm by addition of a term $\sim \mu_c$. For most of the results shown, deviations of the mass scale $\bar{\mu}$ in $\alpha_s$ from the invariant mass $\mu$ of the decaying parton have been ignored: its small effects are shown in fig. 3.

Experiments do not observe final state quarks and gluons; they measure the energies and momentums of hadrons. To compare with experimental data, one must inevitably introduce a model for the formation of hadrons from partons with $\mu \leq \mu_c$. Variations of results over a range of models should allow classes of results insensitive to hadron formation to be identified. An important assumption is that the process of hadron formation depends only on the local structure of the parton system over small spacetime volumes. Suitable sets of partons should thus evolve separately to hadrons in a manner independent of the processes by which they were produced. If the formation of hadrons does not occur in some such local and universal way, there seems little hope of obtaining useful results from QCD without much more detailed knowledge of the structure of hadrons. If, for example, the whole parton system at $\mu_c$ acted cooperatively to generate the final hadrons, then the disposition of the partons at $\mu_c$ would be invisible.

* It is possible that high order divergences in QCD perturbation series may be treated by a procedure analogous to renormalization. A finite number of subdiagrams whose higher-order corrections give numerically divergent series may be identified. The magnitude of each subdiagram must be determined from experiment but definite predictions are obtained for comparisons between processes involving identical subdiagrams. This procedure should be distinguished from the choice of different "renormalization prescriptions" for genuinely renormalized quantities.
One phenomenological approach to hadron formation introduces an arbitrary invariant mass cutoff $\mu_0$ above which parton shower development occurs according to eq. (1), and below which a phenomenological model is used. The simplest phenomenological model takes each parton at $\mu_0$ to evolve and form hadrons independently. This model fails, for example, to account for colour conservation, and is therefore ultimately inadequate. The model may nevertheless be adequate if only some of the parton “decay products” are considered. It is conventional to define fragmentation functions $F_{p \rightarrow h}(z, \mu^2_0)$ which describe inclusive hadron spectra in parton decays, giving the probability for a hadron $h$ to carry a fraction $z$ of the energy of a parton $p$ with invariant mass $<\mu_0$. Changes in hadron spectra with $Q$ (“scaling violations”) may then be estimated by treating only the evolution of the parton shower down to $\mu \sim \mu_0$: the final hadron formation process is assumed to be universal and may be factorized out. The fragmentation function approximation fails when $\mu_0$ or $Q$ are small enough that when individual partons have $\mu \ll \mu_0$ many pairs of partons have $\mu \approx \mu_c$, and may therefore act cooperatively in forming hadrons. Such cooperative processes could in principle be accounted for by “higher-twist” two-parton fragmentation functions. The assumption of locality in hadron formation mentioned above implies that at sufficiently high $\mu_0$, joint fragmentation functions involving more partons are progressively smaller. The validity of the fragmentation function picture at high $Q$ requires that hadron formation occurs when parton invariant masses fall below a fixed critical value $\mu_c$, and thus after a time $\sim Q/\mu^2_c$ in the $\gamma^*$ rest frame. If, instead, hadron formation occurred at a fixed time in the $\gamma^*$ rest frame, a cutoff $\sim \sqrt{Q/\mu^3_0}$ would be imposed on the radiating partons and would exceed $\mu_0$ at high $Q$, yielding asymptotically no scaling violations.

If a $\mu_0 \gg \mu_c$ is chosen, it is plausible that each parton may be taken to decay separately into a jet of hadrons, as in the original parton model, and that the effects of colour compensation may be ignored. The Field-Feynman model might then be used to parametrize the properties of the individual jets. Field-Feynman jets produced in the process $\gamma^* \rightarrow q\bar{q}$ are specified by the energies $E = \frac{1}{2} Q$ of the $q$ and $\bar{q}$, and give rise to hadron jets with invariant masses $\sim \sqrt{E/\mu_c}$. If the partons indeed decay independently, the hadron jets produced must depend only on intrinsic (Lorentz invariant) properties of the decaying partons, and not, for example, on the energies of the partons in the $\gamma^*$ rest frame. If the Field–Feynman model is used, the jets must be generated with appropriate invariant masses $\sim \mu_0$ and zero momentum in the decaying parton rest frame. The results so obtained would be quite different from those found by taking Field–Feynman jets according to the parton energies.

A complete model for hadron formation must identify sets of partons which evolve independently beyond the critical point $\mu = \mu_c$. The presence of long-range colour forces implies that a set of partons can evolve independently only if they form a colour singlet system. It is plausible that appropriate “minimal” colour singlet groups of partons may be taken to form hadrons independently. (A system
is termed "colour neutral" if it has zero eigenvalues of the two commuting generators \((\tau_3, \tau_8)\) of SU\((3)_c\). Colour neutral systems need not be colour singlets: for example, the octet representation of SU\((3)_c\) contains two colour neutral members (analogous to \(\pi^0\) and \(\eta\) for SU\((3)_f\)). To determine whether a colour neutral \(q\bar{q}\) system is a colour singlet requires knowledges of the amplitudes for the various possible arrangements of colour quantum numbers for the individual \(q\) and \(\bar{q}\) (c.f. the state \(\sqrt{2/3}(u\bar{u} + d\bar{d})\) has \(I = 0\) while \(\sqrt{1/3}(u\bar{u} - d\bar{d})\) has \(I = 1\); both have \(I_3 = 0\)). In the classical approximation (1), the phase of the amplitude for each emission is random so that the final partons are statistically distributed among the possible SU\((3)_c\) representations. Hence, for example, a colour neutral \(q\bar{q}\) pair has probability \(\frac{1}{2}\) to be a colour singlet, and probability \(\frac{1}{2}\) to be a colour octet.

There are several distinct classes of colour singlet partons which may be identified.

First, one might collect all colour singlet systems at \(\mu_c\) delimited by a quark and an antiquark. However, the very low multiplicity of secondary \(q\bar{q}\) pairs (evident in fig. 5) at realistic \(Q\) causes such \(qGG\cdots\bar{q}\) systems to have masses \(~Q\) and violates the assumption of locality for hadron formation.

A second scheme selects the lowest invariant mass colour singlet \(q\bar{q}GG\cdots\) or \(GG\cdots\) systems at \(\mu_c\). This scheme again typically yields unacceptably large colour singlet masses because many colour neutral gluon systems must often be combined to form a colour singlet.

Here we shall concentrate on a third scheme, in which each gluon at \(\mu_c\) is forcibly split into a \(q\bar{q}\) pair. Each quark carries one of the spinor colour indices of the

![Fig. 6. Schematic illustration of the procedure used to isolate colour singlet clusters. Each parton line carries a spinor colour index (one for quarks, two for gluons). Final gluons are forcibly split into collinear \(q\bar{q}\) pairs, represented by dashed lines. The quark and antiquark at the ends of each group-theoretical "string" are combined into a colour singlet (strictly, colour neutral) cluster. The clusters thus produced are taken to decay independently into hadrons. Each cluster decay is assumed isotropic in the cluster rest frame, and the final state is determined from a simple phase-space model.](image-url)
gluon; every quark is connected by a group-theoretical string to its colour conjugate antiquark, so as to form a colour neutral pair. The procedure used is illustrated schematically in fig. 6. The $q$ and $\bar{q}$ are taken to have momenta uniformly distributed in the allowed range, parallel to the gluon four-momentum. The splitting of each gluon into a $q\bar{q}$ pair would be justified if gluonium states decayed infinitely rapidly to $q\bar{q}$ mesons. The $q$ and $\bar{q}$ are taken to have flavours $u, d$ or $s$ with equal probabilities so long as clusters carrying strangeness have invariant masses exceeding $m_K$; if not, only $u$ and $d$ flavours are allowed. The invariant mass spectrum of the colour singlet parton systems obtained in this scheme are shown in fig. 7.

Fig. 7. Invariant mass spectra for colour neutral parton clusters formed according to the procedure indicated in fig. 6 from partons produced in decays of virtual photons with $Q = 11$ GeV for various values of $\mu_c$ and $\Lambda$. Masses of intermediate partons in the shower were constrained by the cutoff $\mu_c$, but final partons were taken to have zero invariant mass, thus avoiding manifest violation of gauge invariance but allowing the cluster invariant mass spectrum to extend down to zero mass. In the case $\mu_c = 2.8$ GeV, $\Lambda = 1.4$ GeV, a significant number of events involve no gluon emission and yield a cluster with mass $Q = 11$ GeV. These events contribute a term $\delta(\mu - Q)$ whose integral is shown as a box centred on $\mu = Q$. Except in this pathological case, the invariant mass spectra are seen to be strongly damped above $-2\mu_c$. In addition, the distributions in this region are roughly independent of the initial $Q$ [4]. These features are important in ensuring the "locality" of the hadron production process in the model.
They are seen to be strongly damped at large masses, as required by the locality assumption. They are also roughly independent of the original $Q$ [3], as required by the universality assumption.

Having identified appropriate colour singlet parton clusters, one must introduce a definite model for their transformation into hadrons. The locality assumption implies that the "decays" of the clusters must be determined entirely by their invariant masses, flavours, and possibly total angular momenta. Orbital angular momentum cannot be traced through the development of the parton shower; we nevertheless approximate the final clusters to decay isotropically in their rest frames, thereby implicitly assuming zero total angular momentum. Parton shower development according to eq. (1) leads to clusters with a continuous spectrum of invariant masses, as shown in fig. 7. However, each cluster might perhaps represent a single meson resonance, with definite discrete mass. In this case, the formation of hadrons could no longer be described purely by classical probabilities: destructive interference between quantum mechanical amplitudes must be included to forbid illegal final states. In practice, however, the increase of meson widths with increasing meson mass (phenomenologically $\Gamma \sim 0.1 m$) suggests that the possible meson masses may effectively form a continuous spectrum. A classical model would then suffice to describe the transformation of a cluster into a meson resonance in the band; this resonance would then decay into lighter mesons with definite masses. (The smoothness of the $e^+e^-$ cross section for $Q \gg 1$ GeV suggests that the band extends down to masses $\approx 1$ GeV.) Whether or not cluster decays occur through an intermediate meson, their properties may be estimated from experimental data on the decays of $\gamma^*$ produced in $e^+e^-$ annihilation at the same low energy. All evidence suggests that quasi-two-body final states are universally dominant. For clusters with masses below $\approx 1.5$ GeV, an adequate phenomenological model is to allow decay into pairs of the lowest-lying $0^-$, $1^-$, $1^+$, $2^+$ and $0^+$ mesons, with equal matrix elements for each final spin state (thus weighting a spin-$j$ meson with an overall factor $2j + 1$), yielding decay branching ratios governed solely by available phase space. (The assumption of zero final parton invariant masses allows a few clusters with masses below $2m_\pi$ to be produced. In these rare cases, we treat the clusters as single "pions" or "kaons" with an appropriate mass*. The resulting final hadron multiplicity (in the form of $K$ and $\pi$) is indicated as a function of cluster mass by the dashed line in fig. 8. K meson production is suppressed simply by the larger K mass, and by the larger number of $\pi$ than K produced in decays of low-lying meson resonances. The measured approximate constancy of the total hadron multiplicity in $e^+e^-$ annihilation over the range $1.5 \leq Q \leq 4$ GeV shown in fig. 8 supports the assumption that clusters with masses in this range decay directly to pairs of light mesons, without cascading through clusters of intermediate mass. If the masses of the cluster decay products remain fixed, their momenta should

* An alternative and perhaps more satisfactory procedure would be to combine a very low mass cluster with another cluster before evolving to hadrons.
increase as cluster mass increases. This behaviour suggests a rather smooth transition to the parton decays at larger invariant masses, where daughter partons have masses much smaller than their parents, and thus have large momenta. In as far as the approximation of eq. (1) describes the decay of clusters with sufficiently large masses to lighter clusters, the value of the parameter $\mu_c$ should be irrelevant: changes in $\mu_c$ over a certain range would simply assign a different fraction of the hadron production process to the phenomenological cluster decay stage, leaving results unchanged (as in the fragmentation function approach). In practice, however, only rather small changes in $\mu_c$ exhibit this behaviour.

Detailed comparison of theoretical predictions with experimental data requires that all important effects present in the data be included in the theoretical model. The first potentially important effect is the production and decay of c and b quarks. The formation of hadron jets from such quarks may be treated in analogy with the case of light quarks discussed above, with the modification $\mu_c \rightarrow \mu_c + m_Q$. The weak decays of the resulting colour singlet heavy “mesons” are then treated by a parton model. The heavy quark decays to a lighter one by emitting a virtual $W$. This $W^*$ may either itself decay to a $q\bar{q}$ or lepton pair, or may be absorbed by the spectator
light quark in the heavy meson. In most cases, effects of heavy quark production do not appear to be very large, but a systematic study has yet to be made. A second potentially important effect is emission of hard photons from the e+e− initial state. The magnitude of this effect depends on details of experimental procedure, and we have ignored it throughout. Finally, most of the experimental data shown below is uncorrected for experimental acceptance and efficiency. There are good indications, however, that in most cases, such corrections are at the 10% level.

The primary purpose of our comparisons with experimental data is to establish whether QCD models along the lines described here agree even qualitatively with experiment. Careful treatment of all effects would of course be necessary for a detailed quantitative analysis.

Table 1 gives some average properties of events generated according to the model with various choices of parameters at Q = 10 and 30 GeV. In most cases, several gluons are produced. On average, in only a very small fraction of events are no gluons produced, yielding only one final cluster. For Q ≳ 4 GeV, observed e+e− final states exhibit an approximate two-jet structure. This is reproduced in the model through the rarity of single cluster events. When two or more clusters are produced, each decays isotropically in its rest frame, but when boosted to the γ* rest frame (event c.m.s.), their decay products become collimated*. Fig. 9 shows the resulting hadron $p_T$ distributions, while fig. 10 shows the $\langle p_T^2 \rangle$ as a function of Q. The height of the tail in the $p_T$ distribution, and the magnitude of the resulting $\langle p_T^2 \rangle$, are determined by two effects. First, the higher the invariant mass of the individual hadron cluster, the larger will be the transverse momentum generated in their decays. Second, the larger Λ is, the higher the probability for the emission of large transverse momentum gluons, leading to a larger power-law tail in the parton $p_T$ distribution, evident in fig. 4. The largest $\langle p_T^2 \rangle$ are thus obtained with large Λ (hence large $\alpha_s$) and large $\mu_c$ (hence large cluster masses).

Fig. 11 shows a comparison of data on the transverse momentum distribution of single hadrons “in” and “out” of the plane for e+e− annihilations at a virtual photon invariant mass of Q = 30 GeV with the model results. The $\langle p_T^2 \rangle_{\text{in}}$ and $\langle p_T^2 \rangle_{\text{out}}$ distributions illustrate the planar nature of e+e− events which is reproduced correctly by the model.

The quantities $H_i$ defined above measure the angular flow of energy in an event; in so far as hadron formation is a local process, the values of these parameters for hadron final states should follow those for the partons from which these evolved. Fig. 2 showed the $\langle H_i \rangle$ for parton systems; Fig. 12 shows $H_i$ distributions for hadron as well as parton final states. Even at Q = 30 GeV, the difference between parton and hadron results remains comparatively large. Events with larger $H_2$ and $H_4$ are closer to the two-jet limit $H_2 = H_4 = 1$. The $H_i$ for Q ≳ 10 GeV are comparatively insensitive to the transverse momentum between products of individual cluster

* In fact, the assumption of quasi-two-body phase-space decays implies that even single clusters with sufficiently high masses would exhibit collinear final states.
Properties of events generated according to the model from virtual photons with invariant masses (a) $Q = 11$ GeV and (b) $Q = 30$ GeV, with various values for the QCD mass $\Lambda$ and cutoff invariant mass $\mu_c$

(a) $Q = 11$ GeV

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<td>2.6</td>
<td>4.5</td>
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<td>$\langle m_{\text{cluster}} \rangle$ (GeV)</td>
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<td>0.42</td>
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<td>1.1</td>
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<td>$\langle n_{\text{hadron}}/\text{cluster} \rangle$</td>
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<td>3.4</td>
<td>2.5</td>
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<td>$\langle n_{\text{hadron}} \rangle$</td>
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<td>11.2</td>
</tr>
<tr>
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<td>6.9</td>
<td>5.0</td>
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<tr>
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<tr>
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<td>0.36</td>
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<tr>
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<td>0.11</td>
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<td>0.19</td>
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<tr>
<td>$\langle z^2 \rangle$</td>
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<tr>
<td>$\langle H_2 \rangle$</td>
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<td>0.58</td>
<td>0.55</td>
<td>0.44</td>
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<tr>
<td>$\langle H_4 \rangle$</td>
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<td>0.41</td>
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<td>0.23</td>
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(b) $Q = 30$ GeV

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<td>9.7</td>
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<td>&lt;0.1</td>
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<td>&lt;0.1</td>
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<td>11.8</td>
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<tr>
<td>$\langle p_{\text{L}}\text{sph} \rangle$ (GeV)</td>
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<td>1.2</td>
<td>1.7</td>
<td>1.3</td>
</tr>
<tr>
<td>$\langle p_{\text{T}}\text{sph} \rangle$ (GeV)</td>
<td>0.41</td>
<td>0.26</td>
<td>0.54</td>
<td>0.40</td>
</tr>
<tr>
<td>$\langle p_{\text{T}}\text{sph}^2 \rangle$ (GeV$^2$)</td>
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<td>0.17</td>
<td>0.56</td>
<td>0.30</td>
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<tr>
<td>$\langle z^2 \rangle$</td>
<td>0.29</td>
<td>0.26</td>
<td>0.27</td>
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<td>$\langle H_2 \rangle$</td>
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<td>0.74</td>
<td>0.61</td>
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<td>$\langle H_4 \rangle$</td>
<td>0.55</td>
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<td>0.42</td>
<td>0.44</td>
</tr>
<tr>
<td>$\langle T \rangle$</td>
<td>0.93</td>
<td>0.94</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td>$\langle S \rangle$</td>
<td>0.08</td>
<td>0.06</td>
<td>0.13</td>
<td>0.12</td>
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</tbody>
</table>

Clusters are colour singlet groups of partons formed according to the prescription indicated in fig. 6. The third line gives the fraction of events containing only one cluster. $H_2$, $H_4$ are shape parameters (defined in text), $T$ is thrust and $S$ is sphericity, and $Z$ is hadron fractional momentum.
decays. So long as $\mu_c$ is sufficiently small compared to $Q$ that many clusters are produced in each event, the $H_t$ distribution are roughly independent of $\mu_c$, and depend primarily on the value of $\Lambda$, which determines the rate for gluon production. Larger $\Lambda$ yields broader $p_T$ spectra and softer $H_L$ distributions. Curves at $Q = 12$ GeV with $\mu_c = 2.8$ GeV shown in fig. 12 exhibit a hump close to $H_{2L} = 1$. This hump is a result of events in which only a single hadron cluster was produced and decayed to two very energetic mesons. The abruptness of transition from cluster decay to gluon production suggests that the model is inapplicable for this choice of parameters.
Fig. 10. Mean square charged hadron transverse momentum (with respect to the “sphericity axis”) as a function of initial $\gamma^* \gamma$ invariant mass for various choices of parameters in the model. The discontinuity around $Q = 10$ GeV is associated with the $b$ quark production threshold. Data are from ref. [20].

Fig. 13 shows the single hadron fractional momentum distributions obtained from the model for various choices of parameters. In fig. 14, the resulting distributions with $\Lambda = 1.4$ GeV and $\mu_c = 1.6$ GeV are compared with experimental data for several values of $Q$. Even qualitative agreement is significant, since the model contains no explicit parameters (such as fragmentation functions) which determine

Fig. 11. Comparison of data (ref. [20]) on the transverse momentum distribution of single hadrons “in” and “out” of the plane for $e^+e^-$ annihilations at $Q = 30$ GeV with the results of the model with several choices of the parameters. The dashed, dot-dashed, and solid curves correspond to $\Lambda = 0.5$ GeV, $\mu_c = 1.0$ GeV; $\Lambda = 1.4$ GeV, $\mu_c = 2.8$ GeV; and $\Lambda = 1.4$ GeV, $\mu_c = 1.6$ GeV, respectively.
Fig. 12. Distributions in the shape parameters $H_2$ and $H_4$ (defined in the text) for hadron and parton systems generated according to the model. Curves for hadron final states are labelled as in figs. 8–10; the dash-dot-dot ("d") curve is for parton final states with $\Lambda = 1.4$ GeV, $\mu_c = 1.6$ GeV. The $H_i$ measure the angular flow of energy in the final state, and are expected to be comparatively insensitive to hadron formation. The limit $H_2 = H_4 = 1$ corresponds to a perfectly collinear final state, such as $q\bar{q}$. Notice the comparatively large difference between parton and hadron results. The hump near $H_2 = H_4 = 1$ for large $\mu_c/Q$ is a pathology of the model associated with events in which no gluons are emitted (see fig. 7). Experimental data are taken from refs. [11, 21].

these distributions. The number of hadrons at large $z$ is determined largely by the total number of cluster, and thus by the value of $\mu_c$. The $z$ distributions obtained from our model are not independent of $Q$: the magnitude of scaling violations increases with increasing $\Lambda$. The presence of scaling violation in hadron energy
spectra is an inevitable consequence of the same gluon emission processes which lead to broadening in $p_T$ distributions. The smallness of the scaling violations found in the experimental data is puzzling.

As mentioned above, we take each $q\bar{q}$ pair with sufficient invariant mass to have flavours $u, d$ and $s$ with equal probabilities. Strange meson production is suppressed by two effects: first, a non-strange hadron cluster decays less frequently to $KK$, etc. than to $\pi\pi$, etc. because the available phase space is lower, and second, in meson decays, pions are often produced, while kaons are rarely produced. We assume that baryon-antibaryon pairs may be generated in cluster decays, at a rate governed simply by available phase space, including all $S$-wave baryons [SU(3) octet and decuplet]. At $Q = 30\text{ GeV}$, the model, with $\Lambda = 1.4\text{ GeV}$ and $\mu_c = 1.6\text{ GeV}$, gives an average per event of about $11\pi^\pm, 1K^\pm, 1(K^0 + \bar{K}^0)$ and $0.1(p + \bar{p})$. Experimental measurements yield average multiplicities of $11\pi^\pm, 1.4K^\pm$, and $0.4(p + \bar{p})$ [18]. The agreement for $\pi$ and $K$ is encouraging, since no arbitrary parameters (such as the $\gamma$ of the Field–Feynman model) were introduced. If the larger experimental baryon production is significant, it may suggest direct baryon production mechanisms.

In the early part of this paper, we presented a model for hadron production in high-energy $e^+e^-$ annihilation, and argued that this model followed closely known
features of QCD. At short distances, the model agrees with QCD perturbation theory, while at large distances, it exhibits locality and universality. The model contains essentially only two parameters: the QCD scale $\Lambda$, and an invariant mass cutoff $\mu_c$. In the later part of the paper, we compared results from the model with actual experimental data. In general, we found qualitative agreement between the model and the data for most values of the parameters. Detailed quantitative comparisons are probably premature, but we nevertheless mention that with the choice $\Lambda = 1.4$ GeV, $\mu_c = 1.6$ GeV, the model appeared to agree rather closely with the data. This value of $\Lambda$ is much higher than those deduced from previous investigations. The inclusion of exact kinematics and exact treatment of the cutoff $\mu_c$ yields smaller scaling violations and other effects for a given value of $\Lambda$ than would be obtained from the leading logarithm approximation with the same $\Lambda$. In addition, the assumption of hadron formation from independent colour singlet clusters tends to favour “two-jet” final states [2], and thus to require a larger $\Lambda$. 

Fig. 14. Single hadron fractional momentum distributions obtained from the model with $\Lambda = 1.4$ GeV, $\mu_c = 1.6$ GeV, for several values of the $\gamma^*\gamma^*$ invariant mass $Q$. Notice the inevitable appearance of scaling violations ($Q$ dependence in $z$ distributions). Data are from ref. [19].
to fit observed transverse momenta. It is clear that the value of $\Lambda$ (or $\alpha_s$) deduced from experimental $e^+e^-$ data depends sensitively on the model taken for hadron production, and any complete "tests" of QCD in $e^+e^-$ annihilation must be at best qualitative.

References

[21] T. Gottschalk, in progress